higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

17 November 2022 (X-paper)
09:00–12:00

Scientific calculators may be used.

This question paper consists of 6 pages and a formula sheet of 5 pages.

070Q1E2217

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show all intermediate steps and simplify where possible.
5. All final answers must be rounded off to THREE decimal places.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Sketches must be large, neat and fully labelled

9\. Write neatly and legibly.

QUESTION 1

Determine the following limits:

1.1 $y = \lim_{x \to \infty} \dfrac{\ln[\ln(x^2 + 1)]}{x}$ (4)

1.2 $y = \lim_{x \to 0} \dfrac{x^3}{e^x - x - \frac{1}{2}x^2 - 1}$ (4)

[8]

QUESTION 2

2.1 Derive a formula to determine $\frac{dy}{dx}$; if $y = arc \tan x$. (3)

2.2 Calculate $\frac{dy}{dx}$ of the following, by using the derivatives of $\sin x$ and $\cos x$, as well as the rules of differentiation:

$y = \pi^{\log_\pi \cot x}$

HINT: $a^{\log_a x} = x$ (4)

2.3 Determine $\frac{dy}{dx}$ in each of the following cases:

(Simplification NOT required)

2.3.1 $y = \tan(x^3 \ln^2 x)$ (3)

2.3.2 $y = \dfrac{\sqrt{\ln(2x + 3)}}{4\sqrt{3x - 1}}$ (4)

2.4 Calculate $\frac{dy}{dx}$ if $y = (1 + \sin^{-1} x)^{\ln \cos x}$ with the aid of logarithmic differentiation. (4)

2.5 Determine $\frac{dy}{dx}$ of implicit function,

$\dfrac{2x - 3y}{x + y} = 2x + y^2$ (5)

[23]

QUESTION 3

3.1 Given: $f(x) = 3x^3 + 5x^2 - 7x - 5$

3.1.1 Determine the coordinates of the turning points of $f(x)$. (3)

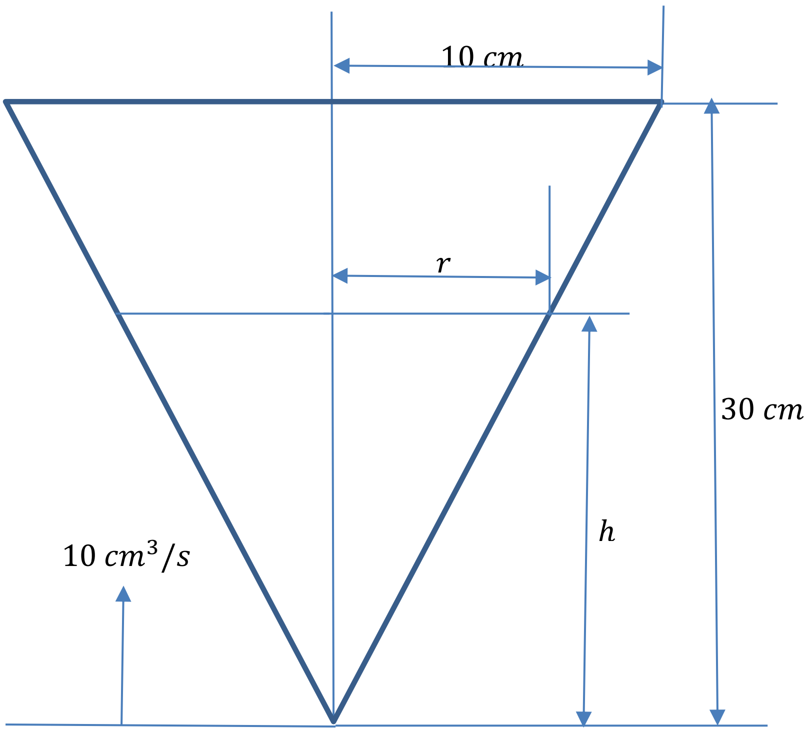
3.1.2 Draw up a table of x and $f(x)$, where x is ranging from $x = -3$ to $x = 2$. (3)

3.1.3 Draw a neat graph of $f(x)$ between these values and show the turning points on it. (2)

3.1.4 Use the table and the graph to estimate a value for the best root between $x = -1$ and $x = 0$ of the equation $3x^3 + 5x^2 - 7x - 5 = 0$ and then use Taylor's/Newton's method ONCE to determine a better approximation of this root (correct to THREE decimal places). (3)

3.2 Water is poured into a conical container at the rate of 10 cm^3/s. The cone points directly downwards, and it has a height of 30 cm and a base radius of 10 cm, as indicated in the sketch below. How fast is the water level rising when the water is 4 cm deep (at its deepest point)? Note the sketch below is not to scale.

