



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N5

30 JULIE 2019

Hierdie nasienriglyn bestaan uit 11 bladsye.

VRAAG 1

1.1 1.1.1

$$\lim_{x \rightarrow 0} \frac{\arcsin 4x}{\arctan 5x}$$

L'Hospital se reël aangesien die toestand $\frac{0}{0}$ voortduur; derhalwe

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{\frac{4}{\sqrt{1-16x^2}} \checkmark}{\frac{5}{1+2x^2} \checkmark} \\ &= \frac{4}{5} \checkmark \end{aligned} \tag{3}$$

1.1.2

$$\begin{aligned} &\lim_{x \rightarrow 0} \sqrt{x} \ln x \\ &= \lim_{x \rightarrow 0} \frac{\ln x}{\frac{1}{\sqrt{x}}} \checkmark \end{aligned}$$

L'Hospital se reël aangesien die toestand $\frac{0}{0}$ voortduur; derhalwe

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{\frac{1}{x} \checkmark}{-\frac{1}{2} x^{-\frac{3}{2}}} \checkmark \\ &= \lim_{x \rightarrow 0} -\frac{2\sqrt{x^3}}{x} \end{aligned}$$

L'Hospital se reël aangesien die toestand $\frac{0}{0}$ voortduur; derhalwe

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{-3\sqrt{x} \checkmark}{1} \checkmark \\ &= 0 \checkmark \end{aligned} \tag{4}$$

1.2 $f(x) = \sec x$

Derhalwe is $f(x)$ kontinuu by $x \neq \frac{\pi}{2} \checkmark$ en $x \neq \frac{3\pi}{2} \checkmark$ dit is $[0; \frac{\pi}{2})$ en $(\frac{\pi}{2}; \frac{3\pi}{2})$ (2)
[9]

VRAAG 2

2.1

$$f(x) = \frac{x+1}{x+2}$$

$$f'(x) = \lim_{h \rightarrow 0} \left[\frac{\frac{(x+h)+1}{(x+h)+2} - \frac{x+1}{x+2}}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\frac{(x+h)+1}{(x+h)+2} \times \frac{x+2}{x+2} - \frac{x+1}{x+2} \times \frac{(x+h)+2}{(x+h)+2}}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\frac{x^2+xh+3x+2h+2-x^2-xh-3x-h-2}{(x+h+2)(x+2)}}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\frac{h}{(x+h+2)(x+2)}}{h} \right]$$

$$= \frac{1}{(x+2)^2}$$

(5)

2.2

$$y = \text{arc cosec } x$$

$$\text{cosec } y = x$$

$$-\text{cosec } y \cot y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = -\frac{1}{\text{cosec } y \cot y}$$

$$= -\frac{1}{\text{cosec } x \sqrt{\text{cosec}^2 x - 1}}$$

$$= -\frac{1}{x\sqrt{x^2 - 1}}$$

(3)

2.3

$$y = \cot x$$

$$y = \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{\sin x (-\sin x) - \cos x (\cos x)}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= -\frac{1}{\sin^2 x}$$

$$= -\text{cosec}^2 x$$

(3)

2.4 2.4.1

$$y = \sqrt{3x + \sqrt{2x + \sqrt{x}}}$$

$$\frac{dy}{dx} = \frac{1}{2} \left[3x + (2x + x^{\frac{1}{2}})^{-\frac{1}{2}} \right]^{\frac{-1}{2}} \times \left[3 + (2x + x^{\frac{1}{2}})^{-\frac{1}{2}} \right]^{\frac{1}{2}} \times \left(2 + \frac{1}{2} x^{-\frac{1}{2}} \right)^{\frac{1}{2}}$$

2.4.2 $y = \left[\left(\frac{1}{x} + 1 \right)^{-1} + 1 \right]^{-1}$

$$\frac{dy}{dx} = - \left[\left(\frac{1}{x} + 1 \right)^{-1} + 1 \right]^{-2} \times \left[- \left(\frac{1}{x} + 1 \right)^{-2} \right] \times \left(- \frac{1}{x^2} \right)$$

(2 × 3) (6)

2.5 $y = (1 - 3x)^{\cos x}$

$$\ln y = \cos x \ln(1 - 3x)$$

$$\frac{1}{y} \frac{dy}{dx} = \cos x \times \frac{1}{1 - 3x} \times -3 - \sin x \times \ln(1 - 3x)$$

$$\frac{dy}{dx} = y \left[- \frac{3 \cos x}{1 - 3x} - \sin x \ln(1 - 3x) \right]$$

$$= (1 - 3x)^{\cos x} \left[- \frac{3 \cos x}{1 - 3x} - \sin x \ln(1 - 3x) \right] \quad (4)$$

