



# higher education & training

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Department:  
Higher Education and Training  
**REPUBLIC OF SOUTH AFRICA**

## **MARKING GUIDELINE**

**NATIONAL CERTIFICATE**

**MATHEMATICS N5**

**23 November 2023**

**This marking guideline consists of 12 pages.**

## QUESTION 1

$$1.1 \quad 1.1.1 \quad y = \lim_{x \rightarrow \infty} \frac{-4x^2 + 7x + 3}{x^2 - 5} : \frac{\infty}{\infty}$$

$$= \lim_{x \rightarrow \infty} \frac{-8x + 7}{2x} \checkmark : \frac{\infty}{\infty}$$

$$= \lim_{x \rightarrow \infty} \frac{-8}{2} \checkmark$$

$$= -4 \checkmark$$

(3)

1.1.2

$$y = \lim_{x \rightarrow 9} \frac{\cos\left(\frac{\pi}{2} + 9 - x\right)}{\ln(x - 8)} \frac{0}{0}$$

$$= \lim_{x \rightarrow 9} \frac{-\sin\left(\frac{\pi}{2} + 9 - x\right) \times (-1)}{\frac{1}{x - 8}} \checkmark$$

$$= 1 \checkmark$$

(2)

$$1.2 \quad y = \frac{2^{-x+3} + 1}{3^{-x} - 1}$$

$$3^{-x} - 1 = 0$$

$$3^{-x} = 1$$

$$3^{-x} = 3^0$$

$$x = 0 \checkmark$$

(1)  
[6]

**QUESTION 2**

2.1  $y = \arcsin x$

$\sin y = x$

$\cos y \times \frac{dy}{dx} = 1 \checkmark$

$\frac{dy}{dx} = \frac{1}{\cos y}$

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 $\cos^2 y = 1 - \sin^2 y$

$\cos y = \sqrt{1 - \sin^2 y} \checkmark$ 

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$= \frac{1}{\sqrt{1 - \sin^2 y}} \checkmark$

$= \frac{1}{\sqrt{1 - x^2}}$

(3)

2.2 2.2.1  $y = \sin^3 \sqrt{x} + \sqrt[3]{\sin x}$

$\frac{dy}{dx} = 3 \sin^2 \sqrt{x} \checkmark \times \cos \sqrt{x} \checkmark \times \frac{1}{2} (x)^{-\frac{1}{2}} \checkmark + \frac{1}{3} (\sin x)^{-\frac{2}{3}} \checkmark \times \cos x \checkmark$  (5)

2.2.2  $y = \tan \left[ \cos \left( \sqrt{\tan x^3} \right) \right]$

$\frac{dy}{dx} = \sec^2 \left[ \cos \left( \sqrt{\tan x^3} \right) \right] \checkmark \times \left[ -\sin \left( \sqrt{\tan x^3} \right) \right] \checkmark \times \frac{1}{2} (\tan x^3)^{-\frac{1}{2}} \checkmark$   
 $\times \sec^2 x^3 \checkmark \times 3x^2 \checkmark$  (5)

2.3  $y = (2x^4 + 1)^{\tan 3x^2}$

$\ln y = \tan 3x^2 \ln(2x^4 + 1)$

$\frac{1}{y} \frac{dy}{dx} \checkmark = \sec^2 3x^2 \times \ln(2x^4 + 1) \times 6x \checkmark + \tan 3x^2 \times \frac{1}{2x^4 + 1} \times 8x^3 \checkmark$

$\frac{dy}{dx} = y \left[ 6x \sec^2 3x^2 \times \ln(2x^4 + 1) + \frac{8x^3 \tan 3x^2}{2x^4 + 1} \right] \checkmark$

