higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

7 April 2021

This marking guideline consists of 12 pages.

QUESTION 1

1.1 1.1.1

$$\lim_{x\to\infty}\left[x\ln\left(1+\frac{3}{x}\right)\right]$$

$$\lim_{x\to\infty}\frac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{x}}\ \checkmark\quad\left(\frac{0}{0}\right)$$

$$=\lim_{x\to\infty}\frac{-3x^{-2}\times\frac{1}{1+\frac{3}{x}}}{-x^{-2}}\checkmark$$

$$=\lim_{x\to\infty}\frac{3}{1+\frac{3}{x}}\checkmark$$

$$=3\checkmark \qquad (4)$$

1.1.2

$$\lim_{x\to7}\frac{\frac{1}{7}-\frac{1}{x}}{x-7}\quad\left(\frac{0}{0}\right)$$

$$=\lim_{x\to7}\frac{-x^{-2}}{1}\checkmark$$

$$=-\frac{1}{49}\checkmark \qquad (2)$$

1.2 1.2.1

$$-\log y=\lim_{x\to0}\frac{\sin 7x}{x}\quad\left(\frac{0}{0}\right)$$

$$=\lim_{x\to0}\frac{7\cos 7x}{1}\ \checkmark$$

$$=7$$

$$\log y=-7\checkmark \qquad (2)$$

1.2.2 $y=10^{-7}\checkmark$ (1)

1.3 $$f(x) = \frac{e^{x^2+1}}{e^x - 2e^{1-x}}$$

$$e^x - 2e^{1-x} = 0$$

$$e^x - \frac{2e}{e^x} = 0$$

$$e^{2x} - 2e = 0 \checkmark$$

$$e^{2x} = 2e$$

$$2x = 1 + \ln 2$$

$$x = \frac{1 + \ln 2}{2} \checkmark$$ (2)

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QUESTION 2

2.1 $$f(x) = \frac{3}{x^5}$$

2.1.1 $$f(x+h) = 3(x+h)^{-5}$$

$$= 3\left[\frac{x^{-5}h^0}{0!} + \frac{(-5)x^{-6}h}{1!} + \frac{30x^{-7}h^2}{2!} + \frac{(-210)x^{-8}h^3}{3!} + \cdots \ldots\right]$$

$$= 3x^{-5} - 15x^{-6}h + 45x^{-7}h^2 - 105x^{-8}h^3 + \cdots \ldots \ldots \ldots \ldots \checkmark\checkmark$$ (2)

2.1.2 $$f(x+h) - f(x) = -15x^{-6}h + 45x^{-7}h^2 - 105x^{-8}h^3 + \cdots \ldots \checkmark$$ (1)

2.1.3 $$\frac{f(x+h) - f(x)}{h} = -15x^{-6} + 45x^{-7}h - 105x^{-8}h^2 + \cdots \ldots \checkmark$$ (1)

2.1.4 $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = -15x^{-6} \checkmark$$ (1)

2.2 $y = e^{\ln(\tan x)}$

$y = \tan x$ ✓

$= \dfrac{\sin x}{\cos x}$

$\dfrac{dy}{dx} = \dfrac{\cos x(\cos x) - \sin x\,(-\sin x)}{\cos^2 x}$ ✓

$= \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x}$ ✓

$= \sec^2 x$ (3)

2.3 2.3.1 $y = [\ln(x^2 + 1) - \tan^{-1}(6x)]^{10}$

$\dfrac{dy}{dx} = 10[\ln(x^2 + 1) - \tan^{-1}(6x)]^9$ ✓ $\times \left[\dfrac{2x}{x^2 + 1}$ ✓ $- \dfrac{6}{36x^2 + 1}$ ✓$\right]$

2.3.2 $y = \dfrac{1 + \sin^{-1} x}{1 - \cos^{-1} x}$

$\dfrac{dy}{dx} = \dfrac{\frac{1-\cos^{-1} x}{\sqrt{1-x^2}}$ ✓ $- \left(-\frac{1+\sin^{-1} x}{\sqrt{1-x^2}}\right)$ ✓$}{(1 - \cos^{-1} x)^2}$ ✓

$= \dfrac{\frac{\sin^{-1} x + \cos^{-1} x}{\sqrt{1-x^2}}}{(1 - \cos^{-1} x)^2}$

(2 × 3) (6)

2.4 $y = (2x - e^{8x})^{\sin 2x}$

$\ln y = \sin 2x \,.\, \ln(2x - e^{8x})$

$\dfrac{1}{y}.\dfrac{dy}{dx}$ ✓ $= 2\cos 2x.\ln(2x - e^{8x})$ ✓ $+ \sin 2x \times \dfrac{1}{2x - e^{8x}} \times (2 - 8e^{8x})$ ✓

