

Western Cape Education Department

MATHEMATICS

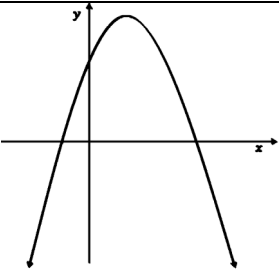
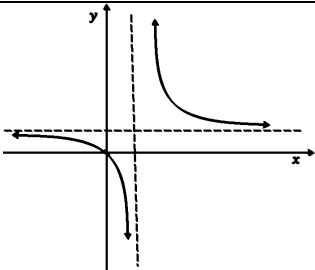
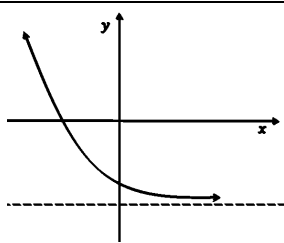
Grade 12 Term 1 2025

Learner notes

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1.1 GRAPHS – SUMMARY

Summary of the grade 11 Graphs

GRAPH NAME	Parabola	Hyperbola	Exponential Graph
EQUATION	$y = ax^2 + bx + c$ $y = a(x + p)^2 + q$	$y = \frac{k}{x + p} + q$	$y = a^{x+p} + q$
WHAT YOU NEED to sketch the graph.	x –intercepts $y = 0$	x –intercepts $y = 0$	x –intercepts $y = 0$
	y –intercepts $x = 0$	y –intercepts $x = 0$	y –intercepts $x = 0$
	TURNING POINT Axes of Symmetry $x = -\frac{b}{2a}$ $f'(x) = 0$	ASYMPTOTES $x = -p$ $y = q$	ASYMPTOTE $y = q$
EXAMPLE of the SKETCH of the graph			

1.2 THE PARABOLA

1.2.1 Sketching a parabola.

EXAMPLE 1:

Equations in the TURNING POINT form $y = (x + p)^2 + q$

Given: $f(x) = (x + 1)^2 - 4$

- 1.1 Write down the turning point of f .
- 1.2 Determine the x -intercepts of f .
- 1.3 Determine the coordinates of the y -intercept of f .
- 1.4 Draw the graph of f . Clearly show the turning point and intercepts with the axes.
- 1.5 Use your graph to answer the following questions:
 - 1.5.1 Give the range of f .
 - 1.5.2 For which values of x will
 - (a) $f(x) > 0$
 - (b) $f'(x) > 0$
 - 1.5.3 Determine the equation of h where h is the graph after f moved 1 unit to the right.
 - 1.5.4 Write down the turning point for g if g is the reflection of f in the x -axis.

EXAMPLE 2:

Equations in the STANDARD FORM $y = ax^2 + bx + c$

Given: $f(x) = -2x^2 + 8x + 10$

- 2.1 Determine the coordinates of the turning point of f .
- 2.2 Determine the x -intercept of f .
- 2.3 Write down the coordinates of the y -intercept of f .
- 2.4 Draw the graph of f . Clearly show the turning point and intercepts with the axes.
- 2.5 Use your graph to answer the following questions:
 - 2.5.1 Give the range of $f(x) - 3$.
 - 2.5.2 For which values of x will
 - (a) $f(x) \leq 0$
 - (b) $f'(x) < 0$
 - 2.5.3 Determine the equation of the axis of symmetry of $f(x - 1)$
 - 2.5.4 Does the graph $-f(x)$ have a maximum or minimum value? Give this value.

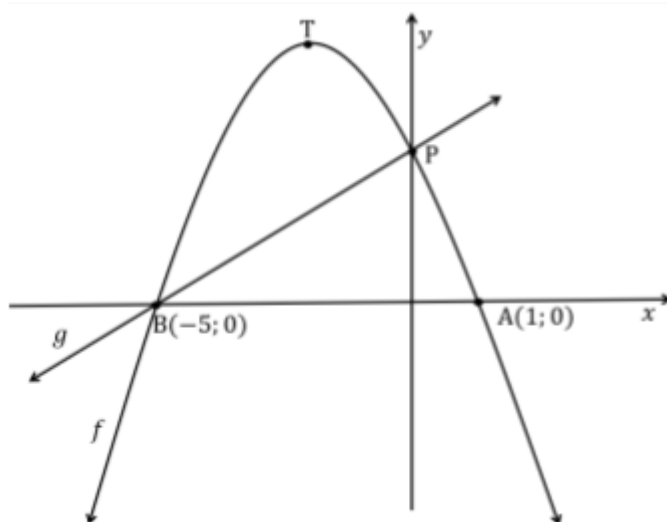
1.2.2 Determine the equation of a parabola.

EXAMPLE 3:

x -INTERCEPTS GIVEN $y = a(x - r_1)(x + r_2)$

The graph of $f(x) = ax^2 + bx + c$ and $k(x) = 2x + 10$ is drawn below.

- The parabola cuts the x -axis at A and B and the y -axis at P.
- B and P are also the intercepts of k .
- T is the turning point of f .



3.1 Write down:

3.1.1 the coordinates of P.

3.1.2 the equation of the axis of symmetry.

3.2 Determine the numerical values of a , b and c .

3.3 For which values of x is,

3.3.1 $f(x) \leq 0$

3.3.2 $f'(x) = 0$

3.4 Give the range of f .

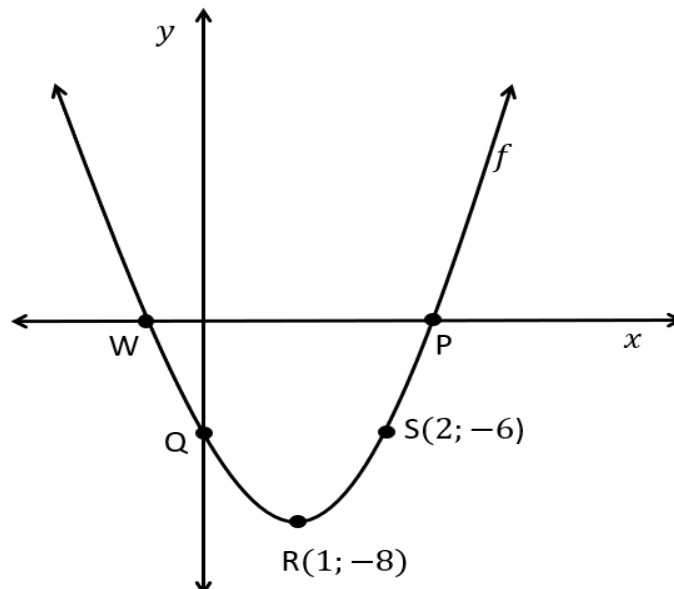
3.5 Determine the values of k for which $f(x) = k$ will have no real roots.

