higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

13 August 2021

This marking guideline consists of 11 pages.

QUESTION 1

1.1 1.1.1 $\lim\limits_{x\to 3}\dfrac{3-x}{\sqrt{x+1}-\sqrt{5x-11}}\ \left(\dfrac{0}{0}\right)$

$\lim\limits_{x\to\infty}\dfrac{-1}{\frac{1}{2\sqrt{x+1}}\checkmark-\frac{5\checkmark}{2\sqrt{5x-11}}}$

$=-1\checkmark$ (3)

1.1.2 $\lim\limits_{x\to -1}\dfrac{\frac{1}{4+3x}+\frac{1}{x}}{x+1}\ \left(\dfrac{0}{0}\right)$

$=\lim\limits_{x\to -1}\dfrac{-\frac{3}{(4+3x)^2}+\frac{1}{x^2}}{1}\checkmark$

$=-2\checkmark$ (2)

1.2 1.2.1 $\ln y=\lim\limits_{x\to 4}\dfrac{\cos(x-4)-1}{2x-8}\ \left(\dfrac{0}{0}\right)$

$=\lim\limits_{x\to 0}\dfrac{-\sin(x-4)}{2}\checkmark$

$=0\checkmark$ (2)

1.2.2 $y=e^0=1\checkmark$ (1)

1.3 $f(x)=\dfrac{\sin 2x}{\cos 2x}$

$\cos 2x=0$

$2x=90^0$

$x=45^0\checkmark\ or\ x=135^0\checkmark$ (2)

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QUESTION 2

2.1 $f(x) = -5x^7$

2.1.1 $f(x+h) = -5(x+h)^7$

$$= -5\left[\frac{x^7h^0}{0!} + \frac{7x^6h}{1!} + \frac{42x^5h^2}{2!} + \frac{210x^4h^3}{3!} + \cdots\ldots\right]$$

$$= -5x^7 - 35x^6h - 105x^5h^2 - 105x^4h^3 + \cdots \ldots\ldots\ldots \checkmark\checkmark \quad (2)$$

2.1.2 $f(x+h) - f(x) = -35x^6h - 105x^5h^2 - 105x^4h^3 + \cdots \ldots\ldots\ldots\ldots\ldots \checkmark$ (1)

2.1.3 $$\frac{f(x+h) - f(x)}{h} = -35x^6 - 105x^5h - 105x^4h^2 + \cdots \ldots\ldots\ldots \checkmark \quad (1)$$

2.1.4 $$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = -35x^6 \checkmark \quad (1)$$

2.2 2.2.1 $y = \tan[(5-x^2)(\ln^2 x)]$

$$\frac{dy}{dx} = \sec^2[(5-x^2)(\ln^2 x)]\checkmark \times\left[-2x(\ln^2 x)\checkmark + (5-x^2)\times 2\ln x\checkmark \times\frac{1}{x}\checkmark\right]$$

$$= \sec^2[(5-x^2)(\ln^2 x)]\times\left[-2x(\ln^2 x) + \frac{2(5-x^2)\times\ln x}{x}\right] \quad (4)$$

2.2.2 $$y = \sqrt[4]{\cos(9-x^2) + \sqrt{\ln x}}$$

$$\frac{dy}{dx} = \frac{1}{4}\left[\cos(9-x^2) + \sqrt{\ln x}\right]^{-\frac{3}{4}}\checkmark \times\left[-\sin(9-x^2)\times 2x\checkmark + \frac{1}{2}(\ln x)^{-\frac{1}{2}}\checkmark \times\frac{1}{x}\checkmark\right]$$

$$= \frac{1}{4}\left[\cos(9-x^2) + \sqrt{\ln x}\right]^{-\frac{3}{4}}\times\left[-2x\sin(9-x^2) + \frac{1}{2x\sqrt{\ln x}}\right] \quad (4)$$

2.3 $$y = \frac{\sin(3x + x^2)}{(6 - x^4)^3}$$

$$\ln y = \ln \sin(3x + x^2) - 3\ln(6 - x^4)$$

$$\frac{1}{y}.\frac{dy}{dx}\checkmark = \frac{1}{\sin(3x + x^2)} \times \cos(3x + x^2)\checkmark \times (3 + 2x) - \frac{3}{6 - x^4} \times 4x^3 \checkmark$$

$$\frac{dy}{dx} = y\left[\frac{(3 + 2x)\cos(3x + x^2)}{\sin(3x + x^2)} + \frac{12x^3}{6 - x^4}\right]$$

$$\frac{dy}{dx} = \frac{\sin(3x + x^2)}{(6 - x^4)^3}\left[\frac{(3 + 2x)\cos(3x + x^2)}{\sin(3x + x^2)} + \frac{12x^3}{6 - x^4}\right]\checkmark$$ (4)

