TEACHERS WITHOUT BORDERS PROGRAMME

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In Bill Gates words, at the Mandela Day 'Living Together' address: "Maintaining the quality of this country's higher education system while expanding access to more students will not be easy. But it's critical to South Africa's future" – working together, we can help achieve this."

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November Examination

Grade 10
Mathematics Paper 2

Time: 2,5 hours

Total: 100

Session: 1

MEMO

Date: 16 November 2013

	Section A				Section B				
Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total**
COG Level	1, 2	1, 2	1, 2	1, 2	3, 4	3, 4	3, 4	3, 4	
Score									
Max Mark	**8**	**8**	**20**	**15**	**7**	**7**	**22**	**13**	**100**
Marker Signature									

** — 1 mark once for rounding
— 1 mark " " units/symbols } indicate on front of script

Instructions:

1. This paper consists of 8 questions and 19 pages including a formula sheet. Please check that your script is complete.
2. Complete all questions neatly and thoroughly.
3. Show all calculations in order to earn marks for method.
4. A calculator may be used unless otherwise stated. Round off your answers to 2 decimal places.

Section A

Question 1

1.1 The table below reflects the size of each of the nine provinces in South Africa, as provided on the site http://www.southafrica.info/about/facts.htm#provinces.

Province	Area (km^2)
Eastern Cape	169 580
Free State	129 480
Gauteng	x
KwaZulu-Natal	92 100
Limpopo	123 910
Mpumalanga	79 490
Northern Cape	361 830
North West	116 320
Western Cape	129 370
TOTAL AREA	1 219 090

1.1.1 Calculate the value of x (2)

$x = 1219090 - (\text{Sum of other values})$ ✓

$= 17010 \text{ km}^2$ ✓

1.1.2 What is the median province in South Africa? (1)

$n = 9$

Median position $= \frac{1}{2}(9+1) = 5^{th}$ largest

$Q_2 = 123910 \text{ km}^2$ ✓

1.1.3 Determine the mean province size. (2)

$\text{Mean} = \frac{1219090}{9} = 135\,454,44 \text{ km}^2$ ✓ ✓

1.2 Complete the table, indicating how to calculate the angle size of the sector representing each eye colour in a pie chart. (Do not draw the pie chart.) (3)

✓ one full calculation is sufficient

EYE COLOUR	FREQUENCY	ANGLE
Blue	12	$\frac{12}{30} \times 360^\circ = 144^\circ$ ✓
Brown	9	$\frac{9}{30} \times 360^\circ = 108^\circ$ ✓
Dark Brown	4	$\frac{4}{30} \times 360^\circ = 48^\circ$ ✓
Green	5	$\frac{5}{30} \times 360^\circ = 60^\circ$ ✓
TOTAL	**30**	360°

Question 2

ALL the answers in this question have to be determined by analytical methods.

2.1.1 Determine midpoint K of the line segment DF, given point D(7; 5) and F(1; 2). (2)

$$K\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$
$$= K\left(\frac{7+1}{2} ; \frac{5+2}{7}\right) \checkmark$$
$$= K(4 ; 3\tfrac{1}{2}) \checkmark$$

2.1.2 What is the gradient of the line $x = 2$? (1)

m does not exist — vertical line

2.2.1 (3)

The figure ABCD is a trapezium. Determine the value of a.

