



higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

17 NOVEMBER 2022

This marking guideline consists of 10 pages.

QUESTION 1

$$\begin{aligned}
 1.1 \quad y &= \lim_{x \rightarrow \infty} \frac{\ln[\ln(x^2 + 1)]}{x} \quad \frac{\infty}{\infty} \\
 &= \lim_{x \rightarrow \infty} \frac{2x}{\ln(x^2 + 1)} \quad \checkmark \quad \frac{\infty}{\infty} \\
 &= \lim_{x \rightarrow \infty} \frac{x^2 + 1}{x} \quad \checkmark \quad \frac{\infty}{\infty} \\
 &= \lim_{x \rightarrow \infty} \frac{2x}{1} \quad \checkmark \\
 &= \infty \checkmark
 \end{aligned}$$

(4)

$$\begin{aligned}
 1.2 \quad y &= \lim_{x \rightarrow 0} \frac{x^3}{e^x - x - \frac{1}{2}x^2 - 1} \quad \frac{0}{0} \\
 &= \lim_{x \rightarrow 0} \frac{3x^2}{e^x - 1 - x} \quad \checkmark \quad \frac{0}{0} \\
 &= \lim_{x \rightarrow 0} \frac{6x}{e^x - 1} \quad \checkmark \quad \frac{0}{0} \\
 &= \lim_{x \rightarrow 0} \frac{6}{e^x} \quad \checkmark \\
 &= 6 \checkmark
 \end{aligned}$$

(4)
 [8]

QUESTION 2

$$\begin{aligned}
 2.1 \quad y &= \arctan x \\
 \tan y &= x \\
 \sec^2 y \frac{dy}{dx} &= 1 \quad \checkmark \\
 \frac{dy}{dx} &= \frac{1}{\sec^2 x} \quad \checkmark \\
 &= \frac{1}{1 + \tan^2 x} \quad \checkmark \\
 &= \frac{1}{1 + x^2}
 \end{aligned}$$

(3)

2.2 $y = \pi^{\log_{\pi} \cot x}$

$y = \cot x$ ✓

$= \frac{\cos x}{\sin x}$

$\frac{dy}{dx} = \frac{\sin x \times -\sin x \checkmark - \cos x \times \cos x \checkmark}{\sin^2 x}$

$= \frac{-1}{\sin^2 x} \checkmark$

$= -\operatorname{cosec}^2 x$ (4)

2.3 2.3.1 $y = \tan(x^3 \ln^2 x)$

$\frac{dy}{dx} = \sec^2(x^3 \ln^2 x) \checkmark \times \left(3x^2 \times \ln^2 x \checkmark + x^3 \times 2 \ln x \times \frac{1}{x} \checkmark \right)$ (3)

2.3.2 $y = \frac{\sqrt{\ln(2x+3)}}{4\sqrt{3x-1}}$

$\frac{dy}{dx} = \frac{4\sqrt{3x-1} \times \frac{1}{2} \times [\ln(2x+3)]^{\frac{1}{2}} \times \frac{1}{2x+3} \times 2 \checkmark \checkmark - \left[\sqrt{\ln(2x+3)} \times 4 \times \frac{1}{2} (3x-1)^{-\frac{1}{2}} \times 3 \right] \checkmark \checkmark}{(4\sqrt{3x-1})^2}$ (4)

2.4 $y = (1 + \sin^{-1} x)^{\ln \cos x}$

$\ln y = \ln \cos x \ln(1 + \sin^{-1} x)$

$\frac{1}{y} \frac{dy}{dx} \checkmark = \ln \cos x \times \frac{1}{1 + \sin^{-1} x} \times \frac{1}{\sqrt{1-x^2}} \checkmark + \ln(1 + \sin^{-1} x) \times \frac{1}{\cos x} \times -\sin x \checkmark$

$\frac{dy}{dx} = y \left[\frac{\ln \cos x}{(1 + \sin^{-1} x)(\sqrt{1-x^2})} - \tan x \cdot \ln(1 + \sin^{-1} x) \right] \checkmark$

$= (1 + \sin^{-1} x)^{\ln \cos x} \left[\frac{\ln \cos x}{(1 + \sin^{-1} x)(\sqrt{1-x^2})} - \tan x \cdot \ln(1 + \sin^{-1} x) \right]$ (4)

$$2.5 \quad \frac{2x - 3y}{x + y} = 2x + y^2$$

$$2x - 3y = (x + y)(2x + y^2)$$

$$2x - 3y = 2x^2 + 2xy + xy^2 + y^3$$

$$2 - 3\frac{dy}{dx} \checkmark = 4x + 2y + 2x\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} \checkmark \checkmark$$

$$3y^2\frac{dy}{dx} + 2xy\frac{dy}{dx} + 2x\frac{dy}{dx} + 3\frac{dy}{dx} = 2 - 4x - 2y - y^2$$

$$(3y^2 + 2xy + 2x + 3)\frac{dy}{dx} = 2 - 4x - 2y - y^2 \checkmark$$

$$\frac{dy}{dx} = \frac{2 - 4x - 2y - y^2}{3y^2 + 2xy + 2x + 3} \checkmark$$

(5)
 [23]

