higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1030**(E)**(J26)T

NATIONAL CERTIFICATE

MATHEMATICS N5

(8120003)

26 July 2018 (X-Paper)
09:00–12:00

This question paper consists of 5 pages and a formula sheet of 5 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. ALL final answers must be rounded off to THREE decimal places.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Questions must be answered in BLUE or BLACK ink.
8. Write neatly and legibly.

QUESTION 1

1.1 Determine the value of the limit:

$$\lim_{x\to 1}\left(\frac{x^2+8x-9}{x^3-2x^2-5x+6}\right)$$ (2)

1.2 Given: $\log y = \lim_{x\to\infty} x^2 e^{-x}$, calculate the numerical value of:

1.2.1 $\log y$ (3)

1.2.2 y (1)

1.3 Determine the value(s) of x for which $f(x)$ is discontinuous if $f(x) = \frac{x}{7-e^x}$. (2)

[8]

QUESTION 2

2.1 Determine the derivative of $f(x) = \frac{1-x}{x+2}$ from first principles. (5)

2.2 Make a neat sketch of the graph $y = arc\ \text{cosec}\, x$ for the range $[0;\pi]$. (2)

2.3 Determine $\frac{dy}{dx}$ in each of the following cases:

(Simplification NOT required)

2.3.1 $y = [\ln(x^2+1) - \tan^{-1}(3x)]^6$ (3)

2.3.2 $$y = \frac{e^{x^2+8x}}{\sqrt{x^4+7}}$$ (4)

2.3.3 $$y = \sqrt{x^2+\sqrt{1+4x}}$$ (3)

2.4 Calculate $\frac{dy}{dx}$ if $y = x^{\sqrt{x}}$ with aid of logarithmic differentiation. (4)

2.5 Determine $\frac{dy}{dx}$ of implicit function $\tan\left(\frac{x}{y}\right) = x + y$. (5)

[26]

QUESTION 3

3.1 Given: $f(x) = 7x^3 - 8x + 4$

3.1.1 Determine the coordinate of the point of inflection of $f(x)$. (3)

3.1.2 Draw up a table of x and $f(x)$, where x is ranging from $x = -2$ to $x = 2$. (2)

3.1.3 Draw a neat graph of $f(x)$ between these values showing the turning points on it. (2)

3.1.4 One root of the equation $f(x) = 7x^3 - 8x + 4$ is close to -1. Use this value and one approximation of Taylor's/ Newton's method to determine a better approximation of this root (Root correct to THREE decimal figures). (3)

3.2 The length of one side of a rectangle is three times the length of the other side. At what rate is the enclosed area decreasing when the shortest side is $6\,m$ long and is decreasing at a rate of $2\,m/s$? (5)

3.3 An object moves in a straight line so that after t seconds its distance is x metres from a fixed point on the line given by $x = t^3 - 7t^2 + 8t + 2$. Obtain an expression for velocity and acceleration of the object after t seconds and then calculate the values of t when the object is at rest. (5)

[20]

QUESTION 4

4.1 Determine $\int y\,dx$ in each of the following cases.

4.1.1 $y = x\sqrt{x+3}$ (3)

4.1.2 $y = \dfrac{x^2 - x - 1}{x - 1}$ (4)

4.1.3 $y = \dfrac{1}{\sqrt{5 - 25x^2}}$ (2)

4.1.4 $y = \cos(4ax)\sin(3bx)$ (3)

4.1.5 $y = \ln x^2$ (3)

4.2 Determine $\int y\,dx$ by resolving the integral into partial fractions:

$$\frac{1}{ax - bx^2}$$ (5)

[20]

QUESTION 5

5.1 Evaluate the definite integral: $\int_0^1 \sqrt{9 - x^2}\, dx$ (3)

5.2 Given: $y = -x + 5$ and $y = x^2 - 6x + 9$

5.2.1 Calculate the coordinates of the points of intersection. (2)

5.2.2 Make a neat sketch to show the enclosed area, the representative strip and the point of intersection. (2)

5.2.3 Calculate the magnitude of the area in QUESTION 5.2.2. (3)

5.2.4 Calculate the volume of the solid of revolution formed when the area in QUESTION 5.2.2 is rotated about the x-axis. (4)

5.3 Calculate the second moment of area of a circular lamina with radius 4 cm and about an axis through its centre and perpendicular to the plane of the lamina. (4)

[18]

QUESTION 6

6.1 If the gradient of a curve is $(1 + x)\, dy = (y + 1)\, dx$, find the equation for the curve if it passes through the point (0,1). (5)

6.2 Determine the general solution of: $\frac{d^2y}{dx^2} = x^3 - e^{-x} + 3$. (3)

[8]

TOTAL: 100

MATHEMATICS N5

FORMULA SHEET

Any other applicable formula may also be used.
Enige ander toepaslike formule kan ook gebruik word.

