higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N3

3 April 2020

This marking guideline consists of 10 pages.

QUESTION 1

1.1 $4(m-n)^2+5(m-n)-6$

Let $k=m-n$

$\therefore 4k^2+5k-6$

$=(4k-3)(k+2)$

$=\left[4(m-n)-3\right]\left[(m-n)+2\right]$ ✓

$=(4m-4n-3)(m-n+2)$ ✓ ✓

or

$4(m-n)^2+5(m-n)-6$

$=\left[4(m-n)-3\right]\left[(m-n)+2\right]$ ✓

$=(4m-4n-3)(m-n+2)$

✓ ✓ (3)

1.2 $f(x)=x^3+6x^2+11x+6$

$f(-1)=(-1)^3+6(-1)^2+11(-1)+6$ ✓

$=0$

$\therefore x+1$ is a factor of $f(x)$

$$x+1\overline{)\,x^3+6x^2+11x+6}\quad \checkmark$$

$$\begin{array}{r} x^2+5x\ +6 \\ \hline x+1\,\big)\,x^3+6x^2+11x+6 \\ x^3+\ x^2 \\ \hline 5x^2+11x \\ 5x^2+5x \\ \hline 6x+6 \\ 6x+6 \\ \hline . \quad . \end{array}$$ ✓

$\therefore f(x)=(x+1)(x^2+5x\ +6)$ ✓

$=(x+1)(x+2)(x+3)$ ✓ (5)

1.3 $\dfrac{3-x}{2x^2-x-3}+\dfrac{2}{9-6x}+\dfrac{4}{5x+5}$

$=\dfrac{3-x}{(2x-3)(x+1)}\ ✓ -\dfrac{2}{3(2x-3)}\ ✓ +\dfrac{4}{5(x+1)}$ ✓

$=\dfrac{15(3-x)-10(x+1)+12(2x-3)}{15(2x-3)(x+1)}$

$=\dfrac{45-15x-10x-10+24x-36}{15(2x-3)(x+1)}$ ✓ ✓

$=\dfrac{-x-1}{15(2x-3)(x+1)}$ ✓

$=\dfrac{-(x+1)}{15(2x-3)(x+1)}$

$=\dfrac{-1}{15(2x-3)}$ ✓ (7)

[15]

QUESTION 2

2.1
$$\frac{1}{(x+y)^{-1}}-\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right)^2$$
$$=x+y-\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right)\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right) \quad \checkmark$$
$$=x+y-x+2x^{\frac{1}{2}}y^{\frac{1}{2}}-y \quad \checkmark$$
$$=2x^{\frac{1}{2}}y^{\frac{1}{2}} \quad \checkmark$$
(3)

2.2 2.2.1
$$\log_a x\times\log_3 a=4$$
$$\therefore \log_a x\times\frac{\log_a a}{\log_a 3}=4 \quad \checkmark$$
$$\therefore \log_a x\times\frac{1}{\log_a 3}=4 \quad \checkmark$$
$$\therefore \log_a x=4\log_a 3 \quad \checkmark$$
$$\therefore \log_a x=\log_a 3^4$$
$$\therefore \log_a x=\log_a 81 \quad \checkmark$$
$$\therefore x=81 \quad \checkmark$$
(5)

2.2.2
$$\frac{6}{\sqrt{3x-1}}-1=\sqrt{3x-1}$$
$$\therefore 6-\sqrt{3x-1}=3x-1 \quad \checkmark$$
$$\therefore 6-3x+1=\sqrt{3x-1}$$
$$\therefore \left(-3x+7\right)^2=\left(\sqrt{3x-1}\right)^2 \quad \checkmark$$
$$\therefore 9x^2-42x+49=3x-1 \quad \checkmark$$
$$\therefore 9x^2-45x+50=0$$
$$\therefore (3x-5)(3x-10)=0 \quad \checkmark$$
$$\therefore x=\frac{5}{3} \quad \checkmark \quad OR \quad x\neq\frac{10}{3} \quad \checkmark$$

Test : $x=\frac{5}{3}$

LHS$=\frac{6}{\sqrt{3\times\frac{5}{3}-1}}-1$ RHS$=\sqrt{3\times\frac{5}{3}-1}$

$=\frac{6}{\sqrt{4}}-1$ $=\sqrt{4}$

$=2$ $=2$

Test : $x=\frac{10}{3}$

LHS$=\frac{6}{\sqrt{3\times\frac{10}{3}-1}}-1$ RHS$=\sqrt{3\times\frac{10}{3}-1}$

$=\frac{6}{\sqrt{9}}-1$ $=\sqrt{9}$

$=1$ $=3$

LHS $\neq$ RHS

Therefore $x=\frac{10}{3}$ is not a solution.

