higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

7 April 2021 (X-paper)
09:00–12:00

Scientific calculators and drawing instruments may be used.

This question paper consists of 5 pages and a formula sheet of 5 pages.

068Q1A2107

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
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NATIONAL CERTIFICATE
MTAHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show all intermediate steps and simplify where possible.
5. All final answers must be rounded off to THREE decimals.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Sketches must be large, neat and fully labelled
8.. Work neatly.

QUESTION 1

1.1 Determine each of the following limits:

1.1.1 $\lim_{x\to\infty}\left[x\ln\left(1+\frac{3}{x}\right)\right]$ (4)

1.1.2 $\lim_{x\to 7}\frac{\frac{1}{7}-\frac{1}{x}}{x-7}$ (2)

1.2 Given: $-\log y = \lim_{x\to 0}\frac{\sin 7x}{x}$

Calculate the numerical value of each of the following:

1.2.1 $\log y$ (2)

1.2.2 y (1)

1.3 Determine the value(s) of x for which $f(x)$ is discontinuous if

$$f(x) = \frac{e^{x^2+1}}{e^x - 2e^{1-x}}$$ (2)

[11]

QUESTION 2

2.1 Given:

$f(x) = \frac{3}{x^5}$

Determine the simplest form of each of the following:

2.1.1 $f(x+h)$ (2)

2.1.2 $f(x+h) - f(x)$ (1)

2.1.3 $\frac{f(x+h)-f(x)}{h}$ (1)

2.1.4 $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ (1)

2.2 Calculate $\frac{dy}{dx}$ of the following by using the derivatives of $\sin x$ and $\cos x$, as well as the rules of differentiation:

$$y = e^{\ln(\tan x)}$$

HINT: $a^{\log_a x} = x$ (3)

2.3 Determine $\frac{dy}{dx}$ in each of the following cases (simplification not required):

2.3.1 $y = [\ln(x^2 + 1) - \tan^{-1}(6x)]^{10}$

2.3.2 $y = \dfrac{1 + \sin^{-1} x}{1 - \cos^{-1} x}$

(2 × 3) (6)

2.4 Calculate, with the aid of logarithmic differentiation, $\frac{dy}{dx}$ if $y = (2x - e^{8x})^{\sin 2x}$ (4)

2.5 Determine $\frac{dy}{dx}$ of the implicit function $\tan(x^2y^4) = 3x + y^2$ (4)

[22]

QUESTION 3

3.1 Given: $f(x) = x^3 - 7x^2 + 8x + 3$

3.1.1 Determine the coordinates of the turning points of $f(x)$. (2)

3.1.2 Draw up a table of x and $f(x)$, where x ranges from $x = 0$ to $x = 5$. (3)

3.1.3 Draw a neat graph of $f(x)$ between the values in QUESTION 3.1.2, showing the turning points on it. (2)

3.1.4 One root of the equation $x^3 - 7x^2 + 8x + 3 = 0$ is close to 1,9.

Use Taylor's/Newton's method once to determine a better approximation of this root (root correct to THREE decimal figures). (3)

3.2 Find the TWO positive numbers whose product is 750, and for which the sum of ONE number and TEN times the other number is a minimum. (5)

[15]

QUESTION 4

4.1 Determine $\int y\,dx$ in each of the following cases:

4.1.1 $y = 4\left(\frac{1}{x} - e^{-x}\right)\cos(e^{-x} + \ln x)$ (3)

4.1.2 $y = \dfrac{1}{\sqrt{4 - 9x^2}}$ (3)

4.1.3 $y = \dfrac{2x^2 - 5x - 1}{x - 3}$ (4)

4.1.4 $y = \sin(\pi x)\sin\left(\frac{x}{5}\right)$ (4)

4.1.5 $y = \frac{\ln x}{x^2}$ (4)

4.2 Determine $\int y\,dx$ by resolving the integral in partial fractions:

$$y = \frac{x+2}{(x-1)^2}$$ (5)

[23]

QUESTION 5

5.1 Evaluate the definite integral:

$$\int_0^2 \left(e^x + \frac{1}{x^2+1}\right)dx$$ (3)

5.2 Given: $y = 6 + x - x^2$ and $3y = 2x^2 - 12x + 18$

5.2.1 Calculate the coordinates of the points of intersection. (2)

5.2.2 Make a neat sketch to show the enclosed area, the representative strip and the point of intersection. (2)

5.2.3 Calculate the magnitude of the area in QUESTION 5.2.2. (3)

5.2.4 Calculate the volume of the solid of revolution formed when the area in QUESTION 5.2.2 is rotated about the x-axis. (5)

5.3 Calculate the moment of inertia of a circular lamina of radius r and mass m about an axis through its centre and perpendicular to the plane of the lamina. (5)

[20]

QUESTION 6

6.1 Determine the general solution of:

$$\frac{dy}{dx} = \frac{x^2}{1-y^2}$$ (3)

