higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N5

26 JULIE 2018

Hierdie nasienriglyn bestaan uit 11 bladsye.

VRAAG 1

1.1 1.1.1

$$\lim_{x\to 1}\left(\frac{x^2+8x-9}{x^3-2x^2-5x+6}\right)$$

$$=\lim_{x\to 0}\left(\frac{2x+8}{3x^2-4x-5}\right)\checkmark\left[\frac{0}{0}\right]$$

$$=-\frac{5}{3}\checkmark$$ (2)

1.2 1.2.1

$$\log y=\lim_{x\to\infty}\frac{x^2}{e^x}\left[\frac{0}{0}\right]$$

$$=\lim_{x\to\infty}\frac{2x}{e^x}\left[\frac{0}{0}\right]\checkmark$$

$$=\lim_{x\to\infty}\frac{2}{e^x}\checkmark$$

$$=0\checkmark$$ (3)

1.2.2

$$y=10^0$$
$$=1\checkmark$$ (1)

1.3

$$f(x)=\frac{x}{7-e^x}$$
$$7-e^x=0$$
$$e^x=7\checkmark$$
$$x=\ln 7\checkmark$$ (2)

[8]

VRAAG 2

2.1 $$f(x) = -\frac{1-x}{x+2}$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{1-x-h}{x+h+2} - \frac{1-x}{x+2}}{h} \checkmark$$

$$= \lim_{h\to 0}\frac{\frac{1-x-h}{x+h+2}\times\frac{x+2}{x+2} - \frac{1-x}{x+2}\times\frac{x+h+2}{x+h+2}}{h} \checkmark$$

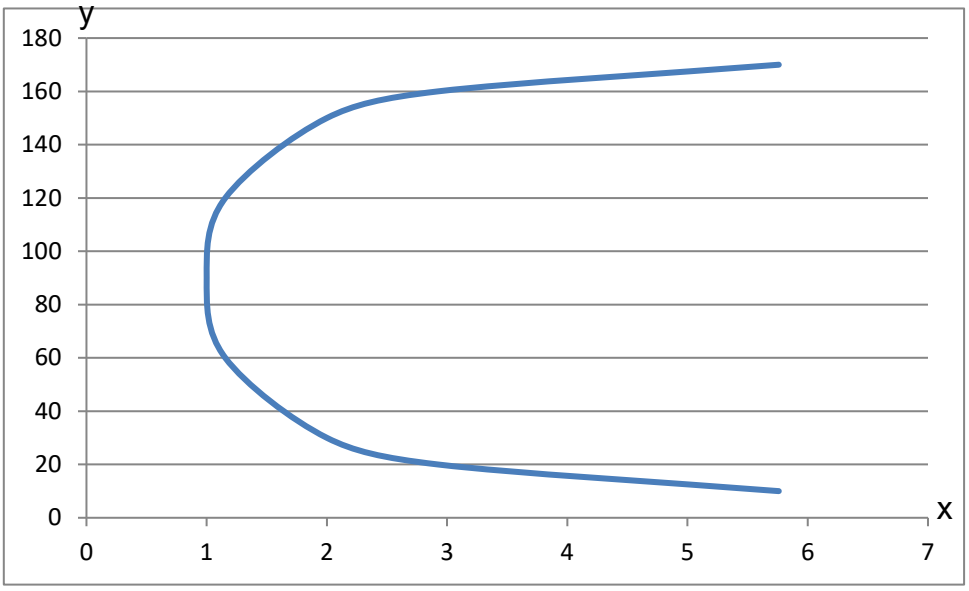
$$= \lim_{h\to 0}\frac{\frac{x-x^2-xh+2-2x-2h-x-h-2+x^2+xh+2x}{(x+2)(x+h+2)}}{h} \checkmark$$

$$= \lim_{h\to 0}\frac{-3h}{h(x+2)(x+h+2)} \checkmark$$

$$= -\frac{3}{(x+2)^2} \checkmark$$

(5)

2.2 $$y = arc\,\text{cosec}\,x$$



✓ = 1 Punt vir die vorm

✓ = 1 Punt vir die reeks en beperkings (2)

2.3 2.3.1 $y = [\ln(x^2+1) - \tan^{-1}(3x)]^6$

$$\frac{dy}{dx} = 6[ln(x^2+1) - tan^{-1}(3x)]^5 \checkmark \times \left[\frac{1}{x^2+1} \times 2x \checkmark - \frac{1}{1+(3x)^2} \times 3 \checkmark\right] \quad (3)$$

2.3.2 $y = \frac{e^{x^2+8x}}{\sqrt{x^4+7}}$

$$\frac{dy}{dx} = \frac{\sqrt{x^4+7} \times e^{x^2+8x} \times (2x+8) \checkmark - e^{x^2+8x} \times \frac{1}{2\sqrt{x^4+7}} \checkmark \times 4x^3 \checkmark}{x^4+7 \checkmark} \quad (4)$$

