



**higher education  
& training**

Department:  
Higher Education and Training  
**REPUBLIC OF SOUTH AFRICA**

# **MARKING GUIDELINE**

**NATIONAL CERTIFICATE**

**MATHEMATICS N5**

**29 MARCH 2018**

**This marking guideline consists of 12 pages.**

**QUESTION 1**

1.1      1.1.1       $\lim_{x \rightarrow 0} \left( \frac{\arcsin 4x}{\arcsin 5x} \right)$

$$= \lim_{x \rightarrow 0} \left( \frac{\frac{1}{\sqrt{1-16x^2}} \checkmark}{\frac{1}{1+25x^2} \checkmark} \right) \left[ \frac{0}{0} \right]$$

$$= 1 \checkmark \quad (3)$$

1.2       $\ln y = \lim_{x \rightarrow 0} \frac{\int_0^x e^t dt}{x}$

1.2.1       $\ln y = \lim_{x \rightarrow 0} \frac{e^x - 1}{x} \checkmark \left[ \frac{0}{0} \right]$

$$= \lim_{x \rightarrow 0} \frac{e^x}{1} \checkmark$$

$$= 1 \checkmark \quad (3)$$

1.2.2       $y = e^1$

$$= ,718 \checkmark \quad (1)$$

1.3       $f(x) = \frac{\sin 3x}{x^3 - 4x}$

$$x^3 - 4x = 0$$

$$x(x-2)(x+2) = 0$$

$$x = 0 \checkmark \text{ or } x = 2 \checkmark \text{ or } x = -2 \checkmark \quad (3)$$

**[10]**

**QUESTION 2**

2.1

$$f(x) = -\frac{3}{x^{-7}} = -3x^7$$

$$f(x+h) = -3(x+h)^7$$

$$= -3x^7 - 21x^6h - 63x^5h^2 + \dots \checkmark \checkmark$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

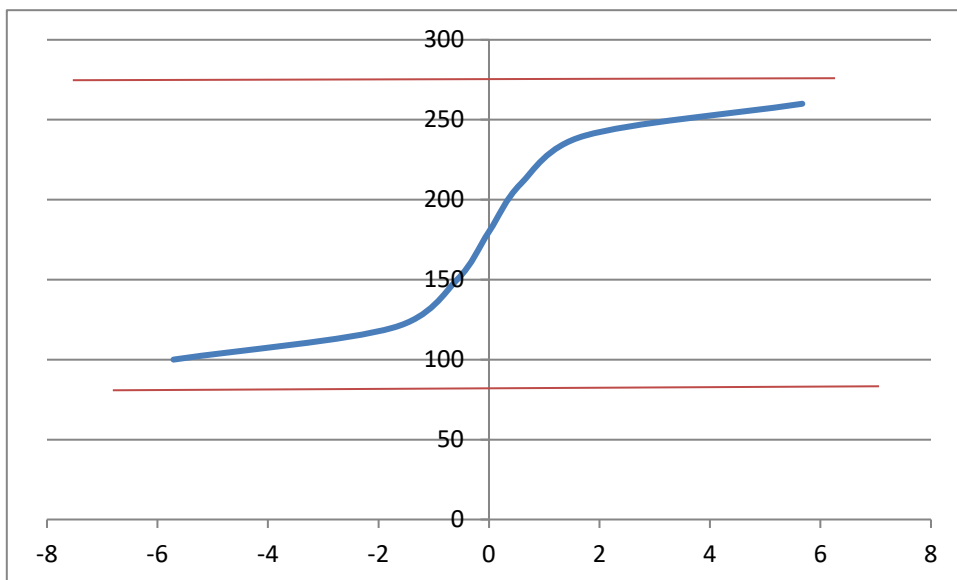
$$= \lim_{h \rightarrow 0} \frac{-3x^7 - 21x^6h - 63x^5h^2 + \dots - (-3x^7)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-21x^6h - 63x^5h^2 + \dots}{h} \checkmark$$

