higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N4

16 November 2022

Hierdie nasienriglyn bestaan uit 8 bladsye.

Kopiereg voorbehou

VRAAG 1

1.1

$$\text{LK} = \frac{1+\sin y}{1-\sin y} - \frac{1-\sin y}{1+\sin y}$$
$$= \frac{(1+\sin y)(1+\sin y)-[(1-\sin y)(1-\sin y)]}{(1-\sin y)(1+\sin y)} \checkmark$$
$$= \frac{1+2\sin y+\sin^2 y-2\sin y-\sin^2 y}{1-\sin^2 y} \checkmark$$
$$= \frac{4\sin y}{\cos^2 y} \checkmark$$
$$= \frac{4\sin y}{\cos y}.\frac{1}{\cos y} \checkmark$$
$$= 4\tan y.\sec y \checkmark$$
$$= \text{RK}$$

(5)

1.2

$$\frac{\tan\theta(\sec\theta+1)+\tan\theta(\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}$$
$$= \frac{\tan\theta.\sec\theta+\tan\theta.\sec\theta-\tan\theta}{\sec^2\theta-1} \checkmark$$
$$= \frac{2\tan\theta.\sec\theta}{\tan^2\theta} \checkmark$$
$$= \frac{2\sec\theta}{\frac{\sin\theta}{\cos\theta}} \checkmark$$
$$= 2\cos ec\theta \checkmark$$

(4)

1.3

$$\frac{\cos ec\text{A}}{1+\cos\text{A}}$$
$$= \frac{\frac{5}{4}}{1+\frac{3}{5}} \checkmark$$
$$= \frac{5}{4}\times\frac{5}{8} \checkmark$$
$$= \frac{25}{32} \checkmark$$

(3)

1.4

$$\frac{1}{sec(-15°)} = cos(-15°)\checkmark$$
$$= cos(60°-45°)$$
$$= cos\,60°\,.\,cos\,45° + sin\,60°\,sin\,45°\checkmark$$
$$= \frac{1+\sqrt{3}}{2\sqrt{2}} \checkmark$$

(3)

1.5 $2\cos^2 2\theta + \cos 2\theta - 1 = 0$

$(2\cos 2\theta - 1)(\cos 2\theta + 1) = 0$ ✓

$1^{ste}: \cos 2\theta = \frac{1}{2}$ Of $\cos 2\theta = -1$ ✓

$1^{ste}: 2\theta = 60°$ en $4^{e}: 2\theta = 360° - 60°$ N/A ✓

$\therefore$ *Algemene oplossing* $\{\pm 30°; \pm 150°\}$✓✓ (5)

[20]

VRAAG 2

2.1 $3^{2x-1} = \ln 8$

$(2x-1)\ln 3 = \ln(\ln 8)$✓

$2{,}197x - 1{,}099 = 0{,}732$ ✓

$2{,}197x = 1{,}831$✓

$x = 0{,}833$✓✓ (5)

2.2 $\left[\frac{1+x}{1-x}\right]^{\frac{1}{m}} = a$

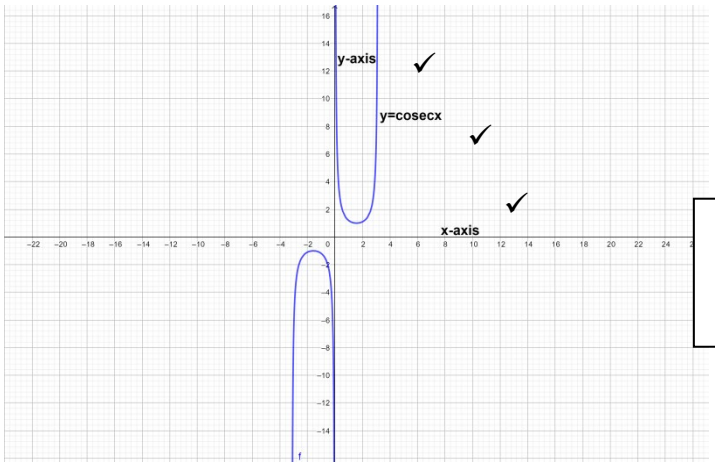
$\log\left(\frac{1+x}{1-x}\right)^{\frac{1}{m}} = \log a$ ✓

$\frac{1}{m}\log\left(\frac{1+x}{1-x}\right) = \log a$ ✓

$\frac{1}{m} = \frac{\log a}{\log\left(\frac{1+x}{1-x}\right)}$ ✓

$\therefore m = \log_a\left(\frac{1+x}{1-x}\right)$✓ (4)