$m_{AB} = m_{DC}$

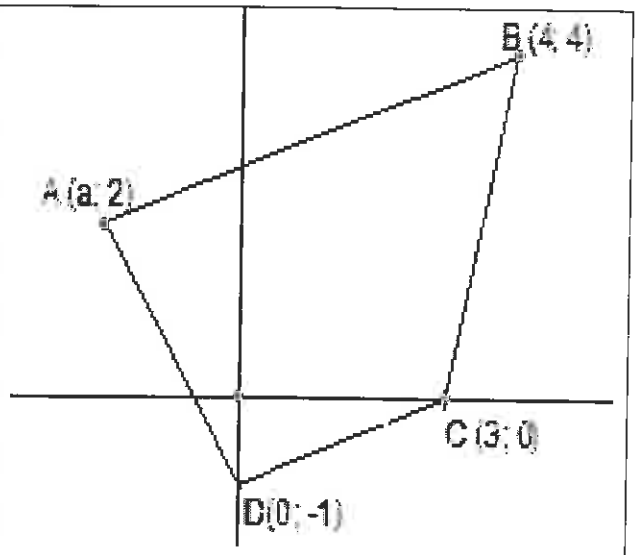
$\frac{2-4}{a-4} = \frac{-1-0}{0-3}$ ✓

$\frac{-2}{a-4} = \frac{-1}{-3}$

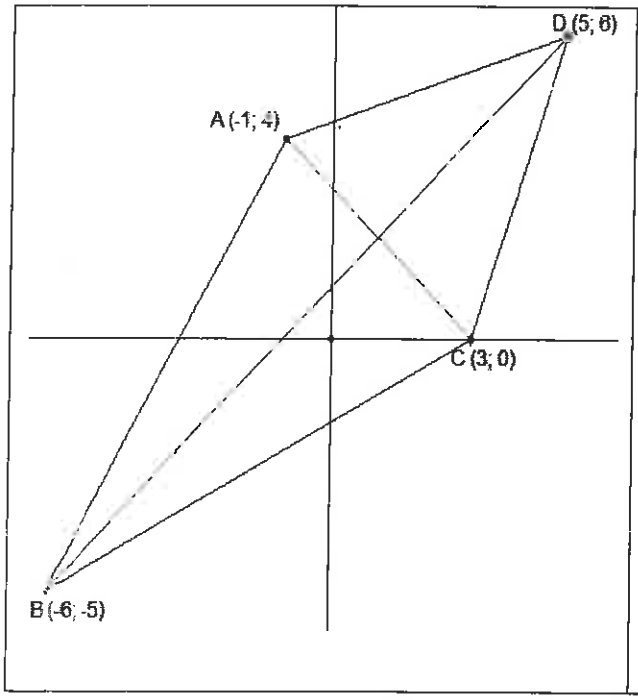
$(-2)(-3) = (-1)(a-4)$ ✓

$6 = -a+4$

$a = -2$ ✓



2.2.2 (2)



Calculate the length of the diagonal BD of the kite, given points B (-6; -5) and D (5; 6). Leave answer in surd form.

$BD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-6-5)^2+(-5-6)^2}$ ✓

$= \sqrt{(-11)^2+(-11)^2}$

$= \sqrt{121+121}$

$= \sqrt{242}$ ✓

$= 2\sqrt{2}$ ✓

Question 3

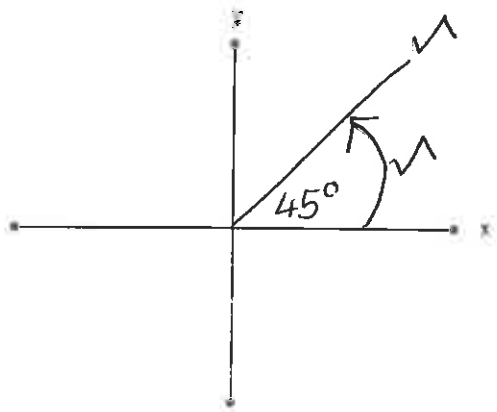
3.1.1a) In which quadrant does the terminal arm of an angle of 182° lie? (1)

3

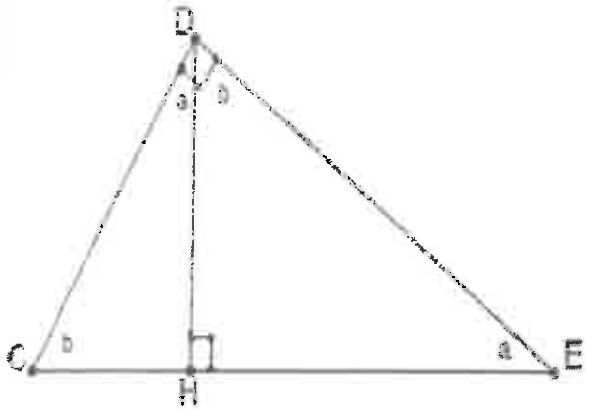
3.1.1b) (1)

Roughly illustrate an angle of 45° on the Cartesian grid alongside.

