



**higher education  
& training**

Department:  
Higher Education and Training  
**REPUBLIC OF SOUTH AFRICA**

# **MARKING GUIDELINE**

**NATIONAL CERTIFICATE**

**MATHEMATICS N5**

**2 April 2020**

**This marking guideline consists of 11 pages.**

### QUESTION 1

1.1 1.1.1  $\lim_{x \rightarrow \frac{1}{9}} \frac{9x - 1}{3\sqrt{x} - 1} \left(\frac{0}{0}\right)$

$$= \lim_{x \rightarrow \frac{1}{9}} \frac{9}{\frac{3}{2\sqrt{x}}} \checkmark$$
$$= \lim_{x \rightarrow \frac{1}{9}} 6\sqrt{x} \checkmark$$
$$= 2 \checkmark \quad (3)$$

1.1.2  $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x) (\infty - \infty)$

$$= \lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) \checkmark$$
$$= \lim_{x \rightarrow 0} \left( \frac{1 - \cos x}{\sin x} \right) \checkmark \left(\frac{0}{0}\right)$$
$$= \lim_{x \rightarrow 0} \left( \frac{\sin x}{\cos x} \right) \checkmark$$
$$= 0 \checkmark \quad (4)$$

1.2  $f(x) = \frac{x^2 + 4x - 12}{x^2 - 2x}$

$$x^2 - 2x = 0$$
$$x(x - 2) = 0$$
$$x = 0 \checkmark \text{ or } x = 2 \checkmark \quad (2)$$

**[9]**

### QUESTION 2

2.1  $y = \operatorname{arc} \cot x$

$$\cot y = x$$
$$-\operatorname{cosec}^2 y \frac{dy}{dx} = 1 \checkmark$$
$$= -\frac{1}{1 + \cot^2 y} \checkmark$$
$$= -\frac{1}{1 + x^2} \quad (2)$$

2.2      2.2.1       $y = \ln(\cos^5(3x^4))$

$$\frac{dy}{dx} = \frac{1}{\cos^5(3x^4)} \checkmark \times 5 \cos^4(3x^4) \checkmark \times (-\sin 3x^4) \checkmark \times 12x^3 \checkmark \quad (4)$$

2.2.2

$$y = \left(\frac{8x - x^6}{x^3}\right)^{-\frac{4}{5}}$$

$$\frac{dy}{dx} = -\frac{4}{5} \left(\frac{8x - x^6}{x^3}\right)^{-\frac{9}{5}} \checkmark \times \frac{x^3(8 - 6x^5) \checkmark - 3x^2(8x - x^6) \checkmark}{(x^3)^2 \checkmark} \quad (4)$$

2.2.3

$$y = \sqrt{1 + \sqrt{1 + x^2}}$$

$$\frac{dy}{dx} = \frac{1}{2} (1 + \sqrt{1 + x^2})^{-\frac{1}{2}} \checkmark \times \frac{1}{2} (1 + x^2)^{-\frac{1}{2}} \checkmark \times 2x \checkmark \quad (3)$$

2.3

$$y = \left(1 + \frac{1}{x}\right)^x$$

$$\ln y = x \ln \left(1 + \frac{1}{x}\right)$$

$$\frac{1}{y} \frac{dy}{dx} \checkmark = \ln \left(1 + \frac{1}{x}\right) \checkmark + x \times \frac{1}{\left(1 + \frac{1}{x}\right)} \times -x^{-2} \checkmark$$

$$\frac{dy}{dx} = y \left[ \ln \left(1 + \frac{1}{x}\right) - \frac{x}{x^2 \left(1 + \frac{1}{x}\right)} \right]$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{x}\right) \left[ \ln \left(1 + \frac{1}{x}\right) - \frac{1}{x \left(1 + \frac{1}{x}\right)} \right] \checkmark \quad (4)$$

2.4      2.4.1       $25x^2 + 9y^2 - 70x - 30y + 49 = 0$

$$50x + 18y \frac{dy}{dx} - 70 - 30 \frac{dy}{dx} + 0 = 0 \checkmark$$

$$18y \frac{dy}{dx} - 30 \frac{dy}{dx} = 70 - 50x \checkmark$$

$$\frac{dy}{dx} (18y - 30) = 70 - 50x$$

$$\frac{dy}{dx} = \frac{30 - 25x}{9y - 15} \checkmark \quad (3)$$

$$2.4.2 \quad \frac{dy}{dx} = -\frac{5}{4} \checkmark$$

$$y = mx + c$$

$$y = -\frac{5}{4}x + c$$

$$3 = -\frac{5}{4}(2) + c$$

$$c = \frac{11}{2} \checkmark$$

$$y = -\frac{5}{4}x + \frac{11}{2} \checkmark$$

(3)  
[23]

