



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NATIONALE SERTIFIKAAT

WISKUNDE N5

13 Augustus 2021

Hierdie nasienriglyn bestaan uit 11 bladsye.

VRAAG 1

1.1 1.1.1
$$\lim_{x \rightarrow 3} \frac{3-x}{\sqrt{x+1} - \sqrt{5x-11}} \left(\frac{0}{0} \right)$$

$$\lim_{x \rightarrow \infty} \frac{-1}{\frac{1}{2\sqrt{x+1}} \checkmark - \frac{5\checkmark}{2\sqrt{5x-11}}}$$

$$= -1\checkmark \quad (3)$$

1.1.2
$$\lim_{x \rightarrow -1} \frac{\frac{1}{4+3x} + \frac{1}{x}}{x+1} \left(\frac{0}{0} \right)$$

$$= \lim_{x \rightarrow -1} \frac{-\frac{3}{(4+3x)^2} + \frac{1}{x^2} \checkmark}{1}$$

$$= -2\checkmark \quad (2)$$

1.2 1.2.1
$$\ln y = \lim_{x \rightarrow 4} \frac{\cos(x-4) - 1}{2x-8} \left(\frac{0}{0} \right)$$

$$= \lim_{x \rightarrow 0} \frac{-\sin(x-4)}{2} \checkmark$$

$$= 0\checkmark \quad (2)$$

1.2.2
$$y = e^0 = 1\checkmark \quad (1)$$

1.3
$$f(x) = \frac{\sin 2x}{\cos 2x}$$

$$\cos 2x = 0$$

$$2x = 90^\circ$$

$$x = 45^\circ \checkmark \text{ of } x = 135^\circ \checkmark \quad (2)$$

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VRAAG 2

2.1 $f(x) = -5x^7$

2.1.1 $f(x + h) = -5(x + h)^7$
 $= -5 \left[\frac{x^7 h^0}{0!} + \frac{7x^6 h}{1!} + \frac{42x^5 h^2}{2!} + \frac{210x^4 h^3}{3!} + \dots \dots \dots \right]$
 $= -5x^7 - 35x^6 h - 105x^5 h^2 - 105x^4 h^3 + \dots \dots \dots \checkmark \checkmark$ (2)

2.1.2 $f(x + h) - f(x) = -35x^6 h - 105x^5 h^2 - 105x^4 h^3 + \dots \dots \dots \checkmark$ (1)

2.1.3 $\frac{f(x + h) - f(x)}{h} = -35x^6 - 105x^5 h - 105x^4 h^2 + \dots \dots \dots \checkmark$ (1)

2.1.4 $\lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} = -35x^6 \checkmark$ (1)

2.2 2.2.1 $y = \tan[(5 - x^2)(\ln^2 x)]$

$\frac{dy}{dx} = \sec^2[(5 - x^2)(\ln^2 x)] \checkmark$
 $\times \left[-2x(\ln^2 x) \checkmark + (5 - x^2) \times 2 \ln x \checkmark \times \frac{1}{x} \checkmark \right]$
 $= \sec^2[(5 - x^2)(\ln^2 x)] \times \left[-2x(\ln^2 x) + \frac{2(5 - x^2) \times \ln x}{x} \right]$ (4)

2.2.2 $y = \sqrt[4]{\cos(9 - x^2) + \sqrt{\ln x}}$

$\frac{dy}{dx} = \frac{1}{4} [\cos(9 - x^2) + \sqrt{\ln x}]^{-\frac{3}{4}} \checkmark$
 $\times \left[-\sin(9 - x^2) \times 2x \checkmark + \frac{1}{2} (\ln x)^{-\frac{1}{2}} \checkmark \times \frac{1}{x} \checkmark \right]$
 $= \frac{1}{4} [\cos(9 - x^2) + \sqrt{\ln x}]^{-\frac{3}{4}} \times \left[-2x \sin(9 - x^2) + \frac{1}{2x\sqrt{\ln x}} \right]$ (4)

2.3

$$y = \frac{\sin(3x + x^2)}{(6 - x^4)^3}$$

$$\ln y = \ln \sin(3x + x^2) - 3 \ln(6 - x^4)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} \checkmark = \frac{1}{\sin(3x + x^2)} \times \cos(3x + x^2) \checkmark \times (3 + 2x) - \frac{3}{6 - x^4} \times 4x^3 \checkmark$$

$$\frac{dy}{dx} = y \left[\frac{(3 + 2x) \cos(3x + x^2)}{\sin(3x + x^2)} + \frac{12x^3}{6 - x^4} \right]$$

$$\frac{dy}{dx} = \frac{\sin(3x + x^2)}{(6 - x^4)^3} \left[\frac{(3 + 2x) \cos(3x + x^2)}{\sin(3x + x^2)} + \frac{12x^3}{6 - x^4} \right] \checkmark \quad (4)$$