Positive angle: anti-clockwise



3.1.1c) (2)



Give 3 answers for the ratio tan a in the figure on the left.

In $\triangle CED$: $\tan a = \frac{CD}{DE}$ ✓

In $\triangle DHE$: $\tan a = \frac{DH}{HE}$ ✓

In $\triangle CHD$: $\tan a = \frac{CH}{DH}$ ✓

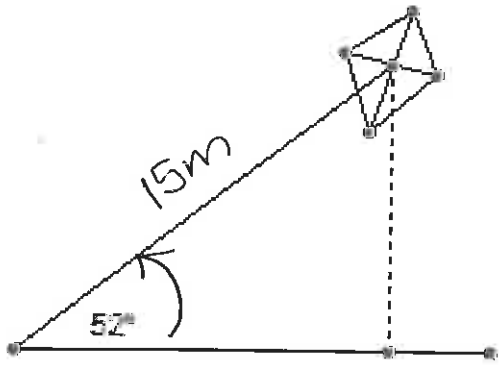
3.1.2 (2)

What is the height of your kite if it is flown at an angle of 52° on a 15m long string?

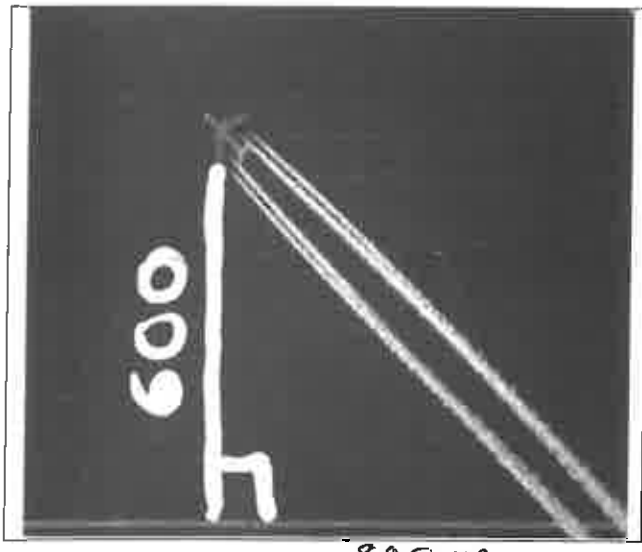
$\sin 52^\circ = \frac{h}{15}$ ✓

$h = 15 \times \sin 52^\circ$

$= 11{,}82\ m$ height



3.1.3 (2)



Calculate the angle of elevation θ of the plane if it is at a height of 600 m when it is a horizontal distance of 800 m from the point of take-off.

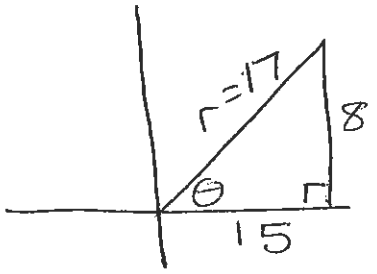
$\tan \theta = \frac{600}{800}$ ✓

$\theta = 36{,}87^\circ$ ✓

3.2 If $\tan\theta = \frac{8}{15}$ and $\theta \in [0°; 90°]$, determine the value of $3\sin\theta + \cos\theta$ **without the use of a calculator**. (4)

$3\sin\theta + \cos\theta$

$= 3 \times \left(\frac{8}{17}\right) + \left(\frac{15}{17}\right)$ ✓ substitution



$r^2 = 15^2 + 8^2$

$= 289$

$r = 17$ ✓

$= \frac{39}{17}$ ✓

$= 2\frac{5}{17}$ ✓

** −1 mark if answer given as 2,29

3.3 Calculate the value of:

a) $\frac{2\tan x + 4\cos y}{7}$ given: $x = 104{,}2°$ and $y = 241{,}7°$. (2)

$\frac{2\tan(104{,}2°) + 4(\cos 241{,}7°)}{7} = -1{,}40$

✓ substitution ✓

b) A given: $\tan A = 2\sin 35°$ and $A \in [180°; 360°]$ (3)

$\tan A = 2\sin 25° \approx 1{,}147$ ✓

Ref $A_g \approx 48{,}92°$ ✓

$\therefore A = 180° + \text{Ref } L = 228{,}92°$ ✓

c) $\cos A + 2\tan(\frac{A}{2})$ given: $A = 229{,}8°$ (3)

$\cos 229{,}8° + 2\tan\left(\frac{229{,}8°}{2}\right)$ ✓

$\approx -0{,}645 + 2\tan(114{,}9°)$ ✓

$= -0{,}645 + 2 \times (-2{,}154)$

$\approx -4{,}95 \rightarrow -4{,}966$ ✓

↳ if only round at end.