### QUESTION 3

3.1 3.1.1

$$f(x) = x^3 - 3x + 1$$

$$f'(x) = 3x^2 - 3 = 0$$

$$3(x^2 - 1) = 0$$

$$x = -1 \text{ or } x = 1$$

$$y = 3 \text{ or } y = -1$$

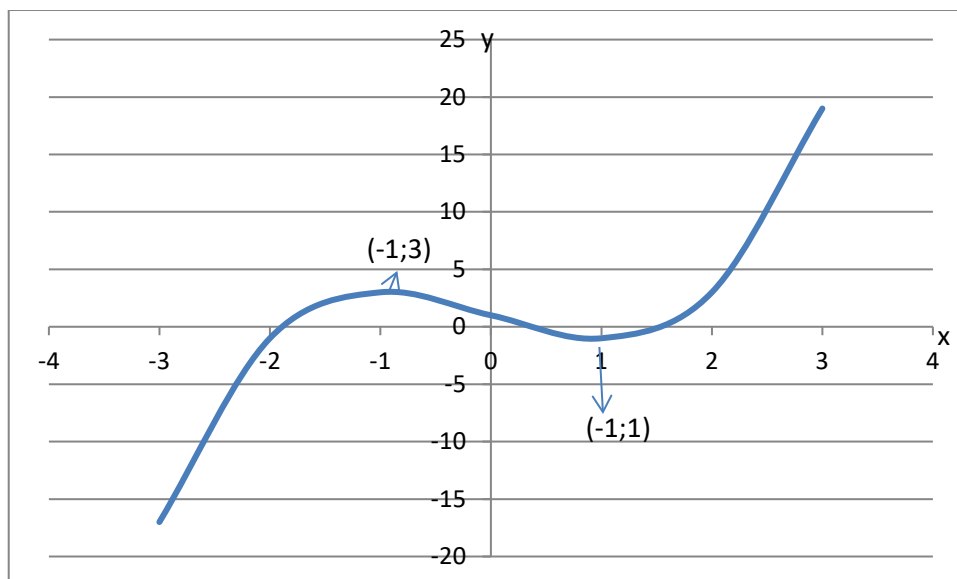
Thus, the coordinates of the turning points are  $(-1; 3) \checkmark$  and  $(1; -1) \checkmark$  (2)

3.1.2

$x$	-2	-1	0	1	2
$y$	-1	3	1	-1	3

$\checkmark = 1$  mark each for any 3 correct answers (3)

3.1.3



✓ = 1 mark for shape

✓ = 1 mark for indication of turning point on graph

(2)

3.1.4 *Let*  $x_0 = 1.5$

$$f(1.5) = -0.125$$

$$f'(1.5) = 3.75$$

$$x_1 = 1.5 - \frac{(-0.125)}{3.75} \checkmark$$

$$= 1.533 \checkmark$$

$$\text{Let } x_1 = 1.533$$

$$f(1.53) = 0.005$$

$$f'(1.533) = 4.053$$

$$x_1 = 1.533 - \frac{0.005}{4.053} \checkmark$$

$$= 1.532 \checkmark$$

(4)

$$\begin{aligned}
 3.2 \quad V &= \frac{1}{3}\pi r^2 h \\
 &= \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h \\
 &= \frac{\pi h^3}{12} \checkmark \\
 \frac{dV}{dt} &= \frac{\pi}{12} \cdot 3h^2 \frac{dh}{dt} \checkmark \\
 \pi &= \frac{\pi}{4}(6)^2 \cdot \frac{dh}{dt} \checkmark \\
 \frac{dh}{dt} &= \frac{1}{9} \text{ m/s} \checkmark
 \end{aligned}$$

(4)  
[15]

**QUESTION 4**

$$\begin{aligned}
 4.1 \quad 4.1.1 \quad & \int \frac{4}{\sec x e^{\sin x}} dx \\
 & \text{let } u = \sin x \\
 & du = \cos x dx \checkmark \\
 & = \int \frac{1}{e^u} du \checkmark \\
 & = \int e^{-u} du \\
 & = -e^{-u} + C \\
 & = e^{-\sin x} \checkmark + C
 \end{aligned}$$

(3)

$$\begin{aligned}
 4.1.2 \quad & \int \frac{(\cos^{-1} 2x)^4}{\sqrt{1-4x^2}} dx \\
 & \text{let } u = \cos^{-1} 2x \\
 & -\frac{du}{2} = \frac{1}{\sqrt{1-4x^2}} dx \checkmark \\
 & = -\frac{1}{2} \int u^4 du \checkmark \\
 & = -\frac{1}{10} u^5 + c \\
 & = -\frac{1}{10} (\cos^{-1} 2x)^5 \checkmark + C
 \end{aligned}$$