EXAMPLE 4:

TURNING POINT GIVEN $y = a(x + p)^2 + q$

The graph of $f(x) = ax^2 + bx + c$ is drawn below. W, P and Q are the intercepts of f .
 $R(1; -8)$ is the turning point of f . $S(2; -6)$ is a point on f .

4.1 Write down the coordinates of Q if S and Q are reflections in the axis of symmetry.



4.2 Write down the range of $f(x) + 8$.

4.3 Determine the numerical values of a , b and c .

4.4 Determine the length of WP

4.5 For which values of x will

4.5.1 $f(x) \leq 0$

4.5.2 $f'(x) > 0$

4.6 Write down the equation of the axis of symmetry for $f(x + 3)$

4.7 Determine the coordinates of the turning point for $f(-x)$.

4.8 Determine the values of k for which $f(x) = k$ will have two unequal, positive real roots.

1.3 THE HYPERBOLA

1.3.1 Sketching a hyperbola.

EXAMPLE 5:

$$y = \frac{k}{x + p} + q$$

Given: $f(x) = \frac{6}{x-3} + 4$

5.1 Write down:

5.1.1 the equation of the horizontal asymptote of f .

5.1.2 the equation of the vertical asymptote of f .

5.1.3 the domain of f .

5.2 Determine:

5.2.1 the x -intercept of f .

5.2.2 the coordinates of the y -intercept of f .

5.3 Draw the graph of f . Clearly show ALL the asymptotes and intercepts with the axes.

5.4 Use your graph to answer the following questions:

5.4.1 Write down the range of f .

5.4.2 For which values of x will

(a) $f(x) \geq 0$

(b) $f(x) < 0$

(c) $f'(x) < 0$

5.5 Determine the equation of h where h is the graph after f moved 1 unit to the right.

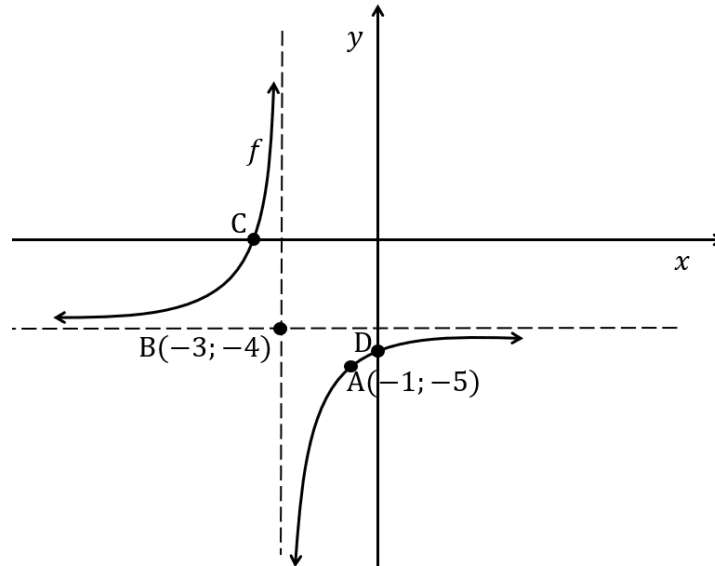
5.6 Determine the equation(s) of the axis of symmetry of f .

1.3.2 Determine the equation of a hyperbola.

EXAMPLE 6: $y = \frac{k}{x+p} + q$

In the diagram is the graph of: $f(x) = \frac{k}{x+p} + q$. C and D are the intercepts of f .

The asymptotes of f intersect at $B(-3; -4)$. A $(-1; -5)$ is a point on f .



6.1 Write down:

6.1.1 the equations of the asymptotes of f .

6.1.2 the range of f .

6.1.3 the domain of $f(x + 3)$.

6.2 Determine:

6.2.1 the numerical values of k, p and q .

6.2.2 the coordinates of the y -intercept of f .

6.2.3 the coordinates of the x -intercept of f .

6.2.4 the equation of the axis of symmetry of f that has a negative gradient.

6.3 For which values of x will:

6.3.1 $f(x) \geq 0$

6.3.2 $f(x) < 0$

6.4 Is the graph increasing or decreasing?

4. THE EXPONENTIAL GRAPH

4.1 Sketching an exponential graph.

EXAMPLE 7: $y = a \cdot b^{x+p} + q$

Given: $f(x) = 2^{-x} - 4$

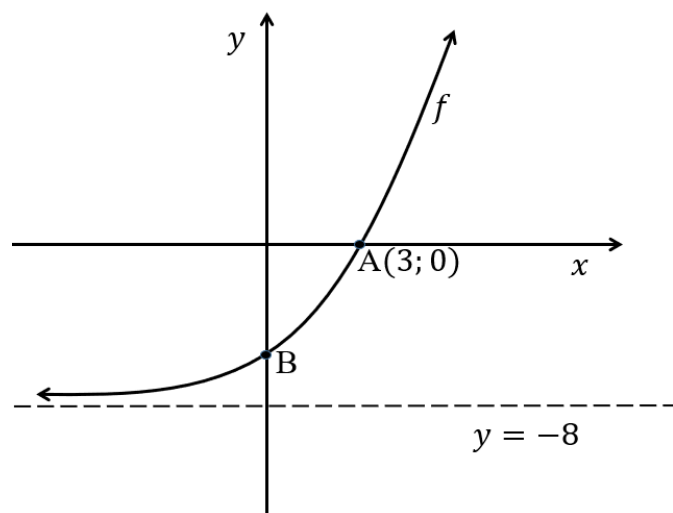
- 7.1 Write down the equation of the asymptote of f .
- 7.2 Determine:
- 7.2.1 the x -intercept of f .
 - 7.2.2 the coordinates of the y -intercept of f .
- 7.3 Draw the graph of f . Clearly show ALL the asymptote(s) and intercepts with the axes.
- 7.4 Use your graph to answer the following questions:
- 7.4.1 Write down the range of f .
 - 7.4.2 For which values of x will
 - (a) $f(x) \geq 0$
 - (b) $f(x) < 0$
- 7.5 Determine the equation of h where h is the graph after f moved 2 unit to the right and 1 unit up.
- 7.6 Is the graph increasing or decreasing?

