higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

30 JULY 2019

This marking guideline consists of 11 pages.

QUESTION 1

1.1 1.1.1 $$\lim_{x\to 0}\frac{arc\sin 4x}{arc\tan 5x}$$

L'hospital's rule since the condition $\frac{0}{0}$ holds; thus

$$=\lim_{x\to 0}\frac{\frac{4}{\sqrt{1-16x^2}}}{\frac{5}{1+2x^2}}\checkmark\checkmark$$

$$=\frac{4}{5}\checkmark$$ (3)

1.1.2 $$\lim_{x\to 0}\sqrt{x}\ln x$$

$$=\lim_{x\to 0}\frac{\ln x}{\frac{1}{\sqrt{x}}}\checkmark$$

L'hospital's rule since the condition $\frac{0}{0}$ holds; thus

$$=\lim_{x\to 0}\frac{\frac{1}{x}}{-\frac{1}{2}x^{-\frac{3}{2}}}\checkmark$$

$$=\lim_{x\to 0}-\frac{2\sqrt{x^3}}{x}$$

L'hospital's rule since the condition $\frac{0}{0}$ holds; thus

$$=\lim_{x\to 0}\frac{-3\sqrt{x}}{1}\checkmark$$

$$=0\checkmark$$ (4)

1.2 $f(x)=\sec x$

Thus, $f(x)$ is continuous at $x\neq\frac{\pi}{2}\checkmark$ and $x\neq\frac{3\pi}{2}\checkmark$ i.e. $[0;\frac{\pi}{2})$ and $\left(\frac{\pi}{2};\frac{3\pi}{2}\right)$ (2)
[9]

QUESTION 2

2.1 $$f(x) = \frac{x+1}{x+2}$$

$$f'(x) = \lim_{h\to 0}\left[\frac{\frac{(x+h)+1}{(x+h)+2} - \frac{x+1}{x+2}}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\frac{(x+h)+1}{(x+h)+2}\times\frac{x+2}{x+2} - \frac{x+1}{x+2}\times\frac{(x+h)+2}{(x+h)+2}\checkmark}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\frac{x^2+xh+3x+2h+2-x^2-xh-3x-h-2}{(x+h+2)(x+2)}\checkmark\checkmark}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\frac{h}{(x+h+2)(x+2)}\checkmark}{h}\right]$$

$$= \frac{1\checkmark}{(x+2)^2}$$ (5)

2.2 $$y = \text{arc cosec}\, x$$

$$\text{cosec}\, y = x$$

$$-\text{cosec}\, y \cot y \frac{dy}{dx} = 1\checkmark$$

$$\frac{dy}{dx} = -\frac{1}{\text{cosec}\, y \cot y}\checkmark$$

$$= -\frac{1}{\text{cosec}\, x\sqrt{\text{cosec}^2 x - 1}}\checkmark$$

$$-\frac{1}{x\sqrt{x^2-1}}$$ (3)

2.3 $$y = \cot x$$

$$y = \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}\checkmark$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}\checkmark$$

$$= -\frac{1}{\sin^2 x}\checkmark$$

$$= -\text{cosec}^2 x$$ (3)

2.4 2.4.1

$$y = \sqrt{3x + \sqrt{2x + \sqrt{x}}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[3x + \left(2x + x^{\frac{1}{2}}\right)^{-\frac{1}{2}}\right]\checkmark^{-\frac{1}{2}} \times \left[3 + \left(2x + x^{\frac{1}{2}}\right)^{-\frac{1}{2}}\right]\checkmark \times \left(2 + \frac{1}{2}x^{-\frac{1}{2}}\right)\checkmark$$

2.4.2

$$y = \left[\left(\frac{1}{x} + 1\right)^{-1} + 1\right]^{-1}$$

$$\frac{dy}{dx} = -\left[\left(\frac{1}{x} + 1\right)^{-1} + 1\right]^{-2}\checkmark \times \left[-\left(\frac{1}{x} + 1\right)^{-2}\right]\checkmark \times \left(-\frac{1}{x^2}\right)\checkmark$$

(2 × 3) (6)

