higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N5

23 November 2023

Hierdie nasienriglyn bestaan uit 12 bladsye.

Blaai om asseblief

VRAAG 1

1.1 1.1.1

$$y = \lim_{x\to\infty} \frac{-4x^2 + 7x + 3}{x^2 - 5} \quad : \quad \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{-8x + 7}{2x} \checkmark \quad : \quad \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{-8}{2} \checkmark$$

$$= -4 \checkmark$$

(3)

1.1.2

$$y = \lim_{x\to 9} \frac{\cos\left(\frac{\pi}{2} + 9 - x\right)}{\ln(x - 8)} \quad \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{-\sin\left(\frac{\pi}{2} + 9 - x\right) \times (-1)}{\frac{1}{x - 8}} \checkmark$$

$$= 1 \checkmark$$

(2)

1.2

$$y = \frac{2^{-x+3} + 1}{3^{-x} - 1}$$

$$3^{-x} - 1 = 0$$

$$3^{-x} = 1$$

$$3^{-x} = 3^0$$

$$x = 0 \checkmark$$

(1)
[6]

VRAAG 2

2.1 $y = arc\sin x$

$\sin y = x$

$\cos y \times \frac{dy}{dx} = 1$ ✓

$\frac{dy}{dx} = \frac{1}{\cos y}$

$cos^2 y = 1 - \sin^2 y$

$\cos y = \sqrt{1 - \sin^2 y}$ ✓

$= \frac{1}{\sqrt{1 - \sin^2 y}}$ ✓

$= \frac{1}{\sqrt{1 - x^2}}$ (3)

2.2 2.2.1 $y = \sin^3 \sqrt{x} + \sqrt[3]{\sin x}$

$\frac{dy}{dx} = 3\sin^2 \sqrt{x}$ ✓ $\times \cos \sqrt{x}$ ✓ $\times \frac{1}{2}(x)^{-\frac{1}{2}}$ ✓ $+ \frac{1}{3}(\sin x)^{-\frac{2}{3}}$ ✓ $\times \cos x$ ✓ (5)

2.2.2 $y = \tan\left[\cos\left(\sqrt{\tan x^3}\right)\right]$

$\frac{dy}{dx} = \sec^2\left[\cos\left(\sqrt{\tan x^3}\right)\right]$ ✓ $\times \left[-\sin\left(\sqrt{\tan x^3}\right)\right]$ ✓ $\times \frac{1}{2}(\tan x^3)^{-\frac{1}{2}}$ ✓
$\times \sec^2 x^3$ ✓ $\times 3x^2$ ✓ (5)

2.3 $y = (2x^4 + 1)^{\tan 3x^2}$

$\ln y = \tan 3x^2 \ln(2x^4 + 1)$

$\frac{1}{y}\frac{dy}{dx}$ ✓ $= \sec^2 3x^2 \times \ln(2x^4 + 1) \times 6x$ ✓ $+ \tan 3x^2 \times \frac{1}{2x^4 + 1} \times 8x^3$ ✓

$\frac{dy}{dx} = y\left[6x \sec^2 3x^2 \times \ln(2x^4 + 1) + \frac{8x^3 \tan 3x^2}{2x^4 + 1}\right]$ ✓

$= (2x^4 + 1)^{\tan 3x^2}\left[6x \sec^2 3x^2 \times \ln(2x^4 + 1) + \frac{8x^3 \tan 3x^2}{2x^4 + 1}\right]$ (4)

2.4 2.4.1 $x^2y + y^4 = 4 + 2x$

$2xy + x^2\frac{dy}{dx} + 4y^3\frac{dy}{dx}$✓ $= 0 + 2$✓

$x^2\frac{dy}{dx} + 4y^3\frac{dy}{dx} = 2 - 2xy$✓

$\frac{dy}{dx}(x^2 + 4y^3) = 2 - 2xy$

$\frac{dy}{dx} = \frac{2 - 2xy}{x^2 + 4y^3}$✓ (4)

