



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE
*NASIONALE
SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 11

MATHEMATICS P2/*WISKUNDE V2*

**NOVEMBER 2022
MEMORANDUM**

MARKS/*PUNTE*: 150

**This memorandum consists of 17 pages.
*Hierdie memorandum bestaan uit 17 bladsye.***

NOTE:

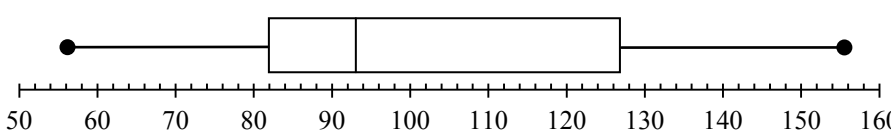
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.*
- *Dit is onaanvaarbaar om waardes/antwoorde aan te neem om 'n probleem op te los.*

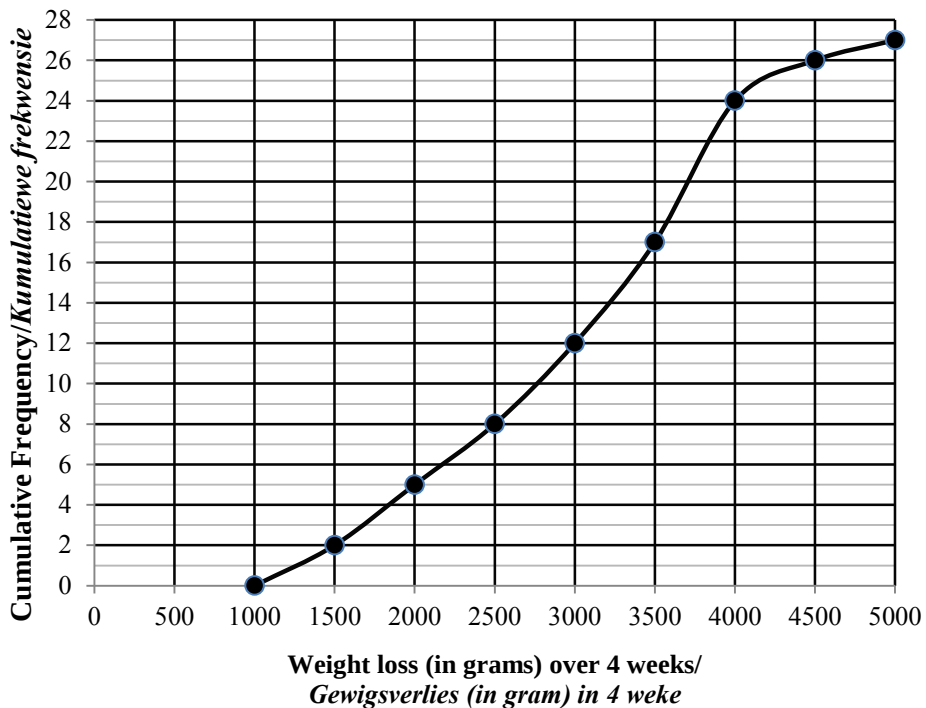
QUESTION/VRAAG 1

56	68	69	71	71	72	82	84	85
88	89	90	92	93	94	96	97	99
102	103	127	128	134	135	137	144	156

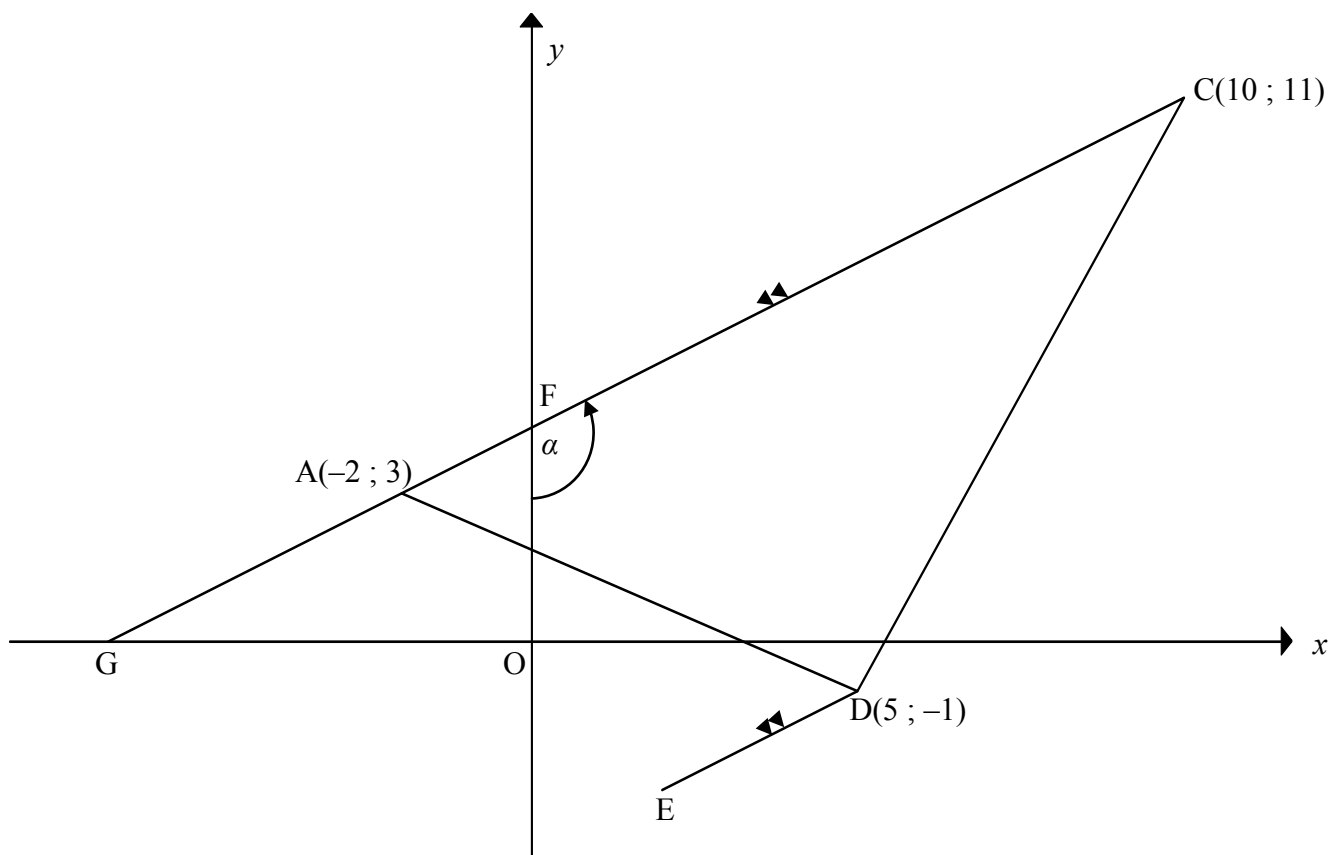
1.1	Range/Omvang = $156 - 56$ = 100 kg	✓ max – min ✓ answer/antw (2)
1.2	Mode/Modus = 71 kg	✓ answer/antw (1)
1.3	Median/Mediaan = $T_{14} = 93$ kg	✓ answer/antw (1)
1.4	$Q_1 = T_7 = 82$ $Q_3 = T_{21} = 127$ $IQR = Q_3 - Q_1$ = $127 - 82$ = 45 kg	✓ $Q_1 = 82$ ✓ $Q_3 = 127$ ✓ answer/antw (3)
1.5		✓ box/mond ✓ whiskers/snor (2)
1.6	SD = $25,838 \approx 25,84$ kg	✓✓ answer/antw (2)
1.7	$\bar{x} = 98,59$ $\bar{x} + 1\sigma = 98,59 + 25,84$ = 124,43 kg $127 > 124,43$ $\therefore$ I agree with this person/ <i>Ek stem met die persoon saam.</i>	✓ $\bar{x} = 98,59$ ✓ 124,43 ✓ conclusion/ gevolgtrekking (3) [14]