2.5 $y = (1 - 3x)^{\cos x}$

$$\ln y = \cos x \ln(1 - 3x)$$

$$\frac{1}{y}\frac{dy}{dx}\checkmark = \cos x \times \frac{1}{1 - 3x}\checkmark \times -3 - \sin x \times \ln(1 - 3x)\checkmark$$

$$\frac{dy}{dx} = y\left[-\frac{3\cos x}{1 - 3x} - \sin x \ln(1 - 3x)\right]$$

$$= (1 - 3x)^{\cos x}\left[-\frac{3\cos x}{1 - 3x} - \sin x \ln(1 - 3x)\right]\checkmark$$

(4)

2.6 $e^{2x+3y} = x^2 - ln(xy^3)$

$$e^{2x+3y}\left[2 + 3\frac{dy}{dx}\right]\checkmark = 2x - \frac{1}{xy^3}\left[y^3 + x \times 3y^2\frac{dy}{dx}\right]\checkmark$$

$$2e^{2x+3y} + 3e^{2x+3y}\frac{dy}{dx} = 2x - \frac{1}{x} - \frac{3}{y}\frac{dy}{dx}$$

$$\left[3e^{2x+3y} + \frac{3}{y}\right]\frac{dy}{dx} = 2x - \frac{1}{x} - 2e^{2x+3y}\checkmark$$

$$\frac{dy}{dx} = \frac{2x - \frac{1}{x} - 2e^{2x+3y}}{3e^{2x+3y} + \frac{3}{y}}\checkmark$$

(4)
[25]

QUESTION 3

3.1 3.1.1 $f(x) = 2x^3 - x^2 + 5$

$f'(x) = 6x^2 - 2x = 0$

$2x(3x - 1) = 0$

$x = 0 \; or \; x = \frac{1}{3}$

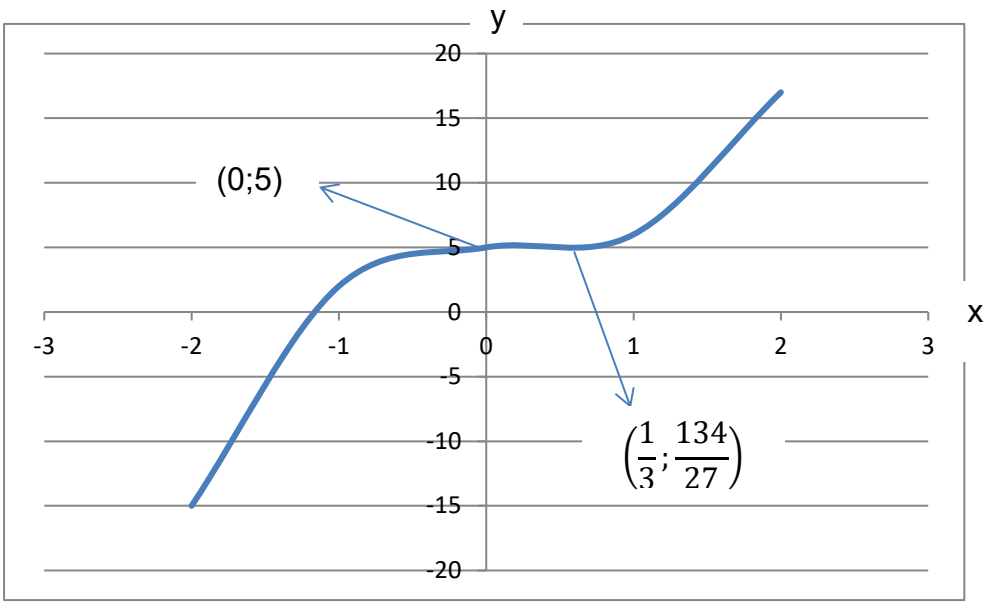
Turning points $\left(\frac{1}{3}; \frac{134}{27}\right)$ ✓ and $(0; 5)$✓ (2)

3.1.2

x	-2	-1	0	1	2
y	-15	2	5	6	17

(ONE mark for any 2 correct answers) (2)

3.1.3



(ONE mark for the shape)
(ONE mark for the indication of the point inflection on the graph) (2)