1.4.2 Determine the equation of an exponential graph.

Example 8: $y = a \cdot b^{x+p} + q$

In the diagram is the graph of: $f(x) = 2^{x+p} + q$. A(3;0) and B are the intercepts of f .

The asymptote of f is at $y = -8$



8.1 Write down:

8.1.1 the range of f .

8.1.2 the range of $f(x) + 3$

8.2 Determine:

8.2.1 the numerical values of p and q .

8.2.2 the coordinates of B, the y-intercept of f .

8.3 For which values of x will:

8.3.1 $f(x) \geq 0$

8.3.2 $f(x) < 0$

8.4 Is the graph increasing or decreasing?

8.5 Describe the transformation from f to $h(x) = \left(\frac{1}{2}\right)^x$

2. INVERSE FUNCTIONS

BACKGROUND KNOWLEDGE

FUNCTIONAL NOTATION

✓ $f(x) = 2x - 1$

$f(3) = 2(3) - 1$

$\therefore (3; 5)$

✓ Given: $f(x) = 3x$

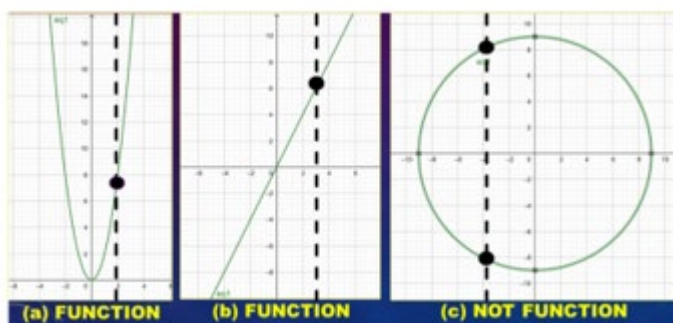
Determine $f(-2)$ and write down the coordinates.

You do!



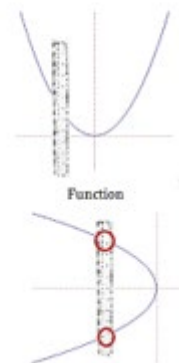
DEFINITION OF A FUNCTION

✓ For every x -value, there is at most one y -value.



✓ The Vertical Line Tests

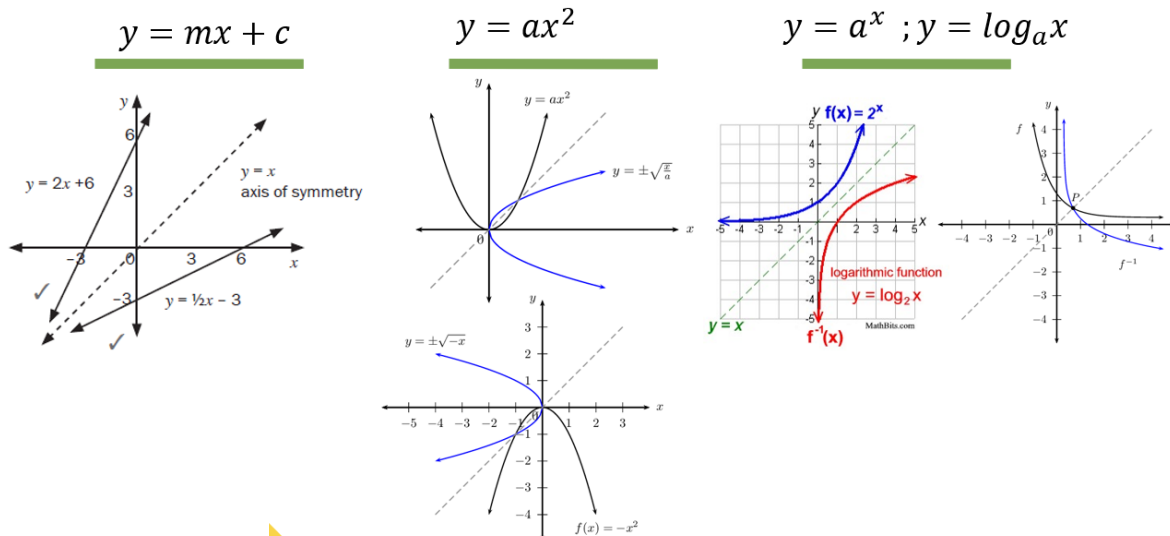
If the ruler only ever cuts the curve in one place, then the graph is a function



INVERSE

1. The inverse of a function takes the y -values (range) of the function to the corresponding x -values (domain) and vice versa. Therefore, the x and y values are interchanged.
2. The function is reflected along the line $y = x$ to form the inverse.
3. The notation for the inverse of a function is f^{-1} .

INVERSE GRAPHS THAT YOU HAVE TO KNOW



THE INVERSE OF A STRAIGHT LINE $y = mx + c$

INVERSE FUNCTION BY SWOPPING x AND y

Example 1:

$$y = 2x - 4$$

← f

← Interchange x and y

← Make y the subject of the formula

← f^{-1}

Example 2:

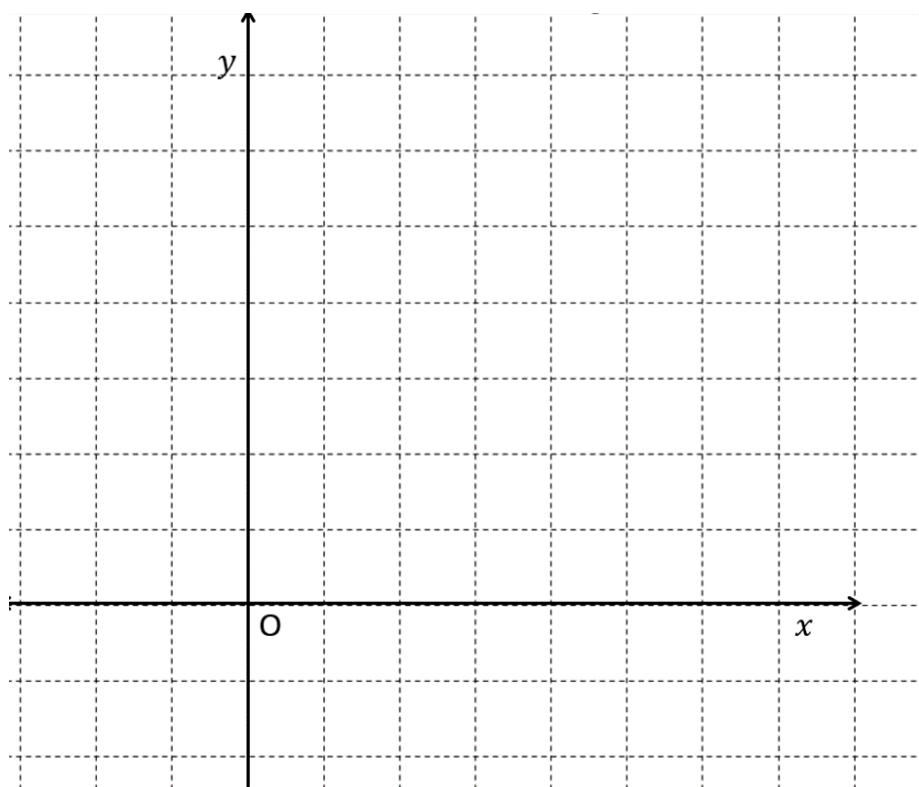
If $f(x) = 2x + 6$

2.1 Determine f^{-1} , that is the inverse of f .

2.2 Sketch the graphs of f , f^{-1} and $y = x$ on the same system of axes.

You do!





WRITING IN LOG FORM

We know that:

$$2^1 = 2$$

$$2^2 = 4$$

If $y = a^x$
then $\log_a y = x$

- But what will the answer be if: $2^x = 3$

Example 1

$$2^x = 3$$

Example 2

$$5^x = 3$$

- To answer this question, we need to understand logarithms.

- The answer to a logarithm is an exponent.

Example 3

$$2^x = 8$$

Example 4

$$10^x = 100$$

NOTE that we do not have to write the base if it is 10.

Writing from exponent form to log form:

Example 1

$$5^2 = 25$$

Example 2

$$3^4 = 81$$

You do!



Example 3

$$10^2 = 100$$

Example 4

$$2^{-2} = \frac{1}{4}$$

Note that the answer of a logarithm can be negative, but

- base of a log cannot be negative and
- you cannot take the log of a negative.

Write the following from log form to exponent form:

You do!



If $\log_a y = x$
then $y = a^x$

Example 1

$$2 = \log_5 25$$

Example 2

$$4 = \log_3 81$$

Example 3

$$-2 = \log_2 \frac{1}{4}$$

Example 4

$$2 = \log 100$$

THE INVERVES OF AN EXPONENTIAL FUNCTION $y = a^x$ AND LOG FUNCTION $y = \log_a x$

Example 3:

Let's investigate the exponential graph and its inverse:

Given: $f(x) = 2^x$

See the completed table for $y = 2^x$:

x	-2	-1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4

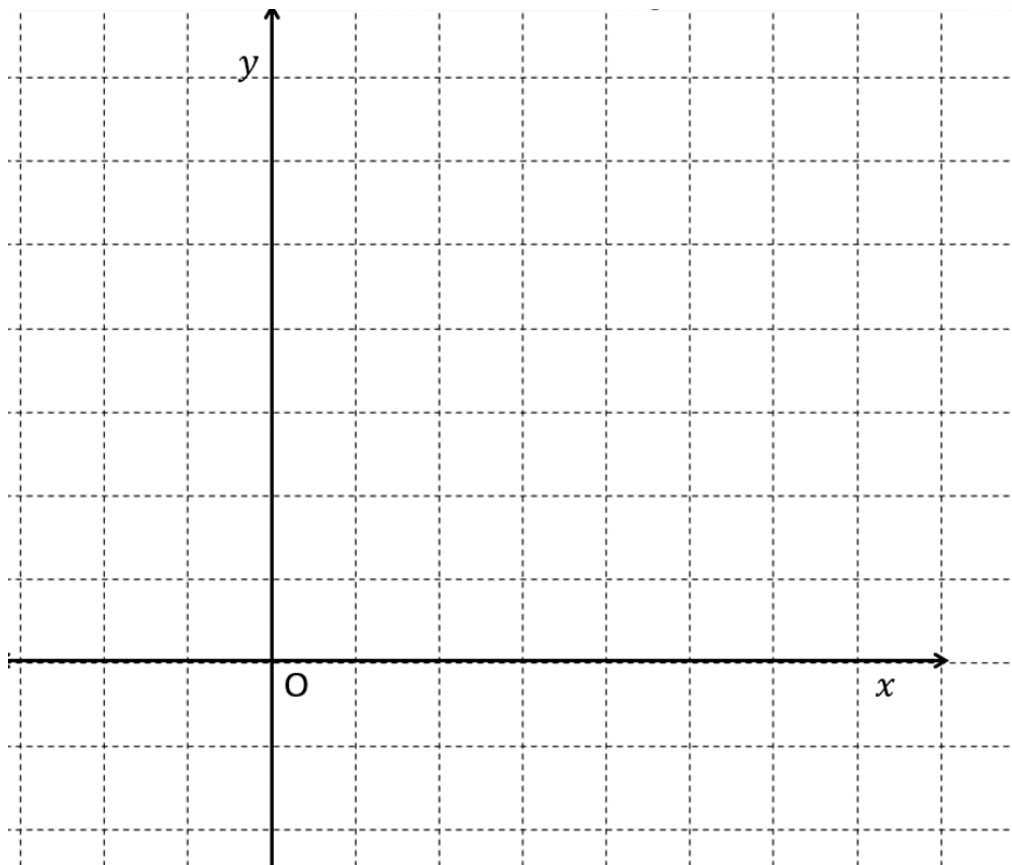
$$f: y = 2^x$$

Let's swop the x and y -values:

$$f^{-1}:$$

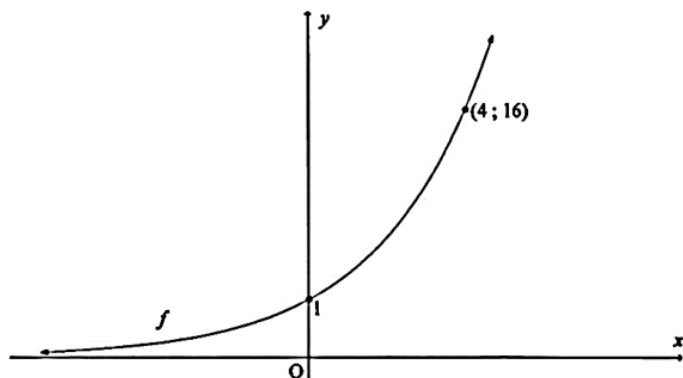
x					
y					

y as subject of the formula

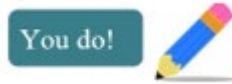


**QUESTION 5**

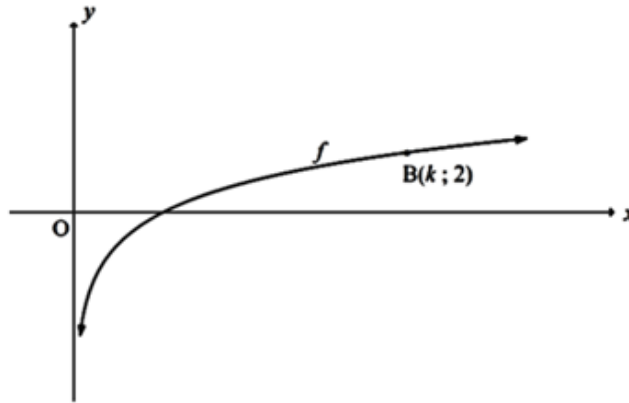
Sketched below is the graph of $f(x) = k^x$; $k > 0$. The point $(4; 16)$ lies on f .



- 5.1 Determine the value of k . (2)
- 5.2 Graph g is obtained by reflecting graph f about the line $y = x$.
Determine the equation of g in the form $y = \dots$ (2)
- 5.3 Sketch the graph g . Indicate on your graph the coordinates of two points on g . (4)

**QUESTION 6**

The graph of $f(x) = \log_4 x$ is drawn below. $B(k; 2)$ is a point on f .



- 6.1 Calculate the value of k . (2)
- 6.2 Determine the values of x for which $-1 \leq f(x) \leq 2$. (2)
- 6.3 Write down the equation of f^{-1} , the inverse of f , in the form $y = \dots$ (2)
- 6.4 For which values of x will $x \cdot f^{-1}(x) < 0$? (2)

THE INVERSE OF A PARABOLA $y = ax^2$

Example 4:

Given: $f(x) = -\frac{1}{2}x^2$

- 4.1 Determine the equation of the inverse function in the form $f^{-1}(x) = \dots$
- 4.2 Sketch the graph of the function and its inverse on the same set of axes
- 4.3 Now restrict the domain of the original function in two different ways to form new one-to-one functions.

Example 5:



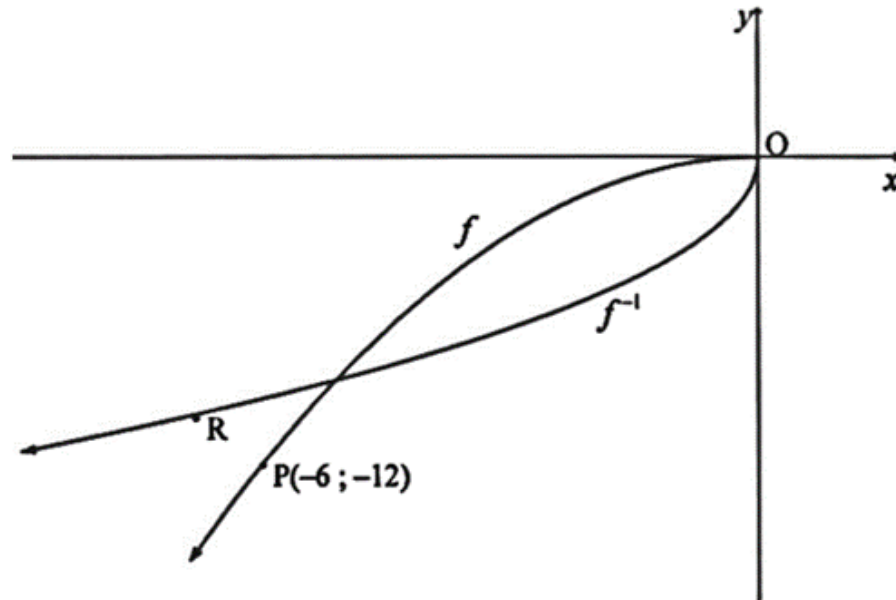
Given $f(x) = 2x^2$

- 5.1 Determine the inverse of f . Hence rewrite the equation of the inverse function in the form $f^{-1}(x) = \dots$
- 5.2 Sketch the graphs of the new function and its inverse function and $y = x$ on the same set of axes.
- 5.3 Is f^{-1} a function? Give a reason for your answer.



QUESTION 4

In the diagram below, the graph of $f(x) = ax^2$ is drawn in the interval $x \leq 0$. The graph f^{-1} is also drawn. $P(-6; -12)$ is a point on f and R is a point on f^{-1} .



4.1 Is f^{-1} a function? Motivate your answer. (2)

4.2 If R is the reflection of P in the line $y = x$, write down the coordinates of R . (1)

4.3 Calculate the value of a . (2)

4.4 Write down the equation of f^{-1} in the form $y = \dots$ (3)

3. Consolidation - Functions

QUESTION 1 [Q5 – DBE/Feb-Mar 2010]