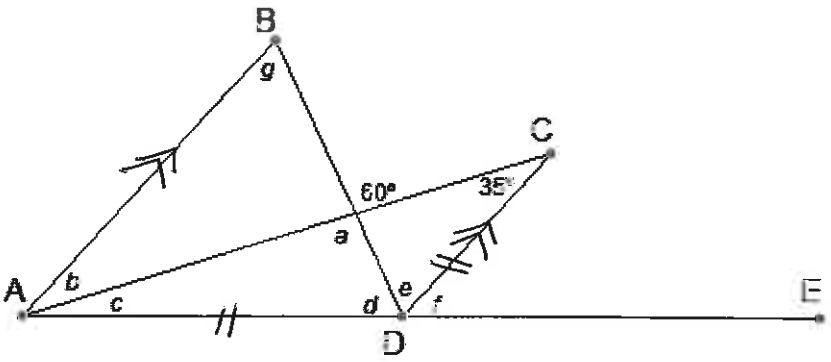
Question 4

4.1 (7)

Find the size of the angles a to g (in that order), giving reasons.

Lines *AB* and *CD* are parallel.

Lines *AD* and *DC* are equal in length.



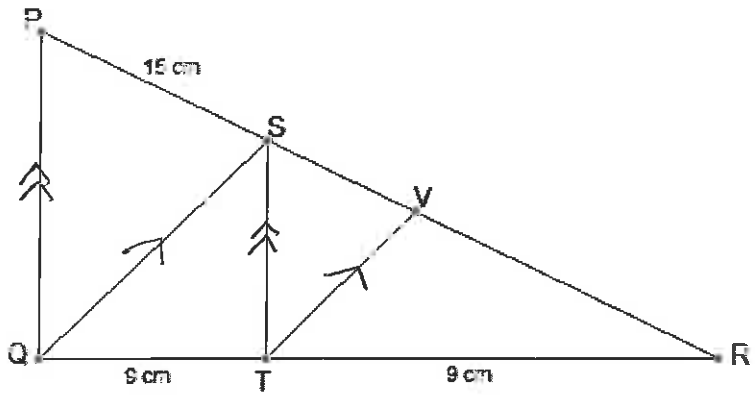
Statement	Reason
$a = 60°$ ✓	vert opp ∠'s = ✓
$b = 35°$ ✓	alt ∠'s =, AB // DC ✓
$c = 35°$ ✓	isos △ ADC base ∠'s = ✓

$d + 60° + 35° = 180°$ (sum int $\angle$s $\triangle = 180°$ ✓)
$d = 85°$ ✓

$e = 60° - 35°$ (ext $\angle$ of $\triangle$)
$= 25°$
(OR)
$e = 180° - (85° + 35° + 35°)$ (sum of $\angle$s of $\triangle$ ✓)
$= 25°$ ✓

$f = 180° - (85° + 25°)$ (sum $\angle$s on str line (OR) adj suppl $\angle$s ✓)
$= 70°$
(OR)
$f = 35° + 35° = 70°$ (corresp $\angle$s =, AB // DC)
$g = 25°$ (alt $\angle$s =, AB // DC.)
✓

4.2 In the diagram below $QS \| TV; PQ \| ST; QT = TR = 9\ cm$ and $PS = 15\ cm$.



4.2.1 Prove $VR = 7\frac{1}{2}\ cm$. (4)

In $\triangle RQP$: T is midpoint of QR and $TS \| PQ$

$\therefore$ S is midpoint of PR ✓

$\therefore$ S = 15 cm ✓

Similarly in $\triangle RQS$: V is midpoint of SR ✓

$\therefore VR = \frac{1}{2}SR = \frac{1}{2}(15) = 7\frac{1}{2}$ cm ✓

4.2.2 a) Calculate PQ if $PQ = \frac{16}{5}VR$ (2)

$PQ = \frac{16}{5} \times 7{,}5$ ✓ substitution

$= 24$ cm ✓

↳ work with answer from 4.2.1

b) Hence prove that *angle PQR* = 90° (2)

Sides of $\triangle PQR$ = 18 : 24 : 30
= 3 : 4 : 5 ✓ OR PROVE TH of Pythag works.

