

Western Cape
Government

QUALITY EDUCATION

for every child | in every classroom | in every school in the province.



Algebraic expressions

Revision of grade 8

- An algebraic expression consists of variables and constants.
- An algebraic expression does not have an equal symbol, unlike a algebraic equation

$$2x^5 - 3x + 6$$

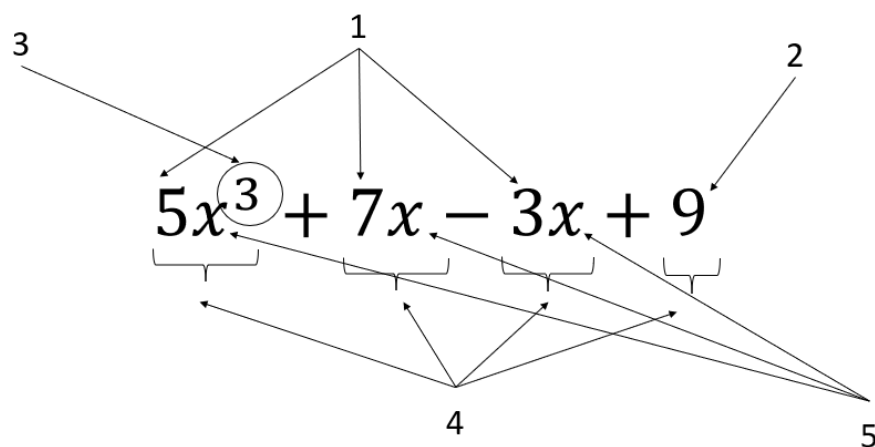
Activity 1

Describe the following terminology

1. Variable: _____
2. Coefficients: _____
3. Constant: _____
4. Term: _____
5. Degree of an expressions: _____

Activity 2

Name the parts of the expression



Type of expressions1. Monomial

- It's an expression that consists of only **one** term.

E.g. $3x$; $5y$; $2xy$; $3x^2$

2. Binomial

- It's an expression that consists of **two** terms that is not similar.

E.g. $3x - 5$; $5y^2 + 4$

3. Trinomial

- It's an expression that consists of **three** terms that is not similar.

E.g. $3x - 5y - 2$; $a^2 + 2a - 3$

4. Polynomial

- It's an expression that consist of **THREE OR MORE** terms that is not similar.

E.g. $3m + 5n - 2a + 4p$

Activity 3

Study the algebraic expression below and answer the following questions:

$$6x^4 - x^3 + 9x - 2x^2 + \frac{6x^2}{12} - 7$$

1. What type of algebraic expression is represented above?
 2. How many terms does the expression have?
 3. Name the exponent of x in term 5?
 4. Give the constant in the algebraic expression.
 5. Give the variables in term 3?
 6. Identify the coefficient of x^3 ?
 7. What is the degree of the expression?
-

Simplifying algebraic expressions**Activity 4**

Group the following expressions the like or unlike terms column below

<u>LIKE TERMS</u>	<u>UNLIKE TERMS</u>

$3x + 10x$

$3m + 2n$

$8a^2 - 6a^2$

$-6c - 8c^2$

$3a^2 - a^2b$

$3xy - 5yx$

$5b + 7b$

$3 - 6b$

Addition and subtraction rules

Signs of integers	Operation to use	Answer sign	Quick example
$\oplus + \oplus$			
$\ominus + \ominus$			
$\oplus + \ominus$			
$\ominus + \oplus$			

Multiplication rules

Signs of integers	Operation to use	Answer sign	Quick example
$\oplus \times \oplus$	Multiply		
$\ominus \times \ominus$	Multiply		
$\oplus \times \ominus$	Multiply		
$\ominus \times \oplus$	Multiply		

- When we have two or more similar terms, we can add or subtract the terms.
- Be on the lookout for exponent law 1.
- Very important!! Know your addition and subtraction rules.

