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GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2016

MEMORANDUM

MARKS/PUNTE: 150

**This memorandum consists of 26 pages.
*Hierdie memorandum bestaan uit 26 bladsye.***

NOTE:

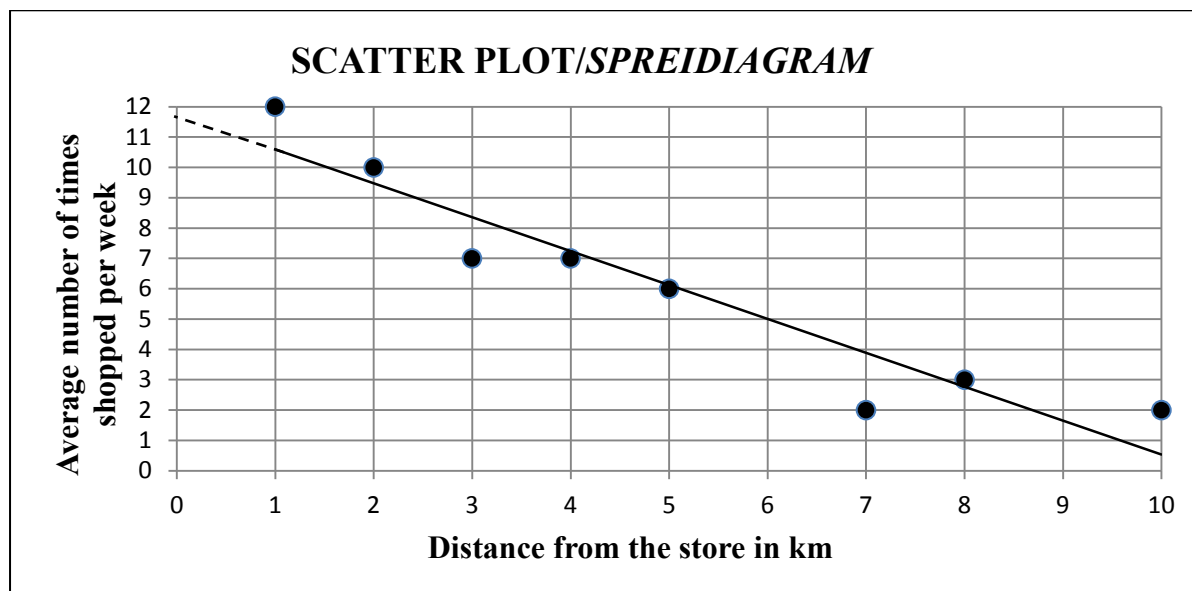
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

QUESTION/VRAAG 1

Distance from the store in km <i>Afstand vanaf die winkel in km</i>	1	2	3	4	5	7	8	10
Average number of times shopped per week <i>Gemiddelde aantal keer wat kopers die winkel per week besoek</i>	12	10	7	7	6	2	3	2



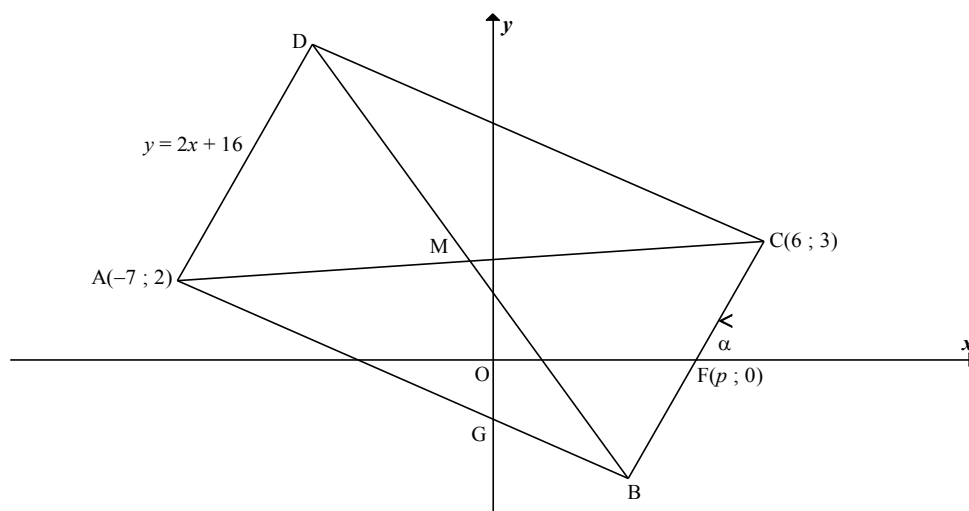
1.1	Strong/Sterk	✓ (1)
1.2	$-0,95$ ($-0,9462\dots$)	✓ (1)
1.3	$a = 11,71$ ($11,7132\dots$) $b = -1,12$ ($-1,1176\dots$) $\hat{y} = -1,12x + 11,71$	✓ value of a ✓ value of b ✓ equation/vgl (3)
1.4	$\hat{y} = -1,12(6) + 11,71$ $= 5$ times	✓ substitution ✓ answer (2)
1.5	On scatter plot/ <i>Op spreidiagram</i>	✓✓ A line close to any 2 of the following points: (5 ; 6) or $(10 ; \frac{1}{2})$ or (6 ; 5) or (0 ; 11,7) (2) [9]

QUESTION/VRAAG 2

2.1	Positively skewed OR skewed to the right/ <i>positief skeef OF skeef na regs</i>	✓ answer (1)												
2.2	Range/ <i>Omvang</i> = $2,21 - 1,39 = 0,82$ m	✓ subtract values ✓ answer (2)												
2.3	<table><thead><tr><th>Intervals <i>Klasse</i></th><th>Cumulative frequency <i>Kumulatiewe frekwensie</i></th></tr></thead><tbody><tr><td>$1,3 \leq x < 1,5$</td><td>24</td></tr><tr><td>$1,5 \leq x < 1,7$</td><td>95</td></tr><tr><td>$1,7 \leq x < 1,9$</td><td>133</td></tr><tr><td>$1,9 \leq x < 2,1$</td><td>156</td></tr><tr><td>$2,1 \leq x < 2,3$</td><td>160</td></tr></tbody></table>	Intervals <i>Klasse</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>	$1,3 \leq x < 1,5$	24	$1,5 \leq x < 1,7$	95	$1,7 \leq x < 1,9$	133	$1,9 \leq x < 2,1$	156	$2,1 \leq x < 2,3$	160	✓ 95 , 133, 156 ✓ 160 (2)
Intervals <i>Klasse</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>													
$1,3 \leq x < 1,5$	24													
$1,5 \leq x < 1,7$	95													
$1,7 \leq x < 1,9$	133													
$1,9 \leq x < 2,1$	156													
$2,1 \leq x < 2,3$	160													
2.4	OGIVE/OGIEF	✓ upper limits / <i>boonste limiete</i> ✓ cum f / <i>kum f</i> ✓ shape / <i>vorm</i> ✓ grounded <i>geanker</i> (4)												
2.5	method (using 80 to determine the height) 1,65 (accept any value between 1,6 and 1,69)	✓ method ✓ answer (2)												
2.6.1	The mean would change by 0,1 m <i>Die gemiddelde sal met 0,1 m verander</i>	✓ answer (1)												
2.6.2	No influence/change as there is no difference in variation of data./ <i>Geen invloed /verandering aangesien daar geen verskil in die variasie van die data is nie.</i>	✓ answer (1)												