2.6 $e^{2x+3y} = x^2 - \ln(xy^3)$

$$e^{2x+3y} \left[2 + 3 \frac{dy}{dx} \right] = 2x - \frac{1}{xy^3} \left[y^3 + x \times 3y^2 \frac{dy}{dx} \right]$$

$$2e^{2x+3y} + 3e^{2x+3y} \frac{dy}{dx} = 2x - \frac{1}{x} - \frac{3}{y} \frac{dy}{dx}$$

$$\left[3e^{2x+3y} + \frac{3}{y} \right] \frac{dy}{dx} = 2x - \frac{1}{x} - 2e^{2x+3y}$$

$$\frac{dy}{dx} = \frac{2x - \frac{1}{x} - 2e^{2x+3y}}{3e^{2x+3y} + \frac{3}{y}}$$

(4)
[25]

VRAAG 3

3.1 3.1.1 $f(x) = 2x^3 - x^2 + 5$

$f'(x) = 6x^2 - 2x = 0$

$2x(3x - 1) = 0$

$x = 0$ or $x = \frac{1}{3}$

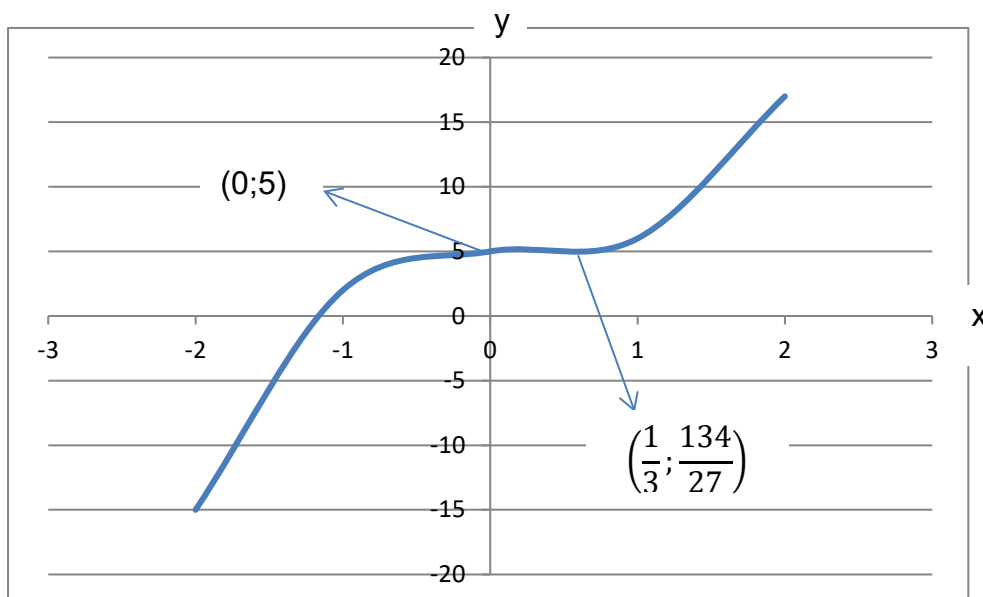
Draaipunte $(\frac{1}{3}; \frac{134}{27})$ ✓ en $(0; 5)$ ✓ (2)

3.1.2

x	-2	-1	0	1	2
y	-15	2	5	6	17

(EEN punt vir enige 2 korrekte antwoorde) (2)

3.1.3



(EEN punt vir die vorm)
 (EEN punt vir die aanduiding van die infleksiepunt op die grafiek) (2)

3.1.4

$$\text{Let } x_0 = -1,5$$

$$f(-1,5) = -4$$

$$f'(-1,5) = 16,5$$

$$x_1 = -1,5 - \frac{(-4)}{16,5} \checkmark$$

$$= -1,257575758 \checkmark$$

$$\text{Laat } x_1 = -1,257575758$$

$$f(-1,257575758) = -0,559200824$$

$$f'(-1,257575758) = 12,00413223$$

$$x_1 = -1,257575758 - \frac{(-0,559200824) \checkmark}{12,00413223}$$

$$= -1,21099173$$

$$\approx -1,211 \checkmark \quad (4)$$

3.2

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dv}{dt} = \frac{4\pi}{3} \times 3r^2 \frac{dr}{dt} \checkmark$$

$$\frac{dr}{dt} = \frac{\frac{dv}{dt}}{4\pi r^2} \checkmark$$

$$\frac{dr}{dt} = \frac{5 \text{ cm}^3/\text{s}}{4\pi(10 \text{ cm})^2} \checkmark$$

$$= 0,004 \text{ cm/s} \checkmark \quad (4)$$

[14]

