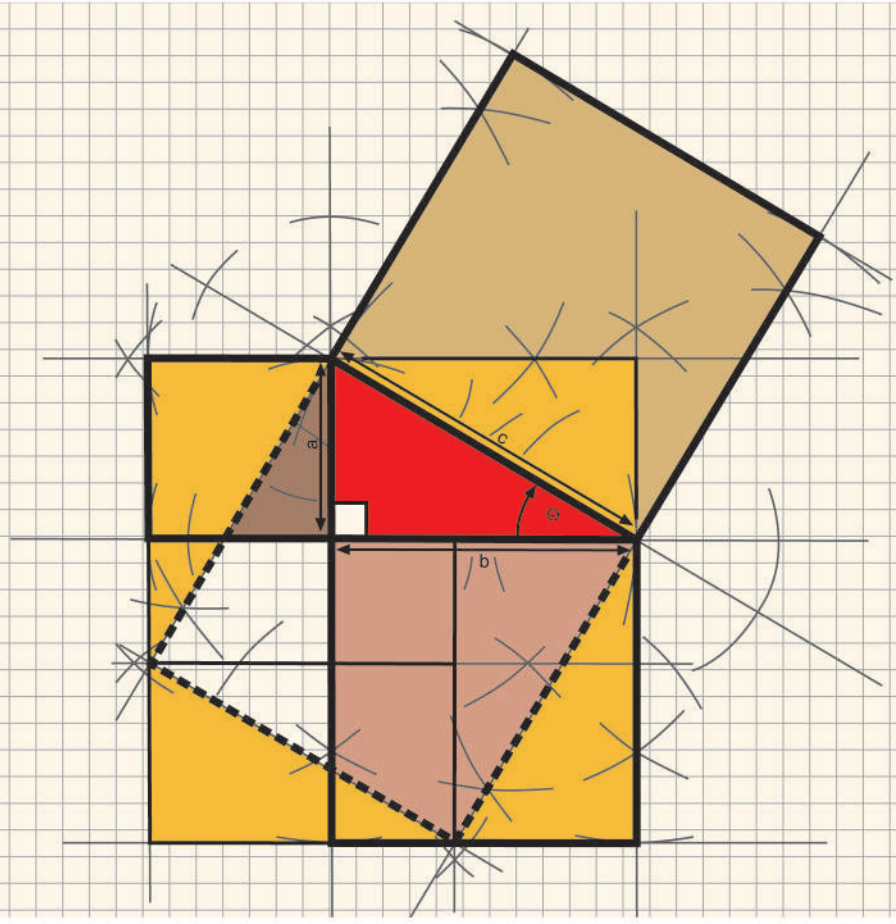
MATHEMATICS

GRADE 8

REVISED EDITION

TEACHER GUIDE



basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SASOL

Mathematics

Grade 8

Teacher Guide

Developed and funded as an ongoing project by the Sasol Inzalo Foundation in partnership with the Ukuqonda Institute

Published by **The Ukuqonda Institute** (http://www.ukuqonda.org.za)

Registered as a Title 21 Company, Reg. No. 2006/026363/08

Public Benefit Organisation, PBO No. 930035134

© This edition published in 2017. Copyright in the work is vested in the publisher. Copyright in the text remains vested in the authors.

Mathematics Teacher Guide Grade 8

ISBN: 978-1-4315-2878-3

This Teacher Guide and its accompanying Learner Book (ISBN: 978-1-4315-2877-6) were developed with the participation of the Department of Basic Education (DBE) of South Africa with funding from the Sasol Inzalo Foundation (SaIF).

Authors and Contributors:

Herholdt Bezuidenhout, Lucinda Cruickshank, Marthinus de Jager, Gudrun Elliott, Andrew Hofmeyr, Piet Human, Louise Keegan, Erna Lampen, Nathi Makae, Enoch Masemola, Alwyn Olivier, Cerenus Pfeiffer, Rika Potgieter, Johan Pretorius, Renate Röhrs, Paul van Koersveld, Therine van Niekerk, Dirk Wessels

Illustrations and computer graphics: Leonora van Staden, Zhandré Stark, Lisa Steyn Illustration

Computer graphics for chapter frontispieces: Piet Human

Cover illustration: Leonora van Staden

Text design: Mike Schramm

Layout and typesetting: Lebone Publishing Services

Subject advisors from the DBE who contributed by means of review: The publisher thanks those subject advisors of the DBE who reviewed this book series on four occasions in 2013-2014, as well as in October 2017. The authors changed the text so as to align with the reviewers' requests/suggestions for improvements, as far as possible, and believe that the books improved as a result of that.

The reviewers were: [names to be supplied]

Rights of other copyright holders: All reasonable efforts have been made to ensure that materials included are not already copyrighted to other entities, or in a small number of cases, to seek permission from and acknowledge copyright holders. In some cases this may not have been possible. The publishers welcome the opportunity for redress with any unacknowledged copyright holders.

Printed by: [printer name and address]

Your freedom to legally copy this book

This work is published under a Creative Commons Attribution-NonCommercial 4.0 Unported License (CC BY-NC 4.0).

http://creativecommons.org/licenses/by-nc/4.0/

You are allowed to freely copy this book subject to the qualifications below. You can photocopy, print and distribute it as you like. You may download it onto any electronic device, distribute it via email, and upload it to your website, at no charge. You may also adapt the text and illustrations.

Attribution: When doing any of the above, you should provide clear acknowledgement of the license/copyright holders ('attribute the original work'). This attribution should include the name(s) of the original book(s) and the publisher, as well as acknowledgement of the SaIF and the DBE of South Africa. You should also provide the Creative Commons web address which explains this type of license: http://creativecommons.org/licenses/by-nc/4.0/

If you made changes to the material, you must indicate what changes you have made. You may not in any way suggest that the licensor endorses you or your use of your material or your changes to the material.

Restrictions: ***You may not make copies*** of this book or a part of this book – in printed or electronic or audio or video or whatever form – ***for a profit-seeking purpose.***

Limited rights on use of photos: A photograph in the book that shows a person or trademark emblem/insignia in an even slightly recognisable manner may not be used out of its learning context, and may not be used for an intended audience other than learners/students and teachers. A photo containing a person or trademark emblem/insignia may only be used in a web page or in print or in a slide show (or in any other media) if it is for a not-for-profit teaching/learning purpose, and if the photo on that page/web page/slide/other media is accompanied by related learning material(s) that is at least as big as the photo itself.

No photograph in the book may be used for promotional or advertising purposes, even if such promotion or advertising is not for a profit-seeking purpose.

No photograph in the book that shows a person may be used on the cover of any derived work, or in any media in a way equally prominent to the cover of a book.

Contents

Term 1

Chapter 1: Whole numbers 3
Chapter 2: Integers 23
Chapter 3: Exponents 39
Chapter 4: Numeric and geometric patterns 57
Chapter 5: Functions and relationships 69
Chapter 6: Algebraic expressions 1 79
Chapter 7: Algebraic equations 1 87

Term 2

Chapter 8: Algebraic expressions 2 93
Chapter 9: Algebraic equations 2 105
Chapter 10: Construction of geometric figures 111
Chapter 11: Geometry of 2D shapes 129
Chapter 12: Geometry of straight lines 147

Term 3

Chapter 13: Common fractions 167
Chapter 14: Fractions in decimal notation 185
Chapter 15: The theorem of Pythagoras 193
Chapter 16: Perimeter and area of 2D shapes 205
Chapter 17: Surface area and volume of 3D objects 219
Chapter 18: Collect, organise and summarise data 231
Chapter 19: Represent data 247
Chapter 20: Interpret, analyse and report on data 261

Term 4

Chapter 21: Functions and relationships 269
Chapter 22: Algebraic equations 279
Chapter 23: Graphs 287
Chapter 24: Transformation geometry 301
Chapter 25: Geometry of 3D objects 317
Chapter 26: Probability 349

Resources: Copiable graph paper 359

Term 1

Chapter 1: Whole numbers 3
1.1 Properties of whole numbers 4
1.2 Calculations with whole numbers 7
1.3 Multiples, factors and prime factors 14
1.4 Solving problems 18

Chapter 2: Integers 23
2.1 What is beyond 0? 24
2.2 Adding and subtracting with integers 27
2.3 Multiplying and dividing with integers 31
2.4 Squares, cubes and roots with integers 35

Chapter 3: Exponents 39
3.1 Revision 40
3.2 Working with integers 43
3.3 Laws of exponents 44
3.4 Calculations 52
3.5 Squares, cubes and roots of rational numbers 52
3.6 Scientific notation 54

Chapter 4: Numeric and geometric patterns 57
4.1 The term-term relationship in a sequence 58
4.2 The position-term relationship in a sequence 61
4.3 Investigating and extending geometric patterns 62
4.4 Describing patterns in different ways 65

Chapter 5: Functions and relationships 69
5.1 Constant and variable quantities 70
5.2 Different ways to describe relationships 74
5.3 Algebraic symbols for variables and relationships 76

Chapter 6: Algebraic expressions 1 79
6.1 Algebraic language 80
6.2 Add and subtract like terms 83

Chapter 7: Algebraic equations 1 87
7.1 Setting up equations 88
7.2 Solving equations by inspection 89
7.3 More examples 90

Grade 8 Term 1 Chapter 1 Whole numbers

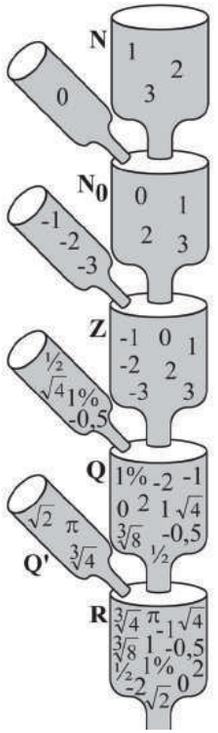
Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
1.1 Properties of whole numbers	The commutative, associative and distributive properties of numbers	Pages 1 to 4
1.2 Calculations with whole numbers	Estimating, approximating, rounding and compensating; adding, subtracting and multiplying in columns; using long division	Pages 4 to 11
1.3 Multiples, factors and prime factors	Multiples versus factors; prime numbers versus composite numbers; prime factorisation; LCM versus HCF	Pages 11 to 15
1.4 Solving problems	Ratio and rate; profit, loss, discount and interest	Pages 15 to 18

CAPS time allocation	6 hours
CAPS content specification	Pages 75 to 77

If natural numbers are added, the answer is also a natural number.
We say: The set of natural numbers is closed under addition.