Given: $f(x) = \frac{2}{x-3} + 1$

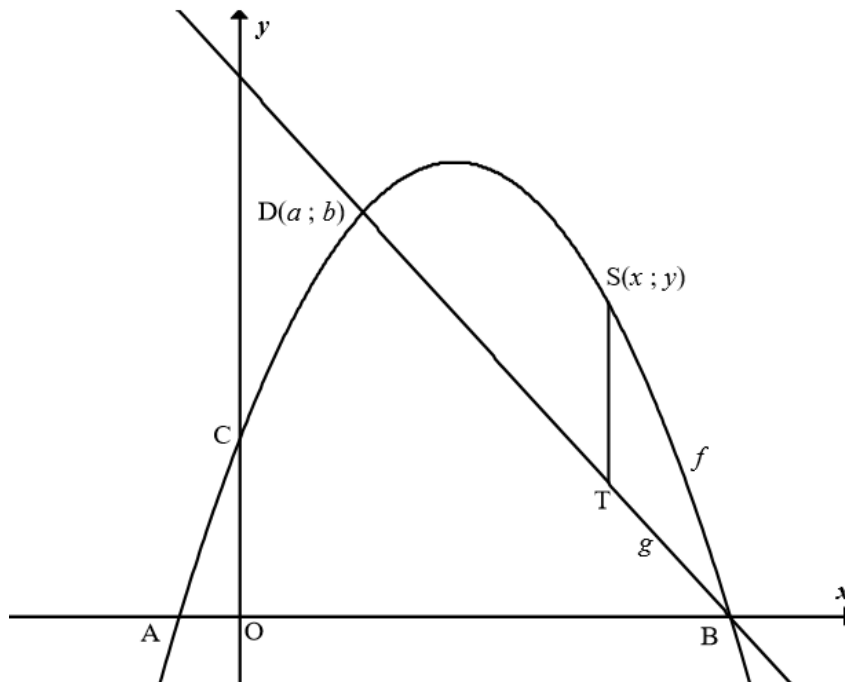
- 1.1 Write down the equation of the asymptotes of f (2)
- 1.2 Calculate the coordinates of the x – and y –intercepts of f . (3)
- 1.3 Sketch f and show all intercepts with the axes and the asymptotes. (3)

[8]

QUESTION 2 [Q6 – DBE/Feb-Mar 2010]

The graphs of $f(x) = -x^2 + 7x + 8$ and $g(x) = -3x + 24$ are sketched below.

f and g intersect in D and B. A and B are the x – intercepts of f .

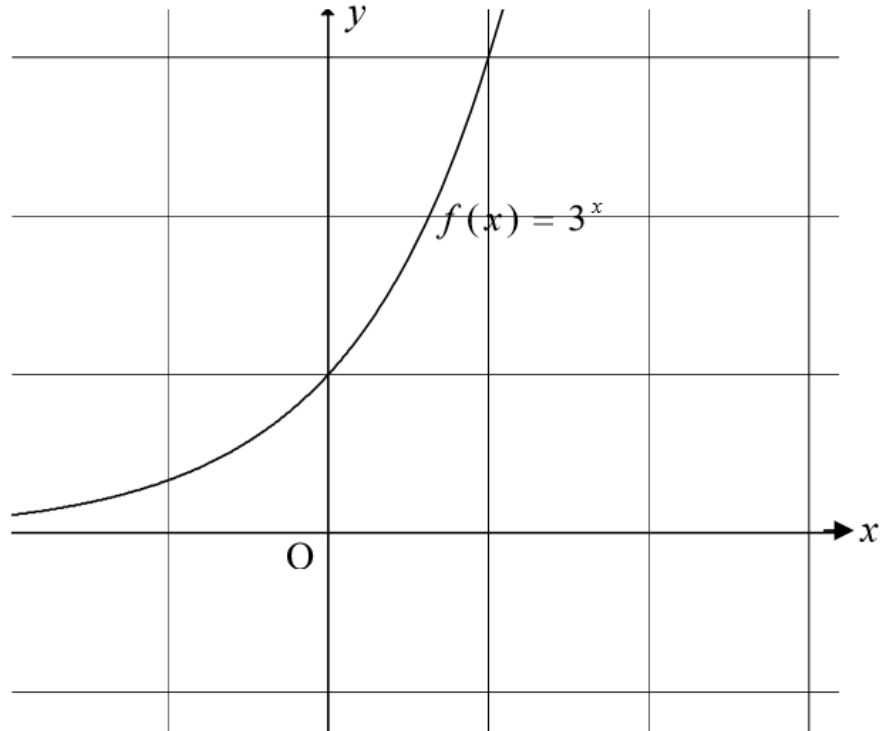


- 2.1 Determine the coordinates of A and B. (4)
- 2.2 Calculate a , the x –coordinate of D. (4)
- 2.3 $S(x; y)$ is a point on the graph of f , where $a \leq x \leq 8$.
ST is drawn parallel to the y –axis with T on the graph of g .
Determine ST in terms of x . (2)
- 2.4 Calculate the maximum length of ST. (2)

[12]

QUESTION 3 [Q7 – DBE/Feb-Mar 2010]

The graphs of $f(x) = 3^x$ is drawn below.

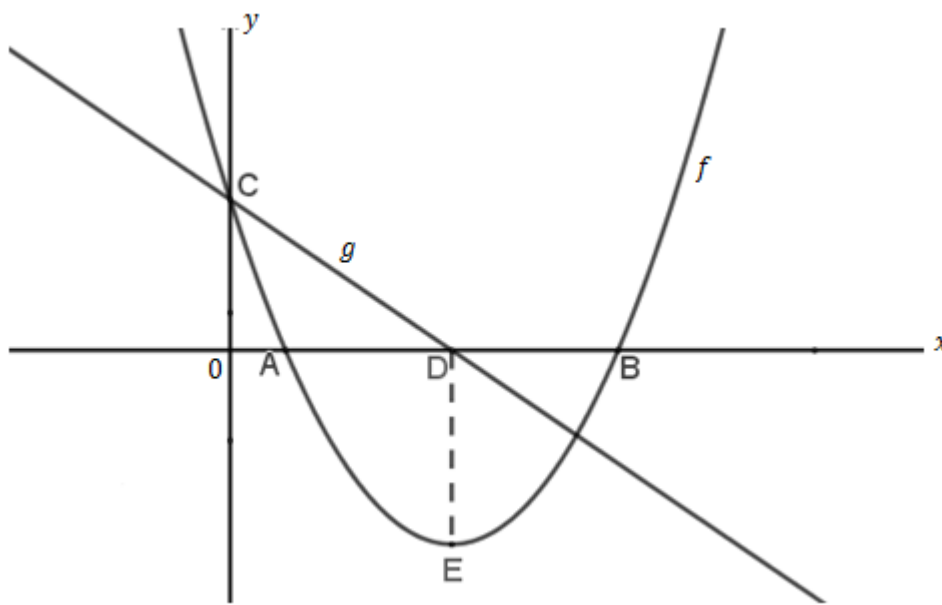


- 3.1 Write f^{-1} in the form $y = \dots$ (1)
- 3.2 Sketch the graph of $y = f^{-1}(x)$ and $y = f^{-1}(x - 2)$. (4)
- 3.3 Use your graphs to solve for x if $\log_3(x - 2) < 1$. (2)
- [7]

QUESTION 4 [Q4 – DBE/Feb-Mar 2018]

Below are the graphs of $f(x) = (x - 4)^2 - 9$ and a straight line g .

