higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

23 November 2023 (X-paper)
09:00–12:00

Scientific calculators and drawing instruments may be used.

This question paper consists of 6 pages and a formula sheet of 5 pages.

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Use only a blue or black pen.
5. Show all intermediate steps and simplify where possible.
6. All final answers must be rounded off to THREE decimal places.
7. Questions may be answered in any order, but subsections of questions must be kept together.
8. Sketches must be large, neat and fully labelled.
9. Scientific calculators may be used.
10. Write neatly and legibly.

QUESTION 1

1.1 Determine the following limits:

1.1.1

$$y = \lim_{x \to \infty} \frac{-4x^2 + 7x + 3}{x^2 - 5}$$ (3)

1.1.2

$$y = \lim_{x \to 9} \frac{\cos\left(\frac{\pi}{2} + 9 - x\right)}{\ln(x - 8)}$$ (2)

1.2 Determine the value(s) of x for which $f(x)$ is discontinuous if

$$y = \frac{2^{-x+3} + 1}{3^{-x} - 1}$$ (1)

[6]

QUESTION 2

2.1 Show that if $y = arc\sin x$, then

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$ (3)

2.2 Determine $\frac{dy}{dx}$ in each of the following cases:
(Simplification NOT required)

2.2.1

$$y = \sin^3 \sqrt{x} + \sqrt[3]{\sin x}$$ (5)

2.2.2

$$y = \tan\left[\cos\left(\sqrt{\tan x^3}\right)\right]$$ (5)

2.3 Calculate $\frac{dy}{dx}$ if $y = (2x^4 + 1)^{\tan 3x^2}$ with aid of logarithmic differentiation. (4)

2.4 Given the implicit function:

$$x^2y + y^4 = 4 + 2x$$

2.4.1 Determine $\frac{dy}{dx}$ (4)

2.4.2 Determine the gradient of the function of the point $(-1;\ 1)$ (1)

[22]

QUESTION 3

3.1 Given: $f(x) = x^3 + 4x^2 + 3x - 5$

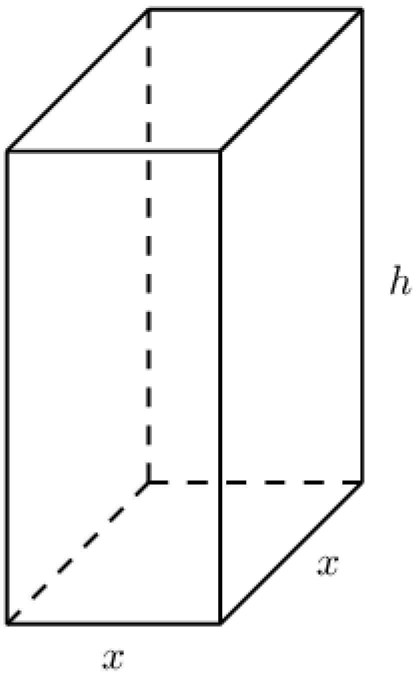
3.1.1 Determine the coordinates of the turning points of $f(x)$. (3)

3.1.2 Draw up a table of x and $f(x)$, where x is ranging from $x = -4$ to $x = 1$. (2)

3.1.3 Draw a neat graph of $f(x)$ between these values. Show the turning points on it. (2)

3.1.4 Use the table and the graph to estimate a value for the best root between $x = 0$ and $x = 1$ of the equation $x^3 + 4x^2 + 3x - 5 = 0$ and then use Taylor's/Newton's method TWICE to determine a better approximation of this root. (Root must be correct to THREE decimal figures.) (4)

3.2 A rectangular juice container, made from cardboard, has a square base and holds 750 cm^3 of juice. The container has a specially designed top that folds to close the container. The cardboard needed to fold the top of the container is twice the cardboard needed for the base, which only needs a single layer of cardboard. Determine the dimensions of the container so that the area of the cardboard used is minimised.



(5)

3.3 Air is being pumped into a spherical balloon at a constant rate of 3 cm^3/s. How fast is the radius of the balloon increasing when the radius reaches 2 cm?

HINT: $V = \frac{4}{3}\pi r^3$

(4)
[20]

QUESTION 4

4.1 Determine $\int y\,dx$ in each of the following cases:

4.1.1
$$y = \frac{\operatorname{cosec} x \cot x}{1 + \operatorname{cosec}^2 x}$$
(3)

4.1.2
$$y = \sin^2\left(\frac{5}{\pi}x\right)$$
(3)

4.1.3
$$y = \cos^5 x \sin^3 x$$
(4)

4.1.3
$$y = \frac{x^3 - 2x^2 - 5x + 6}{x + 3}$$
(5)

4.1.4
$$y = \log_2 x$$
HINT: $u\ or\ f(x) = \log_2 x$
(4)

4.2 Determine $\int y\,dx$ by resolving the integral into partial fractions:

$$y = \frac{x - 10}{x^2 + x - 2}$$
(5)