2.4

$$\cos(x^2 + 2y) + xe^{y^2} = 1$$

$$-\sin(x^2 + 2y) \times \left[2x + 2 \frac{dy}{dx} \right] + e^{y^2} + 2xye^{y^2} \frac{dy}{dx} = 0 \checkmark$$

$$-2x \sin(x^2 + 2y) - 2 \sin(x^2 + 2y) \frac{dy}{dx} + e^{y^2} + 2xye^{y^2} \frac{dy}{dx} = 0 \checkmark$$

$$\frac{dy}{dx} [2xye^{y^2} - 2 \sin(x^2 + 2y)] = 2x \sin(x^2 + 2y) - e^{y^2} \checkmark$$

$$\frac{dy}{dx} = \frac{2x \sin(x^2 + 2y) - e^{y^2}}{2xye^{y^2} - 2 \sin(x^2 + 2y)} \checkmark \quad (5)$$

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VRAAG 3

3.1

3.1.1

$$f(x) = f(x) = 2x^3 + x^2 - x + 2$$

$$f'(x) = 6x^2 + 2x - 1 = 0$$

$$f''(x) = 12x + 2$$

$$x = -\frac{1}{6} \quad y = \frac{59}{27} \checkmark$$

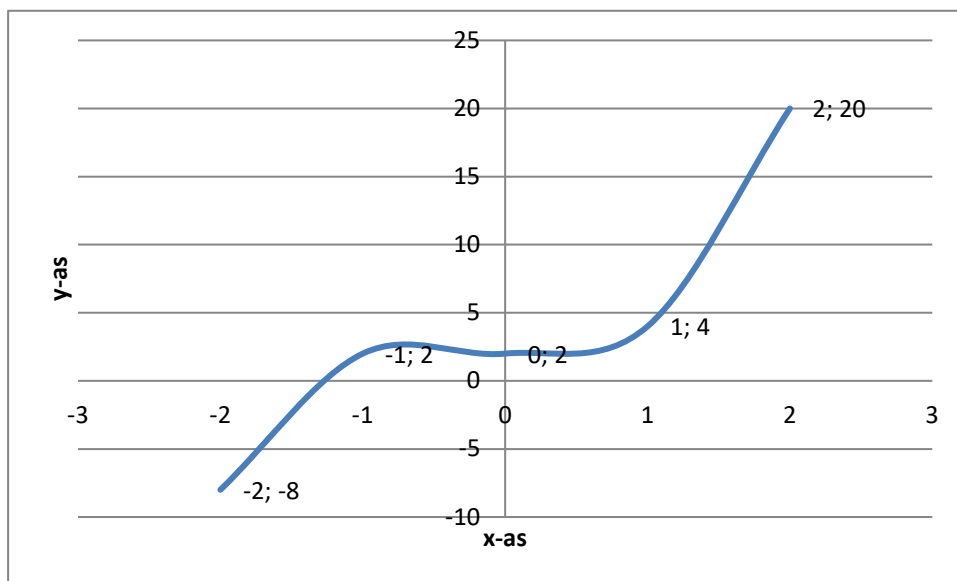
Die koördinaat van die infleksiepunt is dus $\left(-\frac{1}{6}; \frac{59}{27}\right) \checkmark \quad (2)$

3.1.2

x	-2	-1	0	1	2
y	-7	3	3	5	21

$\checkmark = 1$ punt vir enige 3 korrekte antwoorde. (3)

3.1.3



✓ =1 punt vir die vorm.
✓ =1 punt vir die aanduiding van die draaipunt op die grafiek. (2)

3.1.4 Let $x_0 = -1,5$

$$f(-1,5) = -1$$

$$f'(-1,5) = 9,5✓$$

$$x_1 = -1,5 - \frac{(-1)}{9,5}✓$$

$$= -1,395✓$$

$$f(-1,395) = -0,086$$

$$f'(-1,395) = 7,882$$

$$x_1 = -1,395 - \frac{(-0,086)}{7,882}$$

$$= -1,384✓$$

(4)