$= (2x^4 + 1)^{\tan 3x^2} \left[ 6x \sec^2 3x^2 \times \ln(2x^4 + 1) + \frac{8x^3 \tan 3x^2}{2x^4 + 1} \right]$  (4)

$$2.4 \quad 2.4.1 \quad x^2y + y^4 = 4 + 2x$$

$$2xy + x^2 \frac{dy}{dx} + 4y^3 \frac{dy}{dx} \checkmark = 0 + 2 \checkmark$$

$$x^2 \frac{dy}{dx} + 4y^3 \frac{dy}{dx} = 2 - 2xy \checkmark$$

$$\frac{dy}{dx} (x^2 + 4y^3) = 2 - 2xy$$

$$\frac{dy}{dx} = \frac{2 - 2xy}{x^2 + 4y^3} \checkmark$$

(4)

$$2.4.2 \quad \frac{dy}{dx} = \frac{2 - 2(-1)(1)}{(-1)^2 + 4(1)^3} = \frac{4}{5} \checkmark$$

(1)

[22]

**QUESTION 3**

$$3.1 \quad 3.1.1 \quad f(x) = x^3 + 4x^2 + 3x - 5$$

$$f'(x) = 3x^2 + 8x + 3 = 0 \checkmark$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(3)}}{2(3)}$$

$$x = -2,215 \text{ or } x = -0,451$$

$$y = -2,887 \quad y = -5,631$$

Therefore, the coordinates of the turning points are  $(-2,215; -2,887) \checkmark$   
and  $(-0,451; -5,631) \checkmark$

(3)

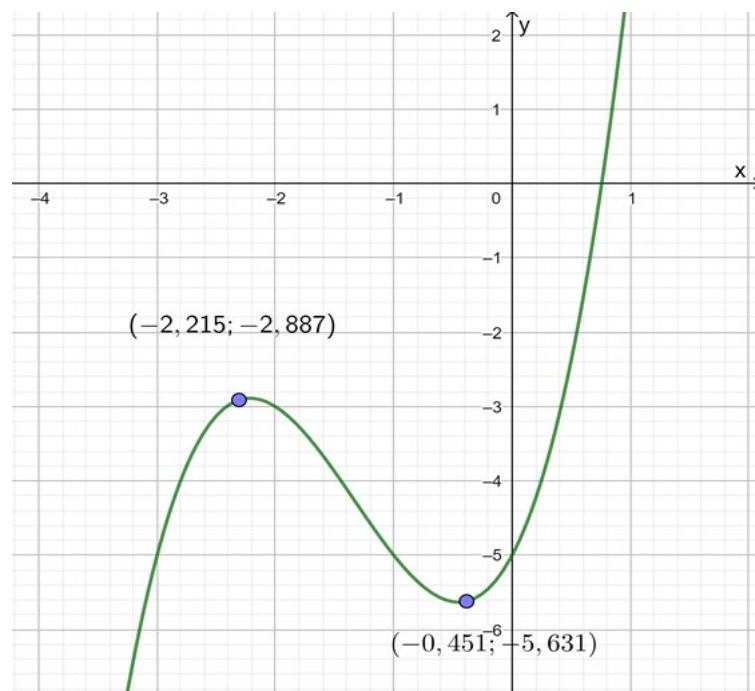
3.1.2

$x$	-4	-3	-2	-1	0	1
$y$	-17	-5	-3	-5	-5	3

1 mark for every three correct answers

(2)

3.1.3



1 mark for the shape

1 mark for the indication of the turning point on the graph

(2)

3.1.4 Let  $x_0 = 0,8$

$$f(0,8) = 0,472$$

$$f'(0,8) = 11,32\checkmark$$

$$x_1 = 0,8 - \frac{0,472}{11,32}\checkmark$$

$$= 0,758\checkmark$$

$$f(0,758) = 0,011$$

$$f'(0,758) = 10,792$$

$$x_2 = 0,758 - \frac{0,011}{10,792}$$

$$= 0,757\checkmark$$

(4)

$$3.2 \quad V = l \times b \times l$$

$$750 = x^2 h$$

$$h = \frac{750}{x^2} \checkmark$$

$A = \text{area of sides} + \text{area of base} + \text{area of top}$

$$A = 4xh + x^2 + 2x^2$$

$$= 4x \left( \frac{750}{x^2} \right) + x^2 + 2x^2$$

$$A = \frac{3\,000}{x} + 3x^2 \checkmark$$

$$A' = -\frac{3\,000}{x^2} + 6x = 0 \checkmark$$

$$-3\,000 + 6x^3 = 0$$

$$x^3 = 500$$

$$x = 7,937 \text{ cm} \checkmark \text{ and } h = 11,906 \text{ cm} \checkmark$$