2.3.3 $y = \sqrt{x^2+\sqrt{1+4x}}$

$$\frac{dy}{dx} = \frac{1}{2}\left(x^2+\sqrt{1+4x}\right)^{-\frac{1}{2}} \checkmark \times \left[2x \checkmark + \frac{1}{2\sqrt{1+4x}} \times 4 \checkmark\right] \quad (3)$$

2.4 $y = x^{\sqrt{x}}$

$$\ln y = \sqrt{x} \ln x$$

$$\frac{1}{y}\frac{dy}{dx} \checkmark = \frac{\sqrt{x}}{x} \checkmark + \frac{\ln x}{2\sqrt{x}} \checkmark$$

$$\frac{dy}{dx} = y\left[\frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}}\right]$$

$$= x^{\sqrt{x}}\left[\frac{\sqrt{x}}{x} + \frac{ln\, x}{2\sqrt{x}}\right] \checkmark \quad (4)$$

2.5 $\tan\left(\frac{x}{y}\right) = x + y$

$$\sec^2\left(\frac{x}{y}\right) \times \frac{1.y - x.\frac{dy}{dx}}{y^2} \checkmark = 1 + \frac{dy}{dx} \checkmark$$

$$y\sec^2\left(\frac{x}{y}\right) - x\sec^2\left(\frac{x}{y}\right)\frac{dy}{dx} = y^2 + y^2\frac{dy}{dx} \checkmark$$

$$x \quad \sec^2\left(\frac{x}{y}\right)\frac{dy}{dx} + y^2\frac{dy}{dx} = y\sec^2\left(\frac{x}{y}\right) - y^2 \checkmark$$

$$\frac{dy}{dx} = \frac{y\,sec^2\left(\frac{x}{y}\right) - y^2}{x\sec^2\left(\frac{x}{y}\right) + y^2} \checkmark \quad (5)$$

[26]

VRAAG 3

3.1 3.1.1 $f'(x) = 21x^2 - 8 = 0$✓

$f''(x) = 42x = 0$✓

$x = 0$

$y = 4$

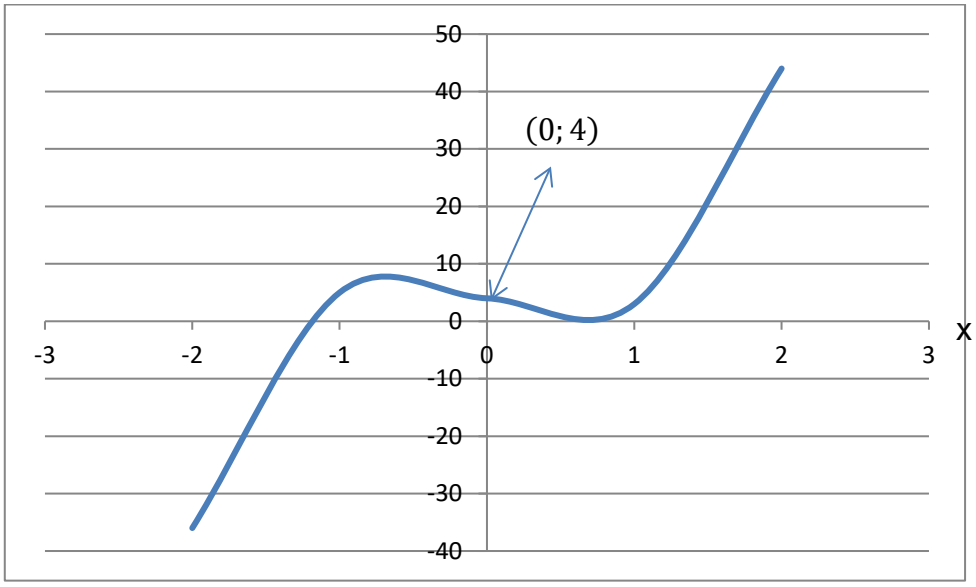
Dus is die koördinaat van die buigpunt; $(0; 4)$✓ (3)

3.1.2

x	-2	-1	0	1	2
y	-36	5	4	3	44

✓ = 1 Punt vir enige 3 korrekte antwoorde (2)

3.1.3



✓ = 1 Punt vir die vorm
✓ = 1 Punt vir die aanduiding van die buigpunt (2)

3.1.4 $Let\ x_0 = -1$

$f(-1) = 5$

$f'(-1) = 13$✓

$x_1 = -1 - \frac{5}{13}$✓

$= -1{,}338462$✓ (3)

