



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N5

7 April 2021

Hierdie nasienriglyn bestaan uit 12 bladsye.

VRAAG 1

1.1 1.1.1 $\lim_{x \rightarrow \infty} \left[x \ln \left(1 + \frac{3}{x} \right) \right]$

$$\lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{3}{x} \right)}{\frac{1}{x}} \quad \checkmark \quad \left(\frac{0}{0} \right)$$

$$= \lim_{x \rightarrow \infty} \frac{-3x^{-2} \times \frac{1}{1 + \frac{3}{x}}}{-x^{-2}} \quad \checkmark$$

$$= \lim_{x \rightarrow \infty} \frac{3}{1 + \frac{3}{x}} \quad \checkmark$$

$$= 3 \quad \checkmark \tag{4}$$

1.1.2 $\lim_{x \rightarrow 7} \frac{\frac{1}{7} - \frac{1}{x}}{x - 7} \quad \left(\frac{0}{0} \right)$

$$= \lim_{x \rightarrow 7} \frac{-x^{-2}}{1} \quad \checkmark$$

$$= -\frac{1}{49} \quad \checkmark \tag{2}$$

1.2 1.2.1 $-\log y = \lim_{x \rightarrow 0} \frac{\sin 7x}{x} \quad \left(\frac{0}{0} \right)$

$$= \lim_{x \rightarrow 0} \frac{7 \cos 7x}{1} \quad \checkmark$$

$$= 7$$

$$\log y = -7 \quad \checkmark \tag{2}$$

1.2.2 $y = 10^{-7} \quad \checkmark \tag{1}$

1.3 $f(x) = \frac{e^{x^2+1}}{e^x - 2e^{1-x}}$

$$e^x - 2e^{1-x} = 0$$

$$e^x - \frac{2e}{e^x} = 0$$

$$e^{2x} - 2e = 0 \checkmark$$

$$e^{2x} = 2e$$

$$2x = 1 + \ln 2$$

$$x = \frac{1 + \ln 2}{2} \checkmark$$

(2)
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VRAAG 2

2.1 $f(x) = \frac{3}{x^5}$

2.1.1 $f(x + h) = 3(x + h)^{-5}$

$$= 3 \left[\frac{x^{-5}h^0}{0!} + \frac{(-5)x^{-6}h}{1!} + \frac{30x^{-7}h^2}{2!} + \frac{(-210)x^{-8}h^3}{3!} + \dots \dots \dots \right]$$

$$= 3x^{-5} - 15x^{-6}h + 45x^{-7}h^2 - 105x^{-8}h^3 + \dots \dots \dots \checkmark \checkmark \quad (2)$$

2.1.2 $f(x + h) - f(x) = -15x^{-6}h + 45x^{-7}h^2 - 105x^{-8}h^3 + \dots \dots \checkmark \quad (1)$

2.1.3 $\frac{f(x + h) - f(x)}{h} = -15x^{-6} + 45x^{-7}h - 105x^{-8}h^2 + \dots \dots \checkmark \quad (1)$

2.1.4 $\lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} = -15x^{-6} \checkmark \quad (1)$

$$\begin{aligned}
 2.2 \quad y &= e^{\ln(\tan x)} \\
 y &= \tan x \checkmark \\
 &= \frac{\sin x}{\cos x} \\
 \frac{dy}{dx} &= \frac{\cos x(\cos x) - \sin x(-\sin x) \checkmark}{\cos^2 x} \\
 &= \frac{\sin^2 x + \cos^2 x}{\cos^2 x} \\
 &= \frac{1 \checkmark}{\cos^2 x} \\
 &= \sec^2 x \tag{3}
 \end{aligned}$$