QUESTION 3

3.1 3.1.1 $f(x) = 3x^3 + 5x^2 - 7x - 5$

$$f'(x) = 9x^2 + 10x - 7 = 0$$

$$= \frac{-10 \pm \sqrt{(10)^2 - 4(9)(-7)}}{2(9)}$$

$$x = -1,598 \quad \text{or} \quad x = 0,487 \checkmark$$

$$y = 6,712 \quad y = -6,877 \checkmark$$

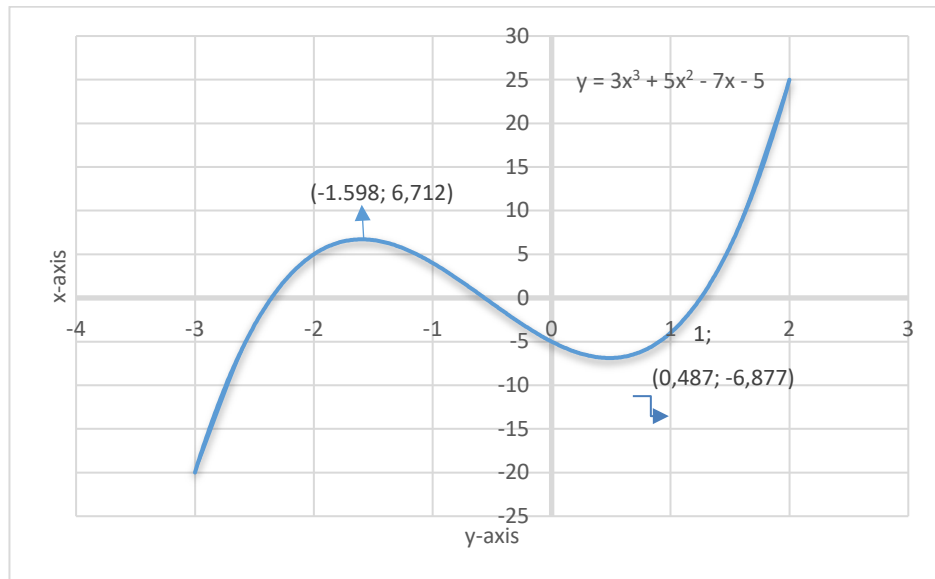
Thus, the coordinates of the turning points are $(-1,598; 6,712)$ and $(0,487; -6,877) \checkmark$ (3)

3.1.2

x	-3	-2	-1	0	1	2
y	-20	5	4	-5	-4	25

1 mark each for any 3 correct answers $\checkmark \checkmark \checkmark$ (3)

3.1.3



✓ = 1 mark for the shape

✓ = 1 mark for the indication of the turning points on the graph

(2)

3.1.4 Let $x_0 = -0,5$

$$f(-0,5) = -0,625$$

$$f'(-0,5) = -9,75 \checkmark$$

$$x_1 = -0,5 - \frac{-0,625}{-9,75} \checkmark$$

$$= -0,564 \checkmark$$

(3)

3.2 Because of similar triangles we have,

$$\frac{r}{h} = \frac{10}{30}$$

$$\therefore r = \frac{h}{3} \checkmark$$

$$V = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi \left(\frac{h}{3}\right)^2 h$$

$$= \frac{\pi h^3}{27} \checkmark$$

$$\frac{dV}{dt} = \frac{\pi h^2}{9} \frac{dh}{dt} \checkmark$$

$$10 \text{ cm}^3/\text{s} = \frac{\pi (4 \text{ cm})^2}{9} \frac{dh}{dt} \checkmark$$

$$\frac{dh}{dt} = \frac{90}{16\pi} \text{ cm/s} \checkmark$$

$$= 1,790 \text{ cm/s}$$

(5)

[15]