3.2 $Laat\ x = kortste\ kant$
$y = langste\ kant$

$Dus,\ y = 3x$✓

$A = xy$

$A = 3x^2$✓

$\frac{dA}{dt} = 6x\frac{dx}{dt}$✓

$\frac{dA}{dt} = 6(6)(-2)$✓

$= -72\ m^2/s$✓ (5)

3.3 $x = t^3 - 7t^2 + 8t + 2$

$snelheid = \frac{dx}{dt} = 3t^2 - 14t + 8$✓

$versnelling = \frac{d^2x}{dt^2} = 6t - 14$✓

$Voorwerp\ by\ ruspunt$

$\frac{dx}{dt} = 3t^2 - 14t + 8 = 0$

$(3t - 2)(t - 4) = 0$✓

$t = 1{,}5s$✓ $en\ t = 4s$✓ (5)
[20]

VRAAG 4

4.1 4.1.1 $\int x\sqrt{x+3}\,dx$

$Laat\ u = x+3$

$du = dx$

$\Rightarrow x = u-3$✓

$= \int (u-3)u^{\frac{1}{2}}du$

$= \int \left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\right)du$✓

$= \frac{2}{5}u^{\frac{5}{2}} + 2u^{\frac{3}{2}}$

$= \frac{2}{5}(x+3)^{\frac{5}{2}} + 2(x+3)^{\frac{3}{2}} + c$✓ (3)

4.1.2 $\int \frac{x^2-x-1}{x-1}dx$

$= \int \left((x^2-x-1)✓ - \frac{6}{x-1}✓\right)dx$

$= \frac{x^3}{3} - \frac{x^2}{2} - x - 6\ln(x-1) + c$✓✓ (4)

4.1.3 $\int \frac{1}{\sqrt{5-25x^2}}dx$

$= \frac{1}{5}✓\int \frac{1}{\sqrt{\frac{1}{5}-x^2}}dx$

$= \frac{1}{5}\sin^{-1}\left(\sqrt{5}x\right) + c$✓ (2)

4.1.4 $\int \cos(4ax)\sin(3bx)\,dx$

$= \frac{1}{2}\int [\sin(4a+3b)x - \sin(4a-3b)x]\,dx$✓

$= \frac{1}{2}\left[-\frac{\cos(4a+3b)x}{4a+3b} + \frac{\cos(4a-3b)x}{4a-3b}\right] + c$

$= \frac{\cos(4a+3b)x}{8a+6b}$✓ $+ \frac{\cos(4a-3b)x}{8a-6b}$✓ $+ c$ (3)

4.1.5 $\int \ln x^2\,dx$

$Let\ u = \ln x^2 \qquad dv = dx$

$du = \frac{2}{x}dx$ ✓ $\qquad v = x$

$= x\,.\ln x^2 - \int x.\frac{2}{x}dx$✓

$= x\,.\ln x^2 - 2x + c$✓ (3)

4.2 $\int \frac{1}{ax - bx^2}dx$

$\frac{1}{x(a-bx)} = \frac{A}{x} + \frac{B}{a-bx}$

$1 = A(a-bx) + Bx$✓

$Let\ x = \frac{a}{b}, B = \frac{b}{a}$✓

$Let\ x = 0, A = \frac{1}{a}$✓

$= \frac{1}{a}\int \frac{1}{x}dx + \frac{b}{a}\int \frac{1}{a-bx}dx$

$= \frac{1}{a}\ln x$✓ $- \frac{1}{a}\ln(a-bx)$✓ $+ c$ (5)

[20]

VRAAG 5

5.1 $\int_0^1 \sqrt{9-x^2}\,dx$

$= \frac{9}{2}\checkmark\left[\sin^{-1}\left(\frac{x}{3}\right)+\frac{x}{2}\sqrt{9-x^2}\right]_0^1\checkmark$

$= 2{,}9435\checkmark$ (3)

