higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T970(**E**)(A4)T
APRIL EXAMINATION

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

4 April 2016 (X-Paper)
09:00–12:00

Scientific calculators may be used.

This question paper consists of 6 pages and 1 formula sheet of 5 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. ALL final answers must be rounded off to THREE decimal places.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Use ONLY blue or black ink.
8. Write neatly and legibly.

QUESTION 1

1.1 Determine the following limits:

1.1.1 $\lim_{x\to 0}\dfrac{e^x}{xe^x - 2x}$ (2)

1.1.2 $\lim_{x\to\frac{\pi}{2}}(\sec x - \tan x)$ (3)

1.2 Determine whether $f(x) = \dfrac{x^3 - 27}{x - 3}$ is continuous at $x = -3$. (2)

[7]

QUESTION 2

2.1 Determine the derivative of $f(x) = \cos x$ from first principles.

HINT: $\lim_{h\to 0}\dfrac{\cosh - 1}{h} = 0$; $\lim_{h\to 0}\dfrac{\sinh}{h} = 1$ (4)

2.2 Determine $\dfrac{dy}{dx}$ in each of the following cases (simplification is not required):

2.2.1 $y = \cos^4(x^2 - 4) + \cos(x^2 - 4)^4$ (4)

2.2.2 $y = \dfrac{3\tan^2\sqrt{1-x}}{2\ln \cos ec 4x}$ (5)

2.2.3 $y = arc\sec(2.4^{3x})$ (2)

2.3 Determine $\dfrac{dy}{dx}$ with the aid of logarithmic differentiation if:

$\arcsin(\sin y) = (x^{e^x})$ (4)

2.4 Given: $3x^4 - xy = 2^{3x}$

2.4.1 Determine the slope $\dfrac{dy}{dx}$ of the tangent at the point: (1;-5). (3)

2.4.2 Hence, determine the equation of the tangent at this point. (2)

[24]

QUESTION 3

3.1 Given: $f(x) = x(x^2 - 5) - 4$

3.1.1 Determine the co-ordinates of the turning points of $f(x)$. (3)

3.1.2 Verify, using a table, that the equation $0 = x(x^2 - 5) - 4$ has a root between the points $x = 2$ and $x = 3$.

Use values on the table: $0 \le x \le 4$ (4)

3.1.3 Hence, make a neat sketch of the graph of the function $f(x)$. (2)

3.1.4 If the positive root of $f(x)$ is estimated as 2,7, use Taylor's/Newton's method to determine a better approximation of this root. (4)

3.2 Two sides of a rectangle are lengthened at a rate of 3 cm/s while the other two sides are being shortened in such a way that the figure remains a rectangle with a constant area of 50 cm^2.

3.2.1 Calculate the rate of change of the perimeter of the rectangle when the length of an increasing side is 7 cm. (5)

3.2.2 Prove that when the rate of change of the perimeter is zero, the figure must be a square. (2)

3.3 A particle moves in a straight line according to the distance formula $s(t) = \sqrt{t}(3 - 3t - t^2)$.

3.3.1 Calculate the velocity of the particle after 3,5 seconds. (4)

3.3.2 Calculate the acceleration after 2 seconds. (3)

[27]

QUESTION 4

4.1 Determine: $\int (e^x + e^{-x})^4 . (e^x - e^{-x})dx$ (2)

4.2 Determine $\int ydx$ in each of the following cases:

4.2.1 $y = \dfrac{\sin x}{\sqrt{1 + \cos x}}$ (3)

4.2.2 $y = x.\sec^2 x$ (3)

4.2.3 $y = \cos 6x . \cos 2x$ (2)

4.2.4 $y = \dfrac{2}{3 + 4x^2}$ (3)

4.3 Determine $\int ydx$ by resolving the integrand into partial fractions:

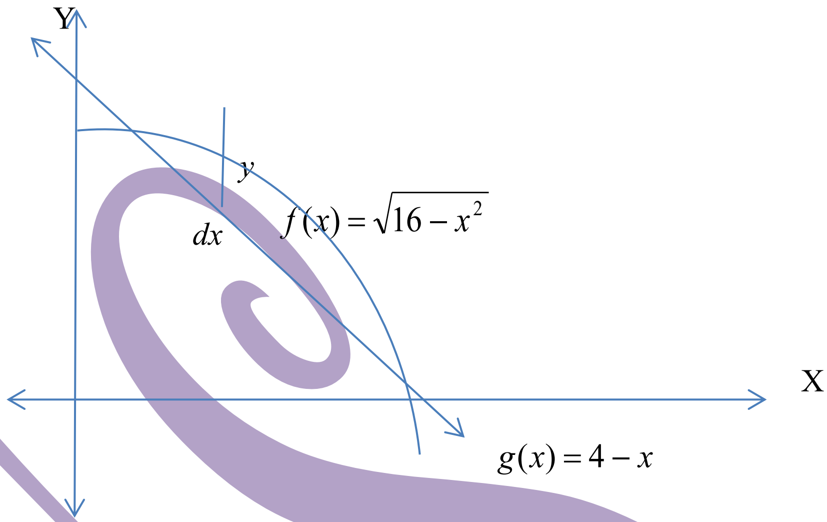
$y = \dfrac{x^3 - 2}{x^4 - 1}$ (5)