Mathematical background

- The illustration on the right shows how the set of real numbers has developed from the set of natural numbers. As additional sets of numbers are added to the natural numbers, the advantages of each new set of numbers increase and its disadvantages decrease.
- The set of **natural numbers** is defined by the set $N = \{1; 2; 3; 4; 5; ...\}$.
  - Advantages: N is closed under addition and multiplication; N contains the identity element for multiplication (1).
  - Disadvantages: N is not closed under subtraction or division; N does not contain the identity element for addition (0).
- The set of **whole numbers** $N_0 = \{0; 1; 2; 3; 4; 5; ...\}$ is formed by adding 0 to the set of natural numbers.
  - Additional advantage: N_0 contains the identity element for addition (0).
- The set of **integers** $Z = \{...; -3; -2; -1; 0; 1; 2; 3; ...\}$ is formed by adding the set $\{...; -4; -3; -2; -1\}$ to the set of whole numbers.
  - Additional advantage: Z is closed under subtraction.
- The set of **rational numbers** Q is formed by adding common fractions (and therefore mixed numbers), decimal fractions (and therefore decimal numbers), percentages and ratios to the set of integers.
  - Additional advantage: Q is closed under division (but division by 0 is undefined).
- The set of **real numbers** R is formed by adding the set of **irrational numbers** Q' (surds and numbers like π) to the set of rational numbers.
- The commutative, associative and distributive properties play a very important role in operations with numbers (refer to section 1.1 below).



1.1 Properties of whole numbers

THE COMMUTATIVE PROPERTY OF ADDITION AND MULTIPLICATION

Background information

- The **commutative property of addition** states that, if a numerical expression involves only addition, calculations can be performed in any order. This property does not work for subtraction.
- The **commutative property of multiplication** states that, if a numerical expression involves only multiplication, calculations can be performed in any order. This property does not work for division.

Teaching guidelines

Demonstrate the commutative property of addition and multiplication:

- 25 + 40 = 65 and 40 + 25 = 65
- 25 × 40 = 1 000 and 40 × 25 = 1 000

Note on question 1

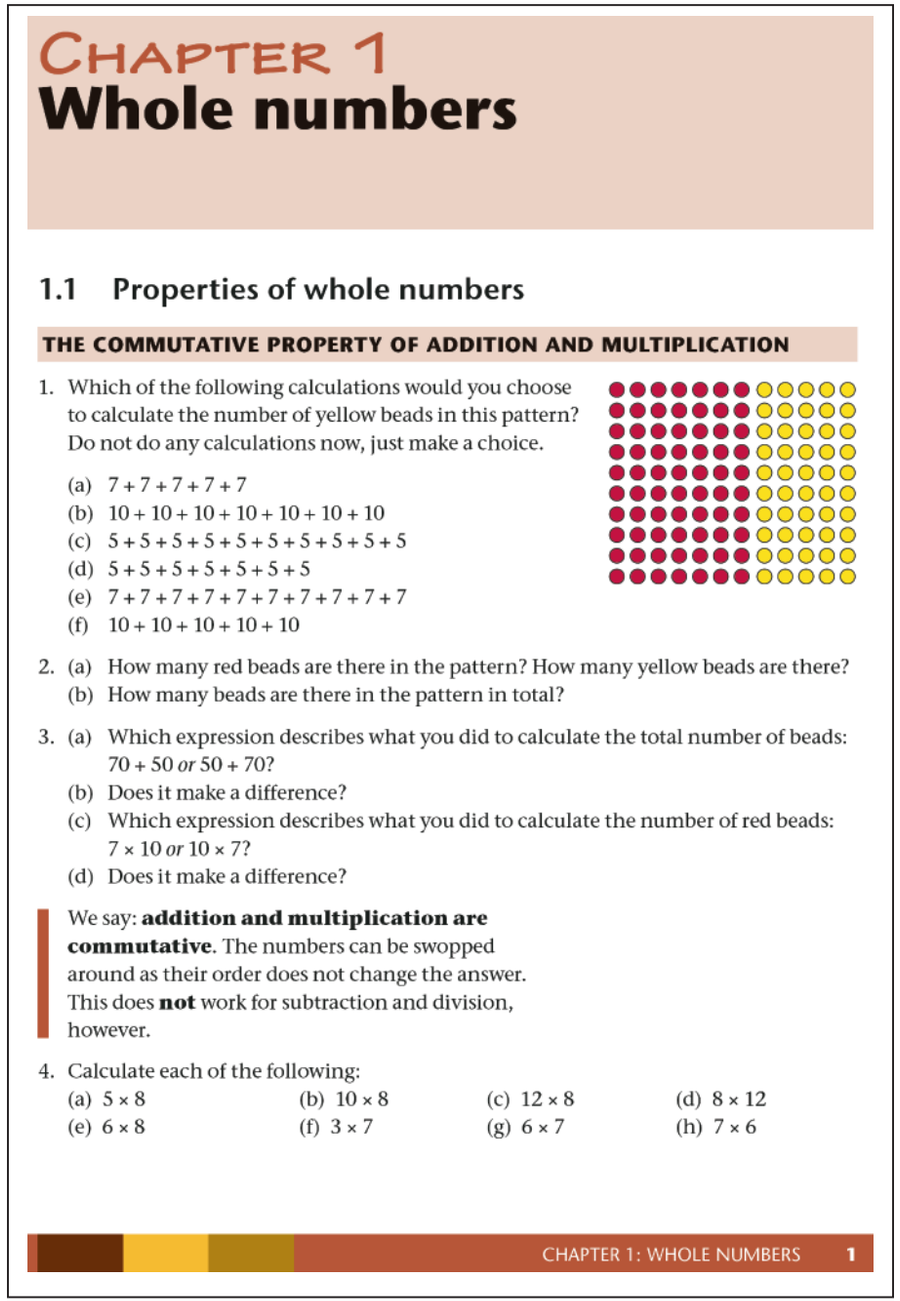
5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 can be written as 10 × 5, which equals 50.

10 + 10 + 10 + 10 + 10 can be written as 5 × 10, which equals 50.

This illustrates the commutative property of multiplication.

Answers

1. Learners' choice, but only (c) and (f) will give the correct answer.
2. (a) 70 red beads, 50 yellow beads
   (b) 120
3. (a) Answers will vary.
   (b) No (70 + 50 = 120 and 50 + 70 = 120)
   (c) Answers will vary.
   (d) No (7 × 10 = 70 and 10 × 7 = 70)
4. (a) 5 × 8 = 40 (b) 10 × 8 = 80 (c) 12 × 8 = 96 (d) 8 × 12 = 96
   (e) 6 × 8 = 48 (f) 3 × 7 = 21 (g) 6 × 7 = 42 (h) 7 × 6 = 42



CHAPTER 1
Whole numbers

1.1 Properties of whole numbers

THE COMMUTATIVE PROPERTY OF ADDITION AND MULTIPLICATION

1. Which of the following calculations would you choose to calculate the number of yellow beads in this pattern? Do not do any calculations now, just make a choice.

   (a) 7 + 7 + 7 + 7 + 7
   (b) 10 + 10 + 10 + 10 + 10 + 10 + 10
   (c) 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5
   (d) 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5
   (e) 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7
   (f) 10 + 10 + 10 + 10 + 10

2. (a) How many red beads are there in the pattern? How many yellow beads are there?
   (b) How many beads are there in the pattern in total?

3. (a) Which expression describes what you did to calculate the total number of beads: 70 + 50 *or* 50 + 70?
   (b) Does it make a difference?
   (c) Which expression describes what you did to calculate the number of red beads: 7 × 10 *or* 10 × 7?
   (d) Does it make a difference?

We say: **addition and multiplication are commutative**. The numbers can be swopped around as their order does not change the answer. This does **not** work for subtraction and division, however.

4. Calculate each of the following:
   (a) 5 × 8 (b) 10 × 8 (c) 12 × 8 (d) 8 × 12
   (e) 6 × 8 (f) 3 × 7 (g) 6 × 7 (h) 7 × 6

CHAPTER 1: WHOLE NUMBERS 1

THE ASSOCIATIVE PROPERTY OF ADDITION AND MULTIPLICATION

Background information

- The **associative property of addition** states that, if three or more numbers have to be added, the numbers can be grouped in any order. This property does not work for subtraction.
- The **associative property of multiplication** states that, if three or more numbers have to be multiplied, the numbers can be grouped in any order. This property does not work for division.

Teaching guidelines

Demonstrate the associative property of addition and multiplication:

- $2 + 3 + 4$, $(2 + 3) + 4$, $2 + (3 + 4)$ and $(2 + 4) + 3$ are all equal to 9
- $2 \times 3 \times 4$, $(2 \times 3) \times 4$, $2 \times (3 \times 4)$ and $(2 \times 4) \times 3$ are all equal to 24

Answers

1. Yes
2. (a) 22 (b) 22 (c) 22 (d) 22
3. (a) Yes (b) E.g. $5 + 10 + 3 = 18$ and $10 + 3 + 5 = 18$
4. (a) $10 \times 17 = 170$ (b) $20 \times 7 = 140$
   (c) $100 + 37 = 137$ (d) $100 + 100 = 200$
5. $82 + \underline{18}$ $44 + \underline{56}$ $56 + \underline{44}$ $78 + \underline{22}$
   $24 + \underline{76}$ $89 + \underline{11}$ $77 + \underline{23}$
6. $250 \times \underline{4}$ $125 \times \underline{8}$ $25 \times \underline{40}$ $500 \times \underline{2}$ $200 \times \underline{5}$ $50 \times \underline{20}$
7. (a) $(82 + 18) + (54 + 46) + 237 = 437$
   (b) $(24 + 76) + (89 + 11) + (44 + 56) = 300$
   (c) $(25 \times 4) \times 86 = 100 \times 86 = 8\,600$
   (d) $(8 \times 125) \times 4 = 1\,000 \times 4 = 4\,000$

MORE CONVENTIONS AND THE DISTRIBUTIVE PROPERTY

Background information

- The **distributive property** allows the decomposition of a product into partial products that can be more easily calculated. For example, 7×648, which means $7 \times (600 + 40 + 8)$, can be broken down into $7 \times 600 + 7 \times 40 + 7 \times 8$. A person who knows basic multiplication facts (tables) can easily calculate these partial products and add them up to evaluate 7×648.

