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Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

20 November 2020 (X-paper)
09:00–12:00

Scientific calculators may be used.

This question paper consists of 5 pages and a formula sheet of 5 pages.

187Q1E2020

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show all intermediate steps and simplify where possible.
5. All final answers must be rounded off to THREE decimals.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Sketches must be large, neat and fully labelled
8. Work neatly.

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QUESTION 1

1.1 Determine the following limit:

$$\lim_{x \to 0} x \cot x$$ (4)

1.2 Given: $\log_4 y = \lim_{x \to 0} \frac{\int_{-x}^{0} e^t \, dt}{2x}$

Calculate the numerical value of each of the following:

HINT: $\int_{-x}^{0} e^t \, dt = 1 - e^{-x}$

1.2.1 $\log_4 y$ (3)

1.2.2 y (1)

1.3 If $f\left(x = \frac{x^3 - 6x + 4}{x^2 - 1}\right)$, determine the value(s) of x for which $f(x)$ is discontinuous. (2)

[10]

QUESTION 2

2.1 Determine the derivative of $f(x) = \frac{1-x}{2-x}$ from first principles. (5)

2.2 Prove that if $y = \arccos x$, then $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$ $y = arc \cos x$, then $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$ (3)

2.3 Determine $\frac{dy}{dx}$ in each of the following cases:
(Simplification not required)

2.3.1 $y = \sqrt{\sin(7x + \ln 5x)}$

2.3.2 $y = 4 \ln(\ln(\sec^2 x))$

(2 × 4) (8)

2.4 Calculate $\frac{dy}{dx}$ if $y = (3x^2 + 2)^{\frac{1}{x}}$ with aid of logarithmic differentiation (4)

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2.5 Given the implicit function: $\frac{x+y}{x-y}=x$

2.5.1 Determine $\frac{dy}{dx}$ (4)

2.5.2 Determine the equation of the tangent to the graph at the point $(0,1)$ (2)

[26]

QUESTION 3

3.1 Given: $f(x)=2x^3+x^2-3x+3$

3.1.1 Determine the coordinate of the point of inflection of $f(x)$ (2)

3.1.2 Draw up a table of x and $f(x)$, where x is ranging from $x=-2$ to $x=2$ (3)

3.13 Draw a neat graph of $f(x)$ between the values in QUESTION 3.1.2 and show the turning points on it. (2)

3.1.4 One root of the equation $x^3-x^2+x+1=0$ is close to $-1{,}6$.

Use Taylor/Newton's method twice to determine a better approximation of this root. (Root correct to THREE decimals) (4)

3.2 The relative displacement in metres of a particle at time t in seconds is defined by the function $f(t)=4t^2 1nt$.

What is the acceleration of the particle at $t=3{,}5\ s$ (4)

[15]

QUESTION 4

4.1 Determine $\int y\ dx$ in each of the following cases:

4.1.1 $y=(e^x+e^{-x})^{\frac{1}{4}}(e^x-e^{-x})$ (3)

4.1.2 $y=\frac{1}{x(1+2\ln x)}$ (3)

4.1.3 $y=\frac{2x^3+2x+7x^2+9}{2x+3}$ (5)

4.1.4 $y=e^x\sqrt{1-e^{2x}}$ (4)

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4.2 Determine $\int y\,dx$ by resolving the integral into partial fractions:

$$y = \frac{9x + 25}{(x + 3)^2}$$ (5)

[20]

QUESTION 5

5.1 Evaluate the definite integral: $\int_1^9 \frac{x-1}{\sqrt{x}}\,dx$ (4)

5.2 Given: $x^2 + 4y^2 = 4$ in the first quadrant

5.2.1 Make a neat sketch to show the enclosed area, the representative strip and the point of intersection. (2)

5.2.2 Calculate the magnitude of the area in QUESTION 5.2.1. (4)

5.2.3 Calculate the volume of the solid of revolution formed when the area in QUESTION 5.2.1 is rotated about the y-axis. (4)

5.3 Calculate the second moment of area of a circular lamina with radius 7 cm and about an axis through its centre and perpendicular to the plane of the lamina (5)

[19]

QUESTION 6

6.1 Determine the general solution of $\frac{dy}{dx} = \left(1 + e^{-x}\right)\left(y^2 - 1\right)$ (3)

6.2 Determine the particular solution of $\frac{d^2y}{dx^2} = \left(x^2 + \pi\right)^2$ if $\frac{dy}{dx} = 3$, $x = 0$ and $y = -4$ (7)

[10]

TOTAL: 100

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MATHEMATICS N5

FORMULA SHEET

Any other applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1 \rightarrow \cos^2 x = 1 - \sin^2 x$