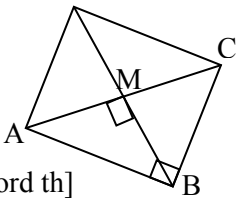
[13]

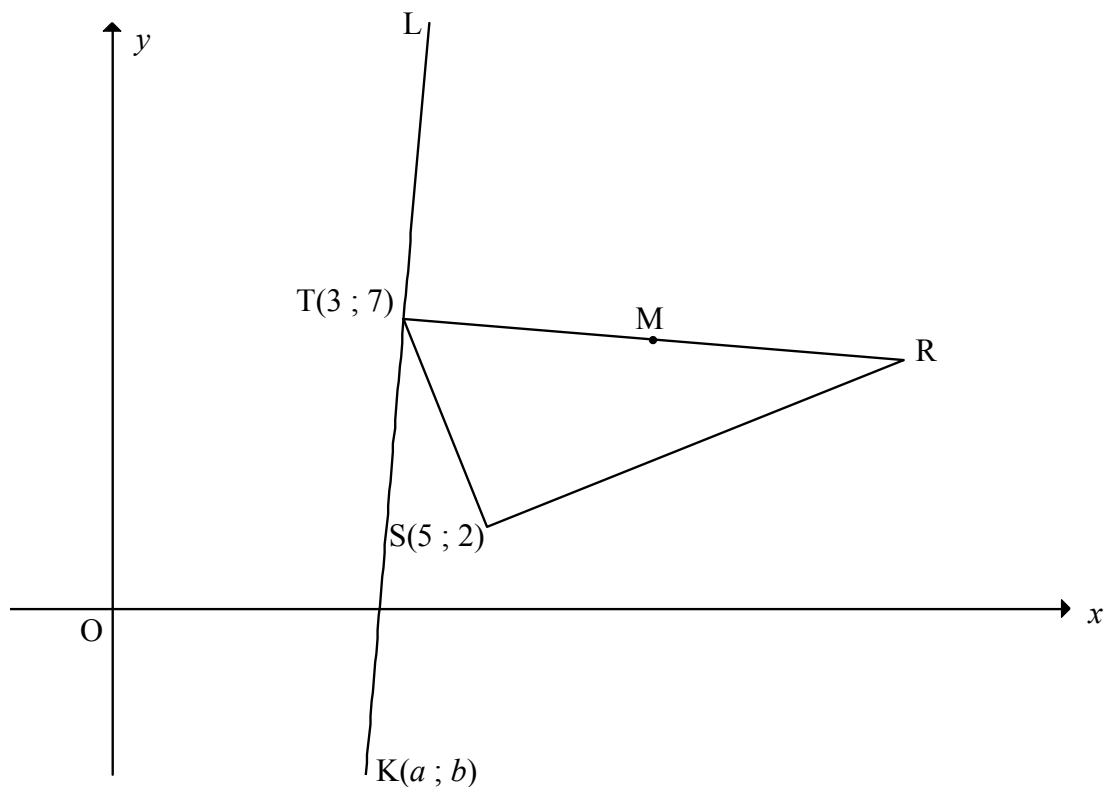
[13]

QUESTION/VRAAG 3

3.1	M = Midpt of AC [diags of rectangle bisect/ hoek v reghoek halveer] $= M\left(\frac{-7+6}{2}; \frac{2+3}{2}\right)$ $= M\left(-\frac{1}{2}; \frac{5}{2}\right)$	✓ x-value of M ✓ y-value of M (2)
3.2	$m_{BC} = \frac{3-0}{6-p} = \frac{3}{6-p}$ OR/OF $m_{BC} = \frac{0-3}{p-6} = \frac{-3}{p-6}$	✓ answer (1) ✓ answer (1)
3.3	$m_{AD} = m_{BC}$ [AD BC] $m_{BC} = 2$ $\frac{3}{6-p} = 2$ $3 = 12 - 2p$ $p = 4\frac{1}{2}$ OR/OF $y - y_1 = 2(x - x_1)$ C(6;3) $y - 3 = 2(x - 6)$ $\therefore y = 2x - 9$ but $y = 0$ $\therefore x = 4\frac{1}{2} = p$ OR/OF	✓ $m_{BC} = 2$ ✓ equating ✓ answer (3) ✓ $m_{BC} = 2$ ✓ substituting (6; 3) ✓ answer (3)

	$y = 2x + c$ $3 = 12 + c$ $-9 = c$ $y = 2x - 9$ $0 = 2x - 9$ $x = \frac{9}{2} \quad \therefore p = \frac{9}{2}$	$\checkmark m_{BC} = 2$ $\checkmark$ substituting $\checkmark$ answer (3)
3.4	$DB = AC$ [diag of rectangle = / <i>hoeke</i> v <i>reghoek</i> =] $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $AC = \sqrt{(6 + 7)^2 + (3 - 2)^2}$ $AC = \sqrt{13^2 + 1^2}$ $AC = \sqrt{170}$ $\therefore DB = \sqrt{170}$ or 13,04	$\checkmark$ substitution $\checkmark$ length of AC $\checkmark AC = BD$ (3)
3.5	$\tan \alpha = m_{BC} = 2$ $\therefore \alpha = 63,43^\circ$	$\checkmark \tan \alpha = m_{BC}$ $\checkmark \alpha = 63,43^\circ$ (2)
3.6	In quadrilateral OFBG: $\hat{O}FB = 63,43^\circ$ [vert opp $\angle$ s/ <i>regoorst</i> $\angle$ e] $\hat{F}OG = \hat{G}BF = 90^\circ$ $\therefore \hat{OGB} = 360^\circ - [90^\circ + 90^\circ + 63,43^\circ]$ [sum $\angle$ s quad/ <i>som</i> $\angle$ e <i>vierh</i> = 360°] $\therefore \hat{OGB} = 116,57^\circ$ OR/OF $m_{AB} = -\frac{1}{2}$ $90^\circ + \hat{OGA} = 153,43^\circ$ $\therefore \hat{OGA} = 63,43^\circ$ $\hat{OGB} = 180^\circ - 63,43^\circ$ $= 116,57^\circ$ OR/OF $\hat{F}OG = \hat{G}BF = 90^\circ$ $\therefore$ GOFB is cyc quad $\hat{OGB} = 180^\circ - 63,43^\circ$ [$\angle$ s of cyc quad = 180°] $= 116,57^\circ$ OR/OF $\hat{OFB} = 63,43^\circ$ $\hat{XOG} = \hat{FBG} = 90^\circ$ $\therefore$ OGBF is a cyclic quad $\therefore \hat{OGB} = 180^\circ - 63,43^\circ$ $\hat{OGB} = 116,57^\circ$	$\checkmark$ size of $\hat{OFB}$ $\checkmark$ S $\checkmark$ answer (3) $\checkmark m_{AB} = -\frac{1}{2}$ $\checkmark$ S $\checkmark$ answer (3) $\checkmark$ S $\checkmark$ S $\checkmark$ answer (3) $\checkmark$ S $\checkmark$ S $\checkmark$ answer (3)