5.2 5.2.1 $x^2-6x+9=-x+5$

$x^2-6x+9+x-5=0$

$x^2-5x+4=0$

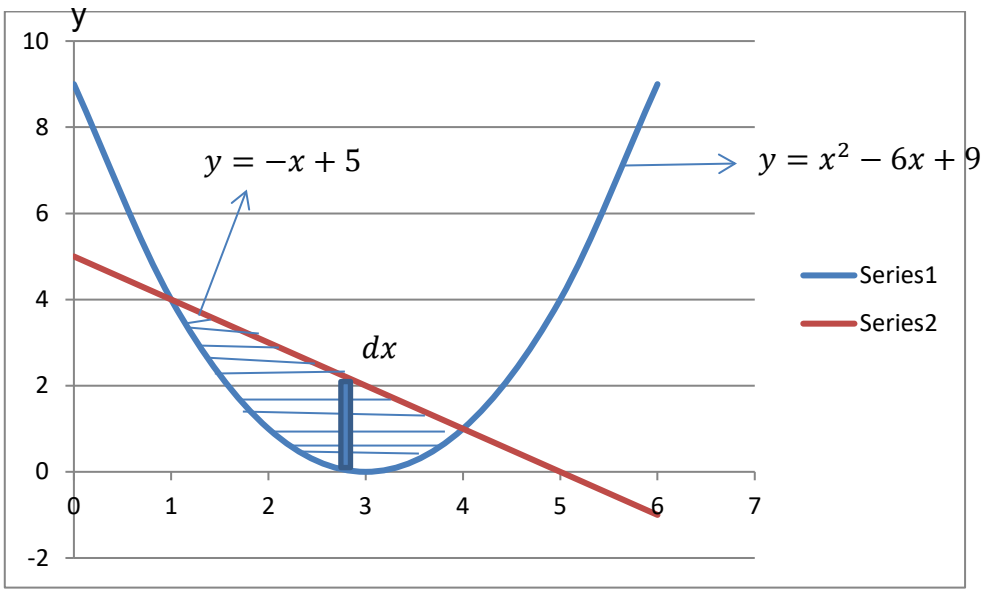
$(x-1)(x-4)=0$

$x=1\ or\ x=4$

$y=4\ or\ y=1$

Dus is die koördinate van die snypunt/kruispunt: $(1;4)\checkmark\ en\ (4;1)\checkmark$ (2)

5.2.2



✓ = 1 Punt vir die aanduiding van die ingeslote area
✓ = 1 Punt vir die aanduiding van die vertikale en horisontale strook (2)

5.2.3

$$A = \int_1^4 (y_1 - y_2)\,dx$$

$$= \int_1^4 [(-x+5) - (x^2 - 6x + 9)]\,dx$$

$$= \int_1^4 (-x^2 + 5x - 4)\,dx \checkmark$$

$$= \left[-\frac{x^3}{3} + \frac{5}{2}x^2 - 4x\right]_1^4 \checkmark$$

$$= 4{,}5\ units^2 \checkmark \qquad (3)$$

5.2.4

$$V = \pi \int_1^4 A(x)\,dx$$

$$= \pi \int_1^2 [(-x+5)^2 - (x^2 - 6x + 9)^2]\,dx$$

$$= \pi \int_1^2 [x^2 - 10x + 25 - (-x^4 - 12x^3 + 18x^2 - 108x + 81)]\,dx \checkmark$$

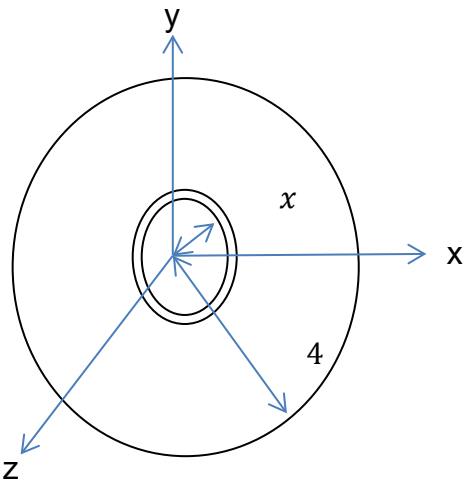
$$= \pi \int_1^2 (-x^4 + 12x^3 - 17x^2 + 98x - 56)\,dx \checkmark$$

$$= \pi \left[-\frac{x^5}{5} + 3x^4 - \frac{17}{3}x^3 + 49x^2 - 56x\right]_1^2 \checkmark$$

$$= 89{,}73\pi\ eenhede^3$$

$$= 281{,}84\ eenhede^3 \checkmark \qquad (4)$$

5.3



$2\pi x$ _____ dx

$d)_z = r^2 dA$

$= x^2 . 2\pi x \, dx$✓

$)^2 = 2\pi \int_0^4 x^3 \, dx$

$= 2\pi \left[\frac{x^4}{4}\right]_0^4$ ✓

$= 2\pi[64 - 0]$✓

$= 128\pi \; cm^4$

$= 402{,}1239 \; cm^4$✓ (4)

[18]

VRAAG 6

6.1 $(1 + x)\, dx = (y + 1)\, dy$

$\frac{dy}{y+1} = \frac{dx}{1+x}$ ✓

$\ln(y + 1) = \ln(x + 1) + c$✓

$c = \ln 2$ ✓

$\ln(y + 1) = \ln(x + 1) + \ln 2$✓

$\ln(y + 1) = \ln[2(1 + x)]$

$y = 2x + 1$✓ (5)

6.2 $\frac{d^2y}{dx^2} = x^3 - e^{-x} + 3$

$\frac{dy}{dx} = \frac{1}{4}x^4 + e^{-x} + 3x + A$✓

$y = \frac{1}{20}x^5 - e^{-x} + \frac{3}{2}x^2 + Ax + B$✓✓ (3)

[8]

TOTAAL: 100