HINT: $V = \frac{1}{3}\pi r^2 h$



(5)
[16]

QUESTION 4

4.1 Determine $\int y\,dx$ in each of the following cases:

4.1.1 $y = \dfrac{\sin\left(\frac{1}{x}\right)}{x^2}$ ☆ (3)

4.1.2 $y = \dfrac{2x^3 - 11x^2 - 9x + 10}{2x - 1}$ (5)

4.1.3 $y = x^2 e^{5x}$ (7)

4.1.4 $y = \dfrac{\sin^3 x}{\cos^3 x}$ (4)

4.2 Determine $\int y\,dx$ by resolving the integral into partial fractions:

$y = \dfrac{2x - 3}{x^2 - 4x}$ ☆ (5)

[24]

QUESTION 5

5.1 If $f(1) = 3$ and $f(0) = 1$, then evaluate $\int_0^1 f'(x)\,[f(x)]^2\,dx$ (4)

5.2 Given: $y = 5x - x^2$ and $y = x^2 - 2x + 1$

5.2.1 Calculate the coordinates of the points of intersection. ☆ (2)

5.2.2 Make a neat sketch to show the enclosed area, the representative strip and the point of intersection. (2)

5.2.3 Calculate the magnitude of the area in QUESTION 5.2.2. (3)

5.2.4 Calculate the volume of the solid of revolution formed when the area in QUESTION 5.2.2 is rotated about the x-axis. (4)

5.3 Find the moment of inertia of a rectangle with sides ***a*** and ***b*** with respect to an axis passing through the side ***b.*** Assume that the density is $\rho = 1$.

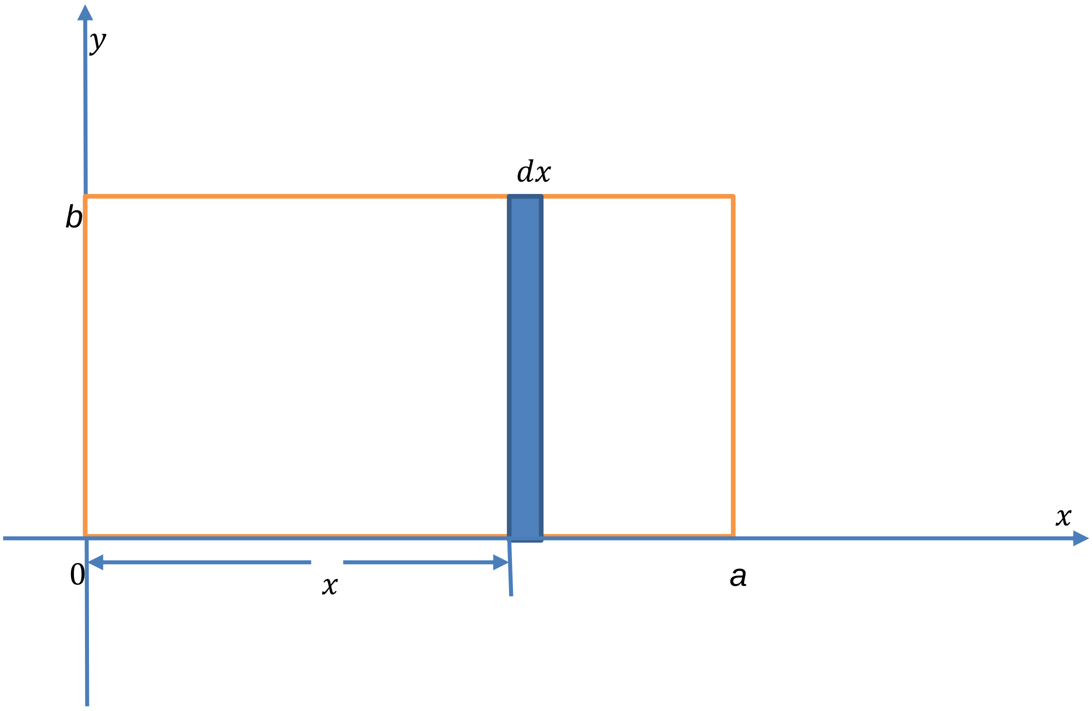


FIGURE 1

(5)
[20]