$\dfrac{dy}{dx} = y\left[2\cos 2x.\ln(2x - e^{8x}) + \dfrac{(2 - 8e^{8x})\sin 2x}{(2x - e^{8x})}\right]$

$\dfrac{dy}{dx} = (2x - e^{8x})^{\sin 2x}\left[2\cos 2x.\ln(2x - e^{8x}) + \dfrac{(2 - 8e^{8x})\sin 2x}{(2x - e^{8x})}\right]$ ✓ (4)

2.5 $\tan(x^2y^4) = 3x + y^2$

$$\sec^2(x^2y^4)\left[2xy^4 + 4x^2y^3.\frac{dy}{dx}\right]\checkmark = 3 + 2y\frac{dy}{dx}\checkmark$$

$$\frac{dy}{dx}[4x^2y^3.\sec^2(x^2y^4) - 2y] = 3 - 2xy^4.\sec^2(x^2y^4)\checkmark$$

$$\frac{dy}{dx} = \frac{3 - 2xy^4.\sec^2(x^2y^4)}{4x^2y^3.\sec^2(x^2y^4) - 2y}\checkmark$$

(4)
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QUESTION 3

3.1 3.1.1 $f(x) = x^3 - 7x^2 + 8x + 3$

$$f'(x) = 3x^2 - 14x + 8 = 0$$

$$(3x - 2)(x - 4) = 0$$

$$x = \frac{3}{2} \quad or \quad x = 4$$

$$y = \frac{21}{8} \qquad y = -13$$

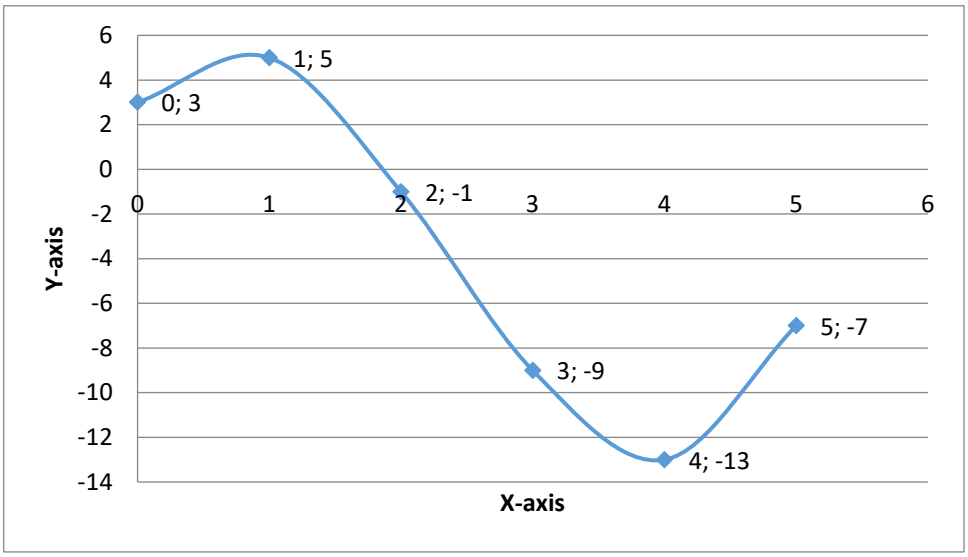
Thus, the coordinates of the turning points are $\left(\frac{3}{2};\frac{21}{8}\right)\checkmark$ and $(4;-13)\checkmark$ (2)

3.1.2

x	0	1	2	3	4	5
y	3	5	-1	-9	-13	-7

(Any 3 × 1) (3)

3.1.3



(1 × 1 for shape and 1 ×1 for indication of turning points) (2)

3.1.4 $Let\ x_0 = 1.9$

$f(1.9) = -0{,}211$

$f'(1.9) = -7{,}77$✓

$x_1 = 1.9 - \dfrac{(-0{,}211)}{(-7{,}77)}$✓

$= 1.872$✓ (3)

3.2 $xy = 750 \ldots\ldots\ldots\ldots (1)$✓

$S = y + 10x \ldots\ldots\ldots\ldots (2)$✓

$y = \dfrac{750}{x} \ldots\ldots\ldots\ldots\ldots (3)$

$S = \dfrac{750}{x} + 10x$

$S' = -\dfrac{750}{x^2} + 10 = 0$✓

$10x^2 = 750$

$x = \pm\sqrt{75} = 5\sqrt{3}$✓

$y = 50\sqrt{3}$✓ (5)

