



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N4

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Hierdie nasienriglyn bestaan uit 8 bladsye.

VRAAG 1

$$\begin{aligned}
 1.1 \quad LK &= \frac{1 + \sin y}{1 - \sin y} - \frac{1 - \sin y}{1 + \sin y} \\
 &= \frac{(1 + \sin y)(1 + \sin y) - [(1 - \sin y)(1 - \sin y)]}{(1 - \sin y)(1 + \sin y)} \checkmark \\
 &= \frac{1 + 2 \sin y + \sin^2 y - 2 \sin y - \sin^2 y}{1 - \sin^2 y} \checkmark \\
 &= \frac{4 \sin y}{\cos^2 y} \checkmark \\
 &= \frac{4 \sin y}{\cos y} \cdot \frac{1}{\cos y} \checkmark \\
 &= 4 \tan y \cdot \sec y \checkmark \\
 &= RK
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 1.2 \quad &\frac{\tan \theta (\sec \theta + 1) + \tan \theta (\sec \theta - 1)}{(\sec \theta - 1)(\sec \theta + 1)} \\
 &= \frac{\tan \theta \cdot \sec \theta + \tan \theta \cdot \sec \theta - \tan \theta}{\sec^2 \theta - 1} \checkmark \\
 &= \frac{2 \tan \theta \cdot \sec \theta}{\tan^2 \theta} \checkmark \\
 &= \frac{2 \sec \theta}{\frac{\sin \theta}{\cos \theta}} \checkmark \\
 &= 2 \operatorname{cosec} \theta \checkmark
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 1.3 \quad &\frac{\operatorname{cosec} A}{1 + \cos A} \\
 &= \frac{\frac{5}{4}}{1 + \frac{3}{5}} \checkmark \\
 &= \frac{5}{4} \times \frac{5}{8} \checkmark \\
 &= \frac{25}{32} \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 1.4 \quad &\frac{1}{\sec(-15^\circ)} = \cos(-15^\circ) \checkmark \\
 &= \cos(60^\circ - 45^\circ) \\
 &= \cos 60^\circ \cdot \cos 45^\circ + \sin 60^\circ \sin 45^\circ \checkmark \\
 &= \frac{1 + \sqrt{3}}{2\sqrt{2}} \checkmark
 \end{aligned} \tag{3}$$

1.5 $2\cos^2 2\theta + \cos 2\theta - 1 = 0$
 $(2\cos 2\theta - 1)(\cos 2\theta + 1) = 0 \checkmark$
 $1^{ste}: \cos 2\theta = \frac{1}{2}$ Of $\cos 2\theta = -1 \checkmark$
 $1^{ste}: 2\theta = 60^\circ$ en $4^{de}: 2\theta = 360^\circ - 60^\circ$ N/A $\checkmark$
 $\therefore$ Algemene oplossing $\{\pm 30^\circ; \pm 150^\circ\} \checkmark \checkmark$

(5)
[20]

VRAAG 2

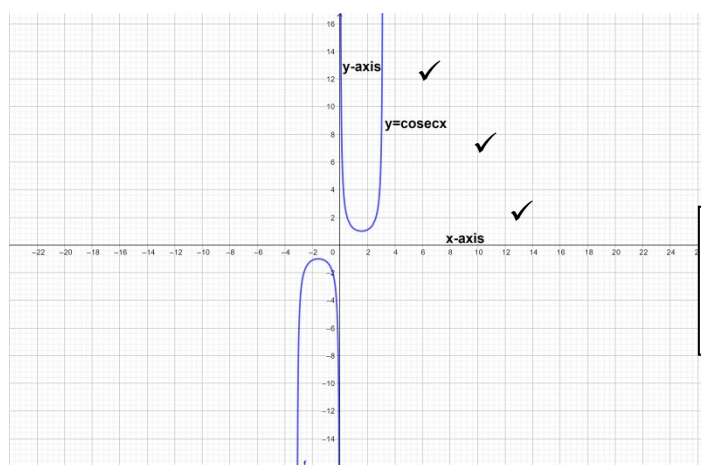
2.1 $3^{2x-1} = \ln 8$
 $(2x-1)\ln 3 = \ln(\ln 8) \checkmark$
 $2,197x - 1,099 = 0,732 \checkmark$
 $2,197x = 1,831 \checkmark$
 $x = 0,833 \checkmark \checkmark$

(5)

2.2 $\left[\frac{1+x}{1-x} \right]^{\frac{1}{m}} = a$
 $\log \left(\frac{1+x}{1-x} \right)^{\frac{1}{m}} = \log a \checkmark$
 $\frac{1}{m} \log \left(\frac{1+x}{1-x} \right) = \log a \checkmark$
 $\frac{1}{m} = \frac{\log a}{\log \left(\frac{1+x}{1-x} \right)} \checkmark$
 $\therefore m = \log_a \left(\frac{1+x}{1-x} \right) \checkmark$

(4)

