Mathematics
Grade 8

TOPIC:

Numeric and Geometric Patterns

Section A: Numeric Patterns

1. Investigate the following numeric patterns and find the next three terms. Describe the rule in words.

   a) 3, 7, 11, 15, 19, ___, ___, ___

   b) 2, 6, 18, 54, 162, ___, ___, ___

   c) 1, 4, 9, 16, 25, ___, ___, ___

   d) 100, 93, 86, 79, 72, ___, ___, ___

2. Investigate the following pattern and answer the questions:

   5, 8, 13, 20, 29, 40, ...

   a) What is the next term in the sequence?

   b) Describe the relationship between consecutive terms in your own words.

   c) Find the 10^{th} term in this pattern.

3. Investigate this pattern and answer the questions: **3, 7, 13, 21, 31, 43, ...**

   a) What is the next term in the sequence?

   b) Describe the relationship between consecutive terms in your own words.

   c) Find the 15^{th} term in this pattern.

4. Investigate this pattern and answer the questions: **50, 47, 42, 35, 26, 15, ...**

   a) What is the next term in the sequence?

   b) Describe the relationship between consecutive terms in your own words.

   c) Find the 100^{th} term in this pattern.

Section B: Geometric Patterns

5. Study the pattern of apples below:

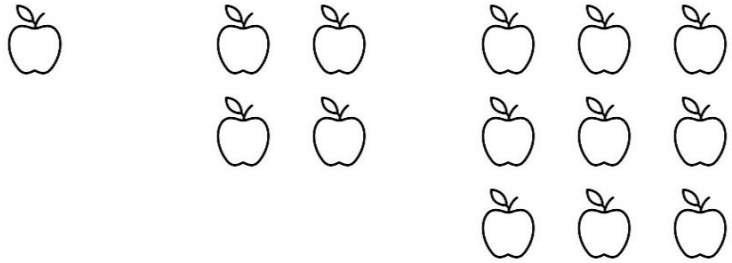


Figure 1 Figure 2 Figure 3

a) Draw Pattern 4.

b) How many apples are in Pattern 5?

c) Complete the table:

Pattern Number (n)	**1**	**2**	**3**	**4**	**5**	**10**
Number of Apples						

d) Write a rule in algebraic form ($T_n =\ldots$) for the number of apples in Pattern n.

6. Investigate this matchstick pattern:

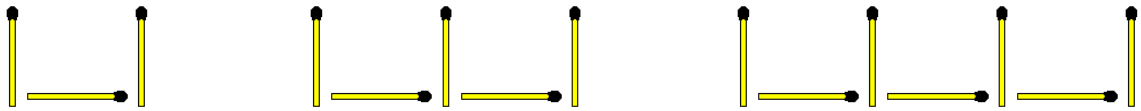


Figure 1 Figure 2 Figure 3

a) How many matchsticks are needed for Figure 5?

b) Complete the table:

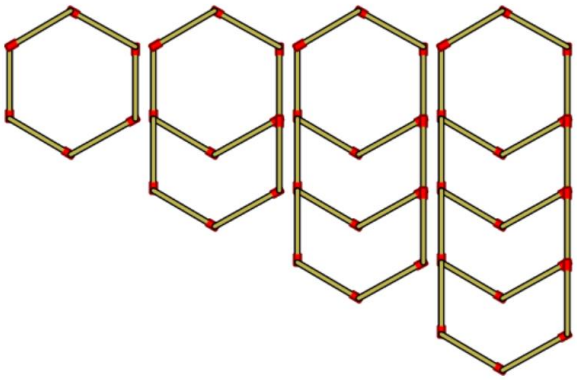
Figure Number (n)	**1**	**2**	**3**	**4**	**5**	***n***
Number of Matchsticks						

c) Describe the pattern in your own words.

d) Write an algebraic rule ($T_n =\ldots$) for the number of matchsticks in Figure n.

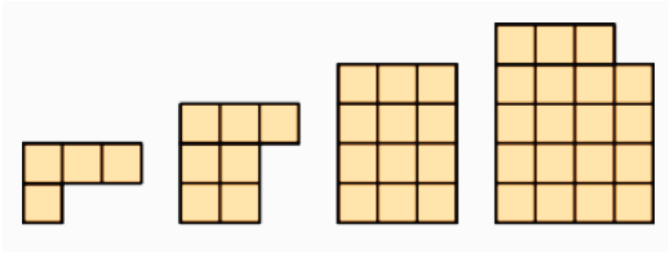
e) How many matchsticks would be needed for Figure 20? Show your working.