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QUESTION 4

4.1 4.1.1

$$\int \left[4\left(\frac{1}{x}-e^{-x}\right)\cos(e^{-x}+\ln x)\right]dx$$

$$let\ u = e^{-x}+\ln x$$

$$du = \left(\frac{1}{x}-e^{-x}\right)dx\checkmark$$

$$= 4\int \cos u\, du\checkmark$$

$$= 4\sin u + C$$

$$= 4\sin(e^{-x}+\ln x)\checkmark + C \quad (3)$$

4.1.2

$$\int \frac{1}{\sqrt{4-9x^2}}dx$$

$$= \frac{1}{2}\checkmark\int \frac{1}{\sqrt{1-\frac{9}{4}x^2}}dx\checkmark$$

$$= \frac{1}{2}\left(\frac{2}{3}\right)\sin^{-1}\left(\frac{3}{2}x\right)+C$$

$$= \frac{1}{3}\sin^{-1}\left(\frac{3}{2}x\right)+C\checkmark \quad (3)$$

4.1.3

$$\int \frac{2x^2-5x-1}{x-3}dx$$

$$= \int \left(2x+1-\frac{2}{x-3}\right)dx\checkmark$$

$$= x^2\checkmark + x\checkmark - 1\ln(x-3)\checkmark + C \quad (4)$$

4.1.4

$$\int \sin(\pi x)\sin\left(\frac{x}{5}\right)dx$$

$$= \int \frac{1}{2}\left[\cos\left(\pi-\frac{1}{5}\right)x - \cos\left(\pi+\frac{1}{5}\right)x\right]dx\checkmark\checkmark$$

$$= \frac{\sin\left(\pi-\frac{1}{5}\right)x}{2\left(\pi-\frac{1}{5}\right)}\checkmark - \frac{\sin\left(\pi+\frac{1}{5}\right)x}{2\left(\pi+\frac{1}{5}\right)}\checkmark + C \quad (4)$$

4.1.5 $$\int \frac{\ln x}{x^2}\,dx$$

$let\ u = \ln x \qquad dv = \frac{1}{x^2}dx$ ✓
$du = \frac{1}{x}dx \qquad v = -\frac{1}{x}$

$$\int \frac{\ln x}{x^2}dx = -\frac{1}{x}.\ln x - \int -\frac{1}{x}.\frac{1}{x}dx$$✓

$$= -\frac{\ln x}{x}✓ - \frac{1}{x}✓ + C$$ (4)

4.2 $$\int \frac{x+2}{(x-1)^2}dx$$

$$\frac{x+2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

$$x + 2 = A(x-1) + B$$✓

$let\ x = 1, \quad B = 3$✓
or
$x = 0, \quad A = 1$✓

$$= \int \left(\frac{1}{x-1} + \frac{3}{(x-1)^2}\right)dx$$

$$= \ln(x-1)✓ - \frac{3}{x-1}✓ + C$$ (5)

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QUESTION 5

5.1 $$\int_0^2 \left(e^x + \frac{1}{x^2+1}\right)dx$$

$$= [e^x + \tan^{-1}x]_0^2$$✓

$$= (e^2 + \tan^{-1}2) - (e^0 + \tan^{-1}0)$$✓

$$= 69.824$$✓ (3)

5.2 5.2.1

$$6 + x - x^2 = \frac{2}{3}x^2 - 4x + 6$$

$$18 + 3x - 3x^2 = 2x^2 - 12x + 18$$

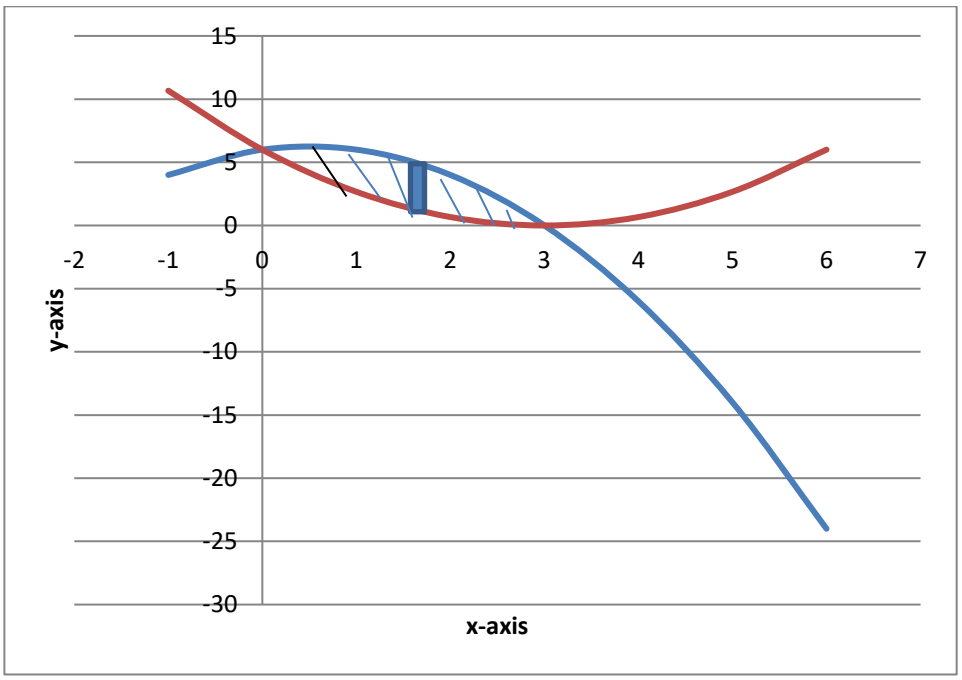
$$-18 - 3x + 3x^2 + 2x^2 - 12x + 18 = 0$$