QUESTION/VRAAG 2

WEIGHT LOSS OVER 4 WEEKS GEWIGSVERLIES IN 4 WEKE (IN GRAMS/GRAM)	FREQUENCY FREKWENSIE f	CUMULATIVE KUMULATIEWE f
$1\ 000 < x \leq 1\ 500$	2	2
$1\ 500 < x \leq 2\ 000$	3	5
$2\ 000 < x \leq 2\ 500$	3	8
$2\ 500 < x \leq 3\ 000$	4	12
$3\ 000 < x \leq 3\ 500$	5	17
$3\ 500 < x \leq 4\ 000$	7	24
$4\ 000 < x \leq 4\ 500$	2	26
$4\ 500 < x \leq 5\ 000$	1	27

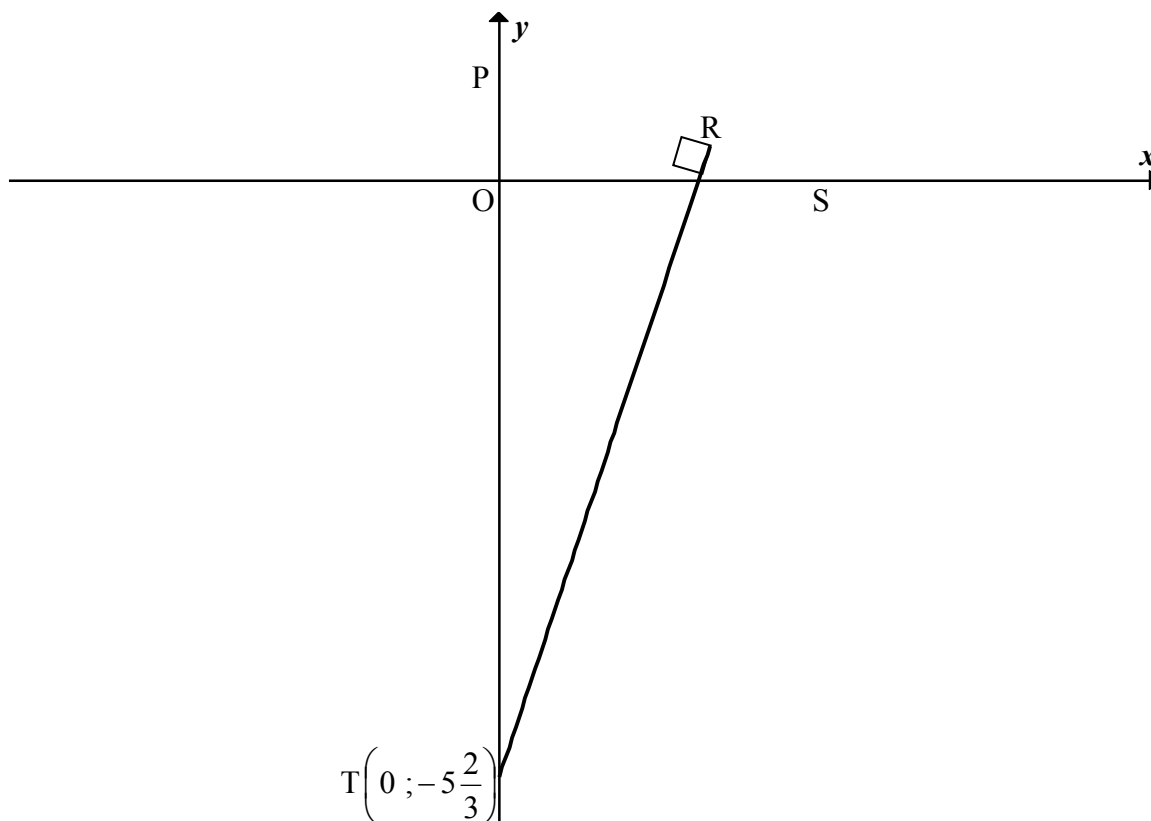
2.1	Average/Gemiddelde = $\frac{1250 \times 2 + 1750 \times 3 + \dots + 4750 \times 1}{27}$ = $\frac{81250}{27}$ = $3009,259 \approx 3009,26\text{ g}$	✓ 81 250 ✓ answer/antw (2)
2.2	OGIVE/OGIEF 	✓ grounded ✓ upper limits ✓ cumulative frequency ✓ smooth curve ✓ geanker ✓ boonste limiete ✓ kumulatiewe frekwensie ✓ gladde kurwe (4)
2.3	(3200 ; 14) $27 - 14 = 13$ participants/deelnemers (accept/aanvaar: 12 – 14)	✓ 14 ✓ 13 (2) [8]