7. Jacob is building patterns with matchsticks. He builds one item each day. Here are the first four constructions:



How many matchsticks does Jacob use on the fifth day?

8. Consider the following pattern:



Picture 1 Picture 2 Picture 3 Picture 4

How many squares will there be in the 7th picture?

Section C: Patterns in Tables

9. The table shows the relationship between two variables:

x	1	2	3	4	5	6
y	5	9	13	17	21	25

a) Describe the relationship between x and y in words.

b) Write an algebraic rule to represent this relationship.

c) What would be the value of y when $x = 10$?

d) If $y = 101$, what is the value of x?

10. Study the following table:

Input (n)	**1**	**2**	**3**	**4**	**5**
Output	1	8	27	64	125

a) What pattern do you observe?

b) What would be the output when $n = 6$?

c) Write an algebraic expression for the output in terms of n.

Section D: Creating Your Own Patterns

11. Create your own numeric pattern with at least 6 terms.

    Pattern: ___, ___, ___, ___, ___, ___

    a) Describe your rule in words:

    b) Write your rule in algebraic form (if possible):

12. Design a geometric pattern using shapes or symbols.

    Draw at least 3 figures of your pattern.

Figure 1	**Figure 2**	**Figure 3**

    a) Explain the rule for your pattern.

    b) If your pattern can be represented algebraically, write the rule.

Section E: Complex Patterns

13. Investigate this pattern:

    2, 5, 10, 17, 26, 37, ...

    a) This pattern does not have a constant difference. Describe what is happening between terms.

    b) Find the next two terms.

    c) Challenge: Can you write a rule for the n^{th} term? (Hint: Look at perfect squares)

14. Study the pattern in this table:

Position (n)	**1**	**2**	**3**	**4**	**5**
Value	3	7	13	21	31

    a) Find the differences between consecutive terms.

    b) Find the second differences (the differences between the differences).

    c) What do you notice about the second differences?

    d) Predict the 6^{th} term and verify using the pattern.

    e) Challenge: Write an algebraic rule ($T_n = \ldots$) for the n^{th} term.

Section F: Justifying Rules

15. For the pattern 4, 9, 14, 19, 24, ...

    a) Write the rule in algebraic form (T_n =...)

    b) Justify why your rule works by explaining how it connects to the pattern.

    c) Use your rule to find the 50th term.

16. A student says the rule for the pattern 1, 4, 9, 16, 25, ... is "add the next odd number each time."

    Another student says the rule is n^2.

    a) Show that both students are correct.

    b) Which rule would be easier to find the 100th term? Explain why.

BONUS QUESTIONS

17. The equation $y = 2x - 1$ is given. Answer the following questions using this given equation:

x	-2	b
y	a	-1

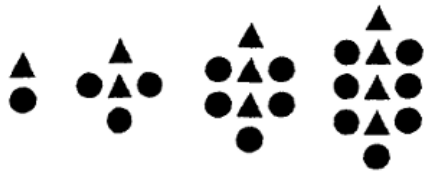
    a) Calculate the values of a and b.

18. A pattern has the rule: $T = 3^n - 2$

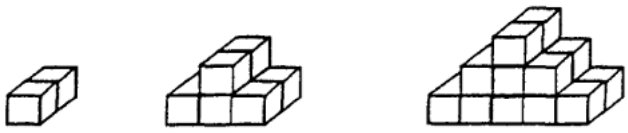
    a) Find the first 3 terms.

    b) Is this a linear pattern?

19. Find an equation that relates the number of circles to the number of triangles. Verify the equation and predict the number of circles that would be in a diagram with 8 triangles.



20. Look at the following pattern:



    Provide a written description of how to build the fourth term in the pattern.
    Describe the pattern.

21. Consider the following sequence:

$$3z+5\,;6z+2\,;9z-1\,;12z-4\,;15z-7\,;18z-10\,;\ldots$$

    Write down the next term.

CAPS CLARRIFICATION:

Investigate and extend patterns

- Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns:
  - represented in physical or diagram form
  - not limited to sequences involving a constant difference or ratio
  - of learner's own creation
  - represented in tables
  - represented algebraically
- Describe and justify the general rules for observed relationships between numbers in own words or in algebraic language