2.4.2 $\frac{dy}{dx} = \frac{2 - 2(-1)(1)}{(-1)^2 + 4(1)^3} = \frac{4}{5}$✓ (1)

[22]

VRAAG 3

3.1 3.1.1 $f(x) = x^3 + 4x^2 + 3x - 5$

$f'(x) = 3x^2 + 8x + 3 = 0$✓

$x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(3)}}{2(3)}$

$x = -2{,}215 \ \ of \ \ x = -0{,}451$

$y = -2{,}887 \quad y = -5{,}631$

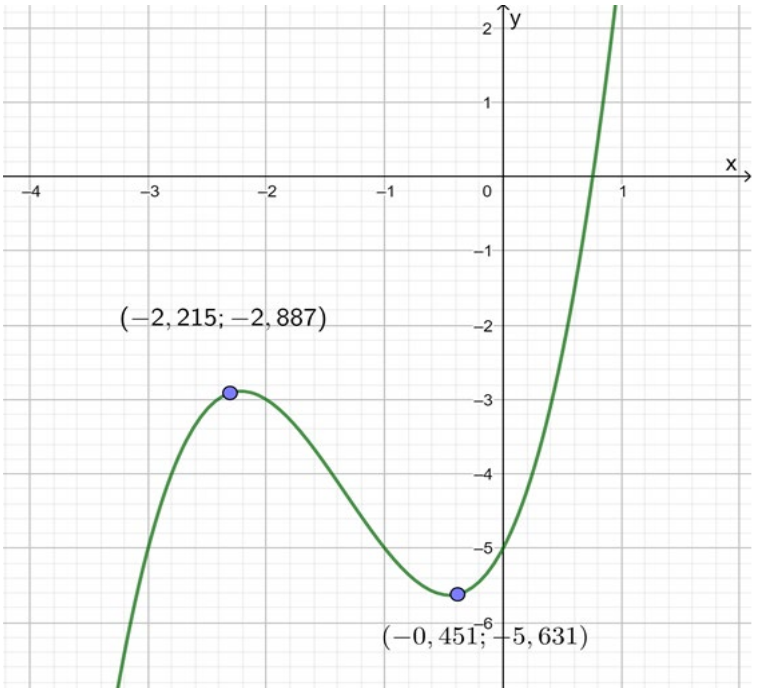
Dus, die koördinate van die draaipunte is $(-2{,}215;\ -2{,}887)$✓ en $(-0{,}451; -5{,}631)$.✓ (3)

3.1.2

x	-4	-3	-2	-1	0	1
y	-17	-5	-3	-5	-5	3

1 punt vir elke drie korrekte antwoorde (2)

3.1.3



1 punt vir die vorm
1 punt vir aanduiding van die draaipunte op die grafiek (2)

3.1.4 $Laat\ x_0 = 0{,}8$

$f(0{,}8) = 0{,}472$

$f'(0{,}8) = 11{,}32$✓

$x_1 = 0{,}8 - \frac{0{,}472}{11{,}32}$✓

$= 0{,}758$✓

$f(0{,}758) = 0{,}011$

$f'(0{,}758) = 10{,}792$

$x_2 = 0{,}758 - \frac{0{,}011}{10{,}792}$

$= 0{,}757$✓ (4)

3.2 $V = l \times b \times l$

$750 = x^2 h$

$h = \frac{750}{x^2}$ ✓

$A = oppervlakte\ van\ sye + oppervlakte\ van\ basis + oppervlakte\ van\ bokant$

$A = 4xh + x^2 + 2x^2$

$= 4x\left(\frac{750}{x^2}\right) + x^2 + 2x^2$

$A = \frac{3\ 000}{x} + 3x^2$ ✓

$A' = -\frac{3\ 000}{x^2} + 6x = 0$ ✓

$-3\ 000 + 6x^3 = 0$

$x^3 = 500$

$x = 7{,}937$ cm ✓ en $h = 11{,}906$ cm ✓ (5)