VRAAG 4

4.1 4.1.1

$$\int 4 \left(\frac{1}{x} - e^{-x} \right) \cos(e^{-x} + \ln x) dx$$

$$= 4 \int \left(\frac{1}{x} - e^{-x} \right) \cos(e^{-x} + \ln x) dx \checkmark$$

$$\text{let } u = e^{-x} + \ln x$$

$$du = \left(\frac{1}{x} - e^{-x} \right) dx \checkmark$$

$$= 4 \int \cos u du$$

$$= 4 \sin(e^{-x} + \ln x) + C \checkmark \quad (3)$$

4.1.2

$$\int \frac{1}{\sqrt{4 - 9x^2}} dx$$

$$= \frac{1}{3} \checkmark \int \frac{1}{\sqrt{\frac{4}{9} - x^2}} dx \checkmark$$

$$= \frac{1}{3} \sin^{-1} \left(\frac{3}{2} x \right) + C \checkmark \quad (3)$$

4.1.3

$$\int \frac{x^3 + x}{x - 1} dx$$

$$= \int \left[x^2 + x + 2 + \frac{2}{x - 1} \right] dx \checkmark$$

$$= \frac{x^3}{3} \checkmark + \frac{x^2}{2} \checkmark + 2x \checkmark + 2 \ln(x - 1) \checkmark + C \quad (5)$$

4.1.4

$$\int \sin 10x \sin 7x dx$$

$$= \int \frac{1}{2} [\cos 3x - \cos 17x] dx \checkmark$$

$$= \frac{1}{2} \int (\cos 3x - \cos 17x) dx$$

$$= \frac{1}{6} \sin 3x \checkmark - \frac{1}{34} \sin 17x \checkmark + C \quad (3)$$

4.1.5

$$\int \frac{x}{\sec 3x} dx$$

$$= \int x \cos 3x dx$$

$let\ u = x$	$dv = \cos 3x\ dx$
$du = dx$	$v = \frac{1}{3} \sin 3x\ \checkmark$

$$\int u\ dv = uv - \int v\ du$$

$$= \frac{x}{3} \sin 3x\ \checkmark - \frac{1}{3} \int \sin 3x\ dx\ \checkmark$$

$$= \frac{x}{3} \sin 3x + \frac{1}{9} \cos 3x + C\ \checkmark \tag{4}$$

4.2

$$\int \frac{x^2}{x^2 - 1} dx$$

$$\int \left[1 + \frac{1}{x^2 - 1} \right] dx\ \checkmark$$

$$\frac{x^2}{(x + 1)(x - 1)} = \frac{A}{x - 1} + \frac{B}{x + 1}$$

$$x^2 = A(x + 1) + B(x - 1)\ \checkmark$$

$$let\ x = -1, \quad then\ B = -\frac{1}{2}\ \checkmark$$

$$x = 1, \quad then\ A = \frac{1}{2}\ \checkmark$$

$$= \int dx + \frac{1}{2} \int \frac{1}{x - 1} dx - \frac{1}{2} \int \frac{1}{x + 1} dx$$

$$= x + \frac{1}{2} \ln(x - 1)\ \checkmark - \frac{1}{2} \ln(x + 1)\ \checkmark + C$$

(6)
 [24]

VRAAG 5

5.1

$$\int_0^{\infty} e^{-st} \cdot f(t) dt$$

$$= -5 \int_0^{\infty} e^{-st} dt \checkmark$$

$$= -5 \left[-\frac{e^{-st}}{s} \right]_0^{\infty} \checkmark$$

$$= -5 \left(0 + \frac{1}{s} \right) \checkmark$$

$$= \frac{-5}{s} \checkmark$$

(4)