3.1.4

$$Let\ x_0 = -1{,}5$$

$$f(-1{,}5) = -4$$

$$f'(-1{,}5) = 16{,}5$$

$$x_1 = -1{,}5 - \frac{(-4)}{16{,}5}\checkmark$$

$$= -1{,}257575758\checkmark$$

$$Let\ x_1 = -1{,}257575758$$

$$f(-1{,}257575758) = -0{,}559200824$$

$$f'(-1{,}257575758) = 12{,}00413223$$

$$x_1 = -1{,}257575758 - \frac{(-0{,}559200824)\checkmark}{12{,}00413223}$$

$$= -1{,}21099173$$

$$\approx -1{,}211\checkmark$$ (4)

3.2

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = \frac{4\pi}{3} \times 3r^2 \frac{dr}{dt}\checkmark$$

$$\frac{dr}{dt} = \frac{\frac{dv}{dt}}{4\pi r^2}\checkmark$$

$$\frac{dr}{dt} = \frac{5\ cm^3/s}{4\pi(10\ cm)^2}\checkmark$$

$$= 0{,}004\ cm/s\checkmark$$ (4)

[14]

QUESTION 4

4.1 4.1.1

$$\int 4\left(\frac{1}{x}-e^{-x}\right)\cos(e^{-x}+\ln x)\ dx$$

$$=4\int \left(\frac{1}{x}-e^{-x}\right)\cos(e^{-x}+\ln x)\ dx\ \checkmark$$

$$let\ u=e^{-x}+\ln x$$

$$du=\left(\frac{1}{x}-e^{-x}\right)dx\ \checkmark$$

$$=4\int \cos u\ du$$

$$=4\sin(e^{-x}+\ln x)+C\checkmark \quad (3)$$

4.1.2

$$\int \frac{1}{\sqrt{4-9x^2}}dx$$

$$=\frac{1}{3}\checkmark\int \frac{1}{\sqrt{\frac{4}{9}-x^2}}dx\ \checkmark$$

$$=\frac{1}{3}\sin^{-1}\left(\frac{3}{2}x\right)+C\checkmark \quad (3)$$

4.1.3

$$\int \frac{x^3+x}{x-1}\ dx$$

$$=\int\left[x^2+x+2+\frac{2}{x-1}\right]dx\checkmark$$

$$=\frac{x^3}{3}\checkmark+\frac{x^2}{2}\checkmark+2x\checkmark+2\ln(x-1)\checkmark+C \quad (5)$$

4.1.4

$$\int \sin 10x\sin 7x\ dx$$

$$=\int\frac{1}{2}[\cos 3x-\cos 17x]\ dx\checkmark$$

$$=\frac{1}{2}\int(\cos 3x-\cos 17x)\ dx$$

$$=\frac{1}{6}\sin 3x\checkmark-\frac{1}{34}\sin 17x\checkmark+C \quad (3)$$

4.1.5 $$\int \frac{x}{\sec 3x}\,dx$$

$$= \int x\cos 3x\,dx$$

$let\ u = x$	$dv = \cos 3x\,dx$
$du = dx$	$v = \frac{1}{3}\sin 3x$ ✓

$$\int u\,dv = uv - \int v du$$

$$= \frac{x}{3}\sin 3x \checkmark - \frac{1}{3}\int \sin 3x\,dx \checkmark$$

$$= \frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x + C \checkmark \quad (4)$$

4.2 $$\int \frac{x^2}{x^2-1}\,dx$$

$$\int \left[1 + \frac{1}{x^2-1}\right] dx \checkmark$$

$$\frac{x^2}{(x+1)(x-1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$x^2 = A(x+1) + B(x-1) \checkmark$$

$$let\ x = -1, \qquad then\ B = -\frac{1}{2} \checkmark$$

$$x = 1, \qquad then\ A = \frac{1}{2} \checkmark$$

$$= \int dx + \frac{1}{2}\int \frac{1}{x-1}dx - \frac{1}{2}\int \frac{1}{x+1}dx$$

$$= x + \frac{1}{2}\ln(x-1) \checkmark - \frac{1}{2}\ln(x+1) \checkmark + C \quad (6)$$

[24]

QUESTION 5

5.1 $\int_0^{\infty} e^{-st}.f(t)dt$

$= -5\int_0^{\infty} e^{-st}dt$ ✓

$= -5\left[-\frac{e^{-st}}{s}\right]_0^{\infty}$ ✓

$= -5\left(0+\frac{1}{s}\right)$ ✓

$= \frac{-5}{s}$ ✓ (4)