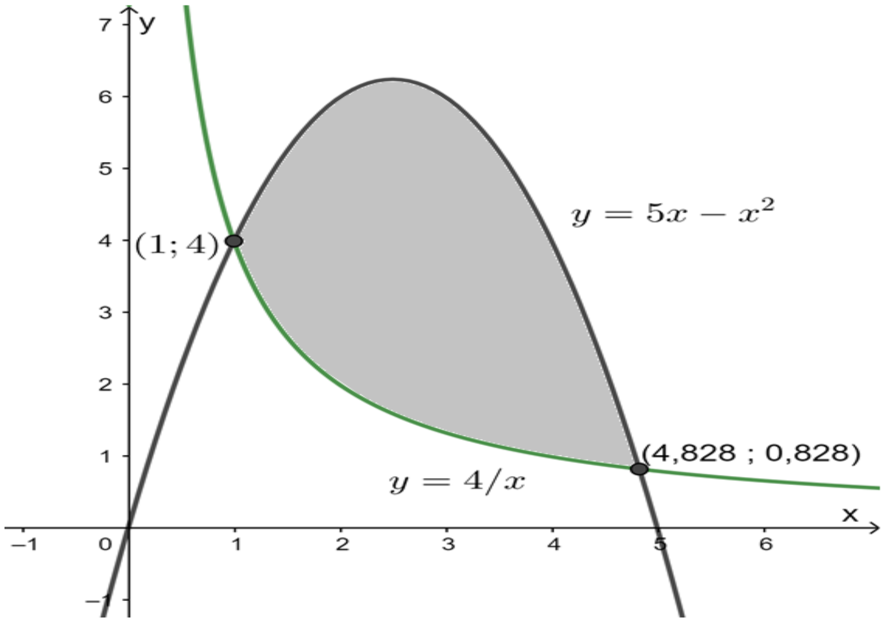
[24]

QUESTION 5

5.1 Given that $F(a) = -3$ and $F(b) = 7$, determine the value of $\int_a^b f(t)dt$. (2)

5.2 By showing all the integration steps, determine $\int_0^\infty e^{-st}.f(t)dt$ if $f(t) = -\pi t$. (5)

5.3 Given below is a graph with the points of intersection and the area bounded by the curves $y = 5x - x^2$ and $y = \frac{4}{x}$.



5.3.1 Calculate the magnitude of the area in QUESTION 5.2.2. (3)

5.3.2 Calculate the volume of the solid of revolution formed when the area in QUESTION 5.3.1 is rotated about the x-axis. (5)

5.4 Determine from first principles the second moment of area of a rectangular lamina with respect to a reference axis parallel to one side of the lamina that bisects it. (5)

[20]

QUESTION 6

6.1 Determine the general solution of:

$$\frac{dy}{dx} = xy - 3x - 2y + 6$$ (3)

6.2 Determine the particular solution of:

$$\tan x + \sec x.\frac{d^2y}{dx^2} = \sec x.3^{-x}, \quad \text{if} \quad \left.\frac{dy}{dx}\right|_{x=0} = \frac{1}{\ln 3} \text{ and } y(0) = -1$$ (5)

[8]

TOTAL: 100

MATHEMATICS N5

FORMULA SHEET

Any applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$

$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan (A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin A \cos B = \frac{1}{2} [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = \frac{1}{2} [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = \frac{1}{2} [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)]$

$\tan x = \dfrac{\sin x}{\cos x}$; $\sin x = \dfrac{1}{\text{cosec}\, x}$; $\cos x = \dfrac{1}{\sec x}$

BINOMIAL THEOREM

$$(x+h)^n = x^n + nx^{n-1}\,h + \frac{n(n-1)}{2!}x^{n-2}\,h^2 + \ldots$$

DIFFERENTIATION

$$e = -\frac{f(a)}{f'(a)}$$

$$r = a + e$$

PRODUCT RULE

$y = u(x)\,.\,v(x)$

$$\frac{dy}{dx} = u\,.\,\frac{dv}{dx} + v\,.\,\frac{du}{dx}$$

$$= u \cdot v' + v \cdot u'$$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\,.\,\frac{du}{dx} - u\,.\,\frac{dv}{dx}}{v^2}$$

$$= \frac{v \cdot u' - u \cdot v'}{v^2}$$

CHAIN RULE

$y = f(u(x))$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
ax^n	nax^{n-1}	$\frac{ax^{n+1}}{n+1} + c$
a	0	$ax + c$
e^x	e^x	$e^x + c$
a^x	$a^x.\ln a$	$\frac{a^x}{\ln a} + c$
$\log_e x$	$\frac{1}{x}$	—
$\log_a x$	$\frac{1}{x \ln a}$	—
$\sin x$	$\cos x$	$-\cos x + c$
$\cos x$	$-\sin x$	$\sin x + c$
$\tan x$	$\sec^2 x$	$\ln(\sec x) + c$
$\cot x$	$-\mathrm{cosec}^2 x$	$\ln(\sin x) + c$
$\sec x$	$\sec x \cdot \tan x$	$\ln[\sec x + \tan x] + c$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cdot \cot x$	$\ln(\mathrm{cosec} x - \cot x) + c$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	—
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	—
$\tan^{-1} x$	$\frac{1}{1+x^2}$	—
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	—
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$	—
$\mathrm{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$	—

$f(x)$	$\frac{d}{dx}\, f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{a^2 - x^2}}$	—	$\sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{a^2 + x^2}$	—	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{x\sqrt{x^2 - a^2}}$	—	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
$\sqrt{a^2 - x^2}$	—	$\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + c$
$\frac{1}{x^2 - a^2}$	—	$\frac{1}{2a}\, ln\left(\frac{x-a}{x+a}\right) + c$
$\frac{1}{a^2 - x^2}$	—	$\frac{1}{2a}\, ln\left(\frac{a+x}{a-x}\right) + c$

INTEGRATION

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln\, f(x) + c$$

$$\int [f(x)]^n\, f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

$$\frac{f(x)}{(x+a)^n} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \cdots \frac{Z}{(x+a)^n}$$

APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y dx\, ;\; A_x = \int_a^b (y_1 - y_2)\, dx$$

$$A_y = \int_a^b x dy\, ;\; A_y = \int_a^b (x_1 - x_2)\, dy$$

VOLUMES

$$V_x = \pi \int_a^b y^2 dx \; ; V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx$$

$$V_y = \pi \int_a^b x^2 dy \; ; V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2 dA \; ; I_y = \int_a^b r^2 dA$$

MOMENTS OF INERTIA

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$