3.7	$M\left(-\frac{1}{2}; \frac{5}{2}\right)$ is the centre/is <i>die middelpunt</i> $r = \frac{\sqrt{170}}{2}$ = radius [BD is diameter/<i>middel lyn</i>] $\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \left(\frac{\sqrt{170}}{2}\right)^2 = \frac{85}{2} = 42,5$	✓ M is centre ✓ $r = \frac{\sqrt{170}}{2}$ ✓ equation (3)
3.8	$\hat{CBM} = \hat{BAM} = 45^\circ$ [diag of square bisect $\angle$s/<i>hoek</i> v <i>vierk halv $\angle$e</i>] $\therefore$ BC will be a tangent [converse tan chord th/<i>omgekeerde raakl-koordst</i>] OR/OF $\hat{AMB} = 90^\circ$ [diag of square bisect $\perp$] $\therefore$ AB is diameter $BC \perp AB$ $\therefore$ BC is tangent [line $\perp$ radius <i>or</i> converse tan-chord th] 	✓ S ✓ R (2) ✓ S ✓ R (2) [19]

QUESTION/VRAAG 4

4.1	$\angle$ in semi circle/ $\angle$ at centre = $2\angle$ on circle $\angle$ in halfsirkel / $\angle$ by middelpt = $2\angle$ op sirkel	$\checkmark$ R (1)
4.2	$m_{TS} = \frac{7-2}{3-5}$ $= -\frac{5}{2}$	$\checkmark$ substitution $\checkmark m_{TS}$ (2)
4.3	$m_{TS} \times m_{RS} = -1$ [TS $\perp$ SR] $\therefore m_{RS} = \frac{2}{5}$ $y = \frac{2}{5}x + c$ $2 = \frac{2}{5}(5) + c$ $c = 0$ $y = \frac{2}{5}x$	$\checkmark m_{RS}$ $\checkmark$ substitution m and (5 ; 2) $\checkmark$ equation (3)
OR/OF		

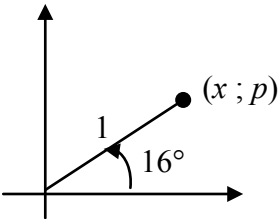
	$m_{TS} \times m_{RS} = -1$ [TS $\perp$ SR] $\therefore m_{RS} = \frac{2}{5}$ $y - y_1 = \frac{2}{5}(x - x_1)$ $y - 2 = \frac{2}{5}(x - 5)$ $y = \frac{2}{5}x$	$\checkmark m_{RS}$ $\checkmark$ substitution m and $(5 ; 2)$ $\checkmark$ equation (3)
4.4.1	$r = \sqrt{36\frac{1}{4}}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$ OR/OF $TM = \sqrt{(3-9)^2 + \left(7-6\frac{1}{2}\right)^2} = \frac{\sqrt{145}}{2}$ $TR = 2.r = 2\left(\sqrt{36\frac{1}{4}}\right) = \sqrt{145}$	$\checkmark r$ $\checkmark$ answer (2) $\checkmark$ substitution $\checkmark$ answer (2)
4.4.2	$M\left(9 ; 6\frac{1}{2}\right)$ $\therefore \frac{x_R + 3}{2} = 9$ and $\frac{y_R + 7}{2} = 6\frac{1}{2}$ $\therefore R(15 ; 6)$ Answer only: full marks Answer only: only 1 coordinate correct (1 mark) OR/OF $M\left(9 ; 6\frac{1}{2}\right)$ $\therefore R\left(9 + 6 ; 6\frac{1}{2} - \frac{1}{2}\right) = R(15 ; 6)$ OR/OF	$\checkmark M$ $\checkmark x$ coordinate $\checkmark y$ coordinate (3) $\checkmark M$ $\checkmark x$ coordinate $\checkmark y$ coordinate (3)

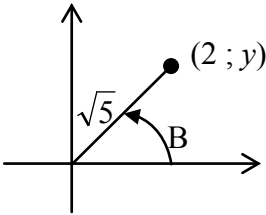
	$m_{TM} = \frac{9-3}{6\frac{1}{2}-7} = -\frac{1}{12}$ $TM: 7 = -\frac{1}{12}(3) + c \quad y = -\frac{1}{12}x + \frac{29}{4} \quad \dots\dots\dots(1)$ $SR: y = \frac{2}{5}x \quad \dots\dots\dots(2)$ $\frac{2}{5}x = -\frac{1}{12}x + \frac{29}{4}$ $\frac{29}{60}x = \frac{29}{4}$ $\therefore x = 15$ $\therefore y = \frac{2}{5}(15) = 6$	✓ equating ✓ x coordinate ✓ y coordinate (3)
4.4.3	$ST = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $ST = \sqrt{(5-3)^2 + (2-7)^2}$ $ST = \sqrt{4+25} = \sqrt{29}$ $\sin R = \frac{TS}{TR} = \frac{\sqrt{29}}{\sqrt{145}} \text{ or } \frac{\sqrt{5}}{5} \text{ or } \frac{1}{\sqrt{5}} \text{ or } 0,45$ OR/OF $TS = \sqrt{29}$ $SR = 2\sqrt{29}$ $\text{area of } \triangle TSR = \frac{1}{2}(\sqrt{29})(2\sqrt{29}) = 29$ $29 = \frac{1}{2}(\sqrt{145})(2\sqrt{29}) \sin R$ $\sin R = \frac{\sqrt{5}}{5} \text{ or } \frac{1}{\sqrt{5}}$	✓ substitution ✓ answer ✓ ratio (3) ✓ area ✓ rule ✓ ratio (3)
4.4.4	$m_{TR} = \frac{7-6}{3-9} = -\frac{1}{12} \quad \quad \quad \textbf{OR/OF} \quad m_{TR} = \frac{7-6}{3-15} = -\frac{1}{12}$ $m_{TR} \times m_{KTL} = -1 \quad [r \perp \text{tangent}]$ $m_{KTL} = 12$ $y - y_1 = 12(x - x_1)$ $y - 7 = 12(x - 3)$ $y = 12x - 29$ substitute K(a;b): $b = 12a - 29$ OR/OF	✓ $m_{TR} = -\frac{1}{12}$ ✓ $m_{KTL} = 12$ ✓ $y = 12x - 29$ (3)