$$\begin{aligned}
 2.3 \quad 2.3.1 \quad y &= [\ln(x^2 + 1) - \tan^{-1}(6x)]^{10} \\
 \frac{dy}{dx} &= 10[\ln(x^2 + 1) - \tan^{-1}(6x)]^9 \checkmark \times \left[\frac{2x}{x^2 + 1} \checkmark - \frac{6}{36x^2 + 1} \checkmark \right] \\
 2.3.2 \quad y &= \frac{1 + \sin^{-1} x}{1 - \cos^{-1} x} \\
 \frac{dy}{dx} &= \frac{\frac{1 - \cos^{-1} x}{\sqrt{1-x^2}} \checkmark - \left(-\frac{1 + \sin^{-1} x}{\sqrt{1-x^2}} \right) \checkmark}{(1 - \cos^{-1} x)^2 \checkmark} \\
 &= \frac{\frac{\sin^{-1} x + \cos^{-1} x}{\sqrt{1-x^2}}}{(1 - \cos^{-1} x)^2} \\
 &= (2 \times 3) \tag{6}
 \end{aligned}$$

$$\begin{aligned}
 2.4 \quad y &= (2x - e^{8x})^{\sin 2x} \\
 \ln y &= \sin 2x \cdot \ln(2x - e^{8x}) \\
 \frac{1}{y} \cdot \frac{dy}{dx} \checkmark &= 2 \cos 2x \cdot \ln(2x - e^{8x}) \checkmark + \sin 2x \times \frac{1}{2x - e^{8x}} \times (2 - 8e^{8x}) \checkmark \\
 \frac{dy}{dx} &= y \left[2 \cos 2x \cdot \ln(2x - e^{8x}) + \frac{(2 - 8e^{8x}) \sin 2x}{(2x - e^{8x})} \right] \\
 \frac{dy}{dx} &= (2x - e^{8x})^{\sin 2x} \left[2 \cos 2x \cdot \ln(2x - e^{8x}) + \frac{(2 - 8e^{8x}) \sin 2x}{(2x - e^{8x})} \right] \checkmark \tag{4}
 \end{aligned}$$

2.5 $\tan(x^2y^4) = 3x + y^2$

$$\sec^2(x^2y^4) \left[2xy^4 + 4x^2y^3 \cdot \frac{dy}{dx} \right] \checkmark = 3 + 2y \frac{dy}{dx} \checkmark$$

$$\frac{dy}{dx} [4x^2y^3 \cdot \sec^2(x^2y^4) - 2y] = 3 - 2xy^4 \cdot \sec^2(x^2y^4) \checkmark$$

$$\frac{dy}{dx} = \frac{3 - 2xy^4 \cdot \sec^2(x^2y^4)}{4x^2y^3 \cdot \sec^2(x^2y^4) - 2y} \checkmark$$

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VRAAG 3

3.1 3.1.1

$$f(x) = x^3 - 7x^2 + 8x + 3$$

$$f'(x) = 3x^2 - 14x + 8 = 0$$

$$(3x - 2)(x - 4) = 0$$

$$x = \frac{3}{2} \text{ of } x = 4$$

$$y = \frac{21}{8} \quad y = -13$$

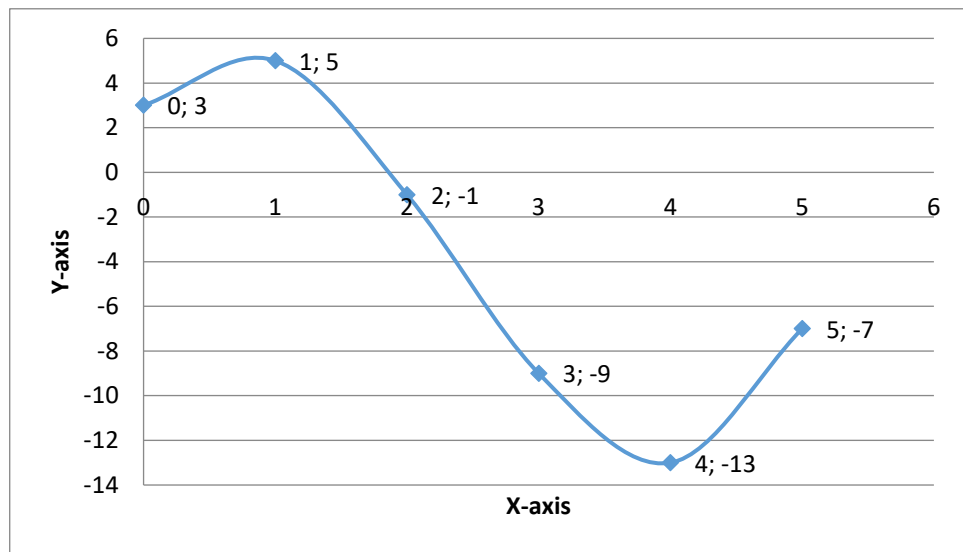
Dus is die koördinate van die draaipunte $\left(\frac{3}{2}; \frac{21}{8}\right) \checkmark$ en $(4; -13) \checkmark$ (2)