QUESTION/VRAAG 3



3.1	$m_{AC} = \frac{11-3}{10-(-2)}$ $= \frac{8}{12} = \frac{2}{3}$	✓ subst into/in gradient form/ gradiëntvorm ✓ answer/antw (2)
3.2	$m_{DE} = m_{AC} = \frac{2}{3} \quad [DE \parallel CA]$ $y - y_1 = m(x - x_1) \quad y = mx + c$ $y - (-1) = \frac{2}{3}(x - 5) \quad -1 = \frac{2}{3}(5) + c$ $y = \frac{2}{3}x - \frac{13}{3} \quad \text{OR/OF} \quad -\frac{13}{3} = c$ $y = \frac{2}{3}x - \frac{13}{3}$	✓ gradients equal/ gradiënte gelyk ✓ subst m & (5 ; -1) into eq of straight line/ in vgl v rt lyn ✓ equation/vgl (3)
3.3	$\tan \hat{FGO} = m_{AC} = \frac{2}{3}$ $\hat{FGO} = 33,69^\circ$ $\therefore \alpha = 90^\circ + 33,69^\circ \quad [\text{ext/buite } \angle \text{ of } \Delta]$ $= 123,69^\circ$	✓ $\tan \hat{FGO} = m_{AC}$ ✓ $33,69^\circ$ ✓ answer/antw (3)

3.4.1	midpoint of BE = midpoint of AD [diagonals of rectangle] <i>midpt v BE = midpt v AD [hoeklyne van reghoek]</i> $M\left(\frac{-2+5}{2}; \frac{3+(-1)}{2}\right)$ $= M\left(\frac{3}{2}; 1\right)$	✓ R ✓ x-value/waarde ✓ y-value/waarde (3)
3.4.2	BE = AD [diagonals of rect /hoeklyne van regh] $BE = \sqrt{(-2-5)^2 + (3-(-1))^2}$ $= \sqrt{49+16}$ $= \sqrt{65} \text{ units/eenh}$	✓ R ✓ subst into distance formula/subst in afstandsformule ✓ answer/antw (3) [14]

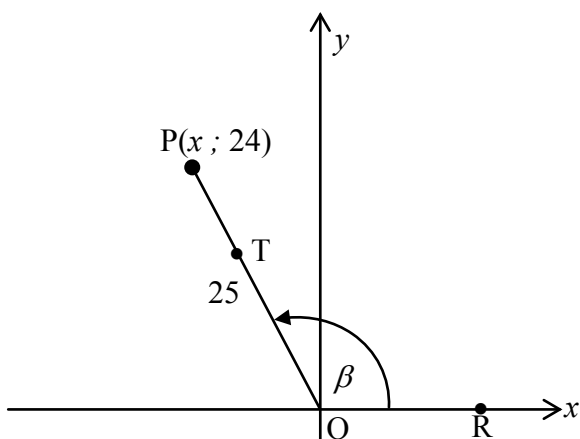
QUESTION/VRAAG 4

4.1	$x + ay - a = 0$ $0 + ay - a = 0$ $y = 1$ $\therefore P(0; 1)$ OR/OF $x + ay - a = 0$ $y = -\frac{x}{a} + 1$ $\therefore P(0; 1)$	$\checkmark x = 0$ $\checkmark y = 1$ $\checkmark x = 0$ $\checkmark y = 1$ (2)
4.2	$OP = 1 \text{ unit/eenh}$ $\therefore SP = 3 \text{ units/eenh}$ $\therefore S(3; 0)$ $3 + a(0) - a = 0$ $\therefore a = 3$ OR/OF $m_{PS} = -\frac{PO}{OS} = -\frac{1}{3}$ $y = -\frac{1}{3}x + 1$ $-\frac{1}{a} = -\frac{1}{3}$ $\therefore a = 3$	$\checkmark S(3; 0)$ $\checkmark \text{answer/antw}$ $\checkmark m_{PS} = -\frac{1}{3}$ $\checkmark \text{answer/antw}$ (2)