THE ASSOCIATIVE PROPERTY OF ADDITION AND MULTIPLICATION

Lebogang and Nathi both have to calculate 25×24.

Lebogang calculates 25×4 and then multiplies by 6.
Nathi calculates 25×6 and then multiplies by 4.

1. Will they get the same answer or not?

If three or more numbers have to be multiplied, it does not matter which two of the numbers are multiplied first.

This is called the **associative property of multiplication**.
We also say **multiplication is associative**.

2. Do the following calculations. **Do not use a calculator now**.
   (a) $4 + 7 + 5 + 6$ (b) $7 + 6 + 5 + 4$
   (c) $6 + 5 + 7 + 4$ (d) $7 + 5 + 4 + 6$
3. (a) Is addition associative?
   (b) Illustrate your answer with an example.
4. Find the value of each expression by working in the easiest possible way:
   (a) $2 \times 17 \times 5$ (b) $4 \times 7 \times 5$
   (c) $75 + 37 + 25$ (d) $60 + 87 + 40 + 13$
5. What must you add to each of the following numbers to get 100?
   82 44 56 78 24 89 77
6. What must you multiply each of these numbers by to get 1 000?
   250 125 25 500 200 50
7. Calculate each of the following. Note that you can make the work very easy by deciding how to group the operations.
   (a) $82 + 54 + 18 + 46 + 237$ (b) $24 + 89 + 44 + 76 + 56 + 11$
   (c) $25 \times (86 \times 4)$ (d) 32×125

MORE CONVENTIONS AND THE DISTRIBUTIVE PROPERTY

The distributive property is a useful property because it allows us to do this:



$3 \times (2 + 4)$ $3 \times 2 + 3 \times 4$

Both answers are 18. We used brackets in the first example to show that the addition operation must be done first. Otherwise, we would have done the multiplication first. For example, the expression $3 \times 2 + 4$ means "multiply 3 by 2; then add 4". It does **not** mean "add 2 and 4; then multiply by 3".

The column format is a convenient way of making the transition from $7 \times (600 + 40 + 8)$ to $7 \times 600 + 7 \times 40 + 7 \times 8$.

- The distributive property enables us to **distribute multiplication over addition and subtraction**. For example:

$7 \times 123 = 7 \times (100 + 20 + 3) = 7 \times 100 + 7 \times 20 + 7 \times 3 = 700 + 140 + 21 = 861$

$7 \times 99 = 7 \times (100 - 1) = 7 \times 100 - 7 \times 1 = 700 - 7 = 693$

Misconceptions

"The distributive property **requires** that 'brackets be removed' before a product expression is evaluated." Although it is true that, for example,

$7 \times (25 + 75) = 7 \times 25 + 7 \times 75$,

there is no "requirement" that the form on the right should be executed to evaluate the expression. In fact, it is much easier to apply the operations specified in the form on the left, i.e. $25 + 75 = 100$ and $7 \times 100 = 700$, than to apply the operations specified on the right, namely $7 \times 25 = 175$ and $7 \times 75 = \ldots$ etc. It is important that learners realise that the distributive property is not a "law" that requires that something be done, it is a proposition that states that two different sets of calculations will consistently produce the same result, hence any of the two sets can be used.

Teaching guidelines

Illustrate how the distributive property enables us to distribute multiplication over addition by calculating the following expressions:

8×234; $8 \times (200 + 30 + 4)$; $8 \times 200 + 8 \times 30 + 8 \times 4$

Illustrate how the distributive property enables us to distribute multiplication over subtraction by calculating the following expressions:

8×394; $8 \times (400 - 6)$; $8 \times 400 - 8 \times 6$

Answers

1. (a) $(8 + 6) \times 5 = 70$ (b) $8 + (6 \times 5) = 38$ (c) $(5 + 8) \times (6 - 2) = 52$
   (d) $(5 + 8) \times 6 - 2 = 76$ (e) $5 + (8 \times 6) - 2 = 51$ (f) $5 + 8 \times (6 - 2) = 37$
2. (a) $100 \times 17 = 1\,700$ (b) $1\,000 + 700 = 1\,700$
   (c) $100 \times 3 = 300$ (d) $1\,000 - 700 = 300$
3. See the answers on LB page 3 alongside.

The expression $4 + 3 \times 2$ also means "multiply 3 by 2; then add 4".

If you wish to specify that addition or subtraction should be **done first**, that part of the expression should be enclosed **in brackets**.

The distributive property can be used to break up a difficult multiplication into smaller parts. For example, it can be used to make it easier to calculate 6×204:

6×204 can be rewritten as $6 \times (200 + 4)$ (Remember the brackets!)
$= 6 \times 200 + 6 \times 4$
$= 1\,200 + 24$
$= 1\,224$

Multiplication can also be distributed over subtraction, for example to calculate 7×96:

$7 \times 96 = 7 \times (100 - 4)$
$= 7 \times 100 - 7 \times 4$
$= 700 - 28$
$= 672$

1. Here are some calculations with answers. Rewrite them with brackets to make all the answers correct.
   (a) $8 + 6 \times 5 = 70$ (b) $8 + 6 \times 5 = 38$ (c) $5 + 8 \times 6 - 2 = 52$
   (d) $5 + 8 \times 6 - 2 = 76$ (e) $5 + 8 \times 6 - 2 = 51$ (f) $5 + 8 \times 6 - 2 = 37$
2. Calculate the following:
   (a) $100 \times (10 + 7)$ (b) $100 \times 10 + 100 \times 7$
   (c) $100 \times (10 - 7)$ (d) $100 \times 10 - 100 \times 7$
3. Copy and complete the table:

×	8	5	4	9	7	3	6	2	10	11	12
7	56	35	28	63	49	21	42	14	70	77	84
3	24	15	12	27	21	9	18	6	30	33	36
9	72	45	36	81	63	27	54	18	90	99	108
5	40	25	20	45	35	15	30	10	50	55	60
8	64	40	32	72	56	24	48	16	80	88	96
6	48	30	24	54	42	18	36	12	60	66	72
4	32	20	16	36	28	12	24	8	40	44	48
2	16	10	8	18	14	6	12	4	20	22	24
10	80	50	40	90	70	30	60	20	100	110	120
12	96	60	48	108	84	36	72	24	120	132	144
11	88	55	44	99	77	33	66	22	110	121	132

Note on the distributive property

Learners can only make good sense of the distributive property if they understand that we use brackets to indicate that the operations within the brackets should be done before the other operations in the expression, unless the expression is replaced by an equivalent expression that does not contain brackets. For example, 4 × (30 + 7) can be replaced by 4 × 30 + 4 × 7.

Answers

4. (a) 50 × (10 + 8) = 500 + 400 = 900
   (b) 125 × 8 + 125 × 20 = 1 000 + 2 500 = 3 500
   (c) (40 × 220) – 220 = 8 800 – 220 = 8 580
   (d) (443 + 557) + 2 100 = 1 000 + 2 100 = 3 100
   (e) (318 + 322) + 650 = 640 + 650 = 1 290
   (f) (522 + 78) + 3 003 = 600 + 3 003 = 3 603

Background information (continued)

- The **additive property of 0** states that, when zero is added to any number, the answer is that number.
- The **multiplicative property of 1** states that, when any number is multiplied by 1, the answer is that number.

Teaching guidelines (continued)

Revise the two properties of numbers listed above.

1.2 Calculations with whole numbers

ESTIMATING, APPROXIMATING AND ROUNDING

Background information

- Estimation is approximation based upon a judgement. However, approximation is not necessarily estimation.
  - **Approximation** is applied to an existing number (actual answer) in order to find a result precise enough for a specific purpose.
  - **Estimation** is the mental skill of making an educated guess of the actual answer of a mathematical problem.
- **Error** is the difference between an estimate and the actual answer.
- **Rounding** is done when a number is truncated so as to minimise error.

4. Use the various mathematical conventions for numerical expressions to make the following calculations easier. Show how you work them out.
   (a) 18 × 50 (b) 125 × 28 (c) 39 × 220
   (d) 443 + 2 100 + 557 (e) 318 + 650 + 322 (f) 522 + 3 003 + 78

Two more properties of numbers are:
- **The additive property of 0:** when we add zero to any number, the answer is that number.
- **The multiplicative property of 1:** when we multiply any number by 1, the answer is that number.

1.2 Calculations with whole numbers

ESTIMATING, APPROXIMATING AND ROUNDING

1. Copy and complete the following statements by giving answers to these questions, without doing any calculations with the given numbers.
   (a) Is 8 × 117 more than 2 000 or less than 2 000? less than 2 000
   (b) Is 27 × 88 more than 3 000 or less than 3 000? less than 3 000
   (c) Is 18 × 117 more than 3 000 or less than 3 000? less than 3 000
   (d) Is 47 × 79 more than 3 000 or less than 3 000? more than 3 000

What you have done when you tried to give answers to questions 1(a) to (d), is called **estimation**. To estimate is to try to get close to an answer without actually doing the calculations with the given numbers.

An estimate may also be called an **approximation**.