(5)

$$3.3 \quad V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \checkmark$$

$$3 \text{ cm}^3/\text{s} = 4\pi(2 \text{ cm})^2 \frac{dr}{dt} \checkmark$$

$$\frac{dr}{dt} = \frac{3 \text{ cm}^3/\text{s}}{4\pi(2 \text{ cm})^2} \checkmark = 0,06 \text{ cm/s} \checkmark$$

(4)

**[20]**

**QUESTION 4**

4.1 4.1.1

$$\int \frac{\operatorname{cosec} x \cot x}{1 + \operatorname{cosec}^2 x} dx$$

$$\text{Let } u = \operatorname{cosec} x$$

$$du = -\operatorname{cosec} x \cot x dx \checkmark$$

$$-du = \operatorname{cosec} x \cot x dx$$

$$= -\int \frac{1}{1 + u^2} du \checkmark$$

$$= \cot^{-1} u + C$$

$$= \cot^{-1}(\operatorname{cosec} x) + C \checkmark$$

(3)

4.1.2

$$\int \sin^2\left(\frac{5}{\pi}x\right) dx$$

$$= \frac{1}{2} \int \left[1 - \cos\left(\frac{10}{\pi}x\right)\right] dx \checkmark$$

$$= \frac{1}{2}x \checkmark - \frac{\pi}{20} \sin\left(\frac{10}{\pi}x\right) \checkmark + C$$

(3)

4.1.3

$$\int \cos^5 x \sin^3 x dx$$

$$= \int \cos^4 x \cdot \cos x \sin^3 x dx$$

$$= \int (1 - \sin^2 x) \cos x \sin^3 x dx \checkmark$$

$$= \int (\cos x \sin^5 x - \cos x \sin^7 x) dx \checkmark$$

$$= \frac{1}{6} \sin^6 x \checkmark - \frac{1}{8} \sin^8 x \checkmark + C$$

(4)

$$\begin{aligned}
 4.1.4 \quad & \int \frac{x^3 - 2x^2 - 5x + 6}{x + 3} dx \\
 & = \int \left( x^2 - 5x + 10 - \frac{24}{x + 3} \right) dx \checkmark \\
 & = \frac{1}{3}x^3 \checkmark - \frac{5}{2}x^2 \checkmark + 10x \checkmark - 24 \ln(x + 3) \checkmark + C \quad (5)
 \end{aligned}$$

$$4.1.5 \quad \int \log_2 x \, dx$$

$Let \, u = \log_2 x \quad dv = dx$ $du = \frac{1}{\ln 2} \frac{1}{x} dx \quad v = x \checkmark$
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$$\begin{aligned}
 \int \log_2 x \, dx & = x \log_2 x - \int \frac{1}{\ln 2} \times \frac{1}{x} \times x dx \checkmark \\
 & = x \log_2 x \checkmark - \frac{1}{\ln 2} x \checkmark + C \quad (4)
 \end{aligned}$$

$$4.2 \quad \int \frac{x - 10}{x^2 + x - 2} dx$$

$$\frac{x - 10}{(x + 2)(x - 1)} = \frac{A}{x + 2} + \frac{B}{x - 1}$$

$$x - 10 = A(x - 1) + B(x + 2) \checkmark$$

$Let \, x = 1; \quad B = -3 \checkmark$ $or$ $x = -2; \quad A = 4 \checkmark$
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$$\begin{aligned}
 & \int \left( \frac{4}{x + 2} + \frac{-3}{x - 1} \right) dx \\
 & = 4 \ln(1 - 2x) \checkmark - 3 \ln(x + 1) \checkmark + C \quad (5)
 \end{aligned}$$

[24]