	$m_{TR} = \frac{7 - 6\frac{1}{2}}{3 - 9} = -\frac{1}{12}$ $m_{TR} \times m_{KTL} = -1 \quad [r \perp \text{tangent}]$ $\frac{b - 7}{a - 3} = 12$ $b - 7 = 12(a - 3)$ $b = 12a - 29$ OR/OF $KR^2 = TR^2 + TK^2$ $(a - 15)^2 + (b - 6)^2 = (15 - 3)^2 + (6 - 7)^2 + (a - 3)^2 + (b - 7)^2$ $-30a + 225 - 12b + 36 = 144 + 1 - 6a + 9 - 14b + 49$ $2b = 24a - 58$ $b = 12a - 29$	$\checkmark m_{TR} = -\frac{1}{12}$ $\checkmark m_{KTL} = 12$ $\checkmark \text{substitution}$ $(3 ; 7) \text{ \& } (a ; b)$ (3) $\checkmark \text{subst into Pyth}$ $\checkmark \text{multiplication}$ $\checkmark \text{simplification}$ (3)
4.4.5	$TK = TR$ $\sqrt{(a - 3)^2 + (b - 7)^2} = \sqrt{145}$ $(a - 3)^2 + (b - 7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a - 3)^2 + (12a - 29 - 7)^2 = 145$ $(a - 3)^2 + (12a - 36)^2 = 145$ $a^2 - 6a + 9 + 144a^2 - 864a + 1296 - 145 = 0$ $145a^2 - 870a + 1160 = 0$ $a = \frac{870 \pm \sqrt{(870)^2 - 4(145)(1160)}}{290}$ $a = 2 \text{ or } a = 4$ $\therefore b = 12(2) - 29 = -5 \quad \text{or} \quad b = 12(4) - 29 = 19$ $\therefore K(2 ; -5)$ OR/OF	$\checkmark \text{substitution into distance formula}$ $\checkmark \text{substitution of}$ $b = 12a - 29$ $\checkmark \text{standard form}$ $\checkmark \text{subst into formula or factorise}$ $\checkmark \text{values of } a$ $\checkmark \text{value of } b$ (6)

	$TK = TR$ $\sqrt{(a-3)^2 + (b-7)^2} = \sqrt{145}$ $(a-3)^2 + (b-7)^2 = 145$ Substitute $b = 12a - 29$ [from 4.4.4] $(a-3)^2 + (12a-29-7)^2 = 145$ $(a-3)^2 + (12a-36)^2 = 145$ $(a-3)^2 + 144(a-3)^2 = 145$ $(a-3)^2 = 1$ $a-3 = \pm 1$ $a = 2 \text{ or } 4$ $\therefore b = 12(2) - 29 = -5 \quad \text{or} \quad b = 12(4) - 29 = 19$ $\therefore K(2; -5)$ OR/OF $KR^2 = TR^2 + TK^2$ $(a-15)^2 + (b-6)^2 = 145 + 145$ $(a-15)^2 + (12a-29-6)^2 = 290$ $(a-15)^2 + (12a-35)^2 = 290$ $a^2 - 30a + 225 + 144a^2 - 840a + 1225 = 290$ $145a^2 - 870a + 1160 = 0$ $a^2 - 6a + 8 = 0$ $\therefore (a-2)(a-4) = 0$ $a = 2 \text{ or } a = 4$ $\therefore b = 12(2) - 29 = -5 \quad \text{or} \quad b = 12(4) - 29 = 19$ $K(2; -5)$	✓ substitution into distance formula ✓ substitution of $b = 12a - 29$ ✓ $(a-3)^2 = 1$ ✓ ± 1 ✓ values of a ✓ value of b (6) ✓ substitution ✓ substitution of $b = 12a - 29$ ✓ standard form ✓ factors ✓ values of a ✓ value of b (6)
		[23]

QUESTION/VRAAG 5

5.1.1	$\sin 196^\circ = -\sin 16^\circ$ $= -p$	✓ reduction ✓ answer (2)
5.1.2	$\cos 16^\circ = \sqrt{1 - \sin^2 16^\circ}$ $= \sqrt{1 - p^2}$ OR/OF $x^2 + p^2 = 1$ $x = \sqrt{1 - p^2}$ $\therefore \cos 16^\circ = \frac{\sqrt{1 - p^2}}{1} = \sqrt{1 - p^2}$ 	✓ statement ✓ answer (2) ✓ x in terms of p ✓ answer (2)
5.2	$\sin(A + B) = \cos[90^\circ - (A + B)]$ $= \cos[(90^\circ - A) - B]$ $= \cos(90^\circ - A)\cos B + \sin(90^\circ - A)\sin B$ $= \sin A \cos B + \cos A \sin B$	✓ co-ratio ✓ correct form ✓ expansion (3)
5.3	$\frac{\sqrt{1 - \cos^2 2A}}{\cos(-A) \cdot \cos(90^\circ + A)}$ $= \frac{\sqrt{\sin^2 2A}}{\cos A \cdot (-\sin A)}$ $= \frac{\sin 2A}{\cos A \cdot (-\sin A)}$ $= \frac{2 \sin A \cos A}{\cos A \cdot (-\sin A)}$ $= -2$ OR/OF $\frac{\sqrt{1 - \cos^2 2A}}{\cos(-A) \cos(90^\circ + A)} = \frac{\sqrt{1 - (2\cos^2 A - 1)^2}}{\cos A \cdot -\sin A}$ $= \frac{\sqrt{1 - (4\cos^4 A - 4\cos^2 A + 1)}}{\cos A \cdot -\sin A} = \frac{\sqrt{4\cos^2 A - 4\cos^4 A}}{\cos A \cdot -\sin A}$ $= \frac{\sqrt{4\cos^2 A(1 - \cos^2 A)}}{\cos A \cdot -\sin A} = \frac{\sqrt{4\cos^2 A \sin^2 A}}{\cos A \cdot -\sin A}$ $= \frac{2\cos A \sin A}{\cos A \cdot -\sin A}$ $= -2$ OR/OF	✓ $\sqrt{\sin^2 2A}$ ✓ $\cos A$ ✓ $-\sin A$ ✓ $2\sin A \cos A$ ✓ answer (5) ✓ $2\cos^2 A - 1$ ✓ $\cos A$ ✓ $-\sin A$ ✓ identity ✓ answer (5)