- A and B are the x – intercepts of f and E is the turning point of f .
- C is the y – intercept of both f and g .
- The x – intercept of g is D. DE is parallel to the y – axis.

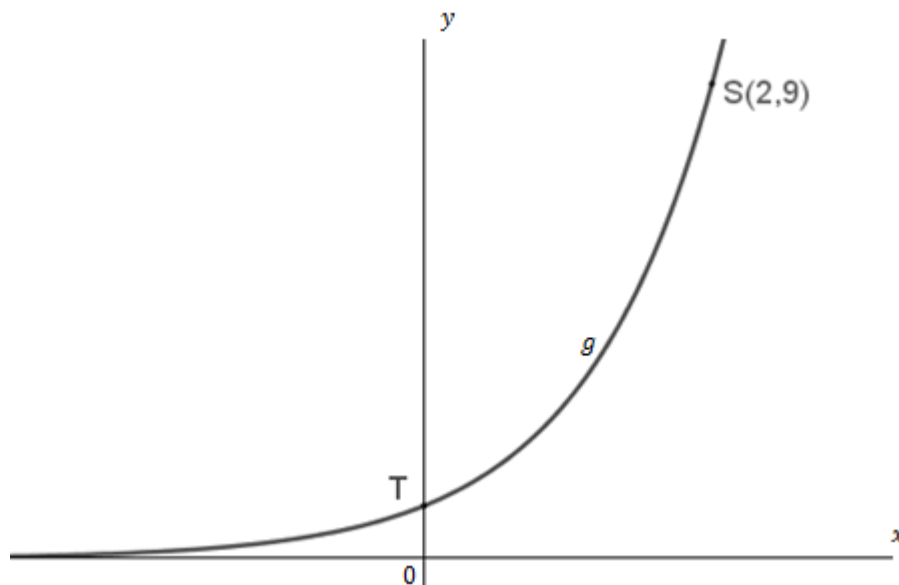


- 4.1 Write down the coordinates of E. (2)
- 4.2 Calculate the coordinates of A. (3)
- 4.3 M is the reflection of C in the axis of symmetry of f . Write down the coordinates of M. (2)
- 4.4 Determine the equation of g in the form $y = mx + c$. (3)
- 4.5 Write down the equation of g^{-1} in the form $y = \dots$ (3)
- 4.6 For which values of x will, $x(f(x)) \leq 0$? (4)

[18]

QUESTION 5 [Q5 – DBE/Feb-Mar 2018]

The graph of $g(x) = a^x$ is drawn in the sketch below. The point $S(2; 9)$ lies on g .
T is the y –intercept of g .



- 5.1 Write down the coordinates of T. (2)
- 5.2 Calculate the value of a . (2)
- 5.3 The graph h is obtained by reflecting g in the y –axis. Write down the equation of h . (2)
- 5.4 Write down the values of x for which, $0 < \log_3 x < 1$. (2)
- [8]**

QUESTION 6 [Q6 – DBE/Feb-Mar 2018]

The function f , defined by $f(x) = \frac{a}{x+p} + q$, has the following properties:

- The range of f is $y \in \mathbb{R}, y \neq 1$.
- The graph f passes through the origin.
- $P(\sqrt{2} + 2; \sqrt{2} + 1)$ lies on the graph f .

- 6.1 Write down the value of q . (1)
- 6.2 Calculate the value of a and p . (5)
- 6.3 Sketch a neat graph of this function. Your graph must include the asymptotes, if any. (4)
- [10]**

QUESTION 7 [Q4 – DBE/Nov 2012]

7.1 Consider the function $f(x) = 3 \cdot 2^x - 6$.

7.1.1 Calculate the coordinates of the y –intercept of the graph of f . (1)

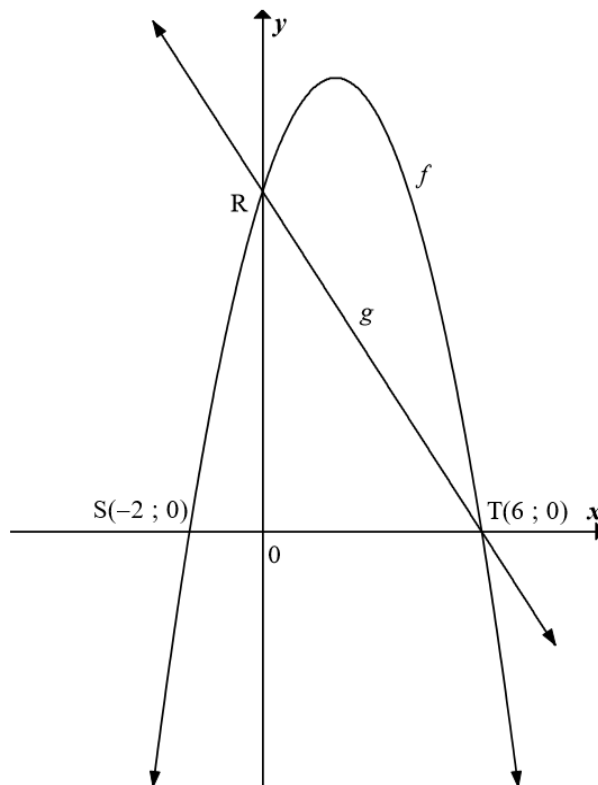
7.1.2 Calculate the x –intercept of the graph of f . (2)

7.1.3 Sketch the graph of f in your ANSWER BOOK. (3)

Clearly show ALL asymptotes and intercepts with the axes.

7.1.4 Write down the range of f . (1)

7.2 $S(-2; 0)$ and $T(6; 0)$ are the x –intercepts of the graphs of $f(x) = ax^2 + bx + c$ and R is the y –intercept. The straight line through R and T represents the graph of $g(x) = -2x + d$.