QUESTION 6

6.1 Determine the general solution of:

$$(x-2)(y+1)\frac{dy}{dx} - \log_5(x-2) = 0$$

(3)

6.2 Determine the particular solution of:

$$\frac{d^2y}{dx^2} = 4 - 6x - 21x^2, \quad \text{if } \frac{dy}{dx} = y'(1) = 2 \text{ and } y(1) = 10$$

(6)
[9]

TOTAL: 100

MATHEMATICS N5

FORMULA SHEET

Any applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec } x}$; $\cos x = \dfrac{1}{\sec x}$

BINOMIAL THEOREM

$$(x+h)^n = x^n + nx^{n-1}\,h + \frac{n(n-1)}{2!}x^{n-2}\,h^2 + \ldots$$

DIFFERENTIATION

$$e = -\frac{f(a)}{f'(a)}$$

$$r = a + e$$

PRODUCT RULE

$$y = u(x)\,.\,v(x)$$

$$\frac{dy}{dx} = u\,.\,\frac{dv}{dx} + v\,.\,\frac{du}{dx}$$

$$= u \cdot v' + v \cdot u'$$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\,.\,\frac{du}{dx} - u\,.\,\frac{dv}{dx}}{v^2}$$

$$= \frac{v \cdot u' - u \cdot v'}{v^2}$$

CHAIN RULE

$$y = f(u(x))$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
ax^n	nax^{n-1}	$\frac{ax^{n+1}}{n+1}+c$
a	0	$ax + c$
e^x	e^x	$e^x + c$
a^x	$a^x.\ln a$	$\frac{a^x}{\ln a}+c$
$\log_e x$	$\frac{1}{x}$	—
$\log_a x$	$\frac{1}{x \ln a}$	—
$\sin x$	$\cos x$	$-\cos x + c$
$\cos x$	$-\sin x$	$\sin x + c$
$\tan x$	$\sec^2 x$	$\ln(\sec x) + c$
$\cot x$	$-\csc^2 x$	$\ln(\sin x) + c$
$\sec x$	$\sec x \cdot \tan x$	$\ln[\sec x + \tan x] + c$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cdot \cot x$	$\ln(\operatorname{cosec} x - \cot x) + c$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	—
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	—
$\tan^{-1} x$	$\frac{1}{1+x^2}$	—
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	—
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$	—
$\operatorname{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$	—

$f(x)$	$\frac{d}{dx}\ f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{a^2 - x^2}}$	—	$\sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{a^2 + x^2}$	—	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{x\sqrt{x^2 - a^2}}$	—	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
$\sqrt{a^2 - x^2}$	—	$\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + c$
$\frac{1}{x^2 - a^2}$	—	$\frac{1}{2a}\ ln\left(\frac{x-a}{x+a}\right) + c$
$\frac{1}{a^2 - x^2}$	—	$\frac{1}{2a}\ ln\left(\frac{a+x}{a-x}\right) + c$

INTEGRATION

$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$

$\int \frac{f'(x)}{f(x)}\, dx = \ln\ f(x) + c$

$\int [f(x)]^n\ f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$

$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$

$\frac{f(x)}{(x+a)^n} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \cdots \frac{Z}{(x+a)^n}$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y\,dx\ ;\ A_x = \int_a^b (y_1 - y_2)\,dx$$

$$A_y = \int_a^b x\,dy\ ;\ A_y = \int_a^b (x_1 - x_2)\,dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx\ ;\ V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx$$

$$V_y = \pi \int_a^b x^2 dy\ ;\ V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA\ ;\ I_y = \int_a^b r^2 dA$$

MOMENTS OF INERTIA

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$