(3)

$$\begin{aligned} 4.1.3 \quad & \int \frac{x^3 + 2x^2 + 9x - 17}{x + 4} dx \\ &= \int \left( x^2 - 2x + 17 + \frac{51}{x + 4} \right) dx \checkmark \\ &= \frac{1}{3}x^3 \checkmark - x^2 \checkmark + 17x \checkmark + 51 \ln(x + 4) \checkmark + C \end{aligned} \quad (5)$$

$$\begin{aligned} 4.1.4 \quad & \int \cos 3\pi x \cdot \cos \pi x \, dx \\ &= \frac{1}{2} \int (\cos 4\pi x + \cos 2\pi x) \, dx \checkmark \\ &= \frac{1}{8\pi} \sin 4\pi x \checkmark + \frac{1}{4\pi} \sin 2\pi x \checkmark + C \end{aligned} \quad (3)$$

$$4.1.5 \quad \int x\sqrt{1 + 2x} \, dx$$

$$\begin{aligned} & \text{let } u = 1 + 2x \\ & \frac{du}{2} = dx \checkmark \\ & x = \frac{u - 1}{2} \end{aligned}$$

$$\begin{aligned} &= \int \frac{u - 1}{2} \sqrt{u} \, du \checkmark \\ &= \frac{1}{4} \int \left( u^{\frac{3}{2}} - u^{\frac{1}{2}} \right) du \\ &= \frac{1}{10} u^{\frac{5}{2}} - \frac{1}{6} u^{\frac{3}{2}} + C \\ &= \frac{1}{10} (1 + 2x)^{\frac{5}{2}} \checkmark - \frac{1}{6} (1 + 2x)^{\frac{3}{2}} \checkmark + C \end{aligned} \quad (4)$$

4.2

$$\int \frac{3x}{(2x+1)(x+4)} dx$$

$$\frac{3x}{(2x+1)(x+4)} = \frac{A}{2x+1} + \frac{B}{x+4}$$

$$3x = A(x+4) + B(2x+1) \checkmark$$

$$\text{let } x = -4, \quad B = \frac{12}{7} \checkmark$$

$$\text{or } x = -\frac{1}{2}, \quad A = -\frac{3}{7} \checkmark$$

$$= -\frac{3}{7} \int \frac{1}{2x+1} dx + \frac{12}{7} \int \frac{1}{x+4} dx$$

$$= -\frac{3}{14} \ln(2x+1) \checkmark + \frac{12}{7} \ln(x+4) \checkmark + C$$

(5)  
[23]**QUESTION 5**

5.1

$$\int_0^{\infty} e^{-st} \cdot f(t) dt$$

$$= \frac{1}{\pi} \int_0^{\infty} t e^{-st} dt \checkmark$$

$$\text{let } u = t \quad dv = e^{-st} dt$$

$$du = dt \quad v = -\frac{1}{s} e^{-st} dt \checkmark$$

$$= \frac{1}{\pi} \left[ -\frac{t}{s} e^{-st} \right]_0^{\infty} + \frac{1}{\pi s} \int_0^{\infty} t e^{-st} dt \checkmark$$

$$= -\frac{1}{s\pi} [t e^{-st}]_0^{\infty} - \frac{1}{s^2\pi} [e^{-st}]_0^{\infty} \checkmark$$

$$= -\frac{1}{s\pi} (0 - 0) - \frac{1}{s^2\pi} (0 - 1)$$

$$= \frac{1}{s^2\pi} \checkmark$$

(5)