Penalise by 1 mark for NOT indicating 'Not a solution'. (6)

[14]

QUESTION 3

3.1

$$\therefore \frac{1}{x}+\frac{1}{x+2}=\frac{12}{35} \quad \checkmark$$
$$\therefore 35(x+2)+35x=12x(x+2) \quad \checkmark$$
$$\therefore 35x+70+35x=12x^2+24x$$
$$\therefore 0=12x^2-46x-70 \quad \checkmark$$
$$\therefore 0=6x^2-23x-35$$
$$\therefore 0=(x-5)(6x+7)$$
$$\therefore x=5$$
The numbers are 5 and 7 ✓✓

(5)

3.2

$$B=ab^{x-1}$$
$$\therefore \frac{B}{a}=b^{x-1} \quad \checkmark$$
$$\therefore \ln\left(\frac{B}{a}\right)=\ln b^{x-1} \quad \checkmark$$
$$\therefore \ln\left(\frac{B}{a}\right)=(x-1)\ln b \quad \checkmark$$
$$\therefore \ln\left(\frac{B}{a}\right)=x\ln b-\ln b \quad \checkmark$$
$$\therefore \ln\left(\frac{B}{a}\right)+\ln b=x\ln b$$
$$\therefore x=\frac{\ln\left(\frac{B}{a}\right)+\ln b}{\ln b} \quad \checkmark$$
or
$$\therefore x=\frac{\ln B-\ln a+\ln b}{\ln b}$$

ALTERNATIVE METHOD

$$B=ab^{x-1}$$
$$\therefore \frac{B}{a}=b^{x-1} \quad \checkmark$$
$$\therefore \log\left(\frac{B}{a}\right)=\log b^{x-1} \checkmark$$
$$\therefore \log\left(\frac{B}{a}\right)=(x-1)\log b \quad \checkmark$$
$$\therefore \log\left(\frac{B}{a}\right)=x\log b-\log b \quad \checkmark$$
$$\therefore \log\left(\frac{B}{a}\right)+\log b=x\log b$$
$$\therefore x=\frac{\log\left(\frac{B}{a}\right)+\log b}{\log b} \quad \checkmark$$
or
$$\therefore x=\frac{\log B-\log a+\log b}{\log b}$$

(5)

3.3 3.3.1

$$7x^2-(2k-1)x+k+9=0$$
Let $x=2$
$$\therefore 7(2)^2-(2k-1)2+k+9=0 \quad \checkmark$$
$$\therefore 28-4k+2+k+9=0$$
$$\therefore -3k=-39$$
$$\therefore k=13 \quad \checkmark$$

Alternative method 1

$7x^2-25x+22=0$

$a=7 \quad b=-25 \quad c=22 \quad \checkmark$

Product is $\frac{c}{a}=\frac{22}{7} \quad \checkmark$

(2)