2. Look at question 1 again.
   (a) The numbers 1 000, 2 000, 3 000, 4 000, 5 000, 6 000, 7 000, 8 000, 9 000 and 10 000 are all multiples of a thousand. In each case, write down the multiple of 1 000 that you think is closest to the answer. The numbers you write down are called **estimates**.
   (b) In some cases you may achieve a **better estimate** by adding 500 to your estimate, or subtracting 500 from it. If so, you may add or subtract 500.
   (c) If you wish, you may write what you believe is an even better estimate by adding or subtracting some hundreds.

3. (a) Use a calculator to find the exact answers for the calculations in question 1. Calculate the **error** in your last approximation of each of the answers in question 1.
   (b) What was your smallest error?

The difference between an estimate and the actual answer is called the **error**.

Teaching guidelines

- We **estimate** the answer to a problem by rounding off the information and guessing a number that can be used to check the actual answer.
- We **approximate** the actual answer by rounding it off so that people find it easier to understand.

Answers

1. See the answers on LB page 4 on the previous page.
2. Learners' own investigations.
3. (a) $8 \times 117 = 936$ $27 \times 88 = 2\,376$ $18 \times 117 = 2\,106$ $47 \times 79 = 3\,713$
   (b) Learners' answers will vary.
4. Sample answer: Estimate the error, for example in 1(d) I knew 4 000 would be too much as $80 \times 50 = 4\,000$.
5. (a) 15 000 (b) 1 300 (c) 12 000 (d) 1 300
   (e) 7 000 (f) 1 300 (g) 19 000 (h) 1 600
   Note: Estimates will vary.
6. (a) 14 952 (b) 1 315 (c) 11 532 (d) 1 358
   (e) 6 636 (f) 1 288 (g) 19 488 (h) 1 593
   Learners calculate the error in each approximation.
7. (a) $80 \times 50 = 4\,000$; 3 588 (b) $70 \times 90 = 6\,300$; 5 896
   (c) $30 \times 280 = 8\,400$; 9 384 (d) $80 \times 180 = 14\,400$; 13 884
   Accept any reasonable approximations.

ROUNDING OFF AND COMPENSATING

Teaching guidelines

Compensation means adding to one number what is taken from the other:
$386 + 3\,435 = (386 + 14) + (3\,435 - 14) = 400 + 3\,421 = 3\,821$

Answers

1. (a) $400 + 3\,400 = 3\,800$ (b) An error of 35
   (c) $(-14) + 35 = 21$ (d) $3\,800 + 21 = 3\,821$

4. Think again about what you did in question 2. In 2(a) you tried to approximate the answers to the nearest **1 000**. In 2(c) you tried to approximate the answers to the nearest **100**. Describe what you tried to achieve in question 2(b).

5. Estimate the answers for each of the following products and sums. Try to approximate the answers for the **products** to the nearest thousand, and for the **sums** to the nearest hundred.

   (a) 84×178 (b) $677 + 638$
   (c) 124×93 (d) $885 + 473$
   (e) 79×84 (f) $921 + 367$
   (g) 56×348 (h) $764 + 829$

6. Use a calculator to find the exact answers for the calculations in question 5. Calculate the error in each of your approximations. Use the second line in each question to do this.

Calculating with "easy" numbers that are close to given numbers is a good way to obtain approximate answers, for example:

- To approximate $764 + 829$, you may calculate $800 + 800$ to get the approximate answer 1 600; with an error of 7.
- To approximate 84×178, you may calculate 80×200 to get the approximate answer 16 000; with an error of 1 048.

7. Calculate with "easy" numbers close to the given numbers to produce approximate answers for each product below. **Do not use a calculator.** When you have made your approximations, use a calculator to work out the precise answers.

   (a) 78×46 (b) 67×88
   (c) 34×276 (d) 78×178

ROUNDING OFF AND COMPENSATING

1. (a) Approximate the answer for $386 + 3\,435$, by rounding both numbers off to the nearest hundred, and then adding the rounded numbers.
   (b) Because you rounded 386 up to 400, you introduced an error of 14 in your approximate answer. What error did you introduce by rounding 3 435 down to 3 400?
   (c) What combined (total) error did you introduce by rounding both numbers off before calculating?
   (d) Use your knowledge of the total error to correct your approximate answer, so that you have the correct answer for $386 + 3\,435$.

The word **compensate** means to do things that will remove damage.

Answers

2. (a) Sample answer: 470 + 640 = 1 110
   1 110 + 3 – 2 = 1 111
   (b) Sample answer: 680 + 920 = 1 600
   1 600 – 3 + 1 = 1 598

ADDING NUMBERS IN PARTS WRITTEN IN COLUMNS

Background information

- For addition and subtraction, the **commutative and associative properties** form the basis of the column methods. Writing the one number below the other one and working in columns is useful because it automatically rearranges the parts of the numbers as is allowed by the commutative and associative properties of addition.

  For example, in the calculation of 278 + 546, writing one number below the other and working in columns means the same as replacing (200 + 70 + 8) + (500 + 40 + 6) by (200 + 500) + (70 + 40) + (8 + 6) for the purposes of calculation. The transition from (200 + 70 + 8) + (500 + 40 + 6) to (200 + 500) + (70 + 40) + (8 + 6) is a direct application of the associative and commutative properties of addition. You could point this out when learners do the work on LB page 6.

- The **distributive property** allows a product to be broken down into partial products, which can be calculated more easily.

  For example, 7 × 648, which means 7 × (600 + 40 + 8), can be broken down into 7 × 600 + 7 × 40 + 7 × 8. If the learner knows the basic multiplication facts (tables), she or he can easily calculate these partial products and add them up to evaluate 7 × 648. The column format is a convenient way of making the transition from 7 × (600 + 40 + 8) to 7 × 600 + 7 × 40 + 7 × 8.

Teaching guidelines

Use the systematic layout on LB page 6 alongside to revise addition in columns.

In question 1, you used **rounding off** and **compensating** to find the correct answer for 386 + 3 435. By rounding the numbers off you introduced errors. You then compensated for the errors by making adjustments to your answer.

2. Round off and compensate to calculate each of the following accurately:
   (a) 473 + 638 (b) 677 + 921

Subtraction can also be done in this way. For example, to work out R5 362 – R2 687, you may round R2 687 up to R3 000. You may do this in the following ways:

- Rounding R2 687 up to R3 000 can be done in two steps: 2 687 + 13 = 2 700, and 2 700 + 300 = 3 000. In total, 313 is added.
- You can now add 313 to 5 362 too: R5 362 + 313 = 5 675.
- Instead of calculating R5 362 – R2 687, which is a bit difficult, you may calculate R5 675 – R3 000. This is easy: R5 675 – R3 000 = R2 675.

This means that R5 362 – R2 687 = R2 675, because
R5 362 – R2 687 = (R5 362 + R313) – (R2 687 + R313).

ADDING NUMBERS IN PARTS WRITTEN IN COLUMNS

Numbers can be added by thinking of their **parts** as we say the numbers.
For example, we say 4 994 as *four thousand nine hundred and ninety-four*.
This can be written in expanded notation as 4 000 + 900 + 90 + 4.

Similarly, we can think of 31 837 as 30 000 + 1 000 + 800 + 30 + 7.

31 837 + 4 994 can be calculated by working with the various kinds of parts separately. To make this easy, you can write the numbers below each other so that the units are below the units, the tens below the tens and so on, as shown on the right.

31 837
4 994

We write only this:	In your mind you can see this:				
31 837	30 000	1 000	800	30	7
4 994		4 000	900	90	4

The numbers in each column can be added to get a new set of numbers.

31 837	30 000	1 000	800	30	7
4 994		4 000	900	90	4
11					11
120				120	
1 700			1 700		
5 000		5 000			
30 000	30 000				
36 831	It is easy to add the new set of numbers to get the answer.				

Answers

1. (a) 4 638
   2 667
   7 305

   (b) 748
   7 246
   7 994

2. 742 480

   Learners can write three or four numbers at a time below each other, add them and then calculate the total budget.

3. (a) 14 112
   (b) 9 081
   (c) 40 089
   (d) 10 000

METHODS OF SUBTRACTION

Background information

There are various methods to subtract numbers. For example:

- **adding on** (refer to questions 1 and 2): To find the difference between two numbers, start at the smaller number and add on up to the larger number. This resembles skipping forward from the smaller number to the larger number on a number line: 26 – 14 = 6 + 6 = 12 (start at 14, skip 6 up to 20 and 6 more up to 26).
- **using compensation**: To calculate 26 – 14, compensation can be used in one of two ways:
  - Create an "easier" number **to subtract from** by adding 4 to both numbers: 26 – 14 = (26 + 4) – (14 + 4) = 30 – 18 = 12
  - Create an "easier" number **to subtract** by adding 6 to both numbers: 26 – 14 = (26 + 6) – (14 + 6) = 32 – 20 = 12
- **using expanded form** (refer to question 3 and the teaching guidelines below): Subtracting in columns resembles writing numbers in expanded form and, if necessary, rearranging the first line so that subtraction in each column is possible.

The work may start with the 10 000s or any other parts. Starting with the units, as shown on page 6, makes it possible to do more of the work mentally, and write less, as shown below.

31 837
4 994
36 831

To achieve this, only the units digit 1 of the 11 is written in the first step. The 10 of the 11 is remembered and added to the 30 and 90 of the tens column, to get 130.

We say the 10 is **carried** from the units column to the tens column. The same is done when the tens parts are added to get 130: only the digit "3" is written (in the tens column, so it means 30), and the 100 is carried to the next step.

1. Calculate each of the following without using a calculator:
   (a) 4 638 + 2 667 (b) 748 + 7 246

2. Impilo Enterprises plans a new computerised training facility in their existing building. The training manager has to keep the total expenditure budget under R1 million. This is what she has written so far:

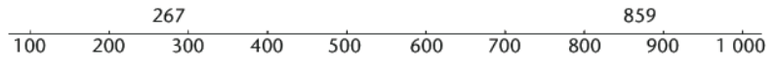
Architects and builders	R 102 700
Painting and carpeting	R 42 600
Security doors and blinds	R 52 000
Data projector	R 4 800
25 new secretary chairs	R 50 400
24 desks for work stations	R123 000
1 desk for presenter	R 28 000
25 new computers	R300 000
12 colour laser printers	R 38 980

   Work out the total cost of all the items for which the training manager has budgeted.