2.4 $$\cos(x^2 + 2y) + xe^{y^2} = 1$$

$$-\sin(x^2 + 2y) \times \left[2x + 2\frac{dy}{dx}\right] + e^{y^2} + 2xye^{y^2}\frac{dy}{dx} = 0\checkmark$$

$$-2x\sin(x^2 + 2y) - 2\sin(x^2 + 2y)\frac{dy}{dx} + e^{y^2} + 2xye^{y^2}\frac{dy}{dx} = 0\checkmark$$

$$\frac{dy}{dx}\left[2xye^{y^2} - 2\sin(x^2 + 2y)\right] = 2x\sin(x^2 + 2y) - e^{y^2}\checkmark$$

$$\frac{dy}{dx} = \frac{2x\sin(x^2 + 2y) - e^{y^2}}{2xye^{y^2} - 2\sin(x^2 + 2y)}\checkmark$$ (5)

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QUESTION 3

3.1 3.1.1 $$f(x) = f(x) = 2x^3 + x^2 - x + 2$$

$$f'(x) = 6x^2 + 2x - 1 = 0$$

$$f''(x) = 12x + 2$$

$$x = -\frac{1}{6} \quad y = \frac{59}{27}\checkmark$$

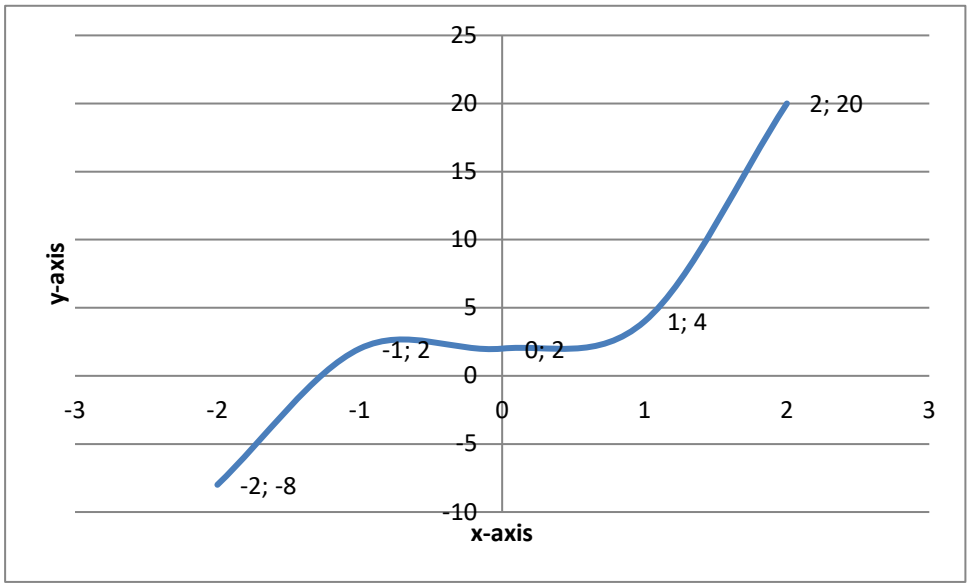
Thus, the coordinate of the point of inflection is $\left(-\frac{1}{6}; \frac{59}{27}\right)\checkmark$ (2)

3.1.2

x	-2	-1	0	1	2
y	-7	3	3	5	21

✓= 1 mark for any 3 correct answers. (3)

3.1.3



✓ =1 mark for the shape.
✓=1 mark for indicating the turning point on the graph. (2)

3.1.4 $Let\ x_0 = -1{,}5$

$f(-1{,}5) = -1$

$f'(-1{,}5) = 9{,}5$✓

$x_1 = -1{,}5 - \frac{(-1)}{9{,}5}$✓

$= -1{,}395$✓

$f(-1{,}395) = -0{,}086$

$f'(-1{,}395) = 7{,}882$

$x_1 = -1{,}395 - \frac{(-0{,}086)}{7{,}882}$

$= -1{,}384$✓ (4)

3.2 $V = l \times w \times h = 6w^2h = 20$

$C = 3(2lh + 2wh) + 15(2lw)$✓

$= 3(12wh + 2wh) + 15(12w^2)$

$= 42wh + 180w^2$

$h = \frac{10}{3w^3}$✓

$C = 4w \times \frac{10}{3w^3} + 180w^2$✓

$= \frac{140}{w} + 180w^2$✓

$C' = -\frac{140}{w^2} + 360w = 0$✓

$360w^3 - 140 = 0$

$w = 0{,}730, \quad l = 4{,}379 \ \ and \ h = 6{,}257$✓ (6)