$\sin^2 x + \cos^2 x = 1 \rightarrow \sin^2 x = 1 - \cos^2 x$

$1 + \tan^2 x = \sec^2 x \longrightarrow \tan^2 x = \sec^2 x - 1$

$1 + \cot^2 x = \text{cosec}^2 x \longrightarrow \cot^2 x = \text{cosec}^2 x - 1$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = ½ - ½ \cos 2A = ½(1 - \cos 2\text{A})$

$\cos^2 A = ½ + ½ \cos 2A = ½ (1 + \cos 2\text{A})$

$\sin (\text{A} \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec}\, x}$; $\cos x = \dfrac{1}{\sec x}$

$\cot x = \dfrac{\cos x}{\sin x}$; $\text{cosec} x = \dfrac{1}{\sin x}$; $\sec x = \dfrac{1}{\cos x}$

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BINOMIAL THEOREM

$$(x+h)^n = x^n + nx^{n-1}h + \frac{n(n-1)}{2!}x^{n-2}h^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}x^{n-3}h^3 + \ldots\ldots$$

DIFFERENTIATION

$$e = -\frac{f(a)}{f'(a)}$$

$$r = a + e$$

PRODUCT RULE

$$y = u(x)\,.\,v(x)$$

$$\frac{dy}{dx} = u\,.\,\frac{dv}{dx} + v\,.\,\frac{du}{dx}$$

$$= u \cdot v' + v \cdot u'$$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\,.\,\frac{du}{dx} - u\,.\,\frac{dv}{dx}}{v^2}$$

$$= \frac{v \cdot u' - u \cdot v'}{v^2}$$

CHAIN RULE

$$y = f(u(x))$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

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$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n dx$
e^{ax+b}	$e^{ax+b} . \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e}.\ln a \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a.\frac{d}{dx}(dx+e)} + C$
$ln(ax)$	$\frac{1}{ax}.\frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)}.\frac{d}{dx} f(x)$	—
$a^{f(x)}$	$a^{f(x)}.\ln a.\frac{d}{dx} f(x)$	—
$ln\ f(x)$	$\frac{1}{f(x)}.\frac{d}{dx} f(x)$	—
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a}.+C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a}.+C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)]+C$
$\cot ax$	$- a\ \text{cosec}^2\ ax$	$\frac{1}{a}\ln[\sin(ax)]+C$
$\sec ax$	$a \sec ax \cdot \tan ax$	$\frac{1}{a}\ln[\sec ax + \tan ax] + C$
$\text{cosec}\ ax$	$- a\ \text{cosec}\ ax \cdot \cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right]+C$

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$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
$\sin f(x)$	$\cos f(x) \cdot f^1(x)$	___
$\cos f(x)$	$-\sin f(x) \cdot f^1(x)$	___
$\tan f(x)$	$\sec^2 f(x) \cdot f^1(x)$	___
$\cot f(x)$	$-\operatorname{cosec}^2 f(x) \cdot f^1(x)$	___
$\sec f(x)$	$\sec f(x) \cdot \tan f(x) \cdot f^1(x)$	___
$\operatorname{cosec} f(x)$	$-\operatorname{cosec} f(x) \cdot \cot f(x) \cdot f^1(x)$	___
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	___
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	___
$\tan^{-1} f(x)$	$\frac{f'(x)}{1+[f(x)]^2}$	___
$\cot^{-1} f(x)$	$\frac{-f'(x)}{1+[f(x)]^2}$	___
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	___
$\operatorname{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	___
$\sin^2(ax)$	___	$\frac{x}{2}-\frac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	___	$\frac{x}{2}+\frac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	___	$\frac{1}{a}\tan(ax)-x+C$
$\cot^2(ax)$	___	$-\frac{1}{a}\cot(ax)-x+C$

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INTEGRATION

$\int f(x)\, g^{'}(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$

$\int [f(x)]^n \bullet f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$

$\int\int \frac{f^{'}(x)}{f(x)} dx = \ln f(x) + C$

APPLICATIONS OF INTEGRATION

AREAS

$A_x = \int_a^b y dx \,;\, A_x = \int_a^b (y_1 - y_2) dx$

$A_y = \int_a^b x dy \,;\, A_y = \int_a^b (x_1 - x_2) dy$

VOLUMES

$V_x = \pi \int_a^b y^2 dx \,;\, V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx$

$V_y = \pi \int_a^b x^2 dy \,;\, V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy$

SECOND MOMENT OF AREA

$I_x = \int_a^b r^2 dA \,;\, I_y = \int_a^b r^2 dA$

MOMENTS OF INERTIA

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$

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