3.3 $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ ✓

$3\ \text{cm}^3/\text{s} = 4\pi(2\ \text{cm})^2 \frac{dr}{dt}$ ✓

$\frac{dr}{dt} = \frac{3\ \text{cm}^3/\text{s}}{4\pi(2\ \text{cm})^2}$ ✓ $= 0{,}06$ cm/s ✓ (4)

[20]

VRAAG 4

4.1 4.1.1

$$\int \frac{\operatorname{cosec} x \cot x}{1+\operatorname{cosec}^2 x}\,dx$$

$Laat\ u = \operatorname{cosec} x$
$du = -\operatorname{cosec} x \cot x\,dx$✓
$-du = \operatorname{cosec} x \cot x\,dx$

$$= -\int \frac{1}{1+u^2}\,du$$✓

$$= \cot^{-1} u + C$$

$$= \cot^{-1}(\operatorname{cosec} x) + C$$✓ (3)

4.1.2 $$\int sin^2\left(\frac{5}{\pi}x\right)dx$$

$$= \frac{1}{2}\int \left[1-\cos\left(\frac{10}{\pi}x\right)\right]dx$$ ✓

$$= \frac{1}{2}x$$✓ $$-\frac{\pi}{20}\sin\left(\frac{10}{\pi}x\right)$$✓ $$+C$$ (3)

4.1.3 $$\int \cos^5 x \sin^3 x\,dx$$

$$= \int \cos^5 x\,.\cos x \sin^3 x\,dx$$

$$= \int (1-\sin^2 x)\cos x \sin^3 x\,dx$$ ✓

$$= \int (\cos x \sin^5 x - \cos x \sin^7 x)\,dx$$ ✓

$$= \frac{1}{6}\sin^6 x$$✓ $$-\frac{1}{8}\sin^8 x$$✓ $$+C$$ (4)

4.1.4 $\int \frac{x^3 - 2x^2 - 5x + 6}{x + 3} dx$

$= \int \left(x^2 - 5x + 10 - \frac{24}{x + 3}\right) dx$ ✓

$= \frac{1}{3}x^3$✓ $- \frac{5}{2}x^2$✓ $+ 10x$✓ $- 24\ln(x + 3)$✓ $+ C$ (5)

4.1.5 $\int \log_2 x\, dx$

$Laat\ u = \log_2 x \qquad dv = dx$ $du = \frac{1}{\ln 2}\frac{1}{x}dx \qquad v = x$ ✓

$\int \log_2 x\, dx = x\log_2 x - \int \frac{1}{\ln 2} \times \frac{1}{x} \times x dx$✓

$= x\log_2 x$ ✓ $- \frac{1}{\ln 2}x$✓ $+ C$ (4)

4.2 $\int \frac{x - 10}{x^2 + x - 2} dx$

$\frac{x - 10}{(x + 2)(x - 1)} = \frac{A}{x + 2} + \frac{B}{x - 1}$

$x - 10 = A(x - 1) + B(x + 2)$✓

$Laat\ x = 1; \qquad B = -3$✓ of $x = -2; \qquad A = 4$✓

$\int \left(\frac{4}{x + 2} + \frac{-3}{x - 1}\right) dx$

$= 4\ln(1 - 2x)$✓ $- 3\ln(x + 1)$ ✓ $+ C$ (5)

[24]

VRAAG 5

5.1
$$\int_a^b f(t)\,dt = F(b) - F(a)\checkmark$$
$$= 7 - -3 = 10\ \checkmark \quad (2)$$

5.2
$$\int_0^\infty e^{-st}.f(t)\,dt$$
$$= -\pi \int_0^\infty te^{-st}\,dt\checkmark$$