3. Calculate each of the following, without using a calculator:
   (a) 7 828 + 6 284 (b) 7 826 + 888 + 367
   (c) 657 + 32 890 + 6 542 (d) 6 666 + 3 333 + 1

METHODS OF SUBTRACTION

There are many ways to find the difference between two numbers. For example, to find the difference between 267 and 859, you may think of the numbers as they would appear on a number line, for example:



Teaching guidelines

Use the following method to explain subtraction in columns. Start with numbers which will not involve any "borrowing" and end with numbers like the following:

- **Subtract without using "borrows"**:

  Step 1: Write both numbers in expanded form:

  - 573 489 = 500 000 + 70 000 + 3 000 + 400 + 80 + 9
  - 379 692 = 300 000 + 70 000 + 9 000 + 600 + 90 + 2

  Step 2: Rearrange the numbers in row 1 so that subtraction is possible:

  - 400 – 100 = 300; 100 + 80 = 180:
    573 489 = 500 000 + 70 000 + 3 000 + 300 + 180 + 9
  - 3 000 – 1 000 = 2 000; 1 000 + 300 = 1 300:
    573 489 = 500 000 + 70 000 + 2 000 + 1 300 + 180 + 9
  - 70 000 – 10 000 = 60 000; 10 000 + 2 000 = 12 000:
    573 489 = 500 000 + 60 000 + 12 000 + 1 300 + 180 + 9
  - 500 000 – 100 000 = 400 000; 100 000 + 60 000 = 160 000:
    573 489 = 400 000 + 160 000 + 12 000 + 1 300 + 180 + 9

  Step 3: Subtract in columns:

  573 489 = 400 000 + 160 000 + 12 000 + 1 300 + 180 + 9
  379 692 = 300 000 + 70 000 + 9 000 + 600 + 90 + 2
  Difference = 100 000 + 90 000 + 3 000 + 700 + 90 + 7 = 193 797

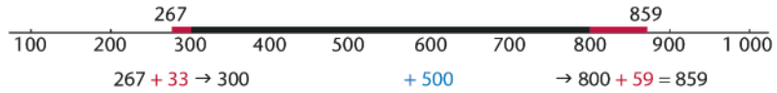
- **Subtract the same numbers using "borrows"**:

```
  4 16
    6 12
      2 13
        3 18
  5 7 3  4 8 9
- 3 7 9  6 9 2
  1 9 3  7 9 7
```

Answers

1. 592
2. The method shown is used below.
   (a) 44 + 300 + 23 = 367 (b) 71 + 800 + 14 = 885
   (c) 28 + 400 + 45 = 473 (d) 33 + 600 + 4 000 + 131 = 4 764

We may think of the distance between 267 and 859 as three steps: from 267 to 300, from 300 to 800, and from 800 to 859. How big are each of these three steps?



This number line shows that 859 – 267 is 33 + 500 + 59.

1. Calculate 33 + 500 + 59 to find the answer for 859 – 267.
2. Calculate each of the following. You may think of working out the distance between the two numbers as shown above, or use any other method you prefer.
   Do not use a calculator now.
   (a) 823 – 456 (b) 1 714 – 829
   (c) 3 045 – 2 572 (d) 5 131 – 367

Like addition, subtraction can also be done by working with the different parts in which we say numbers. For example, 8 764 – 2 352 can be calculated as follows:

8 thousand – 2 thousand = 6 thousand
7 hundred – 3 hundred = 4 hundred
6 tens – 5 tens = 1 ten
4 units – 2 units = 2 units
So, 8 764 – 2 352 = 6 412

Subtraction by parts is more difficult in some cases, for example 6 213 – 2 758:
6 000 – 2 000 = 4 000. This step is easy, but the following steps cause problems:
200 – 700 = ?
10 – 50 = ?
3 – 8 = ?

Fortunately, the parts and sequence of work may be rearranged to overcome these problems, as shown below:

One way to overcome these problems is to work with negative numbers:
200 – 700 = (–500)
10 – 50 = (–40)
3 – 8 = (–5)
4 000 – 500 → 3 500 – 45 =

Instead of		we may do		
3 – 8 =	?	13 – 8 =	5	"borrow" 10 from below
10 – 50 =	?	100 – 50 =	50	"borrow" 100 from below
200 – 700 =	?	1 100 – 700 =	400	"borrow" 1 000 from below
6 000 – 2 000 =	?	5 000 – 2 000 =	3 000	

Answers

3. (a) 3 000 + 400 + 50 + 5 = 3 455

   (b) 13 – 6 = 7; 110 – 70 = 40; 700 – 300 = 400; 400 + 40 + 7 = 447

   11 – 8 = 3; 120 – 60 = 60; 1 300 – 900 = 400; 5 000 – 4 000 = 1 000;

   1 000 + 400 + 60 + 3 = 1 463

4. 447 + 376 = 823 ; 1 463 + 4 968 = 6 431

5. (a) 3 465 (b) 6 797 (c) 1 143

6. (a) R4 000 or R4 300; R4 237 (b) R34 000 or R33 500; R33 511

7. (a) 100 000; 97 316 (b) 120 000; 116 455 (c) 25 000; 25 817

A METHOD OF MULTIPLICATION

Background information

Different methods can be used to multiply numbers:

- Expand the large number into parts, multiply each part by the 2-digit number and add the final products. For example:

  27 × 234 = 27 × (200 + 30 + 4)

  = 27 × 200 + 27 × 30 + 27 × 4

  = 5 400 + 810 + 108

  = 6 318

- Expand the 2-digit number mentally into tens and units and multiply the large number by the two parts separately in columns, first by the units and then by the tens. For example:

```
   2 3 4
×    2 7
     2 8    (7 × 4)
   2 1 0    (7 × 30)
 1 4 0 0    (7 × 200)
     8 0    (20 × 4)
   6 0 0    (20 × 30)
 4 0 0 0    (20 × 200)
 6 3 1 8
```

Teaching guidelines

Discuss the correct layout for long multiplication as illustrated alongside and in the Background information above.

This reasoning can also be set out in columns:

Instead of				we may do				but write only this			
6 000	200	10	3	5 000	1 100	100	13	6	2	1	3
2 000	700	50	8	2 000	700	50	8	2	7	5	8
				3 000	400	50	5	3	4	5	5

3. (a) Complete the above calculations and find the answer for 6 213 – 2 758.

   (b) Use the borrowing technique to calculate 823 – 376 and 6 431 – 4 968.

4. Check your answers in question 3(b) by doing addition.

With some practice, you can learn to subtract using borrowing without writing all the steps. When calculating 6 213 – 2 758, use the column method, as shown on the right.

```
 6 213
 2 758
     5
    50
   400
 3 000
 3 455
```

If you work more mentally, you will write even less, as shown below:

```
 6 213
 2 758
 3 455
```

Do not use a calculator when you do question 5, because the purpose of this work is for you to understand the methods of subtraction. What you learn here will help you to understand **algebra** better at a later stage.

5. Calculate each of the following:

   (a) 7 342 – 3 877 (b) 8 653 – 1 856 (c) 5 671 – 4 528

You may use a calculator to do questions 6 and 7.

6. Estimate the difference between the two car prices in each case, to the nearest R1 000 or closer. Then calculate the difference.

   (a) R102 365 and R98 128 (b) R63 378 and R96 889

7. First estimate the answers to the nearest 100 000, 10 000 or 1 000. Then calculate.

   (a) 238 769 –141 453 (b) 856 333 – 739 878 (c) 65 244 – 39 427

A METHOD OF MULTIPLICATION

You can calculate 7 × 4 598 in parts, as shown here:

7 × 4 000 = 28 000
7 × 500 = 3 500
7 × 90 = 630
7 × 8 = 56

The four partial products can now be added to get the answer, which is 32 186. It is convenient to write the work in vertical columns for units, tens, hundreds and so on, as shown on the right.

```
   4 5 9 8
×        7
       5 6
     6 3 0
   3 5 0 0
 2 8 0 0 0
 3 2 1 8 6
```

Answers

1. (a) 17 523 (b) 131 700 (c) 32 364
2. Learners use a calculator to check their answers in question 1.
3. (a) 18 492 (b) 14 952
4. Learners use a calculator to check their answers in question 3.

LONG DIVISION

Background information

- During division:
  - the **dividend** is the number that gets divided
  - the **divisor** is the number that does the division.

  4 349 ÷ 35
- **Long division** involves subtraction of multiples of the divisor from the dividend until the remainder, if any, is smaller than the divisor.

  Example: Calculate 4 349 ÷ 35.

  Use powers of 10, doubling and halving to find multiples of the divisor:

  35 × 100 = 3 500

  35 × 10 = 350 → 35 × 20 = 700

  35 × 1 = 35 → 35 × 2 = 70 → 35 × 4 = 140

  Subtract multiples of the divisor from the dividend:

```
  4 349
- 3 500      (35 × 100 = 3 500)
    849
-  700      (35 × 20 = 700)
    149
-  140      (35 × 4 = 140)
      9      (100 + 20 + 4 = 124)
```

  Answer: 4 349 ÷ 35 = 124 remainder 9

Teaching guidelines

Explain the correct layout for long division by working through the three steps alongside.