2.3 2.3.1



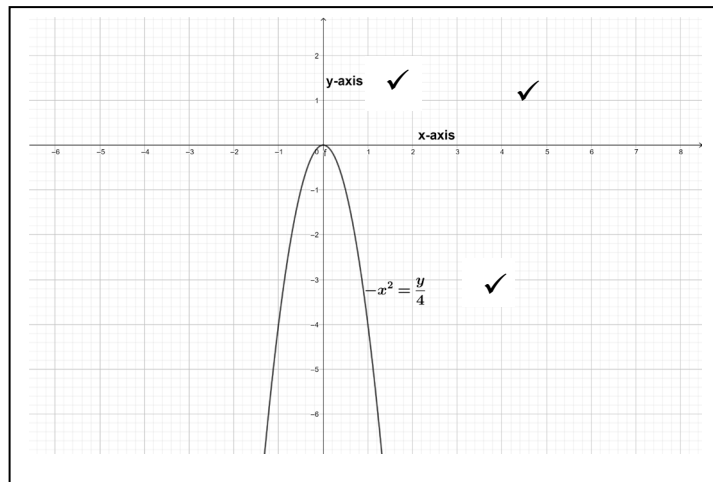
✓ vir die grafiek
 ✓ vir die x-as
 ✓ vir die y-as

(3)

2.3.2 $-180^\circ, 0^\circ, 180^\circ \checkmark \checkmark \checkmark$

(3)

2.4 2.4.1



- ✓ vir die grafiek
- ✓ vir die x-as
- ✓ vir die y-as

(3)

2.4.2 Funksie

(1)

2.4.3 Kontinu

(1)

[20]

VRAAG 3

3.1
$$= \frac{-2}{x} + 3 \ln x + \frac{5^{2x}}{\ln 5} + 4 \ln(\sec x) + 3 \tan x + c$$

(6)

3.2
$$= \left[\sin 2x + c \right]_0^{\frac{\pi}{4}} \checkmark$$

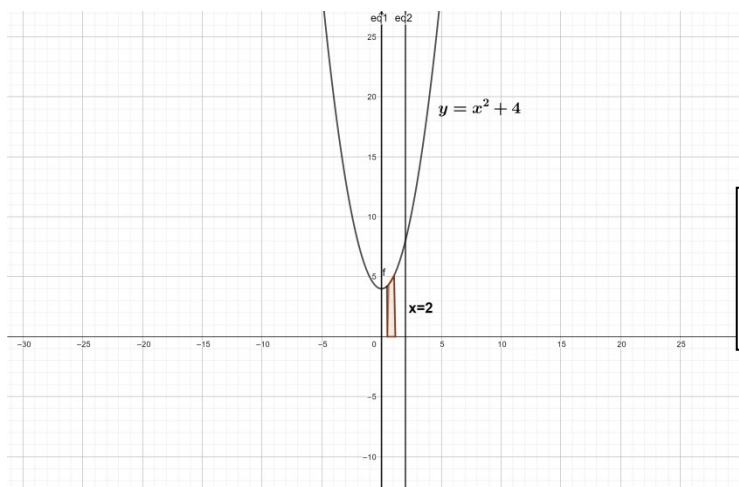
$$= \sin 2\left(\frac{\pi}{4}\right) + c - (\sin 2(0) + c) \checkmark$$

$$= 1 + c - c \checkmark$$

$$= 1 \checkmark$$

(4)

3.3 3.3.1



- ✓ vir elke grafiek
- ✓ vir strook
- ✓ vir oppervlakte

(4)

$$\begin{aligned}
 3.3.2 \quad A &= \int_{x_1}^{x_2} (y_2 - y_1) dx \\
 &= \int_0^2 (x^2 + 4 - 0) dx \checkmark \\
 &= \left[\frac{x^3}{3} + 4x + c \right]_0^2 \checkmark \\
 &= \frac{2^3}{8} + 4(2) + c - \left(\frac{0^3}{3} + 4(0) + c \right) \checkmark \\
 &= 9 \text{ eenhede}^2 \checkmark
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 3.4 \quad &\int \frac{x^2 - x}{x} dx \\
 &= \int (x - 1) dx \checkmark \\
 &= \frac{x^2}{2} - x + c \checkmark
 \end{aligned} \tag{2}$$

[20]

VRAAG 4

$$\begin{aligned}
 4.1 \quad &= \lim_{x \rightarrow 2} \frac{(x-2)(x^2 + 2x + 4)}{(x-2)} \checkmark \\
 &= 2^2 + 2(2) + 4 \checkmark \\
 &= 12 \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 4.2 \quad y &= \log 5x & u &= 5x \\
 y &= \log u & \frac{du}{dx} &= 5 \checkmark \\
 \frac{dy}{du} &= \frac{1}{u} \log e \checkmark \\
 \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\
 &= \frac{1}{u} \log e \times 5 \checkmark \\
 &= \frac{5}{5x} \log e \checkmark \\
 &= \frac{1}{x} \log e \checkmark
 \end{aligned}$$

Alternatiewe metode

$$\begin{aligned}
 y &= \log 5x \\
 y &= \log 5 + \log x \checkmark \checkmark \\
 \frac{dy}{dx} &= \frac{1}{5} \log e + \frac{1}{x} \log e \checkmark \checkmark \\
 &= \frac{1}{x} \log e \checkmark
 \end{aligned} \tag{4}$$