QUESTION 4

4.1 4.1.1

$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2} dx$	<p style="text-align: center; margin: 0;"><u>ALTERNATIVE</u></p> $I = \int \frac{\sin\left(\frac{1}{x}\right)}{x^2} dx$ $\frac{d}{dx} \cos\left(\frac{1}{x}\right) \checkmark = -\sin\left(\frac{1}{x}\right) \times \frac{-1}{x^2} \checkmark$ $= \frac{\sin\left(\frac{1}{x}\right)}{x^2}$ $I = \int \frac{\sin\left(\frac{1}{x}\right)}{x^2} dx = \cos\left(\frac{1}{x}\right) + C \checkmark$
$\text{let } u = \frac{1}{x}$	
$du = -\frac{1}{x^2} dx \checkmark$	
$-du = \frac{1}{x^2} dx$	

$$= -\int \sin u \, du \checkmark$$

$$= \cos u + C$$

$$= \cos\left(\frac{1}{x}\right) + C \checkmark$$

(3)

4.1.2

$$\int \frac{2x^3 - 11x^2 - 9x + 10}{2x - 1} dx$$

$$= \int \left(x^2 - 5x - 7 + \frac{3}{2x - 1}\right) dx \checkmark$$

$$= \frac{1}{3}x^3 \checkmark - \frac{5}{2}x^2 \checkmark - 7x \checkmark + \frac{3}{2} \ln(2x - 1) \checkmark + C$$

(5)

4.1.3

$\int x^2 e^{5x} dx$	$\text{let } f(x) = x^2 \quad g'(x) = e^{5x} dx$ $f'(x) = 2x \quad g(x) = \frac{1}{5} e^{5x} \checkmark$
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$$\int x^2 e^{5x} dx = \frac{x^2 e^{5x}}{5} \checkmark - \frac{2}{5} \int x e^{5x} dx \checkmark$$

$\text{let } f(x) = x \quad g'(x) = e^{5x} dx$ $f'(x) = 1 \quad g(x) = \frac{1}{5} e^{5x} \checkmark$

$$= \frac{x^2 e^{5x}}{5} - \frac{2}{5} \left(\frac{x e^{5x}}{5} \checkmark - \frac{1}{5} \int e^{5x} dx \checkmark \right)$$

$$= \frac{x^2 e^{5x}}{5} - \frac{2x e^{5x}}{25} + \frac{2}{125} e^{5x} \checkmark + C$$

(7)

$$\begin{aligned}
 4.1.4 \quad y &= \frac{\sin^3 x}{\cos^3 x} \\
 &= \int \tan^3 x \, dx \checkmark \\
 &= \int \tan^2 x \tan x \, dx \\
 &= \int (\tan^2 x - 1) \tan x \, dx \checkmark \\
 &= \int \tan^2 x \tan x \, dx - \int \tan x \, dx \\
 &= -\frac{1}{2} \tan^2 x \checkmark - \ln(\sec x) \checkmark + C
 \end{aligned} \tag{4}$$

4.2

$$\begin{aligned}
 &\int \frac{2x - 3}{x^2 - 4x} \, dx \\
 &\frac{2x - 3}{x(x - 4)} = \frac{A}{x} + \frac{B}{x - 4} \\
 &2x - 3 = A(x - 4) + Bx \checkmark
 \end{aligned}$$

$$\begin{aligned}
 &\text{let } x = 4; \quad B = \frac{5}{4} \checkmark \\
 &\quad \quad \quad \text{or} \\
 &x = 0; \quad A = \frac{3}{4} \checkmark
 \end{aligned}$$

$$\begin{aligned}
 &\int \left(\frac{3}{4x} + \frac{5}{4(x - 4)} \right) dx \\
 &= \frac{3}{4} \ln x \checkmark + \frac{5}{4} \ln(x - 4) \checkmark + C
 \end{aligned} \tag{5}$$

[24]

QUESTION 5

5.1

$$\int_0^1 f'(x) [f(x)]^2 dx$$

let $u = f(x)$
 $du = f'(x) \quad \checkmark$
 $u = f(0) = 1 \quad \text{and} \quad u = f(1) = 3$

$$= \int_1^3 u^2 du \quad \checkmark$$

$$= \left[\frac{1}{3} u^3 \right]_1^3 \quad \checkmark$$

$$= \frac{1}{3} \times 3^3 - \frac{1}{3} \times 1^3$$

$$= \frac{26}{3} \quad \checkmark$$

(4)

5.2

5.2.1

$$x^2 - 2x + 1 = 5x - x^2$$

$$2x^2 - 7x + 1 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(1)}}{2(2)}$$