6.2 Determine the particular solution of the following:

$$\frac{d^2y}{dx^2} = \frac{4}{3}x^3 + x, \text{ if } \frac{dy}{dx} = \frac{1}{3}, y = 1 \text{ when } x = 0$$ (6)

[9]

TOTAL: 100

MATHEMATICS N5

FORMULA SHEET

Any other applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$\sin^2 A = {}^1/_2 - {}^1/_2 \cos 2A$

$\cos^2 A = {}^1/_2 + {}^1/_2 \cos 2A$

$\sin(A \pm B) = \sin A\cos B \pm \sin B\cos A$

$\cos(A \pm B) = \cos A\cos B \pm \sin A\sin B$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \pm \tan A\tan B}$$

$\sin A\cos B = {}^1/_2\,[\sin(A + B) + \sin(A - B)]$

$\cos A\sin B = {}^1/_2\,[\sin(A + B) - \sin(A - B)]$

$\cos A\cos B = {}^1/_2\,[\cos(A + B) + \cos(A - B)]$

$\sin A\sin B = {}^1/_2\,[\cos(A - B) - \cos(A + B)]$

$$\tan x = \frac{\sin x}{\cos x};\ \sin x = \frac{1}{cosec\ x};\ \cos x = \frac{1}{\sec x}$$

BINOMIAL THEOREM

$$(x+h)^n = x^n + \mathrm{n}x^{n-1}\,h + \frac{n(n-1)}{2!}x^{n-2}\,h^2 + \cdots$$

DIFFERENTIATION

$$e = -\frac{f(a)}{f'(a)}$$

$$r = a + e$$

PRODUCT RULE

$$\mathrm{y} = \mathrm{u}(x)\,.\,v(x)$$

$$\frac{dy}{dx} = u\,.\frac{dv}{dx} + v\,.\frac{du}{dx}$$

$$= u\,.\,v' + v\,.\,u'$$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\,.\,\frac{du}{dx} - u\,.\,\frac{dv}{dx}}{v^2}$$

$$\frac{v\,.\,u' - u\,.\,v'}{v^2}$$

CHAIN RULE

$$y = f\big(u(x)\big)$$

$$\frac{dy}{dx} = \frac{dy}{du}\,.\frac{du}{dx}$$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
ax^n	nax^{n-1}	$\frac{ax^{n+1}}{n+1}+c$
a	0	$ax+c$
e^x	e^x	e^x+c
a^x	$a^x.lna$	$\frac{a^x}{lna}+c$
$\log_e x$	$\frac{1}{x}$	——
$\log_a x$	$\frac{1}{xlna}$	——
$\sin x$	$\cos x$	$-\cos x+c$
$\cos x$	$-\sin x$	$\sin x+c$
$\tan x$	$\sec^2 x$	$ln(\sec x)+c$
$\cot x$	$-\text{cosec}^2 x$	$ln(\sin x)+c$
$\sec x$	$\sec x.\tan x$	$ln[\sec x+\tan x]+c$
$\text{cosec}\, x$	$-\text{cosec}\, x\ .\cos x$	$ln(\text{cosec}\, x+\cot x)+c$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	——
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	——
$\tan^{-1} x$	$\frac{1}{1+x^2}$	——
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	——
$\sec^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$	——
$\text{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$	——

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{a^2-x^2}}$	—	$\sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{a^2+x^2}$	—	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+ c$
$\frac{1}{x\sqrt{x^2+a^2}}$	—	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
$\sqrt{a^2 - x^2}$	—	$\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + c$
$\frac{1}{x^2-a^2}$	—	$\frac{1}{2a}ln\left(\frac{x-a}{x+a}\right) + c$
$\frac{1}{a^2-x^2}$	—	$\frac{1}{2a}ln\left(\frac{a+x}{a-x}\right) + c$

INTEGRATION

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)g\,(x)dx$$

$$\int \frac{f'(x)}{f(x)}dx = ln\, f(x) + c$$

$$\int [f(x)]^n\, f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

$$\frac{f(x)}{(x+a)^n} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \cdots \frac{Z}{(x+a)^n}$$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y dx\, ;\; A_x = \int_a^b (y_1 - y_1)dx$$

$$A_x = \int_a^b x dy\, ; A_{Y=} \int_a^b (x_1 - x_2)dy$$

VOLUMES

$V_x = \pi \int_a^b y^2\, dx$; $V_{X=\pi} \int_a^b (y_1^2 - y_2^2) dx$

$V_y = \pi \int_a^b x^2\, dy$; $V_{y=\pi} \int_a^b (x_1^2 - x_2^2) dy$

SECOND MOMENT OF AREA

$I_x = \int_a^b r^2\, dA$; $I_{y=} \int_a^b r^2\, dA$

MOMENT OF INERTIA

$Mass = density \times volume$

$M = pv$

$DEFINITION : I = m\, r^2$

$GENERAL : I = \int_a^b r^2 dm = p \int_a^b r^2 dV$