$$3.2 \quad V = l \times w \times h = 6w^2h = 20$$

$$C = 3(2lh + 2wh) + 15(2lw) \checkmark$$

$$= 3(12wh + 2wh) + 15(12w^2)$$

$$= 42wh + 180w^2$$

$$h = \frac{10}{3w^3} \checkmark$$

$$C = 4w \times \frac{10}{3w^3} + 180w^2 \checkmark$$

$$= \frac{140}{w} + 180w^2 \checkmark$$

$$C' = -\frac{140}{w^2} + 360w = 0 \checkmark$$

$$360w^3 - 140 = 0$$

$$w = 0.730, \quad l = 4,379 \text{ en } h = 6,257 \checkmark$$

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VRAAG 4

4.1 4.1.1

$$\int \sec^2 2x(9 + 7 \tan 2x - \tan^2 2x) dx$$

$$\text{laat } u = \tan 2x$$

$$\frac{du}{dx} = 2 \sec^2 2x$$

$$\frac{du}{2} = \sec^2 2x dx \checkmark$$

$$= \frac{1}{2} \int (9 + 7u - u^2) du$$

$$= \frac{1}{2} \left(9u + \frac{7}{2}u^2 - \frac{1}{3}u^3 \right) + C$$

$$= \frac{9}{2} \tan 2x \checkmark + \frac{7}{4} \tan^2 2x \checkmark - \frac{1}{6} \tan^3 2x \checkmark + C$$

(4)

4.1.2

$$\int \frac{7x + 2}{\sqrt{1 - 25x^2}} dx$$

$$= \int \frac{7x}{\sqrt{1 - 25x^2}} dx + \int \frac{2}{\sqrt{1 - (5x)^2}} dx$$

laat $u = 1 - 25x^2$ en $v = 5x$

$$\frac{du}{dx} = -50x$$

$$\frac{du}{-50} = x dx \checkmark$$

$$= -\frac{7}{50} \int u^{-\frac{1}{2}} du + \frac{2}{5} \int \frac{1}{\sqrt{1 - v^2}} dv \checkmark$$

$$= -\frac{2}{25} u^{\frac{1}{2}} + \frac{2}{5} \sin^{-1} v + C$$

$$= -\frac{2}{25} (1 - 25x^2)^{\frac{1}{2}} \checkmark + \frac{2}{5} \sin^{-1} 5x \checkmark + C \tag{4}$$

4.1.3

$$\int \frac{3x^3 - 17x^2 + 18x + 10}{3x + 1} dx$$

$$= \int \left(x^2 - 6x + 8 + \frac{2}{3x + 1} \right) dx \checkmark$$

$$= \frac{x^3}{3} \checkmark - 3x^2 \checkmark + 8x \checkmark + \frac{2}{3} \ln(3x + 1) \checkmark + C \tag{5}$$

4.1.4

$$\int \sin^3 x \cos x dx$$

let $u = \sin x$

$$du = \cos x dx \checkmark$$

$$= - \int u^3 du$$

$$= \frac{1}{4} u^4 + C$$

$$= \frac{1}{4} (\sin x)^4 \checkmark + C \tag{2}$$

4.1.5

$$\int x \sec^2 x \, dx$$

$\begin{aligned} \text{laat } u &= x & dv &= \sec^2 x \, dx \\ du &= dx & v &= \tan x \checkmark \end{aligned}$

$$\int \sec^2 x \, dx = x \tan x - \int \tan x \, dx \checkmark$$

$$= x \tan x - (-\ln|\cos x|) + C$$

$$= x \tan x \checkmark + \ln|\cos x| \checkmark + C \tag{4}$$

4.2

$$\int \frac{x+5}{(x-3)(x+1)} \, dx$$

$$\frac{x+5}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$6x + 12 = A(x+1) + B(x-3) \checkmark$$

$\text{let } x = -1, \quad B = -1 \checkmark$

of

$x = 3, \quad A = 2 \checkmark$

$$= \int \left(\frac{2}{x-3} - \frac{1}{x+1} \right) dx$$

$$= 2 \ln(x-3) \checkmark - \ln(x+1) \checkmark + C$$

(5)
[24]

VRAAG 5

5.1

$$\int_3^7 \left[\frac{9e^x}{e^x + 4} + \frac{(\ln 2x)^2}{x} \right] dx$$

$$\int_3^7 \frac{9e^x}{e^x + 4} dx + \int_3^7 \frac{(\ln 2x)^2}{x} dx$$

$$\text{let } u = e^x + 4 \quad z = \ln 2x \, dx$$

$$du = e^x dx \quad dz = \frac{1}{x} dx \checkmark$$

$$\int \frac{1}{u} du + \int z^2 dz \checkmark$$

$$= \left[9 \ln(e^x + 4) + \frac{1}{3} (\ln 2x)^3 \right]_3^7 \checkmark$$

$$= \left[9 \ln(e^7 + 4) + \frac{1}{3} (\ln 2 \times 7)^3 \right] - \left[9 \ln(e^3 + 4) + \frac{1}{3} (\ln 2 \times 6)^3 \right] \checkmark$$