Answers

1. Yes. Estimate based on 30 × 300 = 9 000
2. (a) R8 100 (b) R1 300 (c) R1 300 – R540 = R760

When working from right to left in the columns, you can "carry" parts of the partial answers to the next column. Instead of writing 56, only the 6 of the product 7 × 8 is written down. The 50 is kept in mind, and added to the 630 obtained when 7 × 90 is calculated in the next step.

```
  4 5 9 8
        7
3 2 1 8 6
```

1. Calculate each of the following. Do not use a calculator now.
   (a) 27 × 649 (b) 75 × 1 756 (c) 348 × 93
2. Use your calculator to check your answers for question 1. Redo the questions for which you had the wrong answers.
3. Calculate each of the following. Do not use a calculator now.
   (a) 67 × 276 (b) 84 × 178
4. Use a calculator to check your answers for question 3. Redo the questions for which you had the wrong answers.

LONG DIVISION

1. The municipal head gardener wants to buy young trees to plant along the main street of the town. The young trees cost R27 each, and an amount of R9 400 has been budgeted for trees. He needs 324 trees. Do you think he has enough money?
2. (a) How much will 300 trees cost?
   (b) How much money will be left if 300 trees are bought?
   (c) How much money will be left if 20 more trees are bought?

The municipal gardener wants to work out exactly how many trees, at R27 each, he can buy with the budgeted amount of R9 400. His thinking and writing are described below.

Step 1:

What he writes:	What he thinks:
27 | 9 400	*I want to find out how many chunks of 27 there are in 9 400.*

Step 2:

What he writes:	What he thinks:
300	*I think there are at least 300 chunks of 27 in 9 400.*
27 | 9 400	
8 100	*300 × 27 = 8 100. I need to know how much is left over.*
1 300	*I want to find out how many chunks of 27 there are in 1 300.*

Step 3: (He has to rub out the one "0" of the 300 on top, to make space.)

What he writes:	What he thinks:
340	*I think there are at least 40 chunks of 27 in 1 300.*
27 | 9 400	
8 100	
1 300	
1 080	*40 × 27 = 1 080. I need to know how much is left over.*
220	*I want to find out how many chunks of 27 there are in 220.*
	Perhaps I can buy some extra trees.

Teaching guidelines (continued)

Explain the correct layout for long division by working through the final step alongside.

Answers

3. (a) R5 440 ÷ 64 = R85 per goat
   (b) R2 850 ÷ R48 = 59 (R18 remaining)
   She can buy 59 candles.
4. (a) 150 with remainder 34 (b) 136 with remainder 3
   (c) 26 with remainder 36 (d) 347 with remainder 19

1.3 Multiples, factors and prime factors

MULTIPLES AND FACTORS

Background information

Multiples, factors and prime numbers are also covered in Grade 7, and learners may already have a good background in this work. It may hence be useful to start this section with the investigation described at the bottom of page 15 of this chapter, with a view to assess and revise prior knowledge and facilitate a more rapid passage through the systematic work on LB pages 11 to 15.

Teaching guidelines

Revise the following concepts:

- A **multiple** is a number made up by multiplying two other numbers together. Example: 5 × 7 = 35 therefore 35 is a multiple of 5 and of 7.
- A **factor** of a number can be divided into that number without leaving a remainder. Example: 3 is a factor of 48 because 48 ÷ 3 = 16 (no remainder).

Answers

1. (a) 600; 700
   (b) Yes (33 × 6 = 198)
   (c) Yes (25 × 7 = 175)

Step 4: (He rubs out another "0".)

What he writes:	What he thinks:
348	*I think there are at least 8 chunks of 27 in 220.*
27 | 9 400	
8 100	
1 300	
1 080	
220	
216	*8 × 27 = 216*
4	*So, I can buy 348 young trees and will have R4 left.*

Do not use a calculator to do questions 3 and 4. The purpose of this work is for you to develop a good understanding of how division can be done. Check all your answers by doing multiplication.

3. (a) Graham bought 64 goats, all at the same price. He paid R5 440 in total. What was the price for each goat? You can start by working out how much he would have paid if he paid R10 per goat. You can start with a bigger step if you wish.
   (b) Mary has R2 850 and she wants to buy candles for her sister's wedding reception. The candles cost R48 each. How many candles can she buy?
4. Calculate each of the following. Do not use a calculator.
   (a) 7 234 ÷ 48 (b) 3 267 ÷ 24
   (c) 9 500 ÷ 364 (d) 8 347 ÷ 24

1.3 Multiples, factors and prime factors

MULTIPLES AND FACTORS

1. The numbers 6; 12; 18; 24; ... are **multiples** of 6.
   The numbers 7; 14; 21; 28; ... are **multiples** of 7.

   If n is a natural number, $6n$ represents the multiples of 6.

   (a) What is the hundredth number in each sequence above?
   (b) Is 198 a number in the first sequence?
   (c) Is 175 a number in the second sequence?

Of which numbers is 20 a multiple?

$20 = 1 \times 20 = 2 \times 10 = 4 \times 5 = 5 \times 4 = 10 \times 2 = 20 \times 1$

Factors come in pairs. The following pairs are factors of 20:

20 is a multiple of 1; 2; 4; 5; 10 and 20, and all of these numbers are factors of 20.



Answers

2. 1 and 30; 2 and 15; 3 and 10; 5 and 6
3. Yes, 4; 8; 12 and 16 are factors of 48. No, Simon is wrong; for example, 28 isn't a factor of 48.
4. (a) 210 can be divided by any of them without leaving a remainder.
   (b) Yes (c) Yes
5. Yes, 20 is a factor of 60. All factors of 20, that is, 1; 2; 4; 5; 10 are also factors of 60.

PRIME NUMBERS AND COMPOSITE NUMBERS

Teaching guidelines

Revise the following concepts:

- A **prime number** can only be expressed as the product of 1 and the number itself. Example: 29 is prime because its only factors are 1 and 29.
- A **composite number** is any natural number with more than two different factors. Example: 8 is composite because its factors are 1, 2, 4 and 8.

Answers

1. (a) 2×33; 3×22; 6×11; $2 \times 3 \times 11$ (b) No other factor
   (c) 2×34; 4×17; $2 \times 2 \times 17$ (d) 3×23
   (e) 2×35; 5×14; 7×10; $2 \times 5 \times 7$ (f) No other factors
   (g) 2×36; 3×24; 4×18; 6×12; 8×9; $2 \times 2 \times 2 \times 3 \times 3$
   (h) No other factors
2. 67; 71; 73
3. 67; 71; 73
4. (a) False. Number 2 is a prime number and it is even.
   (b) False. Number 15, for example, is an odd composite number.
   (c) False. 1 has only one factor.
   (d) True, except for 1, which is neither prime nor composite.
   (e) False. It is a prime number because it has only two factors: 1 and itself.
   (f) False. Any whole number with 5 as the last digit has 5 as a factor.
   (g) False. The prime number 2 is an exception.
   (h) True

2. A rectangle has an area of 30 cm. What are the possible lengths of the sides of the rectangle in centimetres if the lengths of the sides are natural numbers?
3. Are 4, 8, 12 and 16 factors of 48? Simon says that all multiples of 4 smaller than 48 are factors of 48. Is he right?
4. We have defined factors in terms of the product of *two* numbers. What happens if we have a product of *three* or more numbers, for example: $210 = 2 \times 3 \times 5 \times 7$?
   (a) Explain why 2, 3, 5 and 7 are factors of 210.
   (b) Are 2×3, 3×5, 5×7, 2×5 and 2×7 factors of 210?
   (c) Are $2 \times 3 \times 5$, $3 \times 5 \times 7$ and $2 \times 5 \times 7$ factors of 210?
5. Is 20 a factor of 60? What factors of 20 are also factors of 60?

PRIME NUMBERS AND COMPOSITE NUMBERS

1. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.
   (a) 66 (b) 67
   (c) 68 (d) 69
   (e) 70 (f) 71
   (g) 72 (h) 73

The number 36 can be formed as $2 \times 2 \times 3 \times 3$. Because 2 and 3 are used twice, they are called **repeated factors** of 36.

2. Which of the numbers in question 1 cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*?

A number that cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*, is called a **prime number**.

3. Which of the numbers in question 1 are prime numbers?

Composite numbers are natural numbers with more than two different factors. The sequence of composite numbers is 4; 6; 8; 9; 10; 12; ...

4. Are the statements below true or false? If you answer "false", explain why.
   (a) All prime numbers are odd numbers.
   (b) All composite numbers are even numbers.
   (c) 1 is a prime number.
   (d) If a natural number is not prime, then it is composite.
   (e) 2 is a composite number.
   (f) 785 is a prime number.
   (g) A prime number can only end in 1, 3, 7 or 9.
   (h) Every composite number is divisible by at least one prime number.

Answers

5. $13 \times 11 = 143$; already too large
6. (a) Composite: 13; 17
   (b) Composite: 23; 31

PRIME FACTORISATION

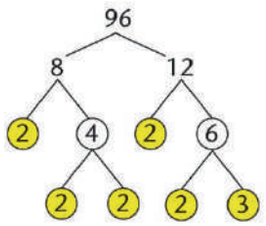
Background information

- **Prime factorisation** means that a number is written as a product of all its prime factors. Example: $72 = 2 \times 2 \times 2 \times 3 \times 3$
- Prime factorisation can be done by using:
  - a **factor tree**, which starts with known factors of the number
  - a **factor ladder**, which starts with the smallest prime factor of the number.

Teaching guidelines

- Discuss how to prime factorise numbers:
  - Use a factor tree by starting with known factors of the number:

Factor tree



$96 = 8 \times 12 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

  - a factor ladder by starting with the smallest prime factor:

Factor ladder

2	96	(96 is divisible by 2)
2	48	(48 is divisible by 2)
2	24	(24 is divisible by 2)
2	12	(12 is divisible by 2)
2	6	(6 is divisible by 2)
3	3	(3 is divisible by 3)
	1	(Stop when this is 1)

$96 = 8 \times 12 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

5. We can find out if a given number is prime by systematically checking whether the primes 2; 3; 5; 7; 11; 13; ... are factors of the given number or not.
   To find possible factors of 131, we need to consider only the primes 2; 3; 5; 7 and 11. Why not 13; 17; 19; ...?