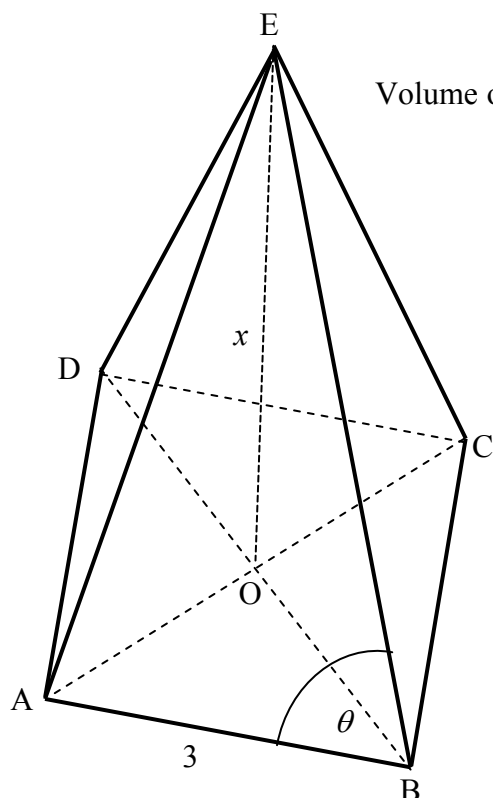
	$\frac{\sqrt{1 - (1 - 2\sin^2 A)^2}}{\cos A - \sin A}$ $= \frac{\sqrt{1 - (1 - 4\sin^2 A + 4\sin^2 A)}}{\cos A - \sin A}$ $= \frac{\sqrt{4\sin^2 A(1 - \sin^2 A)}}{\cos A - \sin A}$ $= \frac{2\sin A \sqrt{\cos^2 A}}{\cos A - \sin A}$ $= -2$	$\checkmark 1 - 2\sin^2 A$ $\checkmark \cos A \checkmark - \sin A$ $\checkmark$ identity $\checkmark$ answer (5)
5.4.1	$\cos 2B = \frac{3}{5}$ $2\cos^2 B - 1 = \frac{3}{5}$ $\cos^2 B = \frac{4}{5}$ $\therefore \cos B = \sqrt{\frac{4}{5}} \text{ or } \frac{2}{\sqrt{5}} \text{ or } \frac{2\sqrt{5}}{5} \quad [0^\circ \leq B \leq 90^\circ]$ OR/OF $\cos B = \frac{\sqrt{\cos 2B + 1}}{2}$ $= \frac{\sqrt{\frac{3}{5} + 1}}{2}$ $= \frac{2\sqrt{5}}{5}$	$\checkmark$ identity $\checkmark$ value of $\cos^2 B$ $\checkmark$ answer (3)
5.4.2	$\sin^2 B = 1 - \cos^2 B$ $= 1 - \left(\frac{2}{\sqrt{5}}\right)^2$ $= \frac{1}{5} \quad \therefore \sin B = \frac{1}{\sqrt{5}} \text{ or } \frac{\sqrt{5}}{5}$ OR/OF $(2)^2 + y^2 = (\sqrt{5})^2$ $4 + y^2 = 5$ $y^2 = 1$ $y = 1$ $\therefore \sin B = \frac{1}{\sqrt{5}} \text{ or } \frac{\sqrt{5}}{5}$ 	$\checkmark \sin^2 B = \frac{1}{5}$ $\checkmark$ answer (2)
		$\checkmark y = 1$ $\checkmark$ answer (2)

	OR/OF $\cos 2B = \frac{3}{5}$ $1 - 2\sin^2 B = \frac{3}{5}$ $\sin^2 B = \frac{1}{5}$ $\therefore \sin B = \frac{1}{\sqrt{5}} \text{ or } \frac{\sqrt{5}}{5}$	$\checkmark \sin^2 B = \frac{1}{5}$ $\checkmark$ answer (2)
5.4.3	$\cos(B + 45^\circ) = \cos B \cdot \cos 45^\circ - \sin B \cdot \sin 45^\circ$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{2}{\sqrt{10}} - \frac{1}{\sqrt{10}}$ $= \frac{1}{\sqrt{10}} \text{ or } \frac{\sqrt{10}}{10}$ OR/OF $\cos(B + 45^\circ) = \cos B \cdot \cos 45^\circ - \sin B \cdot \sin 45^\circ$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{\sqrt{2}}{2}\right)$ $= \frac{2\sqrt{2}}{2\sqrt{5}} - \frac{\sqrt{2}}{2\sqrt{5}}$ $= \frac{\sqrt{2}}{2\sqrt{5}} \text{ or } \frac{\sqrt{10}}{10}$	$\checkmark$ expansion $\checkmark \left(\frac{1}{\sqrt{2}}\right)$ $\checkmark \left(\frac{2}{\sqrt{5}}\right) \& \left(\frac{1}{\sqrt{5}}\right)$ $\checkmark$ answer (4) $\checkmark$ expansion $\checkmark \left(\frac{1}{\sqrt{2}}\right)$ $\checkmark \left(\frac{2}{\sqrt{5}}\right) \& \left(\frac{1}{\sqrt{5}}\right)$ $\checkmark$ answer (4)
		[21]

QUESTION/VRAAG 6

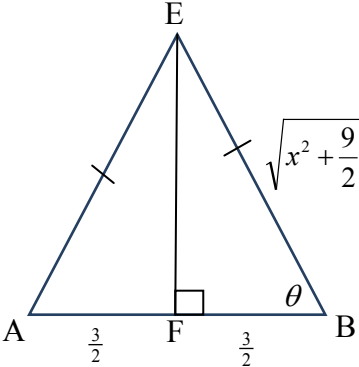
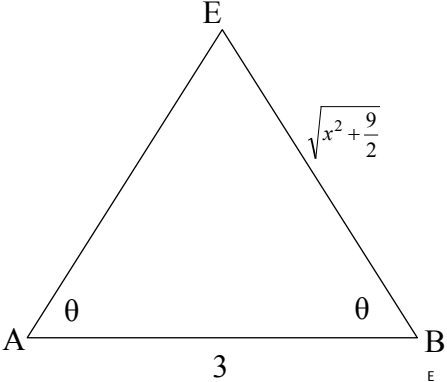
6.1		✓ x- intercepts/ afsnitte ✓ y- intercept/ afsnit ✓ turning pts/ draaipunte (3)
6.2	$f(x) - 3 = 2 \sin 2x - 3$ $\therefore$ maximum value $= 2 - 3 = -1$	✓ ✓ answer (2)
6.3	$2 \sin 2x = -\cos 2x$ $\tan 2x = -\frac{1}{2}$ $\text{ref}\angle = 26,57^\circ$ $2x = 153,43^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$ $x = 76,72^\circ + k \cdot 90^\circ; k \in \mathbb{Z}$ or $x = -13,28^\circ + k \cdot 90^\circ; k \in \mathbb{Z}$ OR/OF $2 \sin 2x = -\cos 2x$ $\tan 2x = -\frac{1}{2}$ $\text{ref}\angle = 26,57^\circ$ $2x = 153,43^\circ + k \cdot 360^\circ$ or $333,43^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $x = 76,72^\circ + k \cdot 180^\circ$ or $166,72^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$	✓ $\tan 2x = -\frac{1}{2}$ ✓ $2x = 153,43^\circ$ or $-26,56^\circ$ ✓ $76,72^\circ$ or $-13,28^\circ$ ✓ $k \cdot 90^\circ; k \in \mathbb{Z}$ (4) ✓ $\tan 2x = -\frac{1}{2}$ ✓ $2x = 153,43^\circ$ & $333,43^\circ$ ✓ $76,72^\circ$ & $166,72^\circ$ ✓ $k \cdot 180^\circ; k \in \mathbb{Z}$ (4)
6.4	$x \in (-103,28^\circ; -13,28^\circ)$ OR/OF $-103,28^\circ < x < -13,28^\circ$	✓ ✓ values ✓ notation (3) ✓ ✓ values ✓ notation (3) [12]