$\therefore \hat{PQR} = 90°$ — ratio of sides Pythagorean "triple" ✓

Section B

Question 5

5.1 The following figures show the daily income (in rand) for a small business in Sunninghill over a 30 day period.

436	520	980	684	798	842	346	472	591	257
745	763	892	385	427	445	676	848	907	605
412	306	584	527	634	701	549	1 002	812	596

5.1.1 Group the data into the 5 equal intervals, and complete the frequency table. (3)

Income (rand)	Frequency
$100 \leq x < 300$	1 ✓
$300 \leq x < 500$	8 ✓
$500 \leq x < 700$	10 ✓
$700 \leq x < 900$	8 ✓
$900 \leq x < 1100$	3 ✓

5.1.2 The owner decides that if he is to keep his business, he has to earn a daily income of R600 or more for at least 60% of the time. By suitable calculation, determine what income the owner is in fact earning 60% of the time. (2)

60% means $\frac{p(n+1)}{100} = \frac{60(30+1)}{100}$ ✓ (no. of days)

$= 18,6$ th position

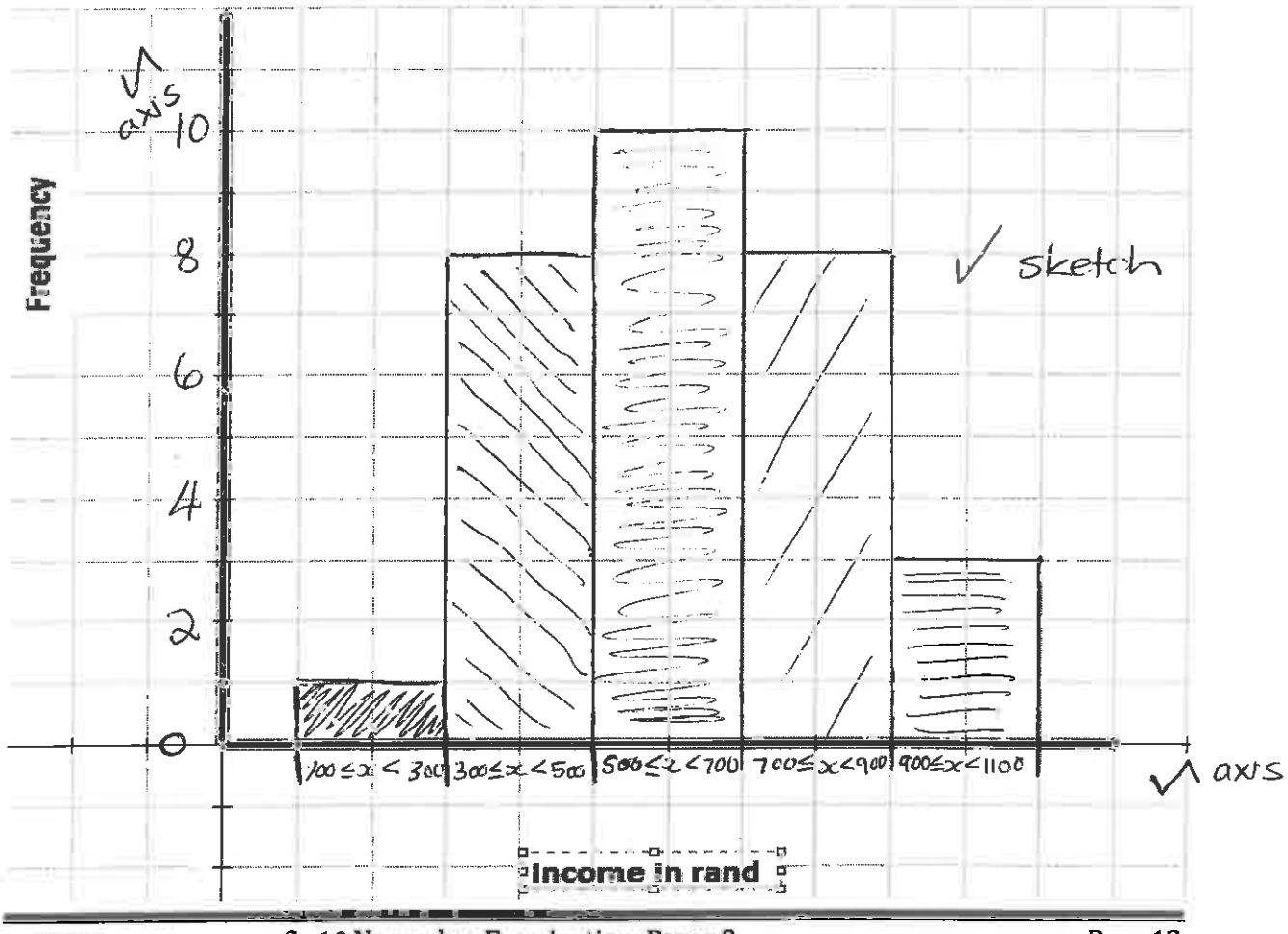
This falls in category $R500 \leq x < R700$