4.3	$m_{PS} = -\frac{PO}{OS} = -\frac{1}{3}$ $m_{PS} \times m_{TR} = -1 \quad (RT \perp PS)$ $\therefore m_{TR} = 3$ $\therefore y = 3x - 5\frac{2}{3}$	$\checkmark m_{PS} = -\frac{1}{3}$ $\checkmark m_{TR} = 3$ $\checkmark \text{equation/vgl}$ (3)
4.4	$x + 3y = 3 \text{ and/en } y = 3x - 5\frac{2}{3}$ $x + 3(3x - 5\frac{2}{3}) = 3$ $x + 9x - 17 = 3$ $10x = 20$ $x = 2$ $\therefore y = 3(2) - 5\frac{2}{3} = \frac{1}{3}$ $\therefore R(2; \frac{1}{3})$	$\checkmark \text{subst of eq of TR into/subst v vgl v TR in } x + 3y = 3$ $\checkmark \text{simplify/vereenv}$ $\checkmark x = 2$ $\checkmark y = \frac{1}{3}$ (4)
4.5	$\text{Area of/Opp van } \Delta PRT = \frac{1}{2} PT \times \perp h$ $= \frac{1}{2} (6\frac{2}{3})(2)$ $= 6\frac{2}{3} \text{ sq units/vk eenh}$	$\checkmark 6\frac{2}{3}$ $\checkmark \perp h$ $\checkmark \text{answer/antw}$ (3)
4.6	PT is the diameter of the circle passing through P, R and T. [PT subtends 90° at circumference] <i>PT is die middellyn vd sirkel wat deur P, R en T gaan. [PT onderspan 90° op omtrek]</i> $r = \frac{1}{2} \times 6\frac{2}{3}$ $= 3\frac{1}{3}$	$\checkmark \text{S/R}$ $\checkmark \text{answer/antw (2)}$ [16]

QUESTION/VRAAG 5

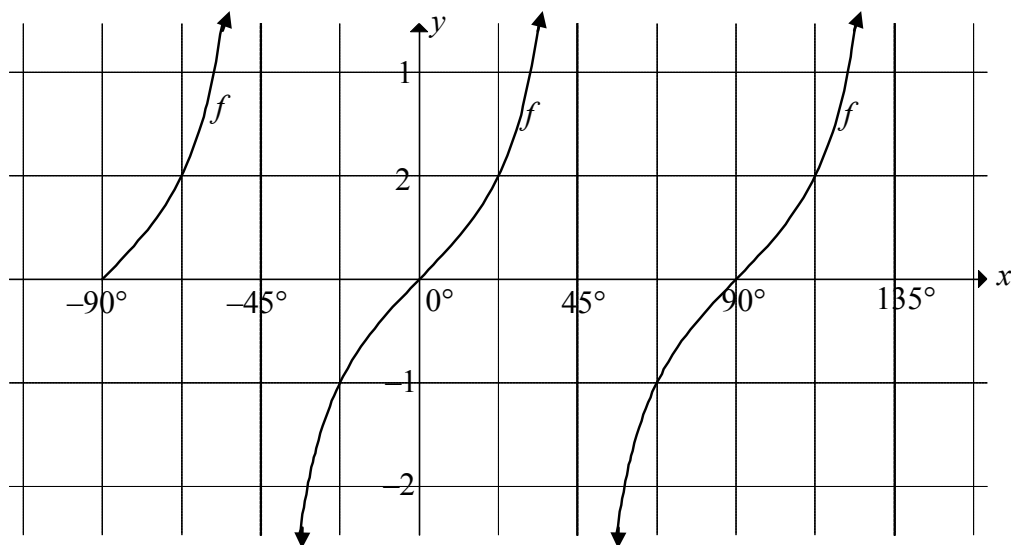
5.1

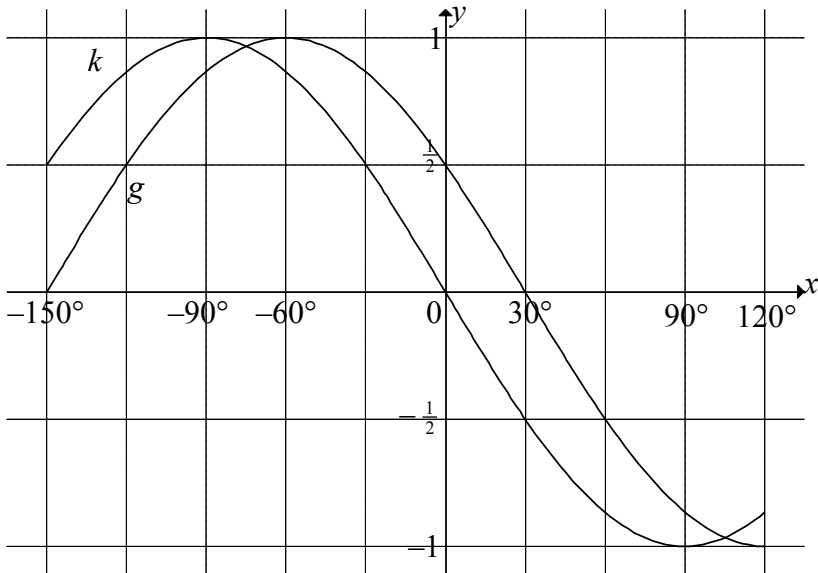


5.1.1	$x^2 + y^2 = r^2$ $x^2 + (24)^2 = 25^2$ $x^2 = 49$ $x = -7$	✓ subst ✓ answer/antw (2)
5.1.2(a)	$\sin \beta = \frac{24}{25}$	✓ answer/antw (1)
5.1.2(b)	$\cos(180^\circ - \beta)$ $= -\cos \beta$ $= -\left(-\frac{7}{25}\right)$ $= \frac{7}{25}$	✓ reduction/reduksie ✓ answer/antw (2)
5.1.2(c)	$\tan(-\beta)$ $= -\tan \beta$ $= -\left(\frac{24}{-7}\right)$ $= \frac{24}{7}$	✓ reduction/reduksie ✓ answer/antw (2)
5.1.3	$\sin \beta = \frac{24}{25} = \frac{y_T}{15}$ $\therefore y_T = 14\frac{2}{5}$ $\cos \beta = -\frac{7}{25} = \frac{x_T}{15}$ $\therefore x_T = -4\frac{1}{5}$ $\therefore T\left(-4\frac{1}{5}; 14\frac{2}{5}\right)$	✓ equating ratios/ stel verh's gelyk ✓ y_T ✓ equating ratios/ stel verh's gelyk ✓ x_T (4)