3.1.2

<i>x</i>	0	1	2	3	4	5
<i>y</i>	3	5	-1	-9	-13	-7

(Enige 3×1) (3)

3.1.3



Y-as
 X-as

(1 punt vir vorm + 1 punt vir aanduiding van draaipunte)

(2)

3.1.4 Laat $x_0 = 1.9$

$$f(1.9) = -0,211$$

$$f'(1.9) = -7,77 \checkmark$$

$$x_1 = 1.9 - \frac{(-0,211)}{(-7,77)} \checkmark$$

$$= 1.872 \checkmark$$

(3)

3.2 $xy = 750 \dots\dots\dots (1) \checkmark$

$$S = y + 10x \dots\dots\dots (2) \checkmark$$

$$y = \frac{750}{x} \dots\dots\dots (3)$$

$$S = \frac{750}{x} + 10x$$

$$S' = -\frac{750}{x^2} + 10 = 0 \checkmark$$

$$10x^2 = 750$$

$$x = \pm\sqrt{75} = 5\sqrt{3} \checkmark$$

$$y = 50\sqrt{3} \checkmark$$

(5)
 [15]

VRAAG 4

4.1 4.1.1

$$\int \left[4 \left(\frac{1}{x} - e^{-x} \right) \cos(e^{-x} + \ln x) \right] dx$$

$$\begin{aligned} \text{laat } u &= e^{-x} + \ln x \\ du &= \left(\frac{1}{x} - e^{-x} \right) dx \checkmark \end{aligned}$$

$$= 4 \int \cos u \, du \checkmark$$

$$= 4 \sin u + C$$

$$= 4 \sin(e^{-x} + \ln x) \checkmark + C \quad (3)$$

4.1.2

$$\int \frac{1}{\sqrt{4 - 9x^2}} dx$$

$$= \frac{1}{2} \checkmark \int \frac{1}{\sqrt{1 - \frac{9}{4}x^2}} dx \checkmark$$

$$= \frac{1}{2} \left(\frac{2}{3} \right) \sin^{-1} \left(\frac{3}{2}x \right) + C$$

$$= \frac{1}{3} \sin^{-1} \left(\frac{3}{2}x \right) + C \checkmark \quad (3)$$

4.1.3

$$\int \frac{2x^2 - 5x - 1}{x - 3} dx$$

$$= \int \left(2x + 1 - \frac{2}{x - 3} \right) dx \checkmark$$

$$= x^2 \checkmark + x \checkmark - 1 \ln(x - 3) \checkmark + C \quad (4)$$

4.1.4

$$\int \sin(\pi x) \sin\left(\frac{x}{5}\right) dx$$

$$= \int \frac{1}{2} \left[\cos\left(\pi - \frac{1}{5}\right)x - \cos\left(\pi + \frac{1}{5}\right)x \right] dx \checkmark \checkmark$$

$$= \frac{\sin\left(\pi - \frac{1}{5}\right)x}{2\left(\pi - \frac{1}{5}\right)} \checkmark - \frac{\sin\left(\pi + \frac{1}{5}\right)x}{2\left(\pi + \frac{1}{5}\right)} \checkmark + C \quad (4)$$