2.3 2.3.1

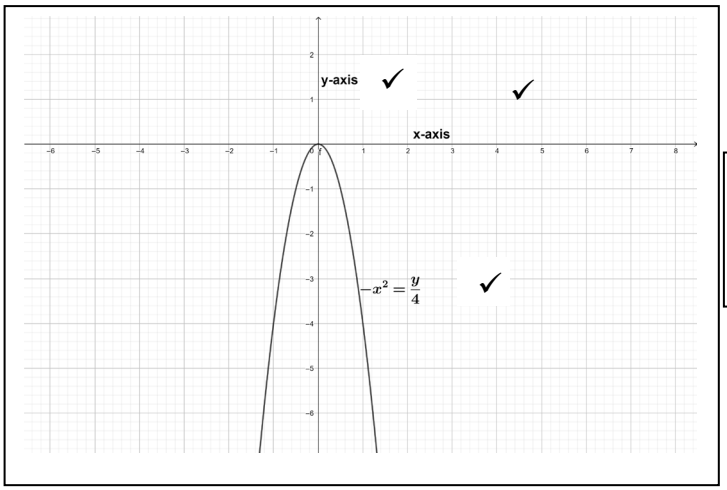


✓ vir die grafiek
✓ vir die x-as
✓ vir die y-as

(3)

2.3.2 $-180°,\ 0°,\ 180°$✓✓✓ (3)

2.4 2.4.1



✓ vir die grafiek
✓ vir die x-as
✓ vir die y-as

(3)

2.4.2 Funksie (1)

2.4.3 Kontinu (1)

[20]

VRAAG 3

3.1

$$= \frac{-2}{x} + 3\ln x + \frac{5^{2x}}{\ln 5} + 4\ln(\sec x) + 3\tan x + c$$

(6)

3.2

$$= \left[\sin 2x + c\right]_0^{\frac{\pi}{4}} \checkmark$$

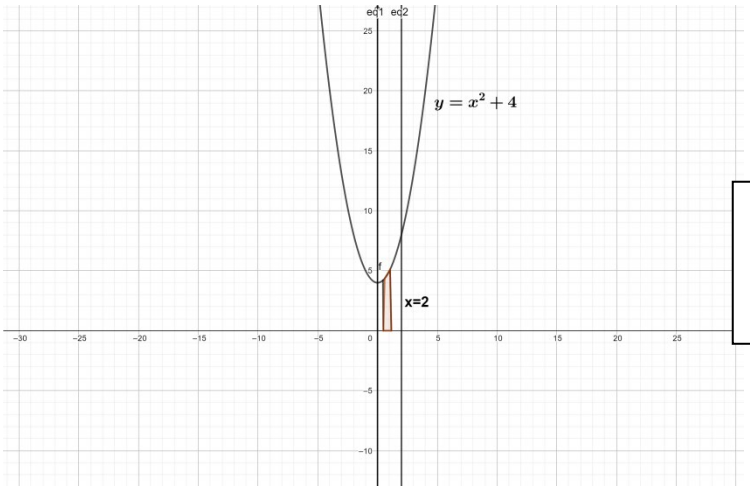
$$= \sin 2\left(\frac{\pi}{4}\right) + c - \left(\sin 2(0) + c\right) \checkmark$$

$$= 1 + c - c \checkmark$$

$$= 1 \checkmark$$

(4)

3.3 3.3.1



✓ vir elke grafiek
✓ vir strook
✓ vir oppervlakte

(4)

3.3.2

$$A = \int_{x_1}^{x_2} (y_2 - y_1)dx$$

$$= \int_0^2 (x^2 + 4 - 0)dx \checkmark$$

$$= \left[\frac{x^3}{3} + 4x + c\right]_0^2 \checkmark$$

$$= \frac{2^3}{8} + 4(2) + c - \left(\frac{0^3}{3} + 4(0) + c\right) \checkmark$$

$$= 9\ eenhede^2 \checkmark$$ (4)

3.4

$$\int \frac{x^2 - x}{x} dx$$

$$= \int (x - 1)dx \checkmark$$

$$= \frac{x^2}{2} - x + c \checkmark$$ (2)

[20]

VRAAG 4

4.1

$$= \lim_{x \to 2} \frac{(x-2)(x^2 + 2x + 4)}{(x-2)} \checkmark$$

$$= 2^2 + 2(2) + 4 \checkmark$$

$$= 12 \checkmark$$ (3)

4.2

$y = \log 5x$ $\qquad$ $u = 5x$

$y = \log u$ $\qquad$ $\frac{du}{dx} = 5 \checkmark$

$$\frac{dy}{du} = \frac{1}{u}\log e \checkmark$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \frac{1}{u}\log e \times 5 \checkmark$$

$$= \frac{5}{5x}\log e \checkmark$$

$$= \frac{1}{x}\log e \checkmark$$

Alternatiewe metode

$$y = \log 5x$$

$$y = \log 5 + \log x \checkmark\checkmark$$

$$\frac{dy}{dx} = \frac{1}{5}\log e + \frac{1}{x}\log e \checkmark\checkmark$$

$$= \frac{1}{x}\log e \checkmark$$ (4)