QUESTION/VRAAG 7

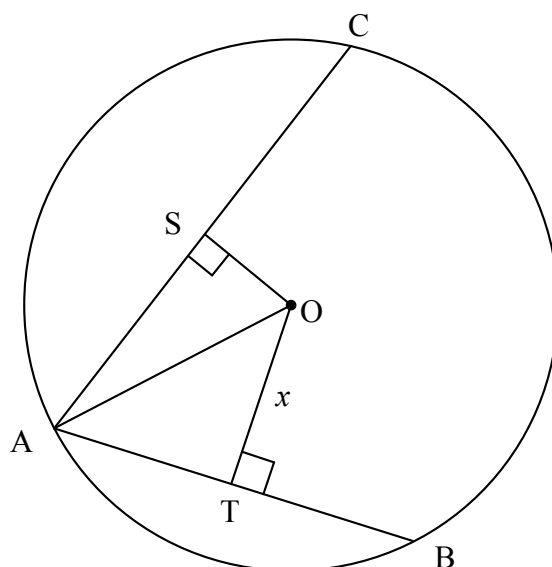


Volume of pyramid = $\frac{1}{3}$ (area of base) $\times$ ($\perp$ height)

7.1	$DB^2 = 3^2 + 3^2$ [Theorem of Pyth] $= 18$ $DB = \sqrt{18}$ $OB = \frac{1}{2} DB = \frac{\sqrt{18}}{2}$ or $\frac{3}{\sqrt{2}}$ or $\frac{3\sqrt{2}}{2}$ or 2,12 OR/OF $\sin 45^\circ = \frac{OB}{3}$ $OB = 3 \sin 45^\circ$ $OB = \frac{3\sqrt{2}}{2}$ or $\frac{3}{\sqrt{2}}$ or 2,12 OF/OR $\cos 45^\circ = \frac{OB}{3}$ $\frac{1}{\sqrt{2}} = \frac{OB}{3}$ $OB = \frac{3}{\sqrt{2}}$ or $\frac{3\sqrt{2}}{2}$ or 2,12	✓ substitution into Pyth ✓ value of DB ✓ answer (3) ✓ correct ratio ✓ OB as subject ✓ answer (3) ✓ correct ratio ✓ special angle ✓ answer (3)
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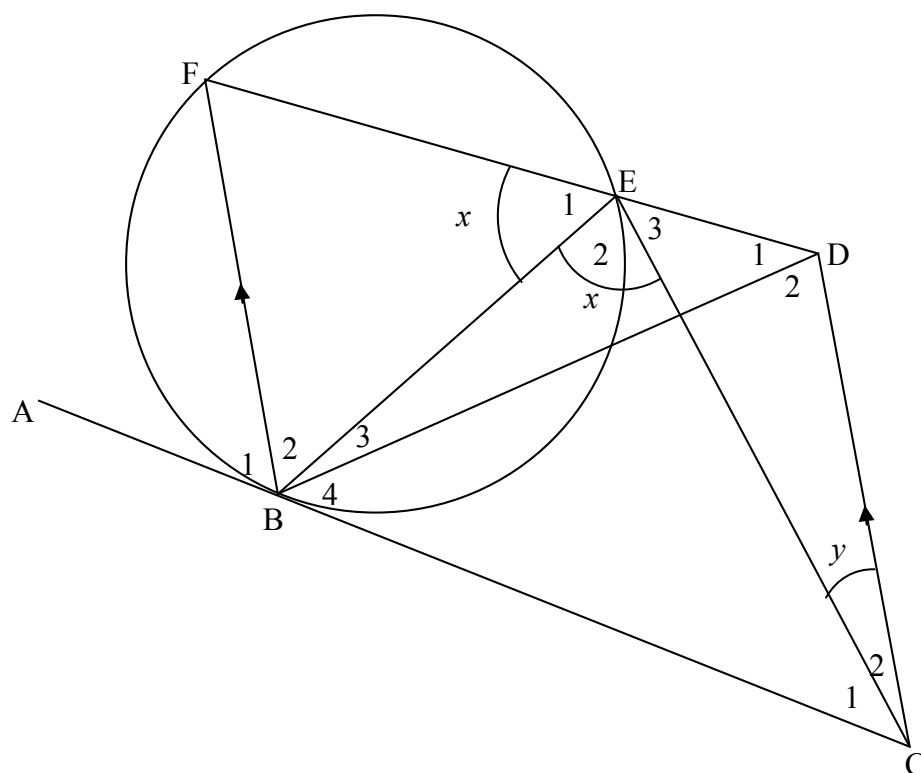
	OR/OF $BE^2 = EO^2 + OB^2 \quad (\text{Pyth})$ $BE^2 = x^2 + \left(\frac{3}{\sqrt{2}}\right)^2$ $BE = \sqrt{x^2 + \frac{9}{2}}$ $\cos \theta = \frac{\frac{3}{2}}{\sqrt{x^2 + \frac{9}{2}}}$ $= \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$  OR/OF $\hat{E} = 180^\circ - 2\theta$ $\sin E = \sin 2\theta$ $\therefore \frac{3}{\sin 2\theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin \theta}$ $\therefore \frac{3}{2 \sin \theta \cos \theta} = \frac{\sqrt{x^2 + \frac{9}{2}}}{\sin \theta}$ $\therefore \frac{3}{2 \cos \theta} = \sqrt{x^2 + \frac{9}{2}}$ $\cos \theta = \frac{3}{2\sqrt{x^2 + \frac{9}{2}}}$ 	✓ substitution into Pyth ✓ length of BE ✓ sketch with values ✓ $\frac{3}{2}$ ✓ substitution (5) ✓ $\hat{E} = 180^\circ - 2\theta$ ✓ $\sin E = \sin 2\theta$ ✓ subst into sine rule ✓ diagram ✓ $2 \sin \theta \cos \theta$ (5)
7.3	Volume = $\frac{1}{3}$(area of base) $\times$ ($\perp$ height) $15 = \frac{1}{3}(9) \times x$ $x = 5$ $\cos \theta = \frac{3}{2\sqrt{25 + \frac{9}{2}}}$ $\therefore \theta = 73,97^\circ$	✓ substitution ✓ x-value ✓ substitution ✓ answer (4) [12]