$$= \lim_{h \rightarrow 0} -21x^6 - 63x^5h \checkmark$$

$$= -21x^6 \checkmark \quad (5)$$

2.2  $y = \arctan x$



✓ =1 mark for the shape

✓ =1 mark for the range or restriction

(2)

$$2.3 \quad y = 10^{\log(\cot x)}$$

$$y = \cot x \checkmark \checkmark$$

$$y = \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{\sin x (-\sin x) - \cos x (\cos x) \checkmark}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x \checkmark}{\sin^2 x}$$

$$= -\frac{1}{\sin^2 x} \checkmark$$

$$= -\operatorname{cosec}^2 x \quad (4)$$

$$2.4 \quad 2.4.1 \quad y = \sin(\arcsin x) - \pi^t$$

$$\frac{dy}{dx} = \cos(\arcsin x) \checkmark \times \frac{1}{\sqrt{1-x^2}} \checkmark \quad (2)$$

$$2.4.2 \quad y = \ln^3(\cos^{-1} x)^{(\cos x)^0}$$

$$= \ln^3(\cos^{-1} x) \checkmark$$

$$\frac{dy}{dx} = 3\ln^2(\cos^{-1} x) \checkmark \times \frac{1}{\cos^{-1} x} \checkmark \times -\frac{1}{\sqrt{1-x^2}} \checkmark \checkmark \quad (4)$$

$$2.5 \quad y = x^{\ln x}$$

$$\ln y = \ln x \ln x$$

$$\frac{1}{y} \frac{dy}{dx} \checkmark = \frac{\ln x}{x} + \frac{\ln x}{x} \checkmark \checkmark$$

$$\frac{dy}{dx} = y \left[ \frac{2\ln x}{x} \right] \checkmark$$

$$= x^{\ln x} \left[ \frac{2\ln x}{x} \right] \checkmark \quad (4)$$

$$2.6 \quad y \sin(x^2) = x \sin(y^2)$$

$$y \cos(x^2) \times 2x + \frac{dy}{dx} \cdot \sin(x^2) \checkmark = \sin(y^2) + x \cos(y^2) \times 2y \cdot \frac{dy}{dx} \checkmark$$

$$[\sin(x^2) - 2xy \cos(y^2)] \times \frac{dy}{dx} = \sin(y^2) - 2xy \cos(x^2) \checkmark$$

$$\frac{dy}{dx} = \frac{\sin(y^2) - 2xy \cos(x^2)}{\sin(x^2) - 2xy \cos(y^2)} \checkmark \quad (4)$$

[25]