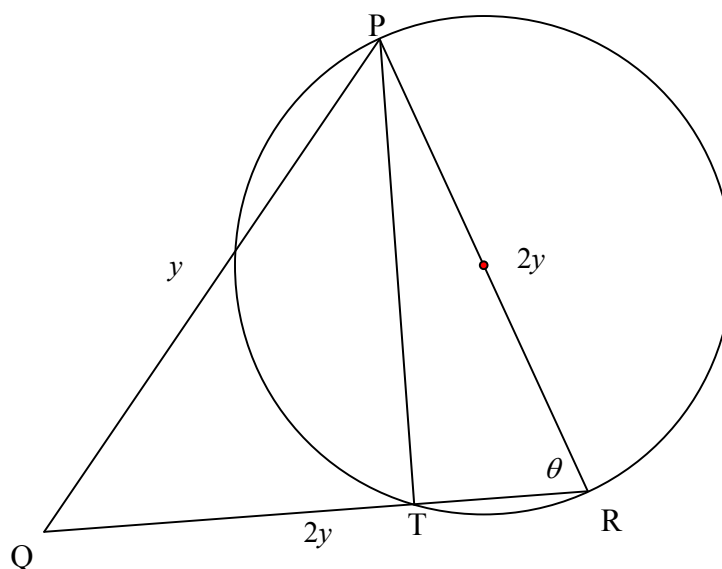
5.2	$\frac{2 \sin x \cdot \cos x \cdot \left(1 + \frac{\sin^2 x}{\cos^2 x}\right)}{\frac{\sin x}{\cos x}}$ $= 2 \sin x \cdot \cos x \cdot \left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right) \times \frac{\cos x}{\sin x}$ $= 2 \cos^2 x \left(\frac{1}{\cos^2 x}\right)$ $= 2$ OR/OF $\frac{2 \sin x \cdot \cos x \cdot \left(1 + \frac{\sin^2 x}{\cos^2 x}\right)}{\frac{\sin x}{\cos x}}$ $= 2 \sin x \cdot \cos x \cdot \left(1 + \frac{\sin^2 x}{\cos^2 x}\right) \times \frac{\cos x}{\sin x}$ $= 2 \cos^2 x \left(1 + \frac{\sin^2 x}{\cos^2 x}\right)$ $= 2(\cos^2 x + \sin^2 x)$ $= 2$	✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ identity/identiteit ✓ simplify/vereenv ✓ answer/antw (4) ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ simplify/vereenv ✓ identity/identiteit ✓ answer/antw (4)
5.3.1	$\frac{1 - \cos^2 A}{4(-\sin A)}$ $= \frac{\sin^2 A}{-4 \sin A}$ $= -\frac{1}{4} \sin A$	✓ $-\sin A$ ✓ identity/identiteit ✓ answer/antw (3)
5.3.2	$-\frac{1}{4} \sin 2x = 0,21$ $\sin 2x = -0,84$ ref/verw $\angle = 57,14^\circ$ $\therefore 2x = 237,14^\circ + k \cdot 360^\circ$ or $2x = 302,86^\circ + k \cdot 360^\circ$ $x = 118,57^\circ + k \cdot 180^\circ$ or $x = 151,43^\circ + k \cdot 180^\circ ; k \in \mathbb{Z}$	✓ correct equation/ korrekte vgl ✓ $-0,84$ ✓ ref/verw $\angle$ $= 57,14^\circ$ ✓ $237,14^\circ$ & $302,86^\circ$ ✓ $118,57^\circ$ & $151,43^\circ$ ✓ $k \cdot 180^\circ ; k \in \mathbb{Z}$ (6) [24]

QUESTION/VRAAG 6



6.1.1	$b = 2$	✓ answer/antw (1)
6.1.2	$45^\circ < x \leq 67,5^\circ$ OR/OF $x \in (45^\circ ; 67,5^\circ]$	✓ extreme values/ grenswaardes ✓ notation/notasie (2)
6.1.3	The graph of f has to shift 55° to the left to form the graph of h . <i>Die grafiek f het 55° na links geskuif om grafiek h te vorm.</i> $x = -10^\circ$ and/en $x = 80^\circ$	✓ $x = -10^\circ$ ✓ $x = 80^\circ$ (2)
6.2.1		✓ TP($-90^\circ ; 1$) ✓ TP($90^\circ ; -1$) ✓ $(-150^\circ ; \frac{1}{2})$ and/en $(120^\circ ; -0,87)$ ✓ y-intercept/afsnit (4)