4.3

$$y = \frac{\cos x}{\ln x}$$

$$\frac{dy}{dx} = \frac{(-\sin x).\ln x - \cos.\left(\frac{1}{x}\right)}{(\ln x)^2} \checkmark\checkmark\checkmark$$

(3)

4.4

$$\therefore \frac{dy}{dx} = -\frac{1}{2}\cos ec^2 x + \sqrt{2}\sin x + \frac{1}{21x}\log e + 0 \checkmark\checkmark\checkmark\checkmark$$

(4)

4.5

$y = x^3 - 9x$

<u>Vir DP laat $y^{/} = 0$</u>

$3x^2 - 9 = 0$

$x^2 = 3$

$\therefore x = \pm\sqrt{3}$ ✓

<u>Ooreenstemmende $y -$ waardes</u>:

$indien\ x = \sqrt{3}$ dan $y = \left(\sqrt{3}\right)^3 - 9\sqrt{3} = -10{,}392$

$indien\ x = -\sqrt{3}$ dan $y = \left(-\sqrt{3}\right)^3 - 9\left(-\sqrt{3}\right) = 10{,}392$✓

$\therefore \mathrm{DP}_1 = \left(\sqrt{3}; -10{,}392\right)$✓

$\therefore \mathrm{DP}_2 = \left(-\sqrt{3}; 10{,}392\right)$✓

$$\frac{d^2y}{dx^2} = 6x$$

$$= 6\left(\sqrt{3}\right)$$

$$= +10{,}392$$

$\therefore \mathrm{DP}_1 = \left(\sqrt{3}; -10{,}392\right)$ Minimum ✓

$$\frac{d^2y}{dx^2} = 6x$$

$$= 6\left(-\sqrt{3}\right)$$

$$= -10{,}392$$

$\therefore \mathrm{DP}_2 = \left(-\sqrt{3}; 10{,}392\right)$Maksimum✓

(6)

[20]

VRAAG 5

5.1

$$2x_1 + 4x_2 = 4$$
$$3x_1 + 7x_2 = 6$$

$$D = \begin{vmatrix} 2 & 4 \\ 3 & 7 \end{vmatrix}$$
$$= 14 - 12$$
$$= 2 \quad ✓$$

$$D_{x_1} = \begin{vmatrix} 4 & 4 \\ 6 & 7 \end{vmatrix}$$
$$= 28 - 24$$
$$= 4 ✓$$

$$D_{x_2} = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$$
$$= 12 - 12$$
$$= 0 ✓$$

$$\therefore x_1 = \frac{D_{x_1}}{D} = \frac{4}{2} = 2 ✓$$
$$\therefore x_2 = \frac{D_{x_2}}{D} = \frac{0}{2} = 0 ✓$$

(5)

5.2 5.2.1

$$\begin{vmatrix} 5 & 3 \\ 2 & -3 \end{vmatrix}$$
$$= -15 - 6 ✓✓$$
$$= -21$$

5.2.2

$$\begin{vmatrix} 5 & 3 \\ 2 & -3 \end{vmatrix}$$
$$= -(-15 - 6) ✓✓$$
$$= 21$$

(2 × 2) (4)

5.3

$$z = z_1 - \frac{z_2 . z_3}{z_2 + z_3}$$
$$= 2 + j5 - \frac{(2 + j2)(2 + j4)}{2 + j2 + 2 + j4}$$
$$= 2 + j5 - \left(\frac{4 + j12 - 8}{4 + j6}\right) \checkmark$$
$$= \frac{6 - j7}{4 + j6} \times \frac{4 - j6}{4 - j6} \checkmark$$
$$= \frac{24 - j36 - j28 + j^2 42}{16 - j24 + j24 - j36} \checkmark$$
$$= \frac{-18}{52} - \frac{j64}{52} \checkmark$$

(4)

5.4

$$(j - 1)(2 + j3) - (3 - j4)j^2$$
$$= j2 + j^2 3 - 2 - j3 + 3 - j4$$
$$= -2 - j5 \checkmark$$

$$|r| = \sqrt{(-2)^2 + (-5)^2}$$
$$= \sqrt{29} \checkmark$$

$$\tan\theta = \frac{-5}{-2} \checkmark$$
$$\theta = \tan^{-1}(2.5)$$
$$= 68{,}199° \checkmark$$
$$\therefore \theta = 180° + 68{,}190° = 248{,}190° \checkmark$$
$$= \sqrt{29}(cos\ 2\ 48{,}199° + j\ sin\ 2\ 48{,}199°) \checkmark$$

(6)

5.5

$$j^{17} = (j^2)^8 . j \checkmark$$
$$= j$$

(1)

[20]

TOTAAL: 100

Kopiereg voorbehou