4.1.5

$$\int \frac{\ln x}{x^2} dx$$

$$\begin{aligned} \text{laat } u &= \ln x & dv &= \frac{1}{x^2} dx \checkmark \\ du &= \frac{1}{x} dx & v &= -\frac{1}{x} \end{aligned}$$

$$\begin{aligned} \int \frac{\ln x}{x^2} dx &= -\frac{1}{x} \cdot \ln x - \int -\frac{1}{x} \cdot \frac{1}{x} dx \checkmark \\ &= -\frac{\ln x}{x} \checkmark - \frac{1}{x} \checkmark + C \end{aligned} \tag{4}$$

4.2

$$\int \frac{x+2}{(x-1)^2} dx$$

$$\frac{x+2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$x+2 = A(x-1) + B \checkmark$$

$$\begin{aligned} \text{laat } x &= 1, & B &= 3 \checkmark \\ & \text{of} \\ x &= 0, & A &= 1 \checkmark \end{aligned}$$

$$\begin{aligned} &= \int \left(\frac{1}{x-1} + \frac{3}{(x-1)^2} \right) dx \\ &= \ln(x-1) \checkmark - \frac{3}{x-1} \checkmark + C \end{aligned} \tag{5}$$

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VRAAG 5

5.1

$$\begin{aligned} &\int_0^2 \left(e^x + \frac{1}{x^2+1} \right) dx \\ &= [e^x + \tan^{-1} x]_0^2 \checkmark \\ &= (e^2 + \tan^{-1} 2) - (e^0 + \tan^{-1} 0) \checkmark \\ &= 69.824 \checkmark \end{aligned} \tag{3}$$

5.2 5.2.1

$$6 + x - x^2 = \frac{2}{3}x^2 - 4x + 6$$

$$18 + 3x - 3x^2 = 2x^2 - 12x + 18$$

$$-18 - 3x + 3x^2 + 2x^2 - 12x + 18 = 0$$

$$5x(x - 3) = 0$$

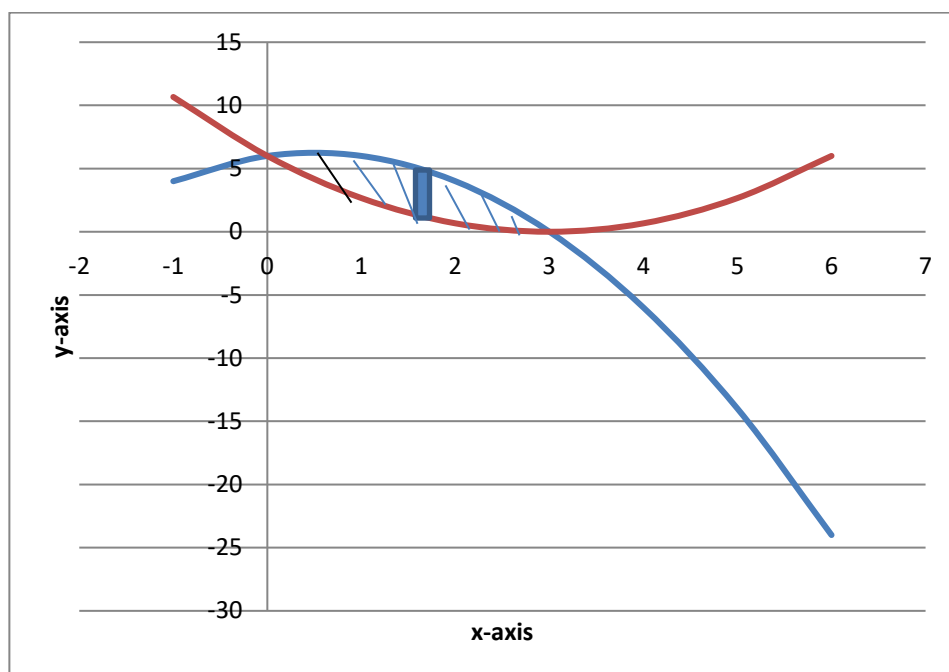
$$x = 0 \text{ of } x = 3$$

$$y = 6 \quad y = 0$$

Dus is die koördinate van die snydingspunte $(0; 6)$ ✓ en $(3; 0)$ ✓.