4.3

$$y = \frac{\cos x}{\ln x}$$

$$\frac{dy}{dx} = \frac{(-\sin x) \cdot \ln x - \cos x \cdot \left(\frac{1}{x}\right)}{(\ln x)^2} \checkmark \checkmark \checkmark$$
(3)

4.4

$$\therefore \frac{dy}{dx} = -\frac{1}{2} \cos e c^2 x + \sqrt{2} \sin x + \frac{1}{21x} \log e + 0 \checkmark \checkmark \checkmark \checkmark$$
(4)

4.5

$$y = x^3 - 9x$$

Vir DP laat $y' = 0$

$$3x^2 - 9 = 0$$

$$x^2 = 3$$

$$\therefore x = \pm\sqrt{3} \checkmark$$

Ooreenstemmende y - waardes:

$$\text{indien } x = \sqrt{3} \text{ dan } y = (\sqrt{3})^3 - 9\sqrt{3} = -10,392$$

$$\text{indien } x = -\sqrt{3} \text{ dan } y = (-\sqrt{3})^3 - 9(-\sqrt{3}) = 10,392 \checkmark$$

$$\therefore DP_1 = (\sqrt{3}; -10,392) \checkmark$$

$$\therefore DP_2 = (-\sqrt{3}; 10,392) \checkmark$$

$$\frac{d^2y}{dx^2} = 6x$$

$$= 6(\sqrt{3})$$

$$= +10,392$$

$$\therefore DP_1 = (\sqrt{3}; -10,392) \text{ Minimum } \checkmark$$

$$\frac{d^2y}{dx^2} = 6x$$

$$= 6(-\sqrt{3})$$

$$= -10,392$$

$$\therefore DP_2 = (-\sqrt{3}; 10,392) \text{ Maksimum } \checkmark$$

(6)
[20]

VRAAG 5

5.1

$$2x_1 + 4x_2 = 4$$

$$3x_1 + 7x_2 = 6$$

$$\begin{aligned} D &= \begin{vmatrix} 2 & 4 \\ 3 & 7 \end{vmatrix} \\ &= 14 - 12 \\ &= 2 \quad \checkmark \end{aligned}$$

$$\begin{aligned} D_{x_1} &= \begin{vmatrix} 4 & 4 \\ 6 & 7 \end{vmatrix} \\ &= 28 - 24 \\ &= 4 \quad \checkmark \end{aligned}$$

$$\begin{aligned} D_{x_2} &= \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} \\ &= 12 - 12 \\ &= 0 \quad \checkmark \end{aligned}$$

$$\therefore x_1 = \frac{D_{x_1}}{D} = \frac{4}{2} = 2 \quad \checkmark$$

$$\therefore x_2 = \frac{D_{x_2}}{D} = \frac{0}{2} = 0 \quad \checkmark$$

(5)

5.2

5.2.1

$$\begin{aligned} &\begin{vmatrix} 5 & 3 \\ 2 & -3 \end{vmatrix} \\ &= -15 - 6 \quad \checkmark \checkmark \\ &= -21 \end{aligned}$$

5.2.2

$$\begin{aligned} &\begin{vmatrix} 5 & 3 \\ 2 & -3 \end{vmatrix} \\ &= -(-15 - 6) \quad \checkmark \checkmark \\ &= 21 \end{aligned}$$

(2 × 2)

(4)

5.3

$$\begin{aligned}
z &= z_1 - \frac{z_2 \cdot z_3}{z_2 + z_3} \\
&= 2 + j5 - \frac{(2 + j2)(2 + j4)}{2 + j2 + 2 + j4} \\
&= 2 + j5 - \left(\frac{4 + j12 - 8}{4 + j6} \right) \checkmark \\
&= \frac{6 - j7}{4 + j6} \times \frac{4 - j6}{4 - j6} \checkmark \\
&= \frac{24 - j36 - j28 + j^2 42}{16 - j24 + j24 - j36} \checkmark \\
&= \frac{-18}{52} - \frac{j64}{52} \checkmark
\end{aligned} \tag{4}$$

5.4

$$\begin{aligned}
&(j-1)(2+j3) - (3-j4)j^2 \\
&= j2 + j^2 3 - 2 - j3 + 3 - j4 \\
&= -2 - j5 \checkmark \\
\\
&|r| = \sqrt{(-2)^2 + (-5)^2} \\
&= \sqrt{29} \checkmark \\
\\
&\tan \theta = \frac{-5}{-2} \checkmark \\
&\theta = \tan^{-1}(2.5) \\
&= 68,199^\circ \checkmark \\
\therefore \theta &= 180^\circ + 68,190^\circ = 248,190^\circ \checkmark \\
&= \sqrt{29}(\cos 248,199^\circ + j \sin 248,199^\circ) \checkmark
\end{aligned} \tag{6}$$

5.5

$$\begin{aligned}
j^{17} &= (j^2)^8 \cdot j \checkmark \\
&= j
\end{aligned} \tag{1}$$

[20]

TOTAAL: 100