higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N3

(16030143)

24 November 2023 (X-Paper)
09:00–12:00

Scientific calculators may be used.

This question paper consists of 6 pages and a formula sheet of 2 pages.

090Q1E2324

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N3
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but keep subsections of questions together.
5. Show all calculations and intermediate steps.
6. All final answers must be accurately rounded off to THREE decimal places.
7. Diagrams are NOT drawn to scale.
8. Use only a black or blue pen.
9. All graph work must be done in the ANSWER BOOK. A pencil may be used.
10. Write neatly and legibly.

QUESTION 1

1.1 Factorise as far as possible in prime factors:

$(p+qn)(p-qn)+2qn-1$ (4)

1.2 Determine the value of a if $f(x)=2x^3+4ax^2-3a^2x-2$ gives a remainder of 22 when divided by $x-a$. (4)

1.3 Simplify the following:

$$\frac{a^2-a-6}{a^2+a-20}\times\frac{2a-8}{2a-6}\div\frac{a^2-3a-10}{a^2-25}$$ (6)

1.4 Solve for x by completing the square:

$4x^2-x-1=0$ (5)

[19]

QUESTION 2

2.1 Simplify the following expression:

$20^{2n+1}\times 100^{2-n}\div 64^{\frac{n}{3}}$ (4)

2.2 Solve for x:

2.2.1 $(\log x)^2-5=\log(10x)$ (5)

2.2.2 $3(x^{\frac{1}{2}}+x^{-\frac{1}{2}})=10$ (5)

[14]

QUESTION 3

3.1 Make A the subject of the formula:

$BA^2+DA=V$ (3)

3.2 The second angle of a triangle is five times as large as the first angle. The measurement of the third angle is 12° greater than that of the first angle.

Determine the size of the angles of the triangle in degrees. (5)

3.3 Solve for x:

$$\frac{x+1}{x-1}=\frac{-2}{x+2}-\frac{x+2}{1-x}$$ (5)

[13]

QUESTION 4

FIGURE A shows $A(6;-2), B(2;6)$ and $C(-2;2)$, the vertices of ΔABC. $P(x;y)$ is the midpoint of line segment AB.

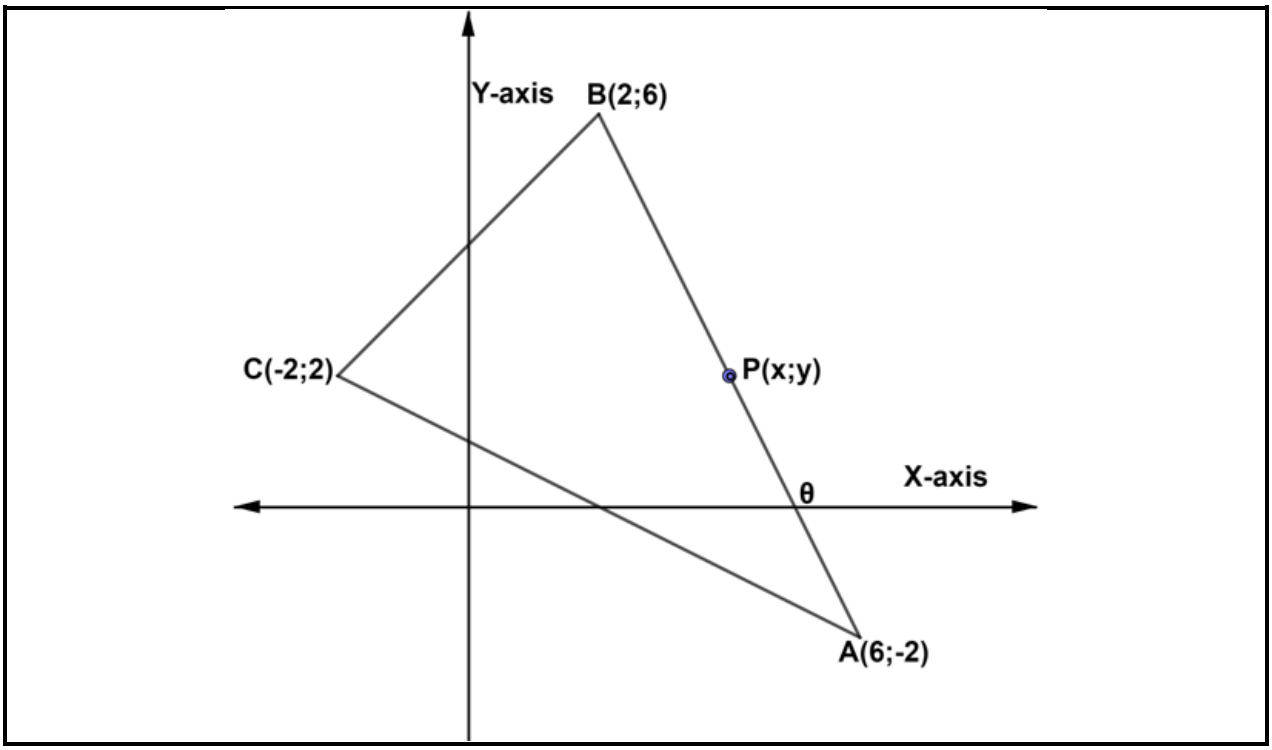

FIGURE A

Determine the following:

4.1 The gradient of line AB (2)

4.2 The size of θ, the angle of inclination between line AB and the *X*-axis (3)

4.3 The coordinates of P, the midpoint of line AB (2)

4.4 The distance between A and C in the simplest surd form (2)

4.5 The equation of the line passing through point A that is parallel to the *x*-axis. (2)

4.6 The equation of the line passing through point B that is parallel to the *y*-axis. (1)

4.7 Determine the co-ordinates of the point of intersection of the two lines: $x = -4$ and $y = 7$. (1)

4.8 Determine the equation of the line through the origin and B. (2)

4.9 The equation of a line passing through point B that is perpendicular to BC. Leave the answer in the gradient-intercept form. (4)

[19]

QUESTION 5

5.1 Draw the graph of $y = 2x^2$ in the ANSWER BOOK, showing the turning point, axis of symmetry and intercepts on both axes (where they exist). (3)

5.2 Determine $\frac{dy}{dx}$ of $y = \frac{2}{x} - 2\sqrt{x} + \frac{1}{3}x^3$ by means of the rules of differentiation. Leave the final answer with positive exponents and in surd form where applicable. Show all the steps. (5)

5.3 Consider the following function: $f(x) = 2x^3 + 6x^2$

5.3.1 Make use of differentiation to determine the coordinates of the turning point of $f(x)$. (5)

5.3.2 Draw the graph of $f(x)$ and show the calculations in QUESTION 5.3.1 on the graph. (3)

[16]

QUESTION 6

6.1 Use basic trigonometric identities to prove that:

$$\frac{\tan A}{1+\tan^2 A} = \sin A \cos A$$ (4)

6.2 If $\cos A = -\frac{5}{13}$ and $0^\circ \leq A \leq 180^\circ$, without using a calculator, find the values of following:

6.2.1 $\tan A$ (2)

6.2.2 $\sec A$ (2)

6.3 Consider FIGURE B below. BD is a vertical cliff and C and A are two points in the same horizontal plane as the foot of the cliff. The angle of elevation from C to B is 47^o and the angle of elevation from A to B is 25^o. A and C are 53 m apart. A, B, C and D are all in the same vertical plane. Calculate the height of the cliff, BD.

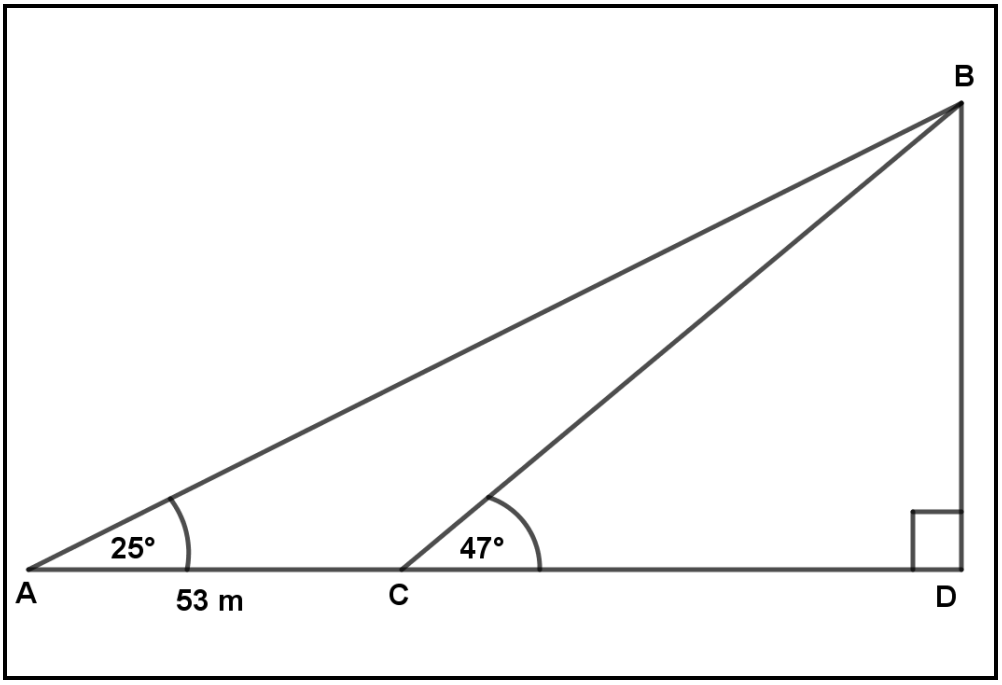

FIGURE B

(6)

6.4 Sketch the graph of $f(x) = 2\cos x + 1$ for $0° \leq x \leq 270°$.

Show the coordinate of the turning points and the intercepts with the axes. (5)

[19]

TOTAL: 100

MATHEMATICS N3

FORMULA SHEET

Any applicable formula may also be used.

1. **Factors**

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

2. **Logarithms**

$$\log ab = \log a + \log b$$

$$\log \frac{a}{b} = \log a - \log b$$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

$$\log a^m = m \log a$$

$$\log_b a = \frac{1}{\log_a b}$$

$$\log_a a = 1 \therefore 1n\ e = 1$$

$$a^{\log_a t} = t \therefore e^{1n\, m} = m$$

3. **Quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. **Parabola**

$$y = ax^2 + bx + c$$

$$y = \frac{4ac - b^2}{4a}$$

$$x = \frac{-b}{2a}$$

5. **Circle**

$$x^2 + y^2 = r^2$$

$$D = \frac{x^2}{4h} + h$$

$$x = \sqrt{4Dh - 4h^2}$$

6. **Straight line**

$$y - y_1 = m(x - x_1)$$

Perpendicular: $m_1 \cdot m_2 = -1$

Parallel lines: $m_1 = m_2$

Distance: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint: $P = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$

Angle of inclination: $\theta = tan^{-1} m$

7. **Differentiation**

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

Max/Min

For turning points: $f'(x) = 0$

8. **Trigonometry**

$$\sin\theta = \frac{y}{r} = \frac{1}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$