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QUESTION 4

4.1 4.1.1 $\int \sec^2 2x(9 + 7\tan 2x - \tan^2 2x)\, dx$

$let\ u = \tan 2x$

$\frac{du}{dx} = 2\sec^2 2x$

$\frac{du}{2} = \sec^2 2x\, dx$✓

$= \frac{1}{2}\int (9 + 7u - u^2)\, du$

$= \frac{1}{2}\left(9u + \frac{7}{2}u^2 - \frac{1}{3}u^3\right) + C$

$= \frac{9}{2}\tan 2x$ ✓ $+ \frac{7}{4}\tan^2 2x$✓ $- \frac{1}{6}\tan^3 2x$✓ $+ C$ (4)

4.1.2 $$\int \frac{7x+2}{\sqrt{1-25x^2}}dx$$

$$= \int \frac{7x}{\sqrt{1-25x^2}}dx + \int \frac{2}{\sqrt{1-(5x)^2}}dx$$

$let\ u = 1-25x^2 \quad and\ v = 5x$
$\frac{du}{dx} = -50x$
$\frac{du}{-50} = x\,dx$ ✓

$$= -\frac{7}{50}\int u^{-\frac{1}{2}}du + \frac{2}{5}\int \frac{1}{\sqrt{1-v^2}}dv$$ ✓

$$= -\frac{2}{25}u^{\frac{1}{2}} + \frac{2}{5}\sin^{-1}v + C$$

$$= -\frac{2}{25}(1-25x^2)^{\frac{1}{2}}\checkmark + \frac{2}{5}\sin^{-1}5x\ \checkmark + C$$ (4)

4.1.3 $$\int \frac{3x^3-17x^2+18x+10}{3x+1}dx$$

$$= \int \left(x^2-6x+8+\frac{2}{3x+1}\right)dx$$ ✓

$$= \frac{x^3}{3}\checkmark - 3x^2\checkmark + 8x\checkmark + \frac{2}{3}\ln(3x+1)\checkmark + C$$ (5)

4.1.4 $$\int \sin^3 x\cos x\,dx$$

$let\ u = \sin x$
$du = \cos x\,dx$ ✓

$$= -\int u^3\,du$$
$$= \frac{1}{4}u^4 + C$$

$$= \frac{1}{4}(\sin x)^4\checkmark + C$$ (2)

4.1.5 $$\int x\sec^2 x\,dx$$

$let\ u = x \qquad dv = \sec^2 x\,dx$ $du = dx \qquad v = \tan x$ ✓

$$\int \sec^2 x\,dx = x\tan x - \int \tan x\,dx$$ ✓

$$= x\tan x - (-\ln|\cos x|) + C$$

$$= x\tan x$$ ✓ $$+ \ln|\cos x|$$ ✓ $$+ C$$ (4)

4.2 $$\int \frac{x+5}{(x-3)(x+1)}dx$$

$$\frac{x+5}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$6x + 12 = A(x+1) + B(x-3)$$ ✓

$let\ x = -1, \quad B = -1$ ✓ or $x = 3, \quad A = 2$ ✓

$$= \int \left(\frac{2}{x-3} - \frac{1}{x+1}\right)dx$$

$$= 2\ln(x-3)$$ ✓ $$- \ln(x+1)$$ ✓ $$+ C$$ (5)

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QUESTION 5

5.1

$$\int_3^7 \left[\frac{9e^x}{e^x+4}+\frac{(\ln 2x)^2}{x}\right]dx$$

$$\int_3^7 \frac{9e^x}{e^x+4}dx+\int_3^7 \frac{(\ln 2x)^2}{x}dx$$

$let\ u = e^x + 4 \quad z = \ln 2x\ dx$
$du = e^x\,dx \qquad dz = \frac{1}{x}dx$ ✓

$$\int \frac{1}{u}du+\int z^2\,dz\checkmark$$

$$=\left[9\ln(e^x+4)+\frac{1}{3}(\ln 2x)^3\right]_3^7\checkmark$$

$$=\left[9\ln(e^7+4)+\frac{1}{3}(\ln 2\times 7)^3\right]-\left[9\ln(e^3+4)+\frac{1}{3}(\ln 2\times 6)^3\right]\checkmark$$