5.2 5.2.1 $3x^2 = 6x$

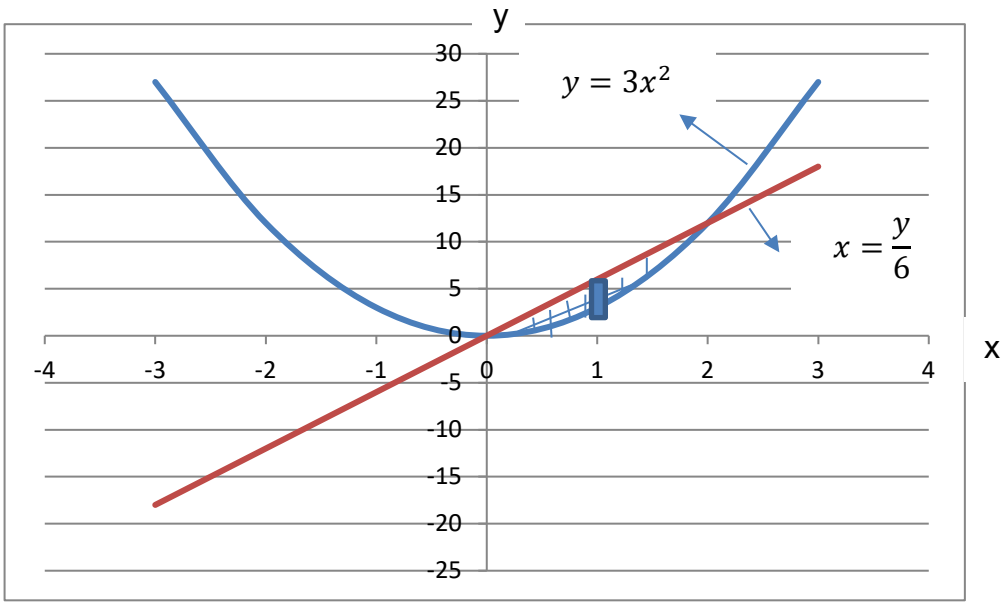
$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \text{ or } x = 2$

Thus the coordinates of the point of intersection are $(0;0)$✓ and $(2;12)$✓ (2)

5.2.2



(ONE mark for indication of enclosed area)
(ONE mark for indication of the vertical or vertical strip) (2)

5.2.3 $A_x = \int_a^b (y_1 - y_2)\,dx$

$= \int_0^2 (6x - 3x^2)\,dx$ ✓

$= [3x^2 - x^3]^2{}_0$ ✓

$= 4\ units^2$ ✓ (3)

5.2.4 $V_x = \int_a^b \left(y^2{}_1 - y^2{}_2\right) dx$

$= \int_0^2 (36x^2 - 9x^4)\,dx$ ✓✓

$= \left[12x^3 + \frac{9}{5}x^5\right]^2{}_0$ ✓

$= 38{,}4\ units^3$ ✓ (4)

5.3 $(d)_y = r^2 A$

$(d)_y = r^2\,dA$

$(d)_y = x^2\,b\,dx$ ✓

$= b\int_{-\frac{a}{2}}^{\frac{a}{2}} x^2\,dx$ ✓

$= ba^2\left[-\frac{x^3}{3}\right]_{-\frac{a}{2}}^{\frac{a}{2}}$ ✓

$= b\left[\frac{a^3}{12}\right]$

$= \frac{a^2 A}{12}$ ✓ (4)

[19]

QUESTION 6

6.1 $(3y^2 + 2)\cos x dx - 6y \sin x\, dy = 0$

$$\frac{6y}{3y^2 + 2} dy = \cot x\, dx \checkmark$$

$$\ln(3y^2 + 2) \checkmark = \ln(\sin x) + C \checkmark \quad (3)$$

6.2 $$\frac{d^2y}{dx^2} = 3x^2 - 2x$$

$$\frac{dy}{dx} = x^3 - x^2 + A \checkmark$$

$$A = -1 \checkmark$$

$$\frac{dy}{dx} = x^3 - x^2 - 1 \checkmark$$

$$y = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x + B \checkmark$$

$$B = \frac{25}{12} \checkmark$$

$$y = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x + \frac{25}{12} \checkmark \quad (6)$$

[9]

TOTAL: 100