6. Determine whether the following numbers are prime or composite. If the number is composite, write down at least two factors of the number (besides 1 and the number itself).
   (a) 221 (b) 713

PRIME FACTORISATION

To find all the factors of a number you can write the number as the product of prime factors; first by writing it as the product of two convenient (composite) factors and then by splitting these factors into smaller factors until all factors are prime. Then you take all the possible combinations of the products of the prime factors.

Every composite number can be expressed as the product of prime factors and this can happen in only one way.

Example: Find the factors of 84.

Write 84 as the product of prime factors by starting with different known factors:

$84 = 4 \times 21 = 2 \times 2 \times 3 \times 7$ or $84 = 7 \times 12 = 7 \times 3 \times 4 = 7 \times 3 \times 2 \times 2$ or $84 = 2 \times 42 = 2 \times 6 \times 7 = 2 \times 2 \times 3 \times 7$

A more systematic way of finding the prime factors of a number would be to start with the prime numbers and try the consecutive prime numbers 2; 3; 5; 7; ... as possible factors. The work may be set out as shown below.

2	1 430
5	715
11	143
13	13
	1

$1\,430 = 2 \times 5 \times 11 \times 13$

3	2 457
3	819
3	273
7	91
13	13
	1

$2\,457 = 3 \times 3 \times 3 \times 7 \times 13$

We can use exponents to write the products of prime factors more compactly as products of powers of prime factors.

$2\,457 = 3 \times 3 \times 3 \times 7 \times 13 = 3^3 \times 7 \times 13$

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

$1\,500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5 = 2^2 \times 3 \times 5^3$

Answers

1. (a) $2^3 \times 3^2 \times 11$

2	792
2	396
2	198
3	99
3	33
11	11
	1

(b) $2^2 \times 3 \times 37$

2	444
2	222
3	111
37	37
	1

2.

2	**28**	2	**32**	2	**124**	2	**36**	2	**42**	3	**345**	2	**182**
2	14	2	16	2	62	2	18	3	21	5	115	7	91
7	7	2	8	31	31	3	9	7	7	23	23	13	13
	1	2	4		1	3	3		1		1		1
		2	2				1						
			1										

$28 = 2 \times 2 \times 7$; $32 = 2 \times 2 \times 2 \times 2 \times 2$; $124 = 2 \times 2 \times 31$; $36 = 2 \times 2 \times 3 \times 3$;
$42 = 2 \times 3 \times 7$; $345 = 3 \times 5 \times 23$; $182 = 2 \times 7 \times 13$

COMMON MULTIPLES AND FACTORS

Background information

- A **common multiple** is a multiple of two or more numbers. Example: 18; 36; 54; ... are common multiples of 6 and 9.
- The **lowest common multiple** (**LCM**) of two or more numbers is the smallest number which is a common multiple of the numbers. Example: 18 is the LCM of 6 and 9.
- A **common factor** is a factor of two or more numbers. Example: $110 = 2 \times \mathbf{5} \times 11$; $105 = 3 \times \mathbf{5} \times 7$; **5** is a common factor of 110 and 105.
- The **highest common factor** (**HCF**) of two or more numbers is the biggest number that is a common factor of the numbers. Example: $96 = \mathbf{2} \times \mathbf{2} \times \mathbf{2} \times 2 \times 2 \times \mathbf{3}$; $120 = \mathbf{2} \times \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times 5$; HCF $= 2 \times 2 \times 2 \times 3 = 24$.

Teaching guidelines

- Revise the concepts listed above.
- Discuss how the LCM is used to add and subtract fractions.
- Discuss how the HCF is used to simplify fractions.

Answers

1. Both answers are Yes.
2. The statement is true.

1. Express the following numbers as the product of powers of primes:
   (a) 792 (b) 444
2. Find the prime factors of the following numbers:
   28 32 124 36 42 345 182

COMMON MULTIPLES AND FACTORS

1. Is 4×5 a multiple of 4? Is 4×5 a multiple of 5?
2. Comment on the following statement:
   The product of numbers is a multiple of each of the numbers in the product.

We use **common multiples** when fractions with different denominators are added. To add $\frac{2}{3} + \frac{3}{4}$, the common denominator is 3×4, so the sum becomes $\frac{8}{12} + \frac{9}{12}$. In the same way, we could use $6 \times 8 = 48$ as a common denominator to add $\frac{1}{6} + \frac{3}{8}$, but 24 is the **lowest common multiple** (**LCM**) of 6 and 8.

Prime factorisation makes it easy to find the **LCM** or highest common factor.

When we simplify a fraction, we divide the same number into the numerator and the denominator. For the simplest fraction, use the **highest common factor** (**HCF**) to divide into both the numerator and denominator.

The HCF is divided into the numerator and the denominator to write the fraction in its **simplest form**.

$$\text{So } \frac{36}{144} = \frac{2 \times 2 \times 3 \times 3}{2 \times 2 \times 2 \times 2 \times 3 \times 3} = \frac{1}{4}$$

Use prime factorisation to determine the LCM and HCF of 32, 48 and 84 in a systematic way:

$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$
$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$
$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$

The LCM is a **multiple** and therefore, all of the factors of all the numbers must divide into it.

All of the factors that are present in the three numbers must also be factors of the LCM, even if it is a factor of only one of the numbers. But because it has to be the lowest common multiple, there are no unnecessary factors in the LCM.

Answers

3. (a) $24 = 23 \times 3$; $28 = 22 \times 7$; $42 = 2 \times 3 \times 7$; HCF = 2; LCM = 168
   (b) 17 is prime; $21 = 3 \times 7$; $35 = 5 \times 7$; No HCF; LCM = 255
   (c) $75 = 3 \times 5^2$; $120 = 2^3 \times 3 \times 5$; $200 = 2^3 \times 5^2$; HCF = 5; LCM = 600
   (d) $18 = 2 \times 3^2$; $30 = 2 \times 3 \times 5$; $45 = 3^2 \times 5$; HCF = 3; LCM = 90

INVESTIGATE PRIME NUMBERS

Answers

1. 113; 127
2. 211; 223; 227; 229
3. 997

1.4 Solving problems

RATE AND RATIO

Background information

- A **ratio** compares the sizes of two (or more) quantities of the **same** kind. For example, if mortar for a brick wall is made by mixing two buckets of cement to seven buckets of sand, the ratio of cement to sand is 2 to 7 (or 2 : 7).
- A **rate** compares quantities of **different** kinds. For example, if a car takes 3 hours to travel 300 km, the car is travelling at a rate of 100 km/h.

Teaching guidelines

Ensure that learners understand the difference between rate and ratio.

Answers

1. 3 000 000 ÷ 15 000 = 200 working days
2. 180 ÷ 2 × 3 = 270 km
3. \$1 is R13,79, so \$56,67 will be 56,67 × 13,79 = R781,48

The highest power of each factor is in the LCM, because all of the other factors can divide into it. In 32, 48 and 84, the highest power of 2 is 2^5, the highest power of 3 is 3 and the highest power of 7 is 7.

LCM = $2^5 \times 3 \times 7 = 672$

The HCF is a common factor. Therefore, for a factor to be in the HCF, it must be a factor of *all* of the numbers. The only number that appears as a factor of all three numbers is 2. The lowest power of 2 is 2^2; therefore, the HCF is 2^2.

3. Determine the LCM and the HCF of the numbers in each case:
   (a) 24; 28; 42
   (b) 17; 21; 35
   (c) 75; 120; 200
   (d) 18; 30; 45

INVESTIGATE PRIME NUMBERS

You may use a calculator for this investigation.

1. Find all the prime numbers between 110 and 130.
2. Find all the prime numbers between 210 and 230.
3. Find the biggest prime number smaller than 1 000.

1.4 Solving problems

RATE AND RATIO

You may use a calculator for the work in this section.

1. Tree plantations in the Western Cape are to be cut down in favour of natural vegetation. There are roughly 3 000 000 trees on plantations in the area and it is possible to cut them down at a **rate** of 15 000 trees per day with the labour available. How many working days will it take before all the trees will be cut down?

Instead of saying "... per day", people often say "at a **rate** of ... per day". Speed is a way in which to describe the rate of movement.

The word **per** is often used to describe a rate and can mean *for every, for, in each, in, out of,* or *every.*

2. A car travels a distance of 180 km in two hours on a straight road. How many kilometres can it travel in three hours at the same speed?
3. Thobeka wants to order a book that costs \$56,67. The rand-dollar exchange rate is R13,79 to a dollar. What is the price of the book in rands?

Note on rate and ratio

The concepts of **rate** and **ratio** (LB pages 15–17) are extremely important. A useful way to clarify the difference between rate and ratio is to consider a situation where two objects move at different constant speeds, say 120 km/h and 80 km/h. Each of the speeds is a rate, and the two speeds are in the ratio 3 : 2. In this case, the ratio is a relationship between two rates.

Background information (continued)

To divide a quantity in the ratio $x : y$, divide the quantity into $(x + y)$ parts.

- Allocate x of these parts to the first entity.
- Allocate y of these parts to the second entity.

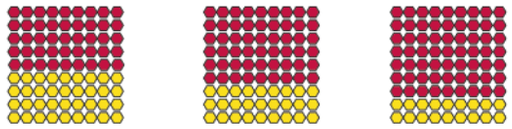
Teaching guidelines (continued)

- An amount of R600 is divided between P and Q in the ratio 5 : 7.
  - P will receive $\frac{5}{12}$ (read as "five twelfths") of R600 = 5 × R50 = R250
  - Q will receive $\frac{7}{12}$ of R600 = 7 × R50 = R350
- An amount of R600 is divided between P, Q and R in the ratio 3 : 4 : 5.
  - P will receive $\frac{3}{12}$ of R600 = 3 × R50 = R150
  - Q will receive $\frac{4}{12}$ of R600 = 4 × R50 = R200
  - R will receive $\frac{5}{12}$ of R600 = 5 × R50 = R250

Answers

4. In pattern B, there are six red beads for every three yellow beads.
   In pattern C, there are seven red beads for every two yellow beads.
5. (a) One and a half times faster (b) 100 × 1,5 = 150
6. In the ratio 5 : 4 : 3 5 + 4 + 3 = 12 600 ÷ 12 = 50
   Nathi gets 5 × 50 = R250; Paul gets 4 × 50 = R200; Tim gets 3 × 50 = R150
7. Six packets for 16 people; 4,5 packets for 12 people.
8. A 6 × 8 rectangle, because $\frac{6}{8}$ is closer to 1 than $\frac{3}{5}$ is.