**QUESTION 3**

3.1 Given:

$$f(x) = x^3 - 7x^2 + 8x - 3$$

3.1.1  $f'(x) = 3x^2 - 14x + 8$

$$(3x - 2)(x - 4) = 0$$

$$x = \frac{2}{3} \text{ or } x = 4$$

$$y = -\frac{13}{27} \text{ or } y = -19$$

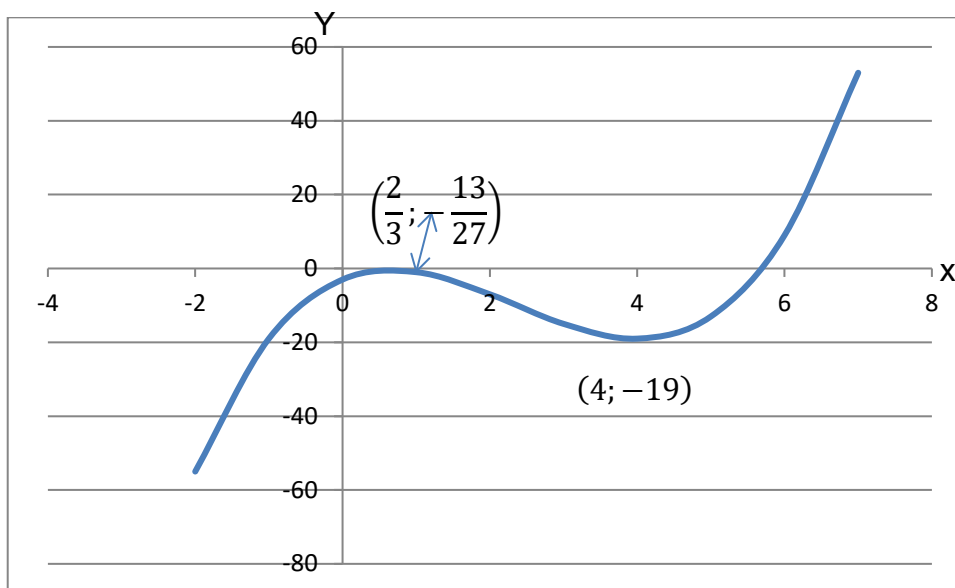
Thus turning points are;  $(\frac{2}{3}; -\frac{13}{27})$  ✓ and  $(4; -19)$  ✓ (2)

3.1.2

x	-2	-1	0	1	2	3	4	5	6
y	-55	-19	-3	-1	-7	-15	-19	-13	9

✓ = 1 mark for any 3 correct answers (2)

3.1.3



✓ = 1 mark for the shape  
 ✓ = 1 mark for the indication of the turning point on the graph (2)

$$\begin{aligned} 3.1.4 \quad & \text{Let } x_0 = 5 \\ & f(2) = -13 \\ & f'(2) = 13 \end{aligned}$$

$$\begin{aligned} x_1 &= 2 - \frac{(-13)}{13} \checkmark \\ &= 6 \checkmark \end{aligned}$$

$$\begin{aligned} f(5) &= 9 \\ f'(5) &= 32 \end{aligned}$$

$$\begin{aligned} x_2 &= 6 - \frac{9}{32} \checkmark \\ &= 5,71875 \checkmark \end{aligned}$$

(4)

$$3.2 \quad A = \pi r^2$$

$$r^2 = \frac{A}{\pi}$$

$$r = \sqrt{\frac{12}{\pi}} \checkmark$$

$$= 1,954 \checkmark$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} \checkmark$$

$$\frac{dr}{dt} = \frac{-0,5}{2\pi(1,954)} \checkmark$$

$$= -0,0407 \checkmark$$

(5)

$$3.3 \quad V = \pi r^2 h$$

$$30 = \pi r^2 h$$

$$h = \frac{30}{\pi r^2} \checkmark$$

$$A = 2\pi r h + \pi r^2$$

$$A = 2\pi r \left( \frac{30}{\pi r^2} \right) + \pi r^2 \checkmark$$

$$= \frac{60}{r} + \pi r^2$$

$$\frac{dA}{dr} = -\frac{60}{r^2} + 2\pi r = 0 \checkmark$$

$$r = 2,1216 \text{ cm} = h \checkmark \checkmark$$

(5)  
[20]

#### QUESTION 4

$$4.1 \quad 4.1.1 \quad \int \frac{\sec^2 \pi x}{1 + \tan \pi x} dx$$

$$\text{Let } u = 1 + \tan \pi x \checkmark$$

$$du = \pi \sec^2 \pi x dx$$

$$= \frac{1}{\pi} \int \frac{1}{u} du \checkmark$$

$$= \frac{1}{\pi} \ln u + c$$

$$= \frac{1}{\pi} \ln (1 + \tan \pi x) + c \checkmark$$

(3)

$$\begin{aligned} 4.1.2 \quad & \int \frac{x}{x+4} dx \\ & \text{Let } u = x + 4 \\ & du = dx \\ & \Rightarrow x = u - 4 \checkmark \\ & = \int \frac{u-4}{u} du \checkmark \\ & = \int \left(1 - \frac{4}{u}\right) du \\ & = u - 4 \ln u + c \\ & = (x + 4) - 4 \ln(x + 4) + c \checkmark \end{aligned} \tag{3}$$