$$=38.608\checkmark \qquad (5)$$

5.2 5.2.1

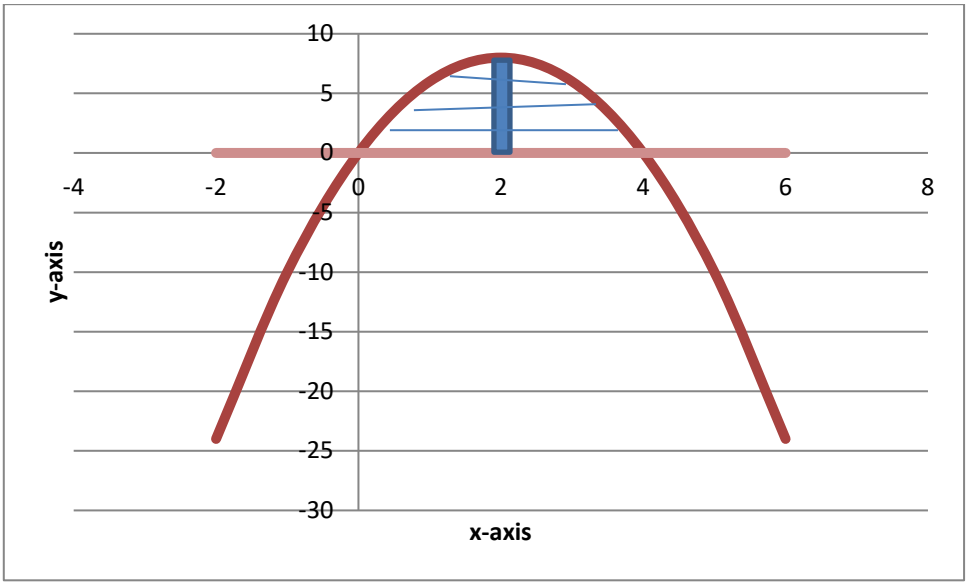
$$8x-2x^2=0$$

$$2x(4-x)=0$$

$x = 0 \ or \ x = 4$
$y = 0 \qquad y = 0$

Thus, the coordinates of the points of intersection are $(0;0)$✓ and $(4;0)$.✓ (2)

5.2.2



✓= 1 mark for indication of enclosed area.
✓= 1 mark for indication of the vertical or horizontal strip. (2)

5.2.3

$$A = \int_a^b y\,dx$$

$$A = \int_0^4 (8x - 2x^2)\,dx \checkmark$$

$$= \left[4x^2 - \frac{2}{3}x^3\right]_0^4 \checkmark$$

$$= \frac{64}{3} \text{ or } 21{,}333\ units^2 \checkmark$$ (3)

5.2.4

$$V = \pi \int_0^4 (8x - 2x^2)^2\,dx$$

$$= \pi \int_0^4 (64x^2 - 32x^3 + 4x^4)\,dx \checkmark\checkmark$$

$$= \pi \left[\frac{64}{3}x^3 - 8x^4 + \frac{4}{5}(4)^5\right]_0^4 \checkmark$$

$$= \frac{2048}{15}\pi \text{ or } 428{,}932\ units^3 \checkmark$$ (4)

5.3 Moment of inertia of a disc is $\frac{1}{2}r^2\,dm$

$$dI_z = \frac{1}{2}r^2\,dm\checkmark$$

$$I_z = \frac{1}{2}r^2\int_0^{0.03} dm\checkmark$$

$$= \frac{1}{2}(0{,}03)^2(15)\checkmark$$

$$= 0{,}007\ kgm^2\checkmark$$

(4)
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QUESTION 6

6.1 $$\frac{dy}{dx} = -\frac{x}{ye^{x^2}}$$

$$y\,dy = -xe^{-x^2}\,dx$$

$$\frac{y^2}{2}\checkmark = \frac{1}{2}e^{-x^2}\checkmark + C$$

$$\frac{1^2}{2} = \frac{1}{2}e^{-(0)^2} + C$$

$$C = 0\checkmark$$

$$\frac{y^2}{2} = \frac{1}{2}e^{-x^2}\checkmark$$

(4)

6.2 $$\operatorname{cosec} x\,.\frac{d^2y}{dx^2} = 1 + \cot x + \frac{x^2}{\sin x}$$

$$\frac{d^2y}{dx^2} = \sin x + \cos x + x^2\checkmark$$

$$\frac{dy}{dx} = \cos x - \sin x + \frac{1}{3}x^3 + A\checkmark$$

$$y = -\sin x - \cos x + \frac{1}{12}x^4 + Ax + B\checkmark$$

(3)
[7]

TOTAL: 100