6.2.2	min value/waarde = $-1 - 3 = -4$	✓✓ answer/antw (2)
6.2.3	$\cos(x + 60^\circ) = -\sin x$ $= \cos(90^\circ + x)$ $x + 60^\circ = -(90^\circ + x) \quad \text{or} \quad x + 60^\circ = 90^\circ + x$ $2x = -150^\circ + k.360^\circ \quad \text{no solution}$ $x = -75^\circ + k.180^\circ$ $\therefore x = -75^\circ \text{ or/of } 105^\circ$ $\cos(x + 60^\circ) = -\sin x$ $= -\cos(90^\circ - x)$ $x + 60^\circ = 180^\circ - (90^\circ - x) \quad \text{or} \quad x + 60^\circ = 180^\circ + (90^\circ - x)$ $x = 90^\circ + x + k.360^\circ \quad 2x = 210^\circ + k.360^\circ$ $\text{no solution/geen oplossing} \quad x = 105^\circ + k.180^\circ$ $\therefore x = -75^\circ \text{ or } 105^\circ$ OR/OF $\cos(x + 60^\circ) = -\sin x$ $\sin[90^\circ - (x + 60^\circ)] = -\sin x$ $150^\circ - x = 180^\circ + x \quad \text{or} \quad 150^\circ - x = 360^\circ - x$ $-2x = 30^\circ + k.360^\circ \quad \text{no solution/geen oplossing}$ $x = -15^\circ - k.180^\circ$ $\therefore x = -75^\circ \text{ or } 105^\circ$	✓ co-ratio/ko-verh ✓✓ correct equations korrekte vgls ✓ $x = -75^\circ + k.180^\circ$ ✓ -75° ✓ 105° (6) ✓ co-ratio/ko-verh ✓✓ correct equations korrekte vgls ✓ $x = 105^\circ + k.180^\circ$ ✓ -75° ✓ 105° (6) ✓ co-ratio/ko-verh ✓✓ correct equations korrekte vgls ✓ x $= -15^\circ - k.180^\circ$ ✓ -75° ✓ 105° (6)
6.2.4	$-75^\circ < x < 105^\circ$ OR/OF $x \in (-75^\circ ; 105^\circ)$	✓ extreme values/ grenswaardes ✓ notation/notasie (2)
6.2.5	$y = \cos(x + 60^\circ)$ $= \sin[90^\circ - (x + 60^\circ)]$ $= \sin(30^\circ - x)$ $= -\sin(x - 30^\circ)$ $\therefore \theta = 30^\circ$ OR/OF By inspections/Deur inspeksie: $\theta = 30^\circ$	✓ co-ratio/ko-verh ✓ answer/antw (2) ✓✓ answer/antw (2) [21]

QUESTION/VRAAG 7

7.1	$PQ^2 = QR^2 + PR^2 - 2QR.PR.\cos\theta$ $y^2 = (2y)^2 + (2y)^2 - 2(2y)(2y).\cos\theta$ $\cos\theta = \frac{4y^2 + 4y^2 - y^2}{2(2y)(2y)}$ $= \frac{7y^2}{8y^2}$ $= \frac{7}{8}$	✓ subst into cosine rule/ <i>in cos-reël</i> ✓ $\cos\theta$ as subject/ <i>as onderwerp</i> ✓ simplify/ <i>vereenv</i> ✓ answer/ <i>antw</i>
7.2	$PT \perp QR$ [$\angle$ in semi-circle/ <i>halfsirkel</i>] $\sin\theta = \frac{PT}{PR} = \frac{PT}{2y}$ $\therefore PT = 2y \sin\theta$ OR/OF $PT \perp QR$ [$\angle$ in semi-circle/ <i>halfsirkel</i>] $\hat{Q} = 90^\circ - \frac{\theta}{2}$ [$\angle$ s opposite = sides/ <i>le to = sye</i>] $\sin(90^\circ - \frac{\theta}{2}) = \frac{PT}{y}$ $PT = y \cos \frac{\theta}{2}$	✓ R ✓ correct ratio/ <i>korrekte verh</i> ✓ answer/ <i>antw</i> (3) ✓ R ✓ correct ratio/ <i>korrekte verh</i> ✓ answer/ <i>antw</i> (3)

[7]

QUESTION/VRAAG 8

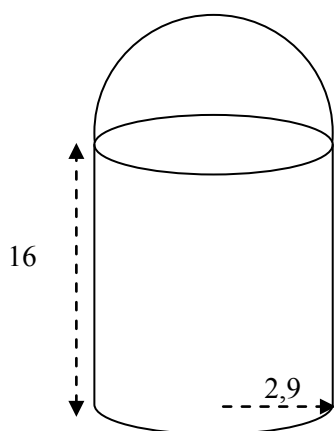


FIGURE 1

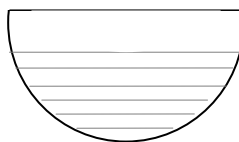
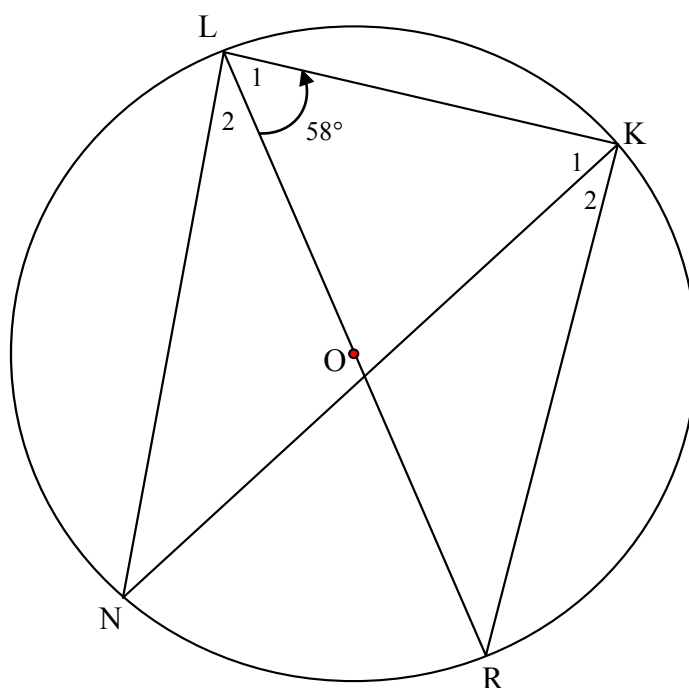