$$= 38,608 \checkmark \quad (5)$$

5.2 5.2.1 $8x - 2x^2 = 0$

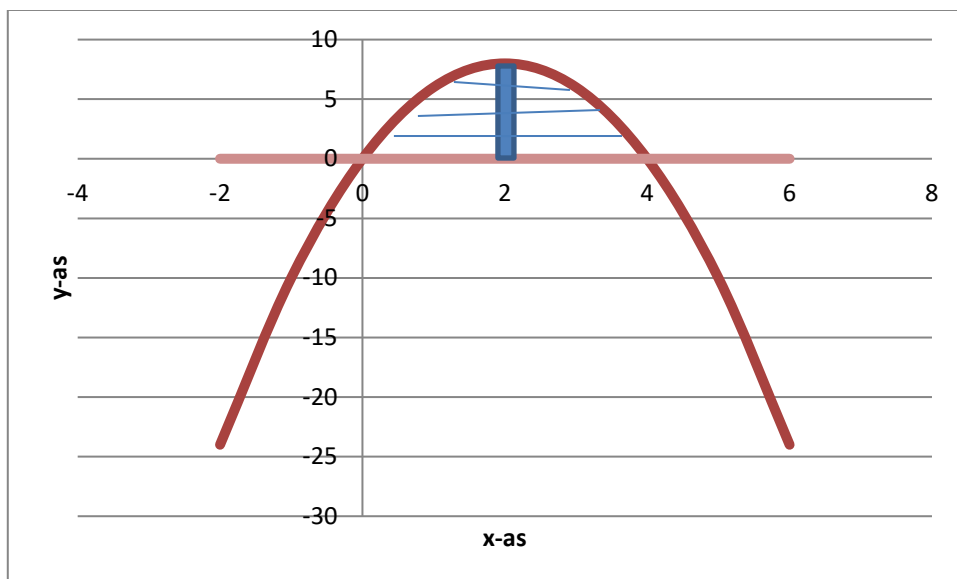
$$2x(4 - x) = 0$$

$$x = 0 \text{ or } x = 4$$

$$y = 0 \quad y = 0$$

Die koördinate van die sny punte is dus (0; 0)✓ en (4; 0).✓ (2)

5.2.2



✓ = 1 punt vir die aanduiding van die ingeslote gebied.
 ✓ = 1 punt vir die aanduiding van die vertikale of horisontale strook. (2)

5.2.3

$$A = \int_a^b y \, dx$$

$$A = \int_0^4 (8x - 2x^2) \, dx \checkmark$$

$$= \left[4x^2 - \frac{2}{3}x^3 \right]_0^4 \checkmark$$

$$= \frac{64}{3} \text{ of } 21,333 \text{ eenhede}^2 \checkmark \quad (3)$$

5.2.4

$$V = \pi \int_0^4 (8x - 2x^2)^2 \, dx$$

$$= \pi \int_0^4 (64x^2 - 32x^3 + 4x^4) \, dx \checkmark \checkmark$$

$$= \pi \left[\frac{64}{3}x^3 - 8x^4 + \frac{4}{5}(4)^5 \right]_0^4 \checkmark$$

$$= \frac{2048}{15} \pi \text{ of } 428,932 \text{ eenhede}^3 \checkmark \quad (4)$$

5.3 Inersiemoment van 'n skyf is $\frac{1}{2}r^2 dm$

$$dI_z = \frac{1}{2}r^2 dm \checkmark$$

$$I_z = \frac{1}{2}r^2 \int_0^{0.03} dm \checkmark$$

$$= \frac{1}{2}(0,03)^2(15) \checkmark$$

$$= 0,007 \text{ kgm}^2 \checkmark$$

(4)
[20]

VRAAG 6

6.1 $\frac{dy}{dx} = -\frac{x}{ye^{x^2}}$

$$y dy = -xe^{-x^2} dx$$

$$\frac{y^2}{2} \checkmark = \frac{1}{2}e^{-x^2} \checkmark + C$$

$$\frac{1^2}{2} = \frac{1}{2}e^{-(0)^2} + C$$

$$C = 0 \checkmark$$

$$\frac{y^2}{2} = \frac{1}{2}e^{-x^2} \checkmark$$

(4)

6.2 $\operatorname{cosec} x \cdot \frac{d^2y}{dx^2} = 1 + \cot x + \frac{x^2}{\sin x}$

$$\frac{d^2y}{dx^2} = \sin x + \cos x + x^2 \checkmark$$

$$\frac{dy}{dx} = \cos x - \sin x + \frac{1}{3}x^3 + A \checkmark$$

$$y = -\sin x - \cos x + \frac{1}{12}x^4 + Ax + B \checkmark$$

(3)
[7]

TOTAAL: 100