3.3.2 $7x^2-(2k-1)x+k+9=0$

Let $k=13$

$\therefore 7x^2-25x+22=0$ ✓

$\therefore (x-2)(7x-11)=0$

$\therefore x-2=0$ or $7x-11=0$

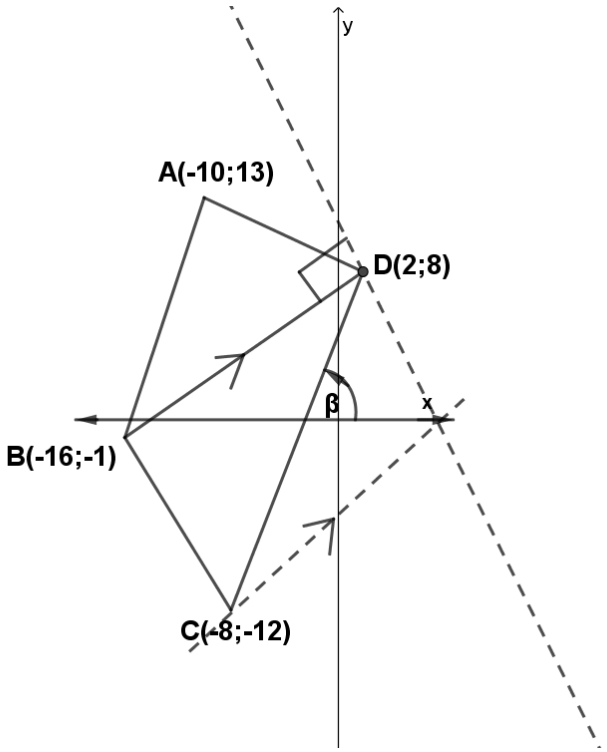
$\therefore x=2$ or $x=\frac{11}{7}$

$\therefore$ The roots: 2 and $\frac{11}{7}$ ✓

Product of the roots $=2\times\frac{11}{7}=\frac{22}{7}$ ✓

(3)
[15]

QUESTION 4



4.1 $M_{CD}=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{8+12}{2+8}$ ✓

$=2$ ✓

(2)

4.2 $\beta=\tan^{-1}(2)$ ✓

$=63,435^{\circ}$ ✓

(2)

4.3 $\text{BD} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2+16)^2 + (8+1)^2}$ ✓

$= \sqrt{324+81}$

$= \sqrt{405}$ ✓

$= 9\sqrt{5}$ ✓ (3)

4.4 $M_{\text{BD}} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{8+1}{2+16}$ ✓

$= \dfrac{1}{2}$ ✓

$M_{\text{BD}} \times M = -1(lines \perp)$

$\therefore \dfrac{1}{2} \times M = -1$

$\therefore M = -2$ ✓

$y - y_1 = m(x - x_1)$

$\therefore y - 8 = -2(x-2)$ ✓

$\therefore y - 8 = -2x + 4$

$\therefore y = -2x + 12$ ✓ (5)

4.5 $M = M_{\text{BD}} = \dfrac{1}{2}(lines \square)$

$\therefore y - y_1 = m(x - x_1)$

$\therefore y + 12 = \dfrac{1}{2}(x+8)$ ✓

$\therefore y + 12 = \dfrac{1}{2}x + 4$ ✓

$\therefore y = \dfrac{1}{2}x - 8$ ✓ (3)

4.6 $y = 8$ ..... (1)

$y = -2x + 2$ .....(2)

From (1) and (2): $8 = -2x + 2$ ✓

$\therefore 2x = -6$

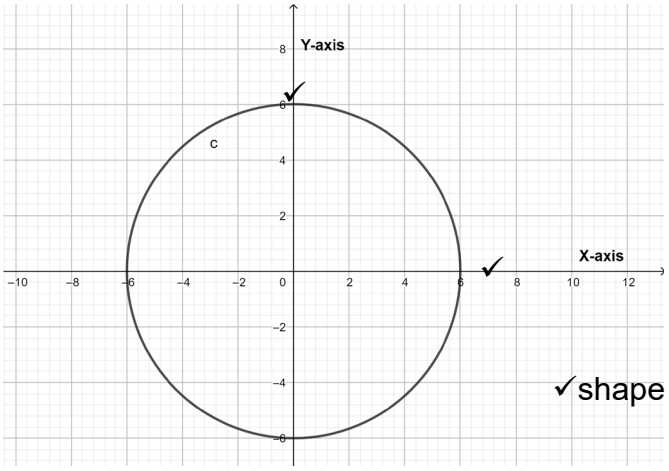
$\therefore x = -3$ ✓

Point of intersection is $(-3;8)$ ✓ (3)

[18]

QUESTION 5

5.1 $y = \pm\sqrt{36 - x^2}$



(Intercepts with axes = 2 marks)
(Shape = 1 mark) (3)

5.2 $y = \dfrac{1}{2x^4} - \dfrac{6}{x^{-\frac{1}{3}}} + ba^2$

$y = \dfrac{x^{-4}}{2} - 6x^{\frac{1}{3}} + ba^2$ ✓

$\dfrac{dy}{dx} = \dfrac{-4x^{-5}}{2} - 6 \times \dfrac{1}{3}x^{-\frac{2}{3}}$ ✓✓

$= \dfrac{-2}{x^5} - \dfrac{2}{x^{\frac{2}{3}}}$

$= \dfrac{-2}{x^5} - \dfrac{2}{\sqrt[3]{x^2}}$ ✓✓

(5)