$$\begin{aligned} 4.1.3 \quad & \int \frac{1}{5 + 25x^2} dx \\ & = \frac{1}{25} \checkmark \\ & = \frac{\sqrt{5}}{25} \tan^{-1}(\sqrt{5}x) + c \checkmark \end{aligned} \tag{2}$$

$$\begin{aligned} 4.1.4 \quad & \int \cot^3 x dx \\ & = \int \cot^2 x \cdot \cot x dx \checkmark \\ & = \int (\operatorname{cosec}^2 x - 1) \cot x dx \checkmark \\ & = \int \operatorname{cosec}^2 x \cot x dx - \int \cot x dx \\ & = -\frac{\cot^2 x}{2} \checkmark - \ln(\sec x + \tan x) \checkmark + c \end{aligned} \tag{4}$$

$$4.1.5 \quad \int \sqrt{x} \ln x \, dx$$

$$\text{Let } u = \ln x \quad dv = \sqrt{x} \, dx$$

$$du = \frac{1}{x} \, dx \checkmark \quad v = \frac{2}{3} x^{\frac{3}{2}}$$

$$= \frac{2}{3} x^{\frac{3}{2}} \ln x - \frac{2}{3} \int \sqrt{x} \, dx \checkmark$$

$$= \frac{2}{3} x^{\frac{3}{2}} \ln x - \frac{4}{9} x^{\frac{3}{2}} + c \checkmark$$

(3)

$$4.2 \quad \int \frac{3-x}{x^2-5x} \, dx$$

$$\frac{3-x}{x(x-5)} = \frac{A}{x} + \frac{B}{x-5}$$

$$3-x = A(x-5) + Bx \checkmark$$

$$\text{Let } x = 5, B = -\frac{2}{5} \checkmark$$

$$\text{Let } x = 0, A = -\frac{3}{5} \checkmark$$

$$= -\frac{3}{5} \int \frac{1}{x} \, dx - \frac{2}{5} \int \frac{1}{x-5} \, dx$$

$$= -\frac{3}{5} \ln x \checkmark - \frac{2}{5} \ln(x-5) \checkmark + c$$

(5)  
 [20]

### QUESTION 5

$$5.1 \quad \int_0^1 \frac{x^2}{x^3+1} \, dx$$

$$\text{let } u = x^3 + 1 \checkmark$$

$$\frac{du}{3} = x^2 \, dx$$

$$= \frac{1}{3} \int_{x=0}^{x=1} \frac{1}{u} \, du \checkmark$$

$$= \frac{1}{3} [\ln u]_1^1 \checkmark$$

$$= 0,2310 \checkmark$$

(4)