There are 10 values in this category, so the value $\frac{8,6}{10}$ of the way through category $= 500 + \frac{8,6}{10}(200)$ (category width)

$= R672$ ✓

∴ owner should keep the business ✓

5.1.3 Plot a histogram from the information in the table in 5.1.1. (2)



Question 6

ALL the answers in this question have to be determined by analytical methods.

6.1 Prove that the points K(2; 3), L(2+4t; 3-5t) and M(2+4u; 3-5u) are collinear for all real values of t and u. (3)

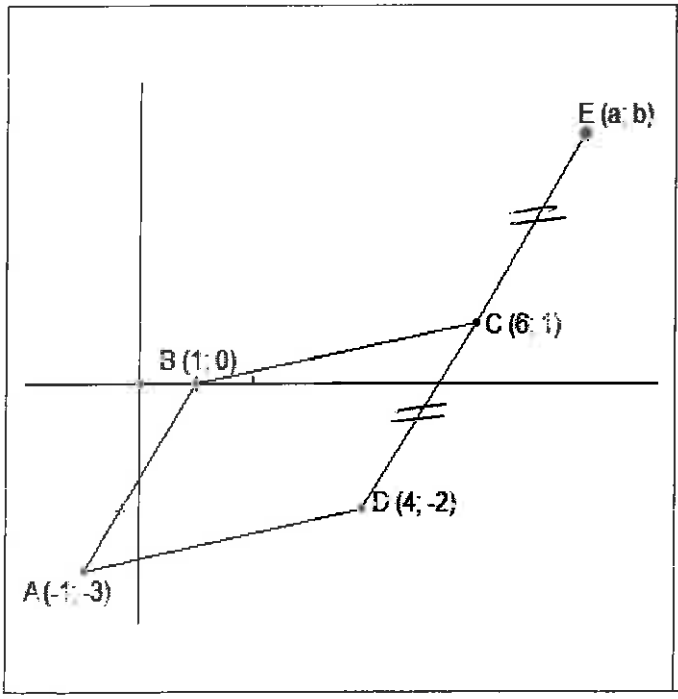
$m = \frac{3-5t-3}{2+4t-2} = \frac{-5t}{4t} = -\frac{5}{4}$ ✓

$m_{KM} = \frac{3-5u-3}{2+4u-2} = \frac{-5u}{4u} = -\frac{5}{4}$ ✓

$\therefore m_{KL} = m_{KM}$ ✓

$\therefore$ Points are collinear ✓

6.2 (2)



If DC is extended to E such that DC = CE, find the values of a and b.