4.4 Determine: $\int \dfrac{x^2}{x - 5} dx$ (4)

[22]

QUESTION 5

5.1 Given: The curves $f(x)=\sqrt{16-x^2}$ and $g(x)=4-x$



5.1.1 Calculate the magnitude of the enclosed area. (3)

5.1.2 Calculate the volume generated when this area rotates about the x-axis. (4)

5.2 Prove that $\int_0^{\infty} -5e^{-st}\,dt = -\frac{5}{s}$ (4)

[11]

QUESTION 6

6.1 Solve the differential equation:

$$\frac{dy}{dx}=\frac{\tan^2 y}{\operatorname{cosec}^2 x}$$ (4)

6.2 Determine the particular solution of the differential equation $\frac{d^2y}{dx^2}=-\frac{1}{2}x^2+\frac{3}{2}x+\pi$ for which $y=2$ and $y'=-3$ when $x=1$. (5)

[9]

TOTAL: 100

FORMULA SHEET

Any other applicable formulas may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \mathrm{cosec}^2 x$

$\sin 2A = 2 \sin A.\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = \dfrac{1}{2} - \dfrac{1}{2}\cos 2A$

$\cos^2 A = \dfrac{1}{2} + \dfrac{1}{2}\cos 2A$

$\sin (A \pm B) = \sin A.\cos B \pm \sin B.\cos A$

$\cos (A \pm B) = \cos A.\cos B \mp \sin A.\sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 - \tan A.\tan B}$

$\sin A.\cos B = ½ [\sin(A + B) + \sin (A - B)]$

$\cos A.\sin B = ½ [\sin(A + B) - \sin(A - B)]$

$\cos A.\cos B = ½ [\cos(A + B) + \cos(A - B)]$

$\sin A.\sin B = ½ [\cos(A - B) - \cos(A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\mathrm{cosec}\ x}$; $\cos x = \dfrac{1}{\sec x}$

BINOMIAL THEOREM

$(x + h)^n = x^n + n.x^{n-1}h + \dfrac{n(n-1)}{2!}x^{n-2}h^2 + \ldots$

DIFFERENTIATION

$$e = -\frac{f(a)}{f^{'}(a)}$$

$$r = a + e$$

PRODUCT RULE

$$y = u(x).v(x)$$

$$\frac{dy}{dx} = u.\frac{dv}{dx} + v.\frac{du}{dx}$$

$$= u.v' + v.u'$$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$$

$$= \frac{v.u' - u.v'}{v^2}$$

CHAIN RULE

$$y = f(u(x))$$

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
ax^n	nax^{n-1}	$\frac{ax^{n+1}}{n+1} + c$
a	0	$ax + c$
e^x	e^x	$e^x + c$
a^x	$a^x.\ln a$	$\frac{a^x}{\ln a} + c$
$\log_e x$	$\frac{1}{x}$	—
$\log_a x$	$\frac{1}{x \ln a}$	—
$\sin x$	$\cos x$	$-\cos x + c$
$\cos x$	$-\sin x$	$\sin x + c$
$\tan x$	$\sec^2 x$	$\ln(\sec x) + c$
$\cot x$	$-\text{cosec}^2 x$	$\ln(\sin x) + c$
$\sec x$	$\sec x.\tan.$	$\ln[\sec x + \tan x] + c$
$\text{cosec}\, x$	$-\text{cosec}\, x.\cot x$	$\ln[\text{cosec}\, x - \cot x] + c$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	—
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	—
$\tan^{-1} x$	$\frac{1}{1+x^2}$	—
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	—
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$	—
$\text{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$	—

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{a^2 - x^2}}$	—	$\sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{a^2 + x^2}$	—	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{x\sqrt{x^2 - a^2}}$	—	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
$\sqrt{a^2 - x^2}$	—	$\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + c$
$\frac{1}{x^2 - a^2}$	—	$\frac{1}{2a} ln\left(\frac{x-a}{x+a}\right) + c$
$\frac{1}{a^2 - x^2}$	—	$\frac{1}{2a} ln\left(\frac{a+x}{a-x}\right) + c$

INTEGRATION

$$\int f(x).g'(x) = f(x).g(x) - \int f'(x).g(x)dx$$

$$\int \frac{f'(x)}{f(x)}dx = \ln f(x) + c$$

$$\int [f(x)]^n . f'(x)dx = \frac{f(x)^{n+1}}{n+1} + c$$

$$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

$$\frac{f(x)}{(x+a)^n} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \ldots + \frac{Z}{(x+a)^n}$$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y dx; \quad A_x = \int_a^b (y_2 - y_1)dx$$

$$A_y = \int_a^b x dy; \quad A_y = \int_a^b (x_2 - x_1)dy$$

VOLUMES

$$V_x = \pi\int_a^b y^2 dx \; ; V_x = \pi\int_a^b \left(y_2{}^2 - y_1{}^2\right) dx$$

$$V_y = \pi\int_a^b x^2 dy \; ; V_y = \pi\int_a^b \left(x_2{}^2 - x_1{}^2\right) dy$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \; ; I_y = \int_a^b r^2 dA$$

MOMENTS OF INERTIA

Mass = density × volume

$M = pV$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho\int_a^b r^2 dV$