5.2 5.2.1

$$2x^2 = x + 1$$

$$2x^2 - x - 1 = 0$$

$$(2x + 1)(x - 1) = 0$$

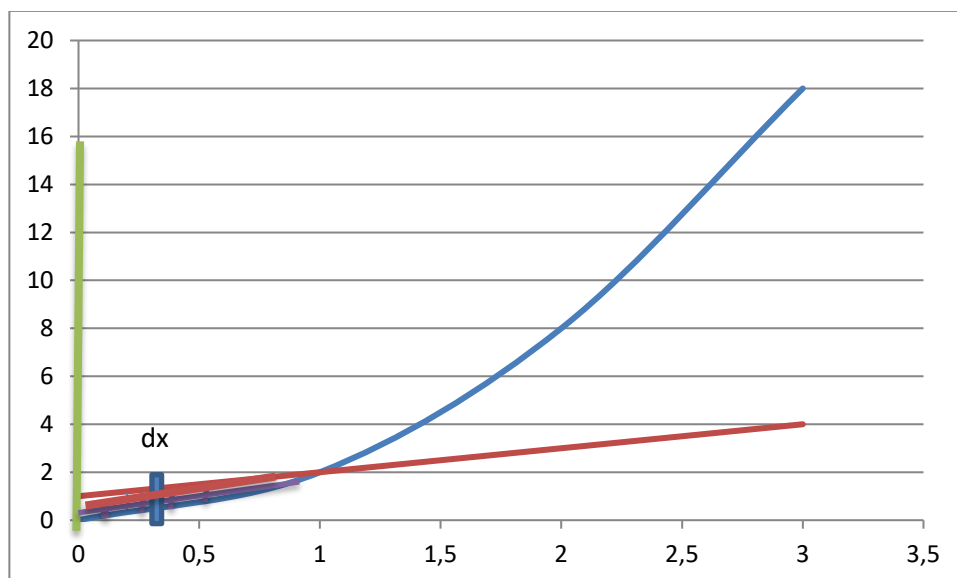
$$x = -\frac{1}{2} \text{ or } x = 1$$

$$y = \frac{1}{2} \text{ or } y = 2$$

Thus, the coordinates of the points of intersection are  $(-\frac{1}{2}; \frac{1}{2})$  ✓ and  $(1; 2)$  ✓

(2)

5.2.2



✓ = 1 mark for indication of enclosed area

✓ = 1 mark for indication of vertical or horizontal strip

(2)

5.2.3

$$A = \int_a^b (y_1 - y_2) dx$$

$$= \int_0^1 (x + 1 - 2x^2) dx \checkmark$$

$$= \left[ \frac{1}{2}x^2 + x - \frac{2}{3}x^3 \right]_0^1 \checkmark$$

$$= \frac{5}{6} \text{ or } 0.833 \text{ units}^2 \checkmark$$

(3)

5.2.4

$$V = \pi \int_a^b (y_1^2 - y_2^2) dx$$

$$V = \pi \int_0^1 [(x+1)^2 - (2x^2)^2] dx$$

$$= \pi \int_0^1 (x^2 + 2x + 1 - 4x^4) dx \checkmark \checkmark$$

$$= \pi \left[ \frac{1}{3}x^3 + x^2 + x - \frac{4}{5}x^5 \right]_0^1 \checkmark$$

$$= \frac{23\pi}{15} \text{ or } 4.817 \text{ units}^3 \checkmark$$

(4)

5.3  $\int y = r^2 dA$

$$= x^2 \cdot 3 dx \checkmark$$

$$= 3 \int_0^6 x^2 dx \checkmark$$

$$= 3 \left[ \frac{x^3}{3} \right]_0^6 \checkmark$$

$$= 216 \text{ units}^4 \checkmark$$

(4)  
[20]

## QUESTION 6

$$6.1 \quad \frac{dy}{dx} = \frac{x(e^{x^2} + 2)}{6y^2}$$

$$6y^2 dy = x(e^{x^2} + 2) dx$$

$$6y^2 dy = (xe^{x^2} + 2x) dx$$

$$2y^3 \checkmark = \frac{1}{2}e^{x^2} + x^2 + C \checkmark$$

$$2(1)^3 = \frac{1}{2}e^{(0)^2} + (0)^2 + c$$

$$c = \frac{3}{2} \checkmark$$

$$2y^3 = \frac{1}{2}e^{x^2} + x^2 + \frac{3}{2} \checkmark$$

(4)

$$6.2 \quad x^2 \cdot \frac{d^2y}{dx^2} = x^6 + x^3 - 2x^2$$

$$\frac{d^2y}{dx^2} = x^4 + x - 2$$

$$\frac{dy}{dx} = \frac{1}{5}x^5 + \frac{1}{2}x^2 - 2x + A \checkmark$$

$$\frac{3}{5} = \frac{1}{5}(1)^5 + \frac{1}{2}(1)^2 - 2(1) + A$$

$$A = 2 \checkmark$$

$$\frac{dy}{dx} = \frac{1}{5}x^5 + \frac{1}{2}x^2 - 2x + 2 \checkmark$$

$$y = \frac{1}{30}x^6 + \frac{1}{6}x^3 - x^2 + 2x + B \checkmark$$

$$\frac{1}{5} = \frac{1}{30}(1)^6 + \frac{1}{6}(1)^3 - (1)^2 + 2(1) + B$$

$$B = -1 \checkmark$$

$$y = \frac{1}{30}x^6 + \frac{1}{6}x^3 - x^2 + 2x - 1 \checkmark$$

(6)

[10]

TOTAL: 100