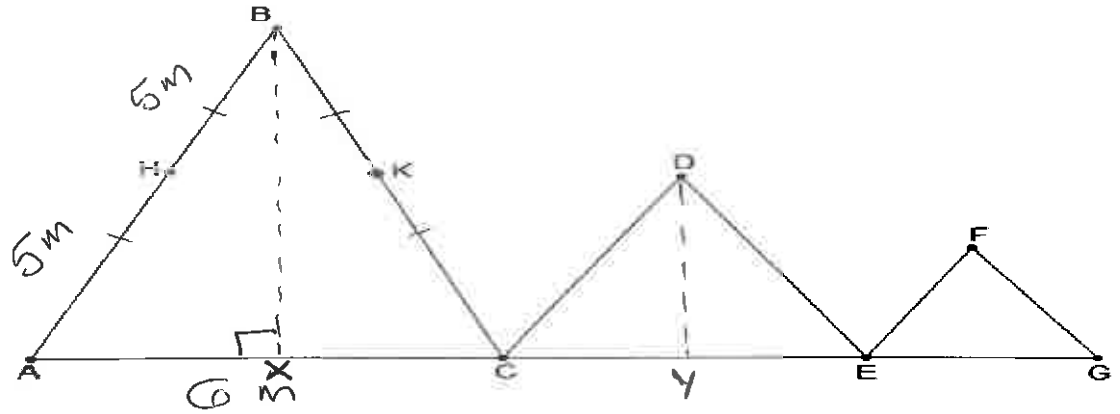
$D \to C$ as $C \to E$

$\therefore (x; y) \to (x+2; y+3)$

$\therefore a = 6+2 = 8$ ✓

$b = 1+3 = 4$ ✓

6.3



Each successive triangle is half the height of the previous one. Find the perpendicular height of ΔDCE, given $AB = BC = 10\ m$ and $AC = 6\ m$. (2)

$BX^2 = AB^2 - AX^2$ Pythag

$= 10^2 - 3^2$ ✓

$= 91$

$BX = \sqrt{91}$ ✓

$\therefore DY = \frac{BX}{2} = \frac{\sqrt{91}}{2} \simeq 4{,}77\ m$ ✓

$AX = 3m$ since $\triangle ABC$ is isosceles

Question 7

7.1 If A and B are complementary angles, and $\sin A = \frac{5}{13}$, make use of an appropriate diagram and determine, **without the use of a calculator**, the value of $3\sin A + 5\cos B$. (6)

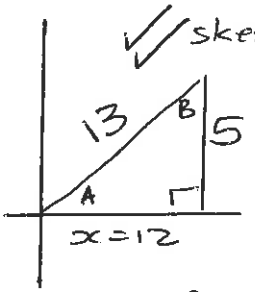
$3\sin A + 5\cos B$

$= 3\left(\frac{5}{13}\right) + 5\left(\frac{12}{13}\right)$ ✓ substitution

$= \frac{15}{13} + \frac{60}{13}$ ✓

$= \frac{75}{13} = 5\frac{10}{13}$ ✓

✓✓ sketch with angles $\hat{A}$ and B



$x^2 = 13^2 - 5^2$ Pythag

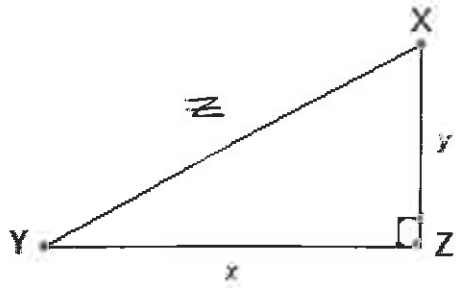
$= 144$

$x = 12$ ✓

* no calculator, so −1 mark for decimal answer 5,77

7.2 (5)

Use the accompanying sketch and **PROVE** the trigonometric identity $\sin^2X + \cos^2X = 1$



$\sin^2 X = \left(\frac{x}{z}\right)^2$ ✓

$\cos^2 X = \left(\frac{y}{z}\right)^2$

$\sin^2 X + \cos^2 X$

$= \left(\frac{x}{z}\right)^2 ✓ + \left(\frac{y}{z}\right)^2 ✓$

$= \frac{x^2}{z^2} + \frac{y^2}{z^2}$

$= \frac{x^2+y^2}{z^2}$ ✓ Pythag

$= \frac{z^2}{z^2}$ ✓

$= 1$ ✓ ∴ LHS = RHS. ✓