$Laat\ u = t \quad dv = e^{-st}\,dt$
$du = dt \ \ v = -\frac{1}{s}e^{-st}\,dt\checkmark$

$$= -\pi\left[-\frac{t}{s}e^{-st}\right]_0^\infty - \frac{\pi}{s}\int_0^\infty e^{-st}\,dt\checkmark$$
$$= \frac{\pi}{s}[te^{-st}]_0^\infty + \frac{\pi}{s^2}[e^{-st}]_0^\infty\checkmark$$
$$= \frac{\pi}{s}(0-0) + \frac{\pi}{s^2}(0-1)$$
$$= -\frac{\pi}{s^2}\checkmark \quad (5)$$

5.3.1
$$A = \int_a^b (y_1 - y_2)\,dx$$
$$= \int_1^{4,828} \left(5x - x^2 - \frac{4}{x}\right)dx\ \checkmark$$
$$= \left[\frac{5}{2}x^2 - \frac{1}{3}x^3 - 4\ln x\right]_1^{4,828}\ \checkmark$$
$$= 12,297\ eenhede^2 \quad \checkmark \quad (3)$$

5.3.2 $$V = \pi \int_a^b (y_1{}^2 - y_2{}^2)\, dx$$

$$= \pi \int_1^{4,828} \left[(5x - x^2)^2 - \left(\frac{4}{x}\right)^2\right] dx \checkmark$$

$$= \pi \int_1^{4,828} \left(x^4 - 10x^3 + 25x^2 - \frac{16}{x^2}\right) dx \checkmark\checkmark$$

$$= \pi \left[\frac{1}{5}x^5 - \frac{5}{2}x^4 + \frac{25}{3}x^3 + \frac{16}{x}\right]_1^{4,828} \checkmark$$

$$= 268{,}314\ eenhede^3 \checkmark$$

(5)

5.4 $I_y = r^2 A$

$dI_y = r^2\, dA; \qquad dA = b dx \checkmark$

$I_y = x^2\, b\, dx \checkmark$

$$= b \int_{-\frac{a}{2}}^{\frac{a}{2}} x^2\, dx \checkmark$$

$$= ba^2 \left[-\frac{x^3}{3}\right]_{-\frac{a}{2}}^{\frac{a}{2}} \checkmark$$

$$= b\left[\frac{a^3}{12}\right]$$

$$= \frac{a^3 b}{12} \checkmark$$

(5)
[20]

VRAAG 6

6.1 $\frac{dy}{dx} = xy - 3x - 2y + 6$

$\frac{dy}{dx} = (y-3)(x-2)$

$\frac{1}{y-3}dy = (x-2)\,dx$✓

$\ln(y-3)$ ✓ $= \frac{1}{2}x^2 - 2x + C$ ✓ (3)

6.2 $\tan x + \sec x.\frac{d^2y}{dx^2} = \sec x.3^{-x}$, as $\frac{dy}{dx}_{|\ x=0} = 1$ en $y(0) = \frac{2}{(\ln 3)^2}$

$\sin x + \frac{d^2y}{dx^2} = 3^{-x}$

$\frac{d^2y}{dx^2} = 3^{-x} - \sin x$

$\frac{dy}{dx} = -\frac{3^{-x}}{\ln 3} + \cos x + A$✓

$1 = -\frac{3^{-0}}{\ln 3} + \cos 0 + A$

$A = \frac{1}{\ln 3}$ ✓

$\frac{dy}{dx} = -\frac{3^{-x}}{\ln 3} + \cos x + \frac{1}{\ln 3}$

$y = \frac{3^{-x}}{(\ln 3)^2} + \sin x + \frac{1}{\ln 3}x + B$✓

$\frac{2}{(\ln 3)^2} = \frac{3^{-0}}{(\ln 3)^2} + \sin 0 + \frac{1}{\ln 3}.0 + B$

$B = \frac{1}{(\ln 3)^2}$ ✓

$y = \frac{3^{-x}}{(\ln 3)^2} + \sin x + \frac{1}{\ln 3}x + \frac{1}{(\ln 3)^2}$ ✓ (5)

[8]

TOTAAL: 100