Example 1

$$2x + 4x$$

Example 2

$$5xy^2 - 7xy^2$$

Example 3

$$5y + 2x^3y^2 - 3x - 12y^2x^3 - 8y - 7x$$

Example 4

$$3x^5 + (3x^2)(5x^3)$$

Activity 5

Simplify the following expressions

1) $4b^2 + 5b^2$

3) $-6c^4 - 4c + 12c^4 - 2c$

5) $m^2b \cdot mb + 5m^3b^2$

7) $(3x^3)^2 \times (-2x)^3$

9) $4(a - b) - 5(2a - 5b)$

2) $-12a + 6b - 4a$

4) $3yx^2 - 7n + 12x^2y - 8n$

6) $-8x^3 \cdot -2x^3$

8) $-(a^3 - 3ab)$

10) $2p - 4p(2p^2 - 1)$

Multiplication of algebraic expressions (Products)

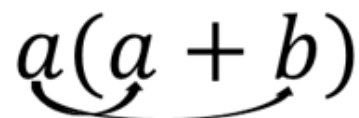
- When multiplying algebraic expressions, it is not necessary for terms to be similar.
- NB!!! Apply exponent law 1.
- Very important!! Know your multiplication rules.

1. *Monomial $\times$ Binomial*

- The term in front of the bracket, must be multiplied with each term inside the bracket.

Example 1

$$3(x^2 - 4)$$


$$a(a + b)$$

Example 2

$$-3m(m^2 - 6n + 5)$$

Activity 6Simplify the following expressions

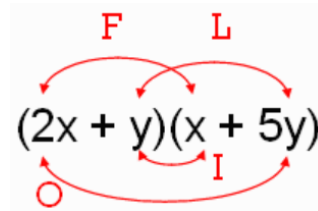
- 1) $5x(-2xm + 3)$
 - 2) $2b(\frac{1}{4}b^3 + 5c^2)$
 - 3) $30z^4(\frac{1}{15}y^2x - \frac{1}{90}z^2 - 3y)$
 - 4) $-(4xz - yx + zx)$
 - 5) $(2n - 7b)(-3)$
-

2. Binomial $\times$ Binomial

- Each term of the first bracket is multiplied with each term of the second bracket.

Example 1

$$(2x + 3)(3x - 5)$$



Example 2

$$(x - 4)(x + 4)$$

Activity 7

Simplify the following expressions

- $(x - 3)(x + 4)$
- $(3b^3 + 4y)(2b^2 - 4y)$
- $(4y + 4z^2)(2y^2 - 5z^4)$
- $(2x - 3)^2$

3. Binomial $\times$ Polinomial

- Each term of the first bracket is multiplied with each term of the second bracket.

Example 1

$$(4x - 5)(2x - 6y + 5z - 3)$$

Activity 8

Simplify

- $(2m - 2n)(2m - 2n + 3k)$
- $(-4c + 5d)(-4c - 5d + 8)$

Activity 9 (Mixed exercise)Simplify the following expressions

1) $\left(2y - \frac{1}{2}\right)\left(2y + \frac{1}{2}\right)$

2) $(8ab + 3)(4ba - 5)$

3) $2(x + 1)(x - 1)$

4) $(y - 3)^2$

5) $(x + 2)(x - 4) + 2(x - 4)$

6) $(2x - 5)(x + 3) + (x + 1)(3x - 2)$

7) $(y - 4)^2 + (2y + 3)^2$

8) $(2a - 3)(3b + 1) - (a + 3)(b - 3)$

Division of algebraic expressions

- When dividing algebraic expressions, its not necessary for terms to be similar.
- The division line of a fraction means you must divide.
- Note!!! Exponent law 2 must be applied if bases are the same.
- Very important!! Know your division rules.

Example 1

$$\frac{10a^9}{5a^4}$$

Example 2

$$\frac{12a^5 + 10a^2}{2a^2}$$

Activity 10Simplify the following expressions

1) $\frac{16x^2y^7}{4y^4}$

2) $\frac{24m^3n^5 - 12n^3}{3n^3}$

3) $\frac{25xy^6z^7 + 10xyz - 20x^3z - 7yzx}{5xz}$

4) $\frac{4a(a+b) + 16a^3b^5 - 28b^2a}{4ab}$

Square roots

Steps to help:

- Find the square root of the coefficient.
- Because you are working with a square root, **divide the exponent underneath the square root with 2.**

Example 1

$$\sqrt{4x^4}$$

Example 2

$$\sqrt{16x^6} + \sqrt{25x^6}$$

Example 3

$$\sqrt{8x^9 \times 2x^5}$$

Cube root

Steps to help:

- Find the cube root of the coefficient.
- Because you are working with a cube root, **divide the exponent underneath the cube root with 3.**

Example 1

$$\sqrt[3]{8x^{12}}$$

Example 2

$$\sqrt[3]{24y^9 + 3y^9}$$

Example 3

$$\sqrt[3]{729x^{27}} \div \sqrt[3]{27x^9}$$

Substitution

- Replace a variable with a constant.