Trigonometry / Trigonometrie

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cos}ec^2 x$

$\sin 2A = 2\sin A.\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\sin^2 A = \dfrac{1}{2} - \dfrac{1}{2}\cos 2A$

$\cos^2 A = \dfrac{1}{2} + \dfrac{1}{2}\cos 2A$

$\sin(A \pm B) = \sin A.\cos B \pm \sin B.\cos A$

$\cos(A \pm B) = \cos A.\cos B \mp \sin A.\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 - \tan A.\tan B}$

$\sin A.\cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\cos A.\sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

$\cos A.\cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\sin A.\sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$\tan x = \dfrac{\sin x}{\cos x}; \sin x = \dfrac{1}{\operatorname{cos}ecx}; \cos x = \dfrac{1}{\sec x}$

BINOMIAL THEOREM / BINOMIAALSTELLING

$(x+h)^n = x^n + n.x^{n-1}h + \dfrac{n(n-1)}{2!}x^{n-2}h^2 + \ldots$

DIFFERENTIATION / DIFFERENSIASIE

$$e = -\frac{f(a)}{f'(a)}$$

$$r = a + e$$

PRODUCT RULE / PRODUKREEL

$$y = u(x).v(x)$$

$$\frac{dy}{dx} = u.\frac{dv}{dx} + v.\frac{du}{dx}$$

$$= u.v' + v.u'$$

QUOTIENT RULE / KWOSIЀNTREEL

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$$

$$= \frac{v.u' - u.v'}{v^2}$$

CHAIN RULE / KETTINGREEL

$$y = f(u(x))$$
$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$

ax^n	nax^{n-1}	$\frac{ax^{n+1}}{n+1}+c$
a	0	$ax+c$
e^x	e^x	e^x+c
a^x	$a^x.\ln a$	$\frac{a^x}{\ln a}+c$
$\log_e x$	$\frac{1}{x}$	—
$\log_a x$	$\frac{1}{x\ln a}$	—
$\sin x$	$\cos x$	$-\cos x+c$
$\cos x$	$-\sin x$	$\sin x+c$
$\tan x$	$\sec^2 x$	$\ln(\sec x)+c$
$\cot x$	$-\cos ec^2 x$	$\ln(\sin x)+c$
$\sec x$	$\sec x.\tan.$	$\ln[\sec x+\tan x]+c$
$\cos ecx$	$-\cos ecx.\cot x$	$\ln[\cos ecx-\cot x\}+c$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	—
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	—
$\tan^{-1} x$	$\frac{1}{1+x^2}$	—
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	—
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$	—
$\cos ec^{-1}x$	$\frac{-1}{x\sqrt{x^2-1}}$	—
$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$

$\frac{1}{\sqrt{a^2 - x^2}}$	— $\sin^{-1}(\frac{x}{a}) + c$
$\frac{1}{a^2 + x^2}$	— $\frac{1}{a}\tan^{-1}(\frac{x}{a}) + c$
$\frac{1}{x\sqrt{x^2 - a^2}}$	— $\frac{1}{a}\sec^{-1}(\frac{x}{a}) + c$
$\sqrt{a^2 - x^2}$	— $\frac{a^2}{2}\sin^{-1}(\frac{x}{a}) + \frac{x}{2}\sqrt{a^2 - x^2} + c$
$\frac{1}{x^2 - a^2}$	— $\frac{1}{2a}\ln(\frac{x-a}{x+a}) + c$
$\frac{1}{a^2 - x^2}$	— $\frac{1}{2a}\ln(\frac{a+x}{a-x}) + c$

INTEGRATION / INTEGRASIE

$$\int f(x).g'(x) = f(x).g(x) - \int f'(x).g(x)dx$$

$$\int \frac{f'(x)}{f(x)}dx = \ln f(x) + c$$

$$\int [f(x)]^n . f'(x)dx = \frac{f(x)^{n+1}}{n+1} + c$$

$$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

$$\frac{f(x)}{(x+a)^n} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \ldots + \frac{Z}{(x+a)^n}$$

Applications of integration / Toepassings van integrasie

AREAS

$$A_x = \int_a^b y\,dx; \quad A_x = \int_a^b (y_2 - y_1)\,dx$$

$$A_y = \int_a^b x\,dy; \quad A_y = \int_a^b (x_2 - x_1)\,dy$$

VOLUMES

$$V_x = \pi\int_a^b y^2\,dx; \quad V_x = \pi\int_a^b ({y_2}^2 - {y_1}^2)\,dx$$

$$V_y = \pi\int_a^b x^2\,dy; \quad V_y = \pi\int_a^b ({x_2}^2 - {x_1}^2)\,dy$$

SECOND MOMENT OF AREA / TWEEDE AREAMOMENT

$$I_x = \int_a^b r^2\,dA; \quad I_y = \int_a^b r^2\,dA$$

MOMENTS OF INERTIA / TRAAGHEIDSMOMENTE

Mass = density x volume / Massa = digtheid x volume

$$M = pV$$

Definition / Definisie: $I = mr^2$

General / Algemeen: $I = \int_a^b r^2\,dm = p\int_a^b r^2\,dV$