5.3 5.3.1 $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$ ✓

For TP's , $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ ✓ or $x = 2$ ✓

If $x = 0$ then $f(x) = 0$ ✓

If $x = 2$ then $f(x) = 2^3 - 3(2)^2$

$= 8 - 12$

$= -4$ ✓

The coordinates of the TP are $(0;0)$ and $(2;-4)$ (5)

5.3.2 x-intercepts: Let $y = 0$

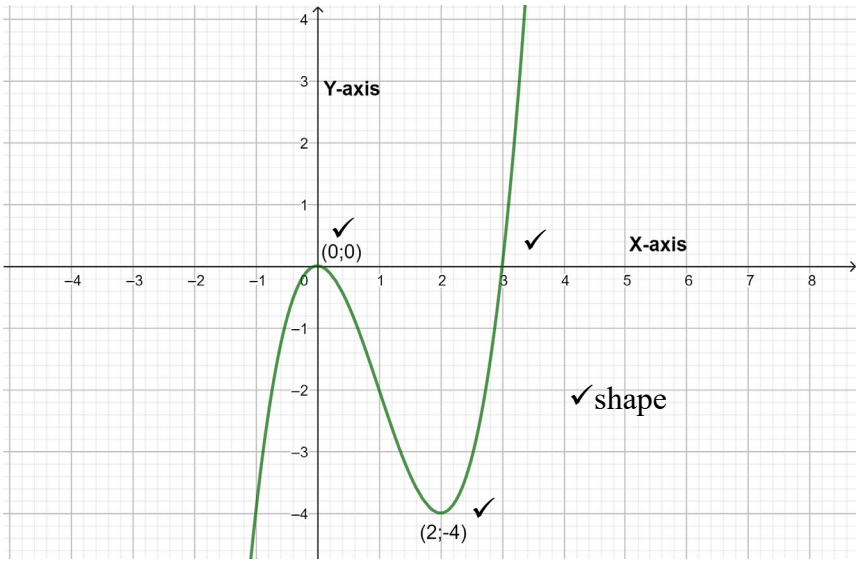
$\therefore x^3 - 3x^2 = 0$

$\therefore x^2(x-3) = 0$

$\therefore x = 0$ or $x = 0$ or $x = 3$

$y-$intercept : Let $x = 0$

$\therefore y = 0$



(4)
[17]

QUESTION 6

6.1 $\dfrac{\sin B}{1+\cos B} + \dfrac{1}{\tan B} = \dfrac{1}{\cos B \tan B}$

$LHS = \dfrac{\sin B}{1+\cos B} + \dfrac{\cos B}{\sin B}$ ✓

$= \dfrac{\sin^2 B + \cos B(1+\cos B)}{\sin B(1+\cos B)}$ ✓

$= \dfrac{\sin^2 B + \cos B + \cos^2 B}{\sin B(1+\cos B)}$

$= \dfrac{1+\cos B}{\sin B(1+\cos B)}$ ✓

$= \dfrac{1}{\sin B}$ ✓

$RHS = \dfrac{1}{\cos B \times \tan B}$

$= \dfrac{1}{\cos B \times \dfrac{\sin B}{\cos B}}$

$= \dfrac{1}{\sin B} = LHS$ ✓

(5)

6.2 $\sin\left(\frac{1}{2}x+10^{o}\right)=1$

$\therefore \frac{x}{2}+10^{o}=90^{o}$ ✓

$\therefore \frac{x}{2}=80^{o}$ ✓

$\therefore x=160^{o}$ ✓ (3)

6.3 $DC = H$

$\therefore DE = H-30$ ✓

$\tan 60^{o}=\frac{H}{x}$ ✓

$\therefore \sqrt{3}=\frac{H}{x}$

$\therefore x=\frac{H}{\sqrt{3}}$ ✓ .....(1)

$\tan 45^{o}=\frac{(H-30)}{x}$ ✓

$\therefore 1=\frac{(H-30)}{x}$ ✓

$x = H-30$ .....(2)

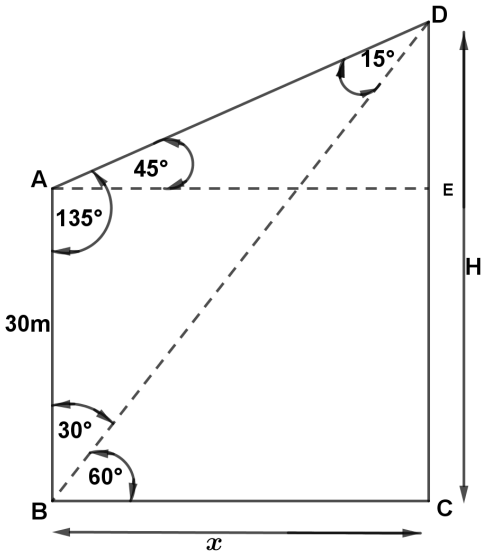
Equate (1) and (2):