5.2 Given:  $y = x - 1$  and  $y = (x - 1)^2$

5.2.1  $(x - 1)^2 = x - 1$

$$x^2 - 2x + 1 - x + 1 = 0$$

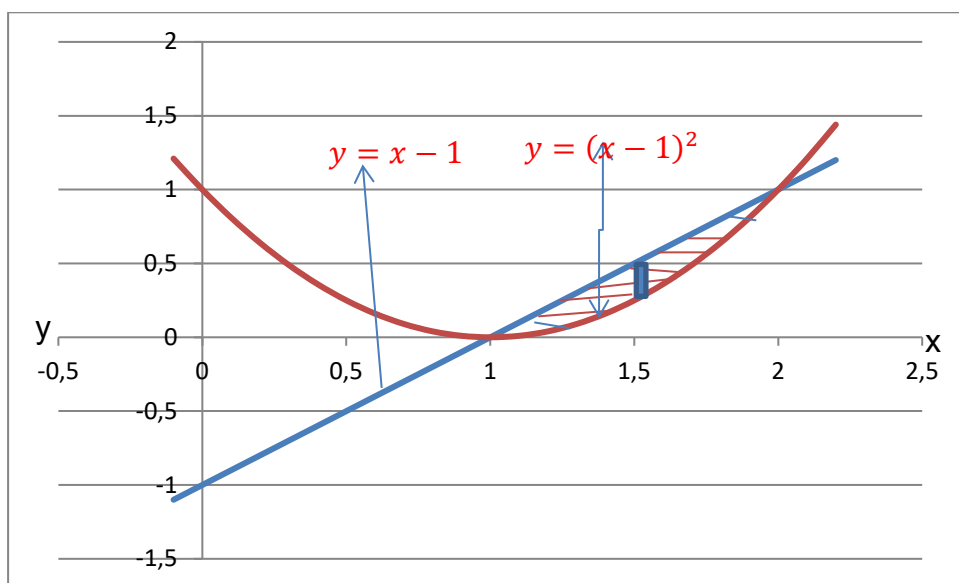
$$(x - 2)(x - 1) = 0$$

$$x = 1 \text{ or } x = 2$$

$$y = 0 \text{ or } y = 1$$

Thus, coordinates of point of intersection are:  $(1; 0)$ ✓ and  $(2; 1)$ ✓ (2)

5.2.2



✓ = 1 mark for indication of enclosed area  
✓ = 1 mark for indication of the vertical or horizontal strip (2)

5.2.3

$$A = \int_1^2 (y_1 - y_2) dx$$

$$= \int_1^2 [(x - 1) - (x - 1)^2] dx$$

$$= \int_1^2 (-x^2 + 3x - 2) dx ✓$$

$$= \left[ -\frac{x^3}{3} + \frac{3}{2}x^2 - 2x \right]_1^2 ✓$$

$$= 0,167 \text{ units}^2 ✓ (3)$$

$$\begin{aligned}
 5.2.4 \quad V &= \pi \int_1^2 A(x) dx \\
 &= \pi \int_1^2 [(x-1)^2 - (x-1)^4] dx \\
 &= \pi \int_1^2 [x^2 - 2x + 1 - (-x^4 - 4x^3 + 6x^2 - 4x + 1)] dx \checkmark \\
 &= \pi \int_1^2 (-x^4 + 4x^3 - 5x^2 + 2x) dx \checkmark \\
 &= \pi \left[ -\frac{x^5}{5} + x^4 - \frac{5}{3}x^3 + x^2 \right]_1^2 \checkmark \\
 &= \frac{2}{15} \pi \text{units}^3 \checkmark \\
 &= 0,4189 \text{units}^3 \checkmark \tag{4}
 \end{aligned}$$

$$\begin{aligned}
 5.3 \quad y &= r^2 dA \\
 &= x^2 \cdot 4 \checkmark \\
 &= 4 \int_0^8 x^2 dx \checkmark \\
 &= 4 \left[ \frac{x^3}{3} \right]_0^8 \checkmark \\
 &= 682,667 \checkmark \tag{4}
 \end{aligned}$$

**[19]**

**QUESTION 6**

6.1  $x dy = y \ln y dx$ , given  $x = 2$  when  $y = e$

$$\frac{dy}{y \ln y} = \frac{dx}{x} \checkmark$$

$$\ln(\ln y) = \ln x + c \checkmark$$

$$\ln(\ln e) = \ln 2 + c$$

$$c = -\ln 2 \checkmark$$

$$\ln(\ln y) = \ln x - \ln 2 \checkmark \quad (4)$$

6.2  $\frac{d^2y}{dx^2} = x^4 - \sin x$

$$\frac{dy}{dx} = \frac{1}{5}x^5 + \cos x + A \checkmark$$

$$y = \frac{1}{30}x^6 + \sin x + Ax + B \checkmark \quad (2)$$

[6]

**TOTAL: 100**