$$5x(x - 3) = 0$$

$x = 0 \ or \ x = 3$
$y = 6 \quad y = 0$

Thus, the coordinates of the points of intersection are $(0; 6)$✓ and $(3; 0)$✓. (2)

5.2.2



(1 × 1 for indication of enclosed area and 1 × 1 for indication of the vertical or horizontal strip) (2)

5.2.3

$$A = \int_a^b (y_1 - y_2)\,dx$$

$$A = \int_0^3 \left[(6 + x - x^2) - \left(\frac{2}{3}x^2 - 4x + 6\right)\right] dx\checkmark$$

$$= \int_0^3 \left(6 + x - x^2 - \frac{2}{3}x^2 + 4x - 6\right) dx$$

$$= \int_0^3 \left(5x - \frac{5}{3}x^2\right) dx$$

$$= \left[\frac{5}{2}x^2 - \frac{5}{9}x^3\right]_0^3 \checkmark$$

$$= \frac{15}{2} \text{ or } 7{,}5 \text{ units}^2\checkmark$$ (3)

5.2.4

$$V = \pi \int_0^3 \left[(6 + x - x^2)^2 - \left(\frac{2}{3}x^2 - 4x + 6\right)^2\right] dx$$

$$= \pi \int_0^3 \left(36 + 12x - 11x^2 - 2x^3 + x^4 - \frac{4}{9}x^4 + \frac{16}{3}x^3 - 24x^2 + 48x - 36\right) dx \checkmark\checkmark$$

$$= \pi \int_0^3 \left(60x - 35x^2 + \frac{10}{3}x^3 + \frac{5}{9}x^4\right) dx\checkmark$$

$$= \pi \left[30x^2 - \frac{35}{3}x^3 + \frac{5}{6}x^4 + \frac{4}{45}x^5\right]_0^3 \checkmark$$

$$= \frac{441}{10}\pi \text{ or } 138{,}544\ units^3\checkmark$$ (5)

5.3 $m = \sigma A$

$dm = \sigma dA$

$dI_z = r^2$

$= x^2 \sigma dm$✓

$= x^2 \sigma 2\pi x\, dx$✓

$= 2\pi\sigma \int_0^r x^3\, dx$ ✓

$= 2\pi\sigma \left[\frac{1}{4}x^4\right]_0^r$ ✓

$= 2\pi\sigma \left[\frac{r^4}{4} - 0\right]$

$= \frac{1}{2}\pi\sigma r^4 units^4$✓ (5)

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QUESTION 6

6.1 $\frac{dy}{dx} = \frac{x^2}{1-y^2}$

$(1-y^2)\, dy = x^2\, dx$✓

$y - \frac{y^3}{3}$✓ $= \frac{x^3}{3} + C$✓ (3)

6.2 $\frac{d^2y}{dx^2} = \frac{4}{3}x^3 + x$

$\frac{dy}{dx} = \frac{1}{3}x^4 + \frac{1}{2}x^2 + A$✓

$\frac{1}{3} = \frac{1}{3}(0)^4 + \frac{1}{2}(0)^2 + A$

$A = \frac{1}{3}$✓

$\frac{dy}{dx} = \frac{1}{3}x^4 + \frac{1}{2}x^2 + \frac{1}{3}$✓

$y = \frac{1}{(3)(5)}x^5 + \frac{1}{(2)(3)}x^3 + \frac{1}{3}x + B$✓

$1 = \frac{1}{15}(0)^5 + \frac{1}{6}(0)^3 + \frac{1}{3}(0) + B$

$B = 1$✓

$y = \frac{1}{15}x^5 + \frac{1}{6}x^3 + \frac{1}{3}x + 1$✓ (6)

[9]

TOTAL: 100