Activity 11

If $x = -\frac{1}{2}$ and $y = 4$, determine the value of the expressions.

2.1 xy

2.2 $y - xy$

2.3 $x^2 + y^2$

2.4 $\sqrt{20x^2 + 5y}$

Factorisation

- With previous chapter we did products. (distributive property).
E.g. $2(b + c) = 2b + 2c$
- The process in reverse is now called factorisation.

The following three forms of factorisation will be done (Keep the order)

1. Common factor
2. Difference of two squares
3. Quadratic trinomial

NB!! When starting with factorisation, ALWAYS try common factor first.

1. Common factor

Steps that could help

- Take out the HCF (highest common factor), that can divide in all the numbers.
- If needed, take out a common letter, with the smallest exponent.
- Divide the original expression with the **common factor** to get the answer between brackets. (Keep exponent laws in mind)

Example 1

$$2x + 4$$

Example 2

$$3xy - 12xb$$

Example 3

$$2y^2 - 5y^4$$

Activity 12

Factorise the following:

1) $3x + 6$

2) $4x - 12$

3) $6ac + 6cd$

4) $7p - 14pq$

5) $15x^2 + 10x^2y$

6) $x^2 + 2x$

7) $9m^2 - 15m$

8) $16y^3z - 12y^2z$

9) $3gh^3 - 33h^5$

10) $18x^6y^4 - 12x^4y^7$

11) $7m^3n^7 - 21m^4n^8 + 35m^5n^9$

12) $-8b - 16$

13) $-27g^6 - 18g^3$

Common factor (WITH BRACKETS)Steps that could help

- The first bracket is an indication of how the other brackets should look like.
- Eg1 – Brackets must look the same, before taking it as the common factor.
- Eg2 – Brackets does not look the same ...change the second bracket...” there is a plus symbol in the second bracket, therefor you can just turn second bracket around.”
- Eg3 – Bracket does not look the same...change second bracket...” there is a plus symbol in the second bracket, therefor you can just turn second bracket around.”
- Eg4 – Brackets does not look the same ... change second bracket ...” there is a negative symbol in second bracket...turn bracket around, **BUT THE SYMBOL INFROUNT OF THE BRACKET MUST CHANGE.**

Example 1

$$x(a + b) + y(a + b)$$

Example 2

$$m(b + 2c) + y(2c + b)$$

Example 3

$$2x(5b + 3m) - 3y(3m + 5b)$$

Example 4Long method

$$2a(b - c) - 3y(c - b)$$

Short method

$$2a(b - c) - 3y(c - b)$$

Activity 13Factorise:

1) $a(b + c) + d(b + c)$

3) $3x(x - 1) + 2y(x - 1)$

5) $b(a - p) - c(p - a)$

7) $7a(2 - 3c) - 8b(3c - 2)$

9) $b(x - 1)^2 - 3(x - 1)$

2) $p(r + q) + m(r + q)$

4) $3m(2 + x) - 4n(x + 2)$

6) $m(3m - 2n) + n(2n - 3m)$

8) $(a - b)^2 - 2(a - b)$

10) $3a(4a - 1) + 15ab(1 - 4a)$

Grouping (optional)

- When there are four terms in the algebraic expression, group terms into pairs and apply the necessary factorisation.

Example 1

$$ax + bx + ay + by$$

Example 2

$$3x^3 + 6x - x^2 - 2$$

Activity 14

Factorise:

1) $ax + bx + 2a + 2b$

2) $3mx + 2nx + 3m + 2n$

3) $15mx + 5my + 6nx + 2ny$

4) $4ax - 4bx + 6ay - 6by$

2. Difference of two squares (DOTS)

How do I know that it's the difference of two squares?

$$x^2 - 4$$

- There must only be two terms (Binomial).
- There must be a minus between the two terms (Binomial).
- Both terms, letters or numbers, **MUST** have a square root.

Steps:

- Answer must have two brackets. () ()
- Get the square root of term 1.
- Get the square root of term 2.
- One of the brackets will get 'n plus symbol and the other bracket will get a minus symbol.