FIGURE 2

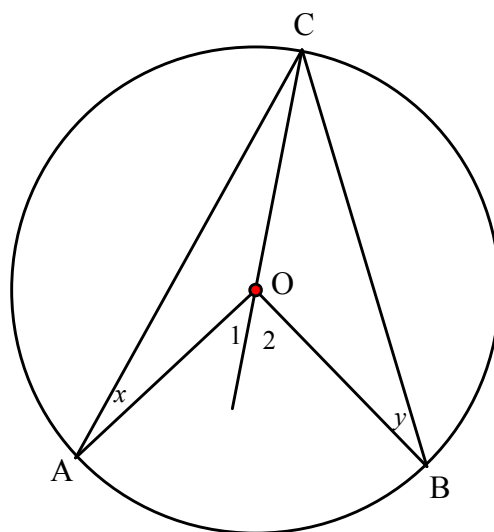
8.1	Surface area(SA) = $2\pi rh + \pi r^2 + 2\pi r^2$ <i>Buite-opp</i> = $2\pi rh + 3\pi r^2$ = $2\pi(2,9)(16) + 3\pi(2,9)^2$ = $118,03\pi$ = $370,80 \text{ cm}^2$	✓ SA of tin/<i>OA v blik</i> ✓ SA of hemisphere/<i>BO v hemisfeer</i> ✓ SA of base/<i>BO v basis</i> ✓ subst ✓ answer/<i>antw</i> (5)
8.2	Volume of hemisphere/<i>v hemisfeer</i> = $\frac{1}{2} \times \frac{4}{3} \pi r^3$ = $\frac{1}{2} \times \frac{4}{3} \pi (2,9)^3$ = $16,26\pi$ 80% of/<i>van</i> volume = $0,8 \times 16,26\pi$ = $40,86 \text{ cm}^3$ OR/OF Volume of hemisphere/<i>v hemisfeer</i> = $\frac{1}{2} \times \frac{4}{3} \pi r^3$ 80% of/<i>van</i> volume = $0,8 \times \frac{1}{2} \times \frac{4}{3} \pi r^3$ = $0,8 \times \frac{1}{2} \times \frac{4}{3} \pi (2,9)^3$ = $40,86 \text{ cm}^3$	✓ V of hemisphere ✓ subst ✓ answer/<i>antw</i> (3) ✓ vol of hemisphere/ <i>vol van hemisfeer</i> ✓ subs ✓ answer/<i>antw</i> (3) [8]

QUESTION/VRAAG 9

9.1	$\hat{LKR} = 90^\circ$ [$\angle$ in semi-circle/ <i>halfsirkel</i>]	✓ S ✓ R (2)
9.2	$\hat{R} = 180^\circ - (90^\circ + 58^\circ) = 32^\circ$ [$\angle$ s/e of/ <i>van</i> Δ]	✓ S ✓ R (2)
9.3	$\hat{N} = 32^\circ$ [$\angle$ s/e in same segment/ <i>dieselfde segment</i>]	✓ S ✓ R (2) [6]