8.2



8.2.1	AT = 20 [line from centre $\perp$ to chord/lyn vanaf midpt $\perp$ koord]	✓S (1)
8.2.2	$AO^2 = OS^2 + AS^2$ [Pyth : ΔAOS] $OT^2 + AT^2 = OS^2 + AS^2$ [Pyth : ΔAOT] But AS = 24 [line from centre $\perp$ to chord/lyn vanaf midpt $\perp$ koord] $OT^2 + 400 = \left(\frac{7}{15}OT\right)^2 + 576$ $176 = \frac{176}{225}OT^2$ $OT^2 = 225$ $OT = 15$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ OR/OF Let OS = 7, then OT = 15 In ΔAOT : $AO^2 = 20^2 + 15^2$ $= 625$ $AO = 25$ In ΔAOS : $AO^2 = 24^2 + 7^2$ $= 625$ $AO = 25$ $\therefore OA = 25$ OR/OF	✓ equating ✓ AS = 24 ✓ substitution $OS = \frac{7}{15}OT$ ✓ OT ✓ radius (5) ✓✓ testing in ΔAOT ✓✓ testing in ΔAOS ✓ conclusion (5)

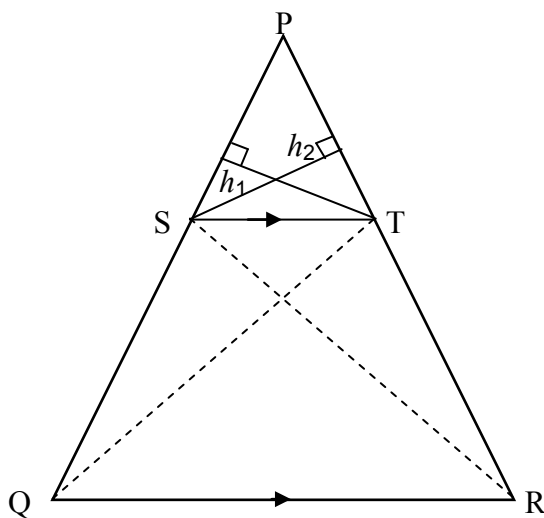
$AO^2 = OS^2 + AS^2$ $OT^2 + AT^2 = OS^2 + AS^2$ Let $OT = 15x$. Then $OS = 7x$ But $AS = 24$ [line from centre $\perp$ to chord/<i>lyn vanaf midpt $\perp$ koord</i>] $(15x)^2 + 400 = (7x)^2 + 576$ $225x^2 + 400 = 49x^2 + 576$ $176x^2 = 176$ $x = 1$ $\therefore AO = \sqrt{225 + 400}$ $= 25$ OR/OF $AS = 24$ [line from centre $\perp$ to chord/<i>lyn vanaf midpt $\perp$ koord</i>] $AO^2 = OS^2 + AS^2$ $= \left(\frac{7}{15}OT\right)^2 + AS^2$ $AO^2 = \frac{49}{225}(AO^2 - 20^2) + 24^2$ [Pyth : ΔAOT] $\frac{176}{225}AO^2 = \frac{4400}{9}$ $AO^2 = 625$ $AO = 25$	✓ equating ✓ $AS = 24$ ✓ substitution ✓ $x = 1$ ✓ radius (5) ✓ $AS = 24$ ✓ substitution $OS = \frac{7}{15}OT$ ✓ equating ✓ subst Pyth ✓ radius (5) [12]
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QUESTION/VRAAG 9

9.1.1	tangent chord theorem/raaklyn-koordstelling	✓ R	(1)
9.1.2	corresponding/ooreenkomstige $\angle$ s/e; $FB \parallel DC$	✓ R	(1)
9.2	$\hat{E}_1 = \hat{B}\hat{C}\hat{D}$ $\therefore BCDE = \text{cyclic quad}$ [converse ext $\angle$ cyc quad/omgek: buite $\angle$ kdvh]	✓ S ✓ R	(2)
9.3	$\hat{D}_2 = \hat{E}_2$ [$\angle$ s in the same segment/ $\angle$ e in dies segment] $\hat{D}_2 = \hat{F}\hat{B}\hat{D}$ [alt $\angle$ s, $BF \parallel CD$ /verwiss $\angle$ e, $BF \parallel CD$]	✓ S ✓ S	(2)
9.4	$\hat{B}_3 = y$ OR $\hat{B}_3 = \hat{C}_2$ [$\angle$ s in the same segment/ $\angle$ e in dies segment] $\hat{B}_2 = x - y$ OR $\hat{B}_3 + \hat{B}_2 = x$ [from 9.3 and 9.4] $\hat{C}_1 = x - y$ [from 9.2 and 9.3] $\therefore \hat{B}_2 = \hat{C}_1$ OR/OF In $\triangle BFE$ and $\triangle BEC$ $\hat{E}_1 = \hat{E}_2$ [= x] $\hat{F} = \hat{B}_3 + \hat{B}_4$ [tan - chord theorem] $\therefore \triangle BFE \sim \triangle CBE$ [$\angle, \angle, \angle$] $\therefore \hat{B}_2 = \hat{C}_1$	✓ S ✓ S ✓ S ✓ identifying Δ 's ✓ S ✓ S	(3) [9]