5.2 5.2.1 $3x^2 = 6x$

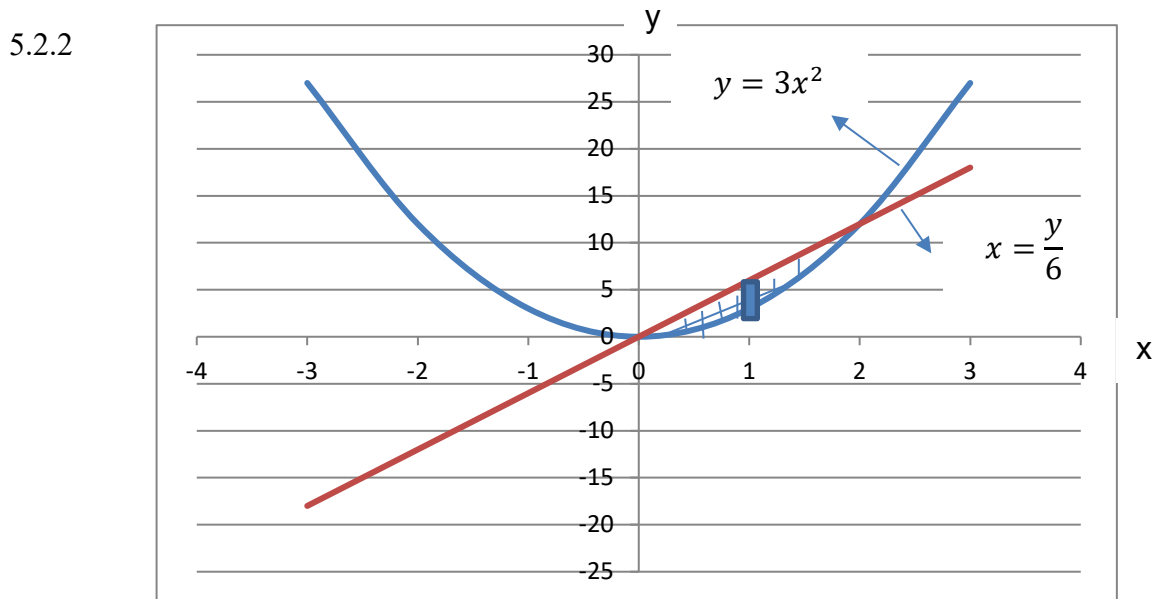
$$3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x = 0 \text{ or } x = 2$$

Derhalwe is die koördinate van die snypunt $(0; 0) \checkmark$ en $(2; 12) \checkmark$

(2)



(EEN punt vir aanduiding van ingeslote oppervlakte)
 (EEN punt vir aanduiding van die vertikaal of vertikale strook)

(2)

$$\begin{aligned}
 5.2.3 \quad A_x &= \int_a^b (y_1 - y_2) dx \\
 &= \int_0^2 (6x - 3x^2) dx \checkmark \\
 &= [3x^2 - x^3]_0^2 \checkmark \\
 &= 4 \text{ eenhede}^2 \checkmark
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 5.2.4 \quad V_x &= \int_a^b (y_1^2 - y_2^2) dx \\
 &= \int_0^2 (36x^2 - 9x^4) dx \checkmark \checkmark \\
 &= \left[12x^3 + \frac{9}{5}x^5 \right]_0^2 \checkmark \\
 &= 38,4 \text{ eenhede}^3 \checkmark
 \end{aligned} \tag{4}$$

$$5.3 \quad (d)_y = r^2 A$$

$$(d)_y = r^2 dA$$

$$(d)_y = x^2 b dx \checkmark$$

$$= b \int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 dx \checkmark$$

$$= ba^2 \left[-\frac{x^3}{3} \right]_{-\frac{a}{2}}^{\frac{a}{2}} \checkmark$$

$$= b \left[\frac{a^3}{12} \right]$$

$$= \frac{a^2 A}{12} \checkmark$$

(4)
 [19]

VRAAG 6

6.1 $(3y^2 + 2) \cos x dx - 6y \sin x dy = 0$

$$\frac{6y}{3y^2 + 2} dy = \cot x dx \checkmark$$

$$\ln(3y^2 + 2) \checkmark = \ln(\sin x) + C \checkmark \quad (3)$$

6.2 $\frac{d^2y}{dx^2} = 3x^2 - 2x$

$$\frac{dy}{dx} = x^3 - x^2 + A \checkmark$$

$$A = -1 \checkmark$$

$$\frac{dy}{dx} = x^3 - x^2 - 1 \checkmark$$

$$y = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x + B \checkmark$$

$$B = \frac{25}{12} \checkmark$$

$$y = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x + \frac{25}{12} \checkmark \quad (6)$$

[9]

TOTAAL: 100