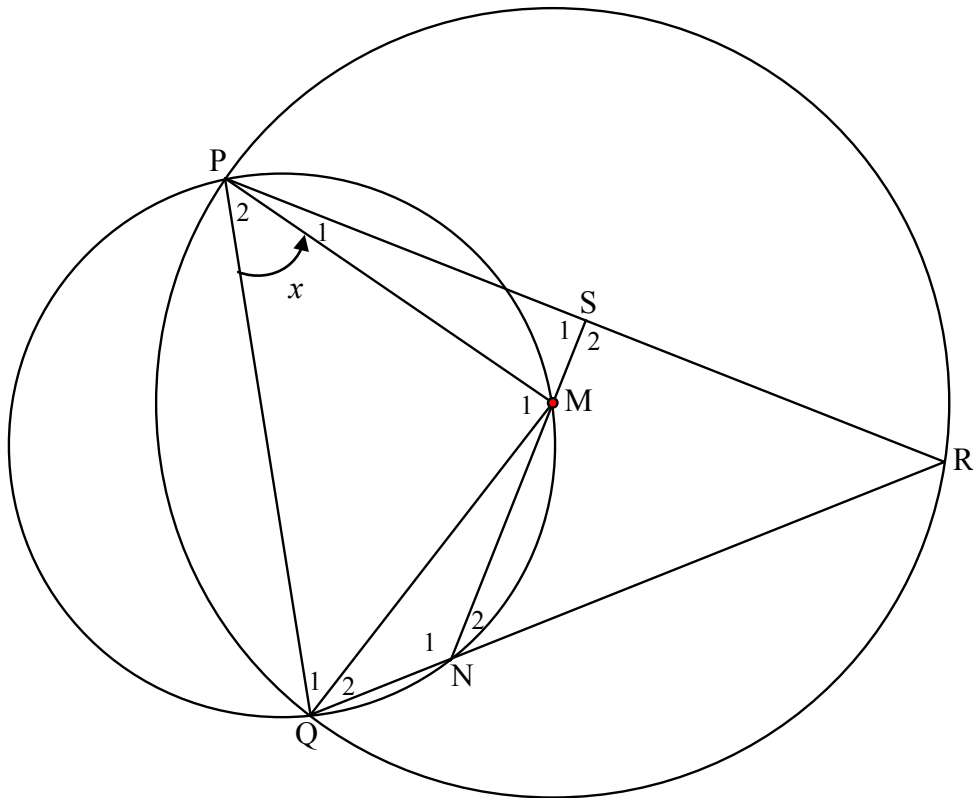
QUESTION/VRAAG 10

10.1



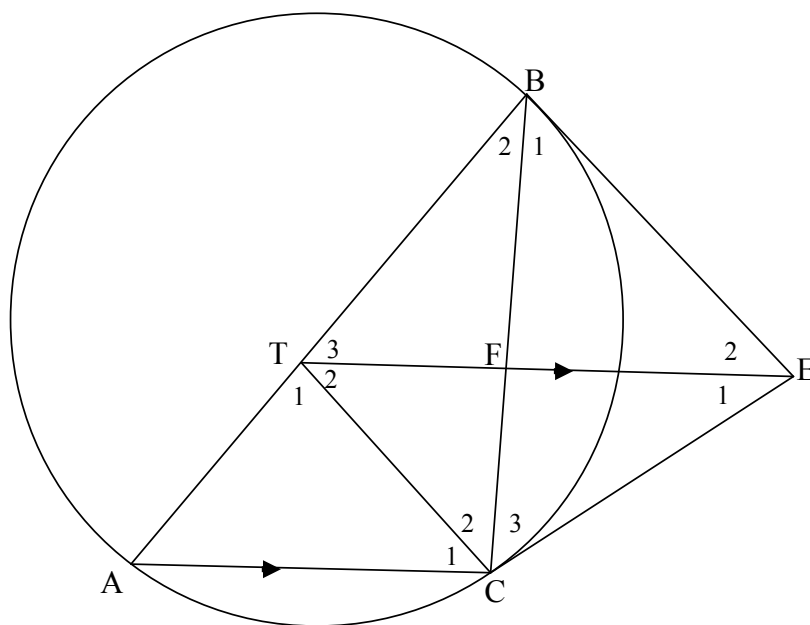
10.1.1	AO and/en CO are radii/is radiusse $\hat{A} = \hat{C}_1 = x$ [$\angle$ s opp equal sides/ $\angle$ e to gelyke sye] $\hat{O}_1 = \hat{A} + \hat{C}_1 = 2x$ [ext/buite $\angle$ of/van Δ]	✓ S ✓ S ✓ R (3)
10.1.2	$\hat{B} = \hat{C}_2 = y$ [$\angle$ s opp equal sides/ $\angle$ e to gelyke sye] $\hat{O}_2 = \hat{B} + \hat{C}_2 = 2y$ [ext/buite $\angle$ of/van Δ] $\hat{AOB} = 2x + 2y$ $= 2(x + y)$ $= 2(\hat{C}_1 + \hat{C}_2)$ $= 2\hat{ACB}$	✓ S ✓ S ✓ S (3)

10.2



10.2.1	ext $\angle$ of cyc quad/ <i>buite $\angle$ v koordevh</i>	✓ R (1)
10.2.2	MP = QM [radii] $\hat{Q}_1 = x$ [$\angle$ s opp equal sides/ <i>$\angle$e to gelyke sye</i>]	✓ S ✓ R (2)
10.2.3	$\hat{M}_1 = 180^\circ - 2x$ [$\angle$ s/e of/ <i>van Δ</i>] $\hat{R} = 90^\circ - x$ [$\angle$ at centre = $2 \times \angle$ at circumference/ <i>midpts $\angle$ = $2 \times$ omtreks $\angle$</i>]	✓ S ✓ S ✓ R (3)
10.2.4	In Δ NSR: $\hat{R} = 90^\circ - x$ and $\hat{N}_2 = x$ $\hat{S}_2 = 180^\circ - (90^\circ - x + x)$ [$\angle$ s/e of/ <i>van Δ</i>] = 90° PS = SR [line from centre $\perp$ chord/ <i>lyn v midpt $\perp$ kd</i>]	✓ S ✓ S ✓ R (3) [15]

QUESTION/VRAAG 11



11.1	$\hat{B}_1 = \hat{A}$ [tangent-chord theorem/ <i>rkl-kdst</i>] $\hat{A} = \hat{T}_3$ [corresp $\angle$ s/ <i>oorenk</i> $\angle$ e ; $TE \parallel AC$] $\therefore \hat{B}_1 = \hat{T}_3$	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (4)
11.2	$BE = CE$ [tangents from same point/ <i>rklyne v dieselfde pt</i>] $\hat{B}_1 = \hat{C}_3$ [$\angle$ s opp equal sides/ $\angle$ e to gelyke sye] $\hat{C}_3 = \hat{T}_3$ [$\hat{B}_1 = \hat{T}_3$] $\therefore$ TBEC a cyclic quad [converse $\angle$ s in the same segment] <i>TBEC is koordevh</i> [<i>omgekeerde</i> $\angle$ e in dieselfde segment]	$\checkmark$ R $\checkmark$ S $\checkmark$ S $\checkmark$ R (4)
11.3	$\hat{B}_1 = \hat{T}_2$ [$\angle$ s in the same segment/ $\angle$ e in dieselfde segment] $\hat{B}_1 = \hat{T}_3$ [proven/ <i>bewys</i> in 11.1] $\therefore \hat{T}_2 = \hat{T}_3$ $\therefore$ ET bisects/ <i>halveer</i> $\hat{B}\hat{T}C$	$\checkmark$ S $\checkmark$ R (2)
11.4	$\hat{B}_2 = \hat{E}_2$ [tangent-chord theorem/ <i>rkl-kdst</i>] $\hat{C}_2 = \hat{E}_2$ [$\angle$ s in the same segment/ $\angle$ e in dieselfde segment] $\therefore TB = TC$ [sides opposite equal $\angle$ s/ <i>syte to gelyke</i> $\angle$ e]	$\checkmark$ S $\checkmark$ R $\checkmark$ R $\checkmark$ R (4)
11.5	$\hat{C}_1 = \hat{T}_2$ [alternate/ <i>verwiss</i> $\angle$ s/ <i>e</i> ; $TE \parallel AC$] $\therefore \hat{C}_1 = \hat{A}$ $\therefore AT = TC$ [sides opposite equal $\angle$ s/ <i>syte to gelyke</i> $\angle$ e] T is a point that is equidistant from A, B and C on the circle/ <i>T is 'n punt wat ewever is vanaf A, B en C op die sirkel.</i> $\therefore$ T is the centre of the circle/ <i>T is die middelpunt van die sirkel.</i>	$\checkmark$ S $\checkmark$ S $\checkmark$ S (3) [17]

TOTAL/TOTAAL: 150