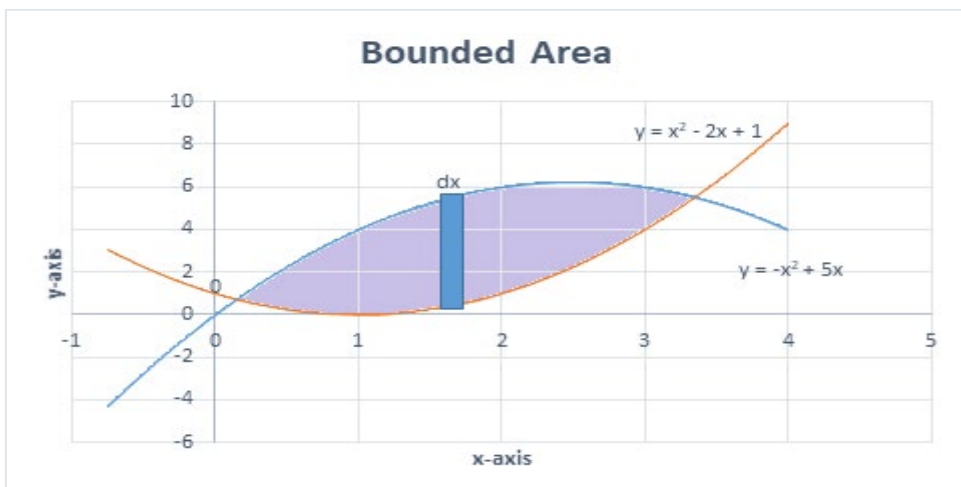
$$x = 0,149 \quad \text{or} \quad x = 3,351 \quad \checkmark$$

$$y = 0,723 \quad y = 5,526$$

Thus, the coordinates of the points of intersection are (0,149 ; 0,723) and (3,351; 5,526). \checkmark

(2)

5.2.2



\checkmark = 1 mark for indication of enclosed area

\checkmark = 1 mark for indication of the vertical or horizontal strip

(2)

$$\begin{aligned}
 5.2.3 \quad A &= \int_a^b (y_1 - y_2) dx \\
 &= \int_{0,149}^{3,351} [(5x - x^2) - (x^2 - 2x + 1)] dx \\
 &= \int_{0,149}^{3,351} (-2x^2 + 7x + 1) dx \checkmark \\
 &= \left[-\frac{2}{3}x^3 + \frac{7}{2}x^2 - x \right]_{0,149}^{3,351} \checkmark \\
 &= 10,939 \text{units}^2 \quad \checkmark \qquad (3)
 \end{aligned}$$

$$\begin{aligned}
 5.2.4 \quad V &= \pi \int_a^b (y_1^2 - y_2^2) dx \\
 &= \pi \int_{0,149}^{3,351} [(5x - x^2)^2 - (x^2 - 2x + 1)^2] dx \\
 &= \int_{0,149}^{3,351} (-6x^3 + 19x^2 + 4x - 1) dx \checkmark \checkmark \\
 &= \left[-\frac{3}{2}x^4 + \frac{19}{3}x^3 + x^2 - x \right]_{0,149}^{3,351} \checkmark \\
 &= 214,780 \text{units}^3 \quad \checkmark \qquad (4)
 \end{aligned}$$

$$\begin{aligned}
 5.3 \quad dI &= x^2 dm \\
 &= x^2 \rho b dx \checkmark \\
 &= 1bx^2 dx \checkmark \\
 I &= \int_0^a bx^2 dx \checkmark \\
 &= \left[\frac{b}{3}x^3 \right]_0^a \checkmark \\
 &= \frac{ba^3}{3} \checkmark \qquad (5)
 \end{aligned}$$

[20]

QUESTION 6

6.1 $(x - 2)(y + 1) \frac{dy}{dx} - \log_5(x - 2) = 0$

$$(y + 1) dy = \frac{\log_5(x - 2)}{x - 2} dx$$

$$(y + 1) dy = \frac{1}{\ln 5} \frac{\ln(x - 2)}{x - 2} dx \checkmark$$

$$\frac{1}{2} y^2 + y \checkmark = \frac{1}{2 \ln 5} \ln^2(x - 2) + C \checkmark$$

(3)

6.2 $\frac{d^2y}{dx^2} = 4 - 6x - 21x^2$

$$\frac{dy}{dx} = A + 4x - 3x^2 - 7x^3 \checkmark$$

$$2 = A + 4(1) - 3(1)^2 - 7(1)^3$$

$$A = 8 \checkmark$$

$$\frac{dy}{dx} = 8 + 4x - 3x^2 - 7x^3 \checkmark$$

$$y = B + 8x + 2x^2 - x^3 - \frac{7}{4}x^4 \checkmark$$

$$10 = B + 8(1) + 2(1)^2 - (1)^3 - \frac{7}{4}(1)^4$$

$$B = \frac{11}{4} \checkmark$$

$$y = \frac{11}{4} + 8x + 2x^2 - x^3 - \frac{7}{4}x^4 \checkmark$$

(6)

[9]

TOTAL: 100