(2)

5.2.2



y-as x-as

(1 punt vir aanduiding van geslote area + 1 punt vir aanduiding van die vertikale of horisontale strook)

(2)

5.2.3

$$\begin{aligned}
 A &= \int_a^b (y_1 - y_2) dx \\
 A &= \int_0^3 \left[(6 + x - x^2) - \left(\frac{2}{3}x^2 - 4x + 6 \right) \right] dx \checkmark \\
 &= \int_0^3 \left(6 + x - x^2 - \frac{2}{3}x^2 + 4x - 6 \right) dx \\
 &= \int_0^3 \left(5x - \frac{5}{3}x^2 \right) dx \\
 &= \left[\frac{5}{2}x^2 - \frac{5}{9}x^3 \right]_0^3 \checkmark \\
 &= \frac{15}{2} \text{ of } 7,5 \text{ eenhede}^2 \checkmark
 \end{aligned} \tag{3}$$

5.2.4

$$\begin{aligned}
 V &= \pi \int_0^3 \left[(6 + x - x^2)^2 - \left(\frac{2}{3}x^2 - 4x + 6 \right)^2 \right] dx \\
 &= \pi \int_0^3 \left(36 + 12x - 11x^2 - 2x^3 + x^4 - \frac{4}{9}x^4 + \frac{16}{3}x^3 - 24x^2 + 48x - 36 \right) dx \checkmark \checkmark \\
 &= \pi \int_0^3 \left(60x - 35x^2 + \frac{10}{3}x^3 + \frac{5}{9}x^4 \right) dx \checkmark \\
 &= \pi \left[30x^2 - \frac{35}{3}x^3 + \frac{5}{6}x^4 + \frac{4}{45}x^5 \right]_0^3 \checkmark \\
 &= \frac{441}{10} \pi \text{ of } 138,544 \text{ eenhede}^3 \checkmark
 \end{aligned} \tag{5}$$

5.3 $m = \sigma A$

$$dm = \sigma dA$$

$$dI_z = r^2$$

$$= x^2 \sigma dm \checkmark$$

$$= x^2 \sigma 2\pi x dx \checkmark$$

$$= 2\pi\sigma \int_0^r x^3 dx \checkmark$$

$$= 2\pi\sigma \left[\frac{1}{4} x^4 \right]_0^r \checkmark$$

$$= 2\pi\sigma \left[\frac{r^4}{4} - 0 \right]$$

$$= \frac{1}{2} \pi \sigma r^4 \text{ eenhede}^4 \checkmark$$

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 [20]

VRAAG 6

6.1 $\frac{dy}{dx} = \frac{x^2}{1-y^2}$

$$(1-y^2) dy = x^2 dx \checkmark$$

$$y - \frac{y^3}{3} \checkmark = \frac{x^3}{3} + C \checkmark$$

(3)

$$6.2 \quad \frac{d^2y}{dx^2} = \frac{4}{3}x^3 + x$$

$$\frac{dy}{dx} = \frac{1}{3}x^4 + \frac{1}{2}x^2 + A \checkmark$$

$$\frac{1}{3} = \frac{1}{3}(0)^4 + \frac{1}{2}(0)^2 + A$$

$$A = \frac{1}{3} \checkmark$$

$$\frac{dy}{dx} = \frac{1}{3}x^4 + \frac{1}{2}x^2 + \frac{1}{3} \checkmark$$

$$y = \frac{1}{(3)(5)}x^5 + \frac{1}{(2)(3)}x^3 + \frac{1}{3}x + B \checkmark$$

$$1 = \frac{1}{15}(0)^5 + \frac{1}{6}(0)^3 + \frac{1}{3}(0) + B$$

$$B = 1 \checkmark$$

$$y = \frac{1}{15}x^5 + \frac{1}{6}x^3 + \frac{1}{3}x + 1 \checkmark$$

(6)
 [9]

TOTAAL: 100