7.2.1 Determine the value of d . (2)

7.2.2 Determine the equation of f in the form $f(x) = ax^2 + bx + c$. (4)

7.2.3 If $f(x) = -x^2 + 4x + 12$, calculate the coordinates of the turning point of f . (2)

7.2.4 For which values of k , will $f(x) = k$ have two distinct roots? (2)

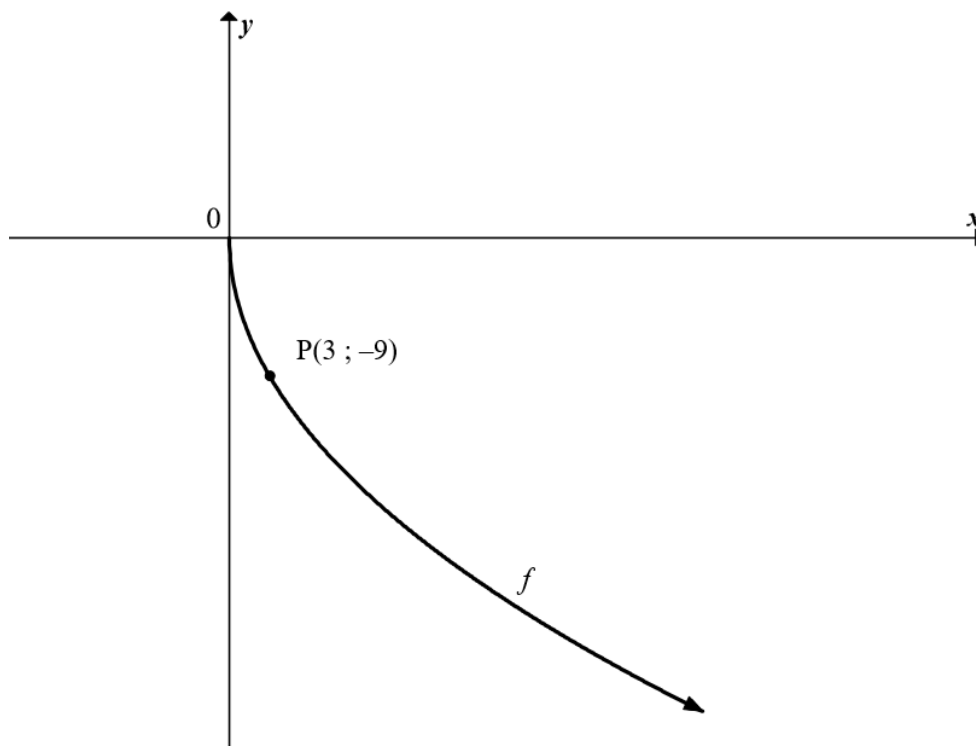
7.2.5 Determine the maximum value of $h(x) = 3^{f(x)-12}$. (3)

[20]

QUESTION 8 [Q5 – DBE/Nov 2012]

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketch below.

The point $P(3; -9)$ lies on the graph of f .



8.1 Use the graph to determine the values of x for which $f(x) \geq -9$. (2)

8.2 Write down the equation of f^{-1} in the form $y = \dots$ Include ALL restrictions. (3)

8.3 Sketch f^{-1} , the inverse of f , in your ANSWER BOOK.

Indicate the intercept(s) with the axes and the coordinates of ONE other point. (3)

8.4 Describe the transformation from f to g if $g(x) = \sqrt{27x}$, where $x \geq 0$. (1)

[9]

QUESTION 9 [Q6 – DBE/Nov 2012]

The graph of a hyperbola with the equation $y = f(x)$ has the following properties:

- Domain: $x \in \mathbb{R}, x \neq 5$
- Range: $y \in \mathbb{R}, y \neq 1$
- Passes through point $(2; 0)$

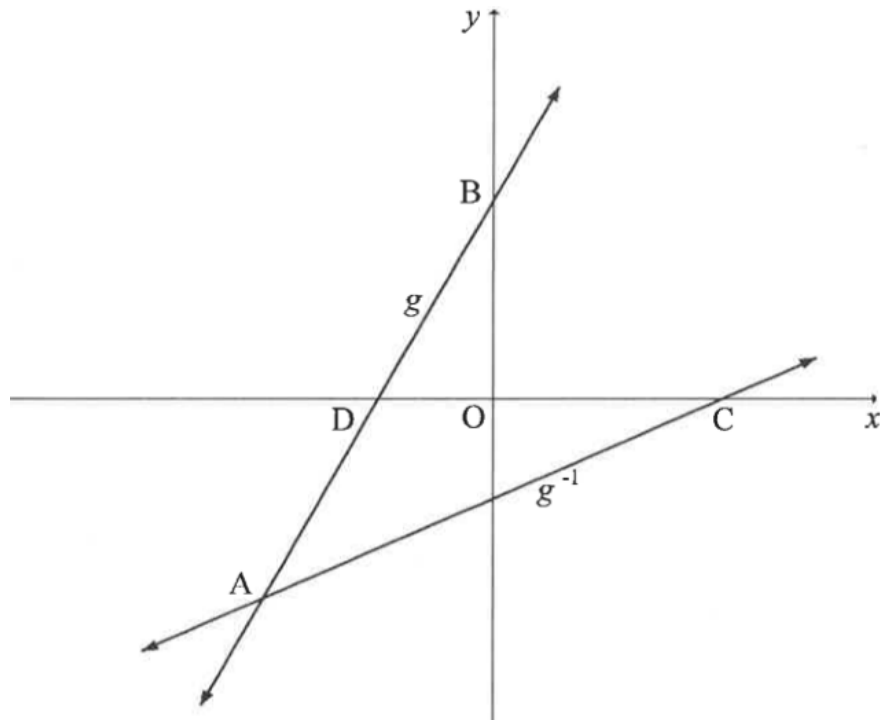
Determine $f(x)$.

[4]

QUESTION 10 [Q5 – DBE/Nov 2022]

The graphs of $g(x) = 2x + 6$ and g^{-1} , the inverse of g , are shown in the diagram below.

- D and B are the x – and y – intercepts respectively of g .
- C is the x – intercept of g^{-1} .
- The graphs of g and g^{-1} intersect at A.



- 10.1 Write down the y – coordinate of B. (1)
- 10.2 Determine the equation of g^{-1} in the form $g^{-1}(x) = mx + n$. (2)
- 10.3 Determine the coordinates of A. (3)
- 10.4 Calculate the length of AB. (2)
- 10.5 Calculate the area of $\triangle ABC$ (5)

[13]