$\therefore H-30=\frac{H}{\sqrt{3}}$ ✓

$\therefore \sqrt{3}H-30\sqrt{3}=H$

$\therefore \sqrt{3}H-H=30\sqrt{3}$

$\therefore H=70,98m$ ✓



ALTERNATIVE METHOD 2

In Δ ABD

$B\hat{A}D=135^{o}$ and $A\hat{D}B=15^{o}$ ✓

$\therefore \frac{BD}{\sin 135^{o}}=\frac{30}{\sin 15^{o}}$ ✓

$\therefore BD=\frac{30\sin 135^{o}}{\sin 15^{o}}$ ✓

$\therefore BD = 81,962m$ ✓

In ΔDBC

$\frac{CD}{BD}=\sin 60^{o}$ ✓

$\therefore \frac{CD}{81,962}=\sin 60^{o}$ ✓

$\therefore CD = 81,962\sin 60^{o}$

$\therefore CD = 70,981m$ ✓

ALTERNATIVE METHOD 1

In Δ ABD

$B\hat{A}D=135^o$ and $A\hat{B}D=30^o$ ✓

$\therefore \frac{AD}{\sin 30^o}=\frac{30}{\sin 15^o}$ ✓

$\therefore AD=\frac{30\sin 30^o}{\sin 15^o}$

$\therefore AD=57,96m$ ✓

In ΔADE

$\frac{DE}{AD}=\sin 45^o$ ✓

$\therefore \frac{DE}{57,96}=\sin 45^o$

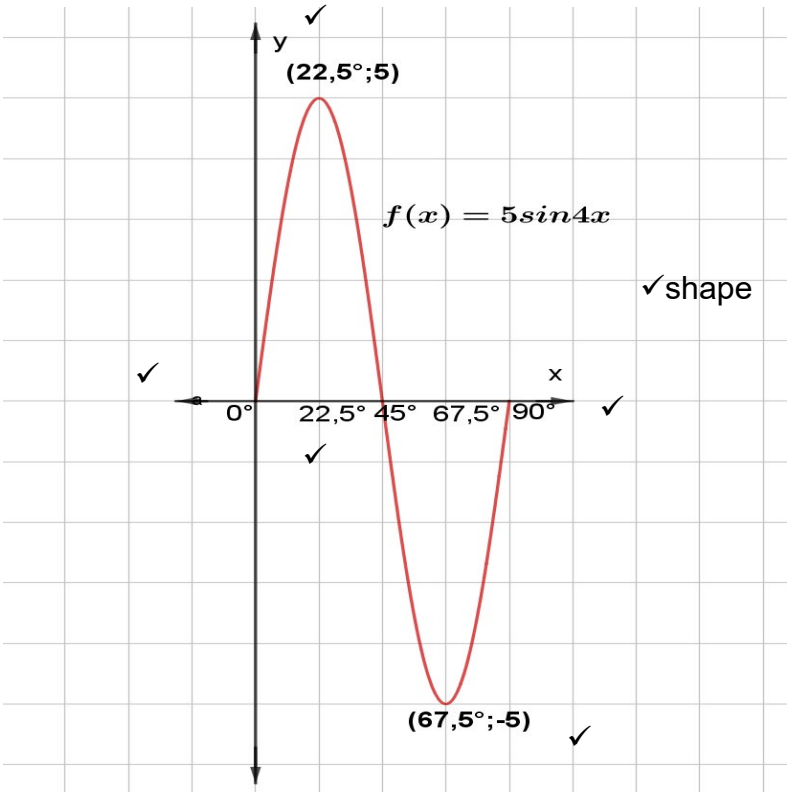
$\therefore DE=57,96\sin 45^o$ ✓

$\therefore DE=40,984m$

$\therefore CD=30+40,984$ ✓

$\therefore CD=70,984m$ ✓ (7)

6.4 $f(x)=5\sin 4x$ for $0^0 \le x \le 90^0$



(6)
[21]

TOTAL: 100