## QUESTION 5

5.1

$$\int_a^b f(t) dt = F(b) - F(a) \checkmark$$

$$= 7 - -3 = 10 \checkmark \quad (2)$$

5.2

$$\int_0^{\infty} e^{-st} \cdot f(t) dt$$

$$= -\pi \int_0^{\infty} t e^{-st} dt \checkmark$$

Let  $u = t$     $dv = e^{-st} dt$   
 $du = dt$     $v = -\frac{1}{s} e^{-st} dt \checkmark$

$$= -\pi \left[ -\frac{t}{s} e^{-st} \right]_0^{\infty} - \frac{\pi}{s} \int_0^{\infty} e^{-st} dt \checkmark$$

$$= \frac{\pi}{s} [t e^{-st}]_0^{\infty} + \frac{\pi}{s^2} [e^{-st}]_0^{\infty} \checkmark$$

$$= \frac{\pi}{s} (0 - 0) + \frac{\pi}{s^2} (0 - 1)$$

$$= -\frac{\pi}{s^2} \checkmark \quad (5)$$

5.3.1  $A = \int_a^b (y_1 - y_2) dx$

$$= \int_1^{4,828} \left( 5x - x^2 - \frac{4}{x} \right) dx \checkmark$$

$$= \left[ \frac{5}{2} x^2 - \frac{1}{3} x^3 - 4 \ln x \right]_1^{4,828} \checkmark$$

$$= 12,297 \text{ units}^2 \quad \checkmark \quad (3)$$

$$\begin{aligned}
 5.3.2 \quad V &= \pi \int_a^b (y_1^2 - y_2^2) dx \\
 &= \pi \int_1^{4,828} \left[ (5x - x^2)^2 - \left(\frac{4}{x}\right)^2 \right] dx \checkmark \\
 &= \pi \int_1^{4,828} \left( x^4 - 10x^3 + 25x^2 - \frac{16}{x^2} \right) dx \checkmark \checkmark \\
 &= \pi \left[ \frac{1}{5}x^5 - \frac{5}{2}x^4 + \frac{25}{3}x^3 + \frac{16}{x} \right]_1^{4,828} \checkmark \\
 &= 268,314 \text{ units}^3 \checkmark
 \end{aligned}$$

(5)

$$5.4 \quad I_y = r^2 A$$

$$dI_y = r^2 dA; \quad dA = b dx \checkmark$$

$$I_y = x^2 b dx \checkmark$$

$$= b \int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 dx \checkmark$$

$$= ba^2 \left[ -\frac{x^3}{3} \right]_{-\frac{a}{2}}^{\frac{a}{2}} \checkmark$$

$$= b \left[ \frac{a^3}{12} \right]$$

$$= \frac{a^3 b}{12} \checkmark$$

(5)

**[20]**

**QUESTION 6**

6.1  $\frac{dy}{dx} = xy - 3x - 2y + 6$

$$\frac{dy}{dx} = (y - 3)(x - 2)$$

$$\frac{1}{y - 3} dy = (x - 2) dx \checkmark$$

$$\ln(y - 3) \checkmark = \frac{1}{2}x^2 - 2x + C \checkmark$$

(3)

6.2  $\tan x + \sec x \cdot \frac{d^2y}{dx^2} = \sec x \cdot 3^{-x}$ , if  $\frac{dy}{dx} \Big|_{x=0} = 1$  and  $y(0) = \frac{2}{(\ln 3)^2}$

$$\sin x + \frac{d^2y}{dx^2} = 3^{-x}$$

$$\frac{d^2y}{dx^2} = 3^{-x} - \sin x$$

$$\frac{dy}{dx} = -\frac{3^{-x}}{\ln 3} + \cos x + A \checkmark$$

$$1 = -\frac{3^{-0}}{\ln 3} + \cos 0 + A$$

$$A = \frac{1}{\ln 3} \checkmark$$

$$\frac{dy}{dx} = -\frac{3^{-x}}{\ln 3} + \cos x + \frac{1}{\ln 3}$$

$$y = \frac{3^{-x}}{(\ln 3)^2} + \sin x + \frac{1}{\ln 3}x + B \checkmark$$

$$\frac{2}{(\ln 3)^2} = \frac{3^{-0}}{(\ln 3)^2} + \sin 0 + \frac{1}{\ln 3} \cdot 0 + B$$

$$B = \frac{1}{(\ln 3)^2} \checkmark$$

$$y = \frac{3^{-x}}{(\ln 3)^2} + \sin x + \frac{1}{\ln 3}x + \frac{1}{(\ln 3)^2} \checkmark$$

(5)

[8]

**TOTAL: 100**