Example 1

$$a^2 - 4$$

Example 2

$$x^2 - y^2$$

Example 3

$$9x^2 - \frac{1}{4}$$

Example 4

$$x^4 - 16$$

Activity 15

Factorise the following

4.1 $x^2 - 1$

4.3 $16x^2 - 9$

4.5 $x^2 - \frac{4}{49}$

4.7 $5y^2 - 5$

4.9 $-x^2 + 4$

4.11 $-7a^2b^2 + 28$

4.13 $4(m - 2n)^4 - 25$

4.2 $p^2 - 36$

4.4 $49n^2 - 121$

4.6 $a^2b^2 - 4e^4$

4.8 $3p^2q - 48q^3$

4.10 $-25p^4 + 16$

4.12 $(x - 2)^2 - 9$

3. Quadratic trinomial

- It consists of three terms.

$$x^2 + 7x + 12$$

Steps to help:

- Answer must have two brackets. () ()
- Get the square root of term 1.
- Write down all the factors of term 3
- The symbol of term 3, determines how you will get to term 2. (keeping factors of term 3 in mind)
- The symbol of term 2, will go next to the biggest factor.
- Lastly, multiply the factors to justify the value of term 3.

Example 1

$$x^2 + 3x + 2$$

Example 2

$$x^2 - 5x - 6$$

Example 3

$$x^2 - 8x + 12$$

Example 4

$$x^2 + 2x - 15$$

Activity 16Factorise completely

5.1 $x^2 + 5x + 4$

5.3 $p^2 + 7p + 12$

5.5 $x^2 - 10x + 9$

5.7 $x^2 - 5x - 24$

5.9 $27x + x^2 - 90$

5.11 $k(k - 9) - 52$

5.13 $-a^2 - 22a + 75$

5.2 $a^2 + 6a + 5$

5.4 $w^2 - 8w + 15$

5.6 $n^2 + n - 20$

5.8 $x^2 - 13x + 30$

5.10 $-64 + 30y + y^2$

5.12 $x^3 - 12x^2 + 27x$

5.14 $-a^2b + 19ab - 84b$

Use factorisation to simplify algebraic fractions**Example 1**

$$\frac{2a^2b^4c^2}{6a^5b^3c^3}$$

Example 2

$$\frac{2x - 8}{2x}$$

Example 3

$$\frac{6x^2 + 18x - 60}{8x^2 + 40x} \div \frac{x - 2}{8x}$$

Activity 17Simplify the following:

1) $\frac{x^2+2x}{x}$

3) $\frac{7b^3-21b^2}{7b}$

5) $\frac{k^2-4}{k^2-2k}$

7) $\frac{6p^2q-9pq^2}{12p^3q^2-18p^2}$

9) $\frac{x^2-5x+6}{x^2-7x+12}$

11) $\frac{3x(x+1)-2(x+1)}{(x^2-1)}$

2) $\frac{10a^2-30a}{10}$

4) $\frac{2xh^2-4x^2h}{hx}$

6) $\frac{8b^8-16b^9}{8b^9-4b^8}$

8) $\frac{h^4-4}{3h^2+6}$

10) $\frac{a^2-9a+20}{a^2-a-20}$

12) $\frac{x^2-3x-18}{x^2-6x} \div \frac{6x^2+18x}{(-2x)^2}$

Activity 18 (Mixed exercise)

1) $7x - 14$

3) $6a^2 - 6ab$

5) $4n^2 - 4$

7) $3x^2 - 243y^2$

9) $b^2 + 6a - 27$

11) $-a^2 + 5a - 6$

13) $2x^2 - 4x - 30$

15) $2x(a + b) - 6(b + a)$

17) $2ax^2 + 4ax + xy + 2y$

19) $16b^2x - 40b^2 - 8bx + 20b$

2) $-5x - 25$

4) $9m^2 - 1$

6) $1 - x^2$

8) $x^2 + 14x + 40$

10) $c^2 - 6c - 27$

12) $6 - 5x - x^2$

14) $b(a - 1) - 4(1 - a)$

16) $ax + 3a + bx + 3b$

18) $x^3 - 3x^2 - 9x + 27$

20) $\frac{x^2-9x}{x}$

22) $\frac{x^2-14x+24}{x^2-11x-12}$

24) $\frac{2x^2-32}{4x^2-12x-16}$

21) $\frac{x^2-9}{x^2-3x}$

23) $\frac{x^2-14x}{x^2-16x+28}$