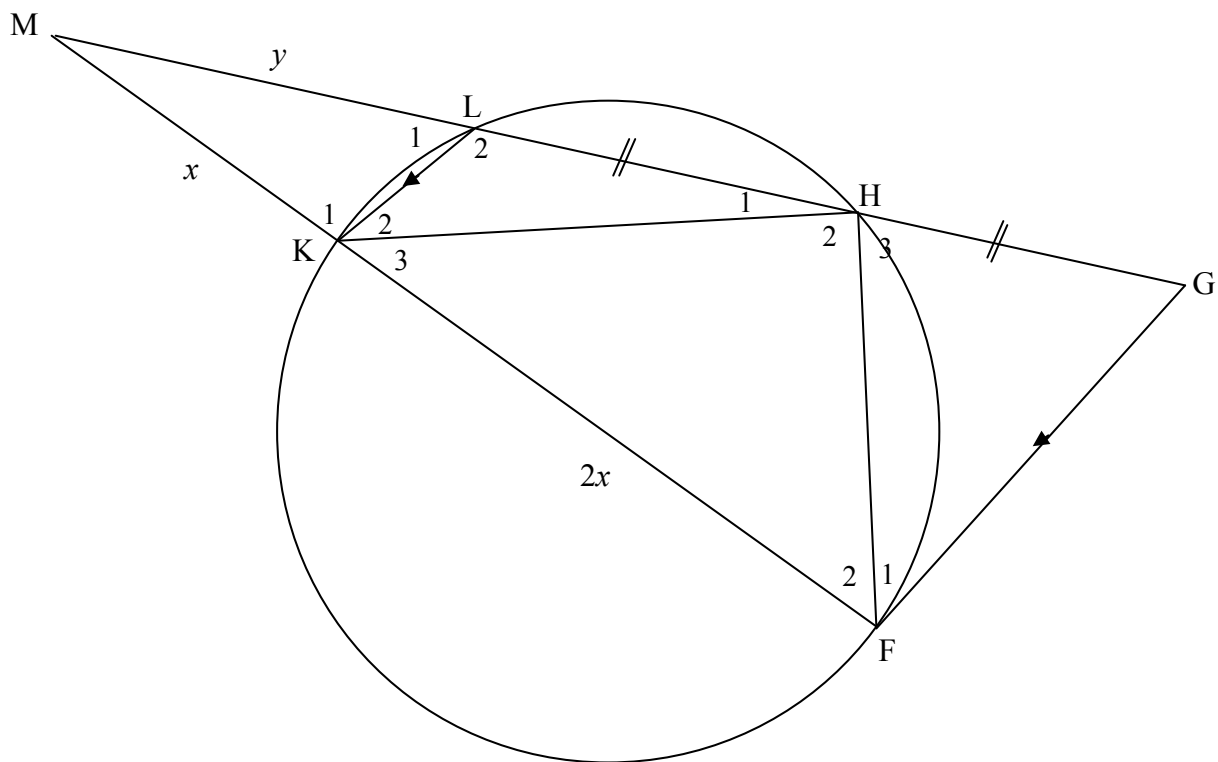
QUESTION/VRAAG 10

10.1



10.1	Constr : Join S to R and T to Q and draw h_1 from S $\perp$ PT and h_2 from T $\perp$ PS/ Verbind SR en TQ en trek h_1 van S $\perp$ PT en h_2 van T $\perp$ PS] Proof : $\frac{\text{area } \triangle PST}{\text{area } \triangle QST} = \frac{\frac{1}{2} PS \times h_2}{\frac{1}{2} SQ \times h_2} = \frac{PS}{SQ} \quad \text{equal altitudes}$ $\frac{\text{area } \triangle PST}{\text{area } \triangle STR} = \frac{\frac{1}{2} PT \times h_1}{\frac{1}{2} TR \times h_1} = \frac{PT}{TR} \quad \text{equal altitudes}$ area $\triangle PST$ = area $\triangle PST$ [common] But area $\triangle QST$ = area $\triangle STR$ [same base, height; $ST \parallel QR$] $\therefore \frac{\text{area } \triangle PST}{\text{area } \triangle QST} = \frac{\text{area } \triangle PST}{\text{area } \triangle STR}$ $\therefore \frac{PS}{SQ} = \frac{PT}{TR}$	✓ constr/konstruksie ✓ $\frac{\text{area } \triangle PST}{\text{area } \triangle QST} = \frac{\frac{1}{2} PS \times h_2}{\frac{1}{2} SQ \times h_2} = \frac{\text{area } \triangle PST}{\text{area } \triangle STR} = \frac{PT}{TR}$ ✓ S ✓ R ✓ S (6)
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10.2



10.2.1	Corresponding/Ooreenkomstige $\angle$ s/e; $GF \parallel LK$	✓ R (1)
10.2.2(a)	$\frac{GL}{LM} = \frac{FK}{KM}$ OR $\frac{GL}{y} = \frac{2x}{x}$ [prop theorem/eweredighst; $GF \parallel LK$] $\frac{2GH}{y} = \frac{2x}{x}$ [LH = HG] $\therefore GH = y$	✓ S ✓ R ✓ $GL = 2GH$ (3)

10.2.2(b)	$\tilde{K}_1 = G\hat{F}M$ $L\hat{K}M$ or $\tilde{K}_1 = M\hat{H}F$ $M\hat{H}F = G\hat{F}M$ In ΔMFH and ΔMGF : $\hat{M} = \hat{M}$ $M\hat{H}F = G\hat{F}M$ $\therefore \Delta MFH \parallel \parallel \Delta MGF$ OR/OR $\tilde{K}_1 = G\hat{F}M$ $L\hat{K}M$ or $\tilde{K}_1 = M\hat{H}F$ $M\hat{H}F = G\hat{F}M$ In ΔMFH and ΔMGF : $\hat{M} = \hat{M}$ $M\hat{H}F = G\hat{F}M$ $\hat{F}_2 = \hat{G}$ $\therefore \Delta MFH \parallel \parallel \Delta MGF$	[corresponding/ooreenkomst $\angle$ s; $GF \parallel LK$] [ext $\angle$ cyclic quad/buite $\angle$ koordevh] [common/gemeen] [proven/bewys] [$\angle \angle \angle$] [corresponding/ooreenkomst $\angle$ s; $GF \parallel LK$] [ext $\angle$ cyclic quad/buite $\angle$ koordevh] [common/gemeen] [proven/bewys] [$\angle$ s of $\Delta = 180^\circ$]	$\checkmark S \checkmark R$ $\checkmark S$ $\checkmark S$ $\checkmark R$ (5) $\checkmark S \checkmark R$ $\checkmark S$ $\checkmark S$ $\checkmark S$ (5)
10.2.2(c)	$\therefore \frac{GF}{FH} = \frac{MF}{MH}$ $= \frac{3x}{2y}$	[$\parallel \parallel \Delta$ s]	$\checkmark S \checkmark R$ (2)
10.2.3	$\frac{MF}{MH} = \frac{MG}{MF}$ $\frac{3x}{2y} = \frac{3y}{3x}$ $\frac{y^2}{x^2} = \frac{9}{6} = \frac{3}{2}$ $\frac{y}{x} = \sqrt{\frac{3}{2}}$	[$\parallel \parallel \Delta$ s] [from 10.2.2(c)]	$\checkmark S$ $\checkmark$ substitution $\checkmark$ simplification (3) [20]
TOTAL MARKS			150