4. In pattern A below, there are five red beads for every four yellow beads.



Pattern A *Pattern B* *Pattern C*

Describe patterns B and C in the same way.

5. Copy and complete the following table to show how many screws are produced by two machines in different periods of time:

Number of hours	1	2	3	5	8
Number of screws at machine A	1 800	3 600	5 400	9 000	14 400
Number of screws at machine B	2 700	5 400	8 100	13 500	21 600

(a) How much faster is machine B than machine A?
(b) How many screws will machine B produce in the same time that it takes machine A to make 100 screws?

The patterns in question 4 can be described like this: In pattern A, the **ratio** of yellow beads to red beads is 4 to 5. This is written as 4 : 5. In pattern B, the ratio between yellow beads and red beads is 3 : 6. In pattern C the ratio is 2 : 7.

In question 5, machine A produces two screws for every three screws that machine B produces. This can be described by saying that the ratio between the production speeds of machines A and B is 2 : 3.

6. Nathi, Paul and Tim worked in Mr Setati's garden. Nathi worked for five hours, Paul for four hours and Tim for three hours. Mr Setati gave the boys R600 for their work. How should they divide the R600 among the three of them?

We use **ratios** to show how many times more, or less, one quantity is than another.

A **ratio** is a comparison of two (or more) quantities. The number of hours that Nathi, Paul and Tim worked are in the ratio 5 : 4 : 3. To be fair, the money should also be shared in that ratio. Therefore, Nathi should receive five parts, Paul four parts and Tim three parts of the money. There were 12 parts, which means Nathi should receive $\frac{5}{12}$ of the total amount, Paul should get $\frac{4}{12}$ and Tim should get $\frac{3}{12}$.

7. Ntabi uses three packets of jelly to make a pudding for eight people. How many packets of jelly does she need to make a pudding for 16 people? And for 12 people?
8. Which rectangle is more like a square: a 3 × 5 rectangle or a 6 × 8 rectangle? Explain.

Answers

9. (a) $56 \div 2 = 28$; $28 \times 3 = 84$ (b) $72 \div 4 = 18$; $18 \times 3 = 54$
10. (a) $840 \div 7 = 120$; $3 \times 120 = 360$; $4 \times 120 = 480$; **360 : 480**
    (b) $360 \div 6 = 60$; $1 \times 60 = 60$; $2 \times 60 = 120$; $3 \times 60 = 180$; **60 : 120 : 180**
11. (a) A: $2\,500 \div 20 = 125$, make it 150 B: $4\,000 \div 40 = 100$
    C: $3\,600 \div 30 = 120$, make it 130 D: $6\,000 \div 50 = 120$
    E: $3\,000 \div 25 = 120$, make it 130 F: $5\,000 \div 50 = 100$, make it 120
    (b) A: $2\,480 \div 17 \approx 145{,}88$ m/min B: $4\,283 \div 43 \approx 99{,}6$ m/min
    C: $3\,729 \div 28 \approx 133{,}18$ m/min D: $6\,209 \div 53 \approx 117{,}15$ m/min
    E: $3\,112 \div 24 \approx 129{,}67$ m/min F: $5\,638 \div 45 \approx 125{,}29$ m/min
    Athletes arranged from fastest to slowest: A, C, E, F, D, B.

PROFIT, LOSS, DISCOUNT AND INTEREST

Background information

- **Cost price** is the amount for which a dealer buys an article from a producer or manufacturer.
- **Marked price** is the amount marked on the article and usually includes the dealer's profit (or loss).
- **Discount** is the amount which is sometimes taken off the marked price of the article and is usually stated as a percentage.
- **Selling price** is the price of the article after discount (if any).

Teaching guidelines

- Discuss the concepts listed above.
- Revise that percentage (%) is another word for hundredths ($\frac{}{100}$).

Answers

1. (a) R100 (b) R8 (c) R56
2. (a) R2 420 (b) R3 200
   (c) R3 200 – R320 = R2 880 (d) R320

To increase 40 in the ratio 2 : 3 means that the 40 represents two parts and must be increased so that the new number represents three parts. If 40 represents two parts, 20 represents one part. The increased number will therefore be $20 \times 3 = 60$.

Remember that if you multiply by 1, the number does not change. If you multiply by a number greater than 1, the number increases. If you multiply by a number smaller than 1, the number decreases.

9. (a) Increase 56 in the ratio 2 : 3.
   (b) Decrease 72 in the ratio 4 : 3.
10. (a) Divide 840 in the ratio 3 : 4.
    (b) Divide 360 in the ratio 1 : 2 : 3.
11. Look at the following data about the performance of different athletes during a walking event. Investigate the data to find out who walks the fastest and who walks the slowest. Arrange the athletes from the fastest walker to the slowest walker.
    (a) First make estimates to do the investigation.
    (b) Then use your calculator to do the investigation.

Athlete	A	B	C	D	E	F
Distance walked in metres	2 480	4 283	3 729	6 209	3 112	5 638
Time taken in minutes	17	43	28	53	24	45

PROFIT, LOSS, DISCOUNT AND INTEREST

1. (a) How much is one eighth of R800?
   (b) How much is one hundredth of R800?
   (c) How much is seven hundredths of R800?

Rashid is a furniture dealer. He buys a couch for R2 420. He displays the couch in his showroom with the price marked as R3 200. Rashid offers a discount of R320 to customers who pay cash.

The amount for which a dealer buys an article from a producer or manufacturer is called the **cost price**. The price marked on the article is called the **marked price** and the price of the article after discount is the **selling price**.

2. (a) What is the cost price of the couch in Rashid's furniture shop?
   (b) What is the marked price?
   (c) What is the selling price for a customer who pays cash?
   (d) How much is ten hundredths of R3 200?

Background information (continued)

- **Profit** is made when something is sold for more than what it originally cost. For example, if you buy a pair of trousers for R400 and sell them for R480 you have made a profit of R480 – R400 = R80.
- **Percentage profit** is the profit expressed as a percentage of the cost price. For example, if a profit of R80 is made on an article that costs R400, the percentage profit = $\frac{R80}{R400} \times 100\% = 20\%$.
- **Loss** is made when something is sold for less than what it originally cost. For example, if you buy a mobile phone for R699 and sell it for R549, you have made a loss of R699 – R549 = R150.
- **Interest** is the money one has to pay for using another person's money.

Teaching guidelines

Discuss the concepts listed above.

Answers

3. (a) R3 600 ÷ 100 = R36; R36 × 6 = R216
   (b) R9 360 ÷ 100 = R93,60; R93,60 × 6 = R561,60
4. (a) R7 (b) 9 hundredths (c) 9%
5. One hundredth of R300 is R3; R36 = 12 hundredths of R300
   The percentage discount is 12%.
6. (a) $\frac{30}{100}$ of 7 500 = R2 250; Marked price = R7 500 + R2 250 = R9 750
   Selling price = $\frac{80}{100}$ of R9 750 = R7 800
   (b) R7 800 – R7 500 = R300; $\frac{R300}{R7\,500} \times \frac{100}{1} = 4\%$
7. R7 000 × 1,14 = R7 980 (or 14% of R7 000 = R980; R980 + R7 000 = R7 980)
8. (a) R5 600 × 1,08 = R6 048 or
   R5 600 ÷ 100 × 8 = R448; R5 600 + R448 = R6 048
   (b) R6 048 × 1,08 = R6 531,84 or
   R6 048 ÷ 100 × 8 = R483,84; R6 048 + R483,84 = R6 531,84
   (c) 6 531,84 × 1,08 → 7 054,39 × 1,08 → 7 618,74 × 1,08 → R 8 228,24

The **discount** on an article is always less than the marked price of the article. In fact, it is only a fraction of the marked price. The discount of R320 that Rashid offers on the couch is ten hundredths of the marked price.

Another word for hundredths is **percentage**, and the symbol for percentage is %. We can therefore say that Rashid offers a discount of 10%.

A percentage is a number of hundredths.

18% is 18 hundredths and 25% is 25 hundredths.

% is a symbol for hundredths. 8% means eight hundredths and 15% means 15 hundredths.

The symbol % is just a variation of the $\frac{}{100}$ that is used in the common fraction notation for hundredths.

For example: 8% is $\frac{8}{100}$.

A discount of 6% on an article can be calculated in two steps:

Step 1: Calculate one hundredth of the marked price (divide by 100).

Step 2: Calculate six hundredths of the marked price (multiply by six).

3. Calculate a discount of 6% on each of the following marked prices of articles:
   (a) R3 600 (b) R9 360
4. (a) How much is one hundredth of R700?
   (b) A customer pays cash for a coat marked at R700.
   He is given a R63 discount. How many hundredths of R700 is this?
   (c) What is the percentage discount?
5. A client buys a blouse marked at R300 and she is given R36 discount for paying cash. Work as in question 4 to determine what percentage discount she was given.

You may use a calculator to do questions 6, 7 and 8.

6. A dealer buys an article for R7 500 and makes the price 30% higher. The article is sold at a 20% discount.
   (a) What is the selling price of the article?
   (b) What is the dealer's percentage profit?

When a person borrows money from a bank or some other institution, he or she normally has to pay for the use of the money. This is called **interest**.

7. Sam borrows R7 000 from a bank at 14% interest for one year. How much does he have to pay back to the bank at the end of the period?
8. Jabu invests R5 600 for one year at 8% interest.
   (a) What will the value of his investment be at the end of that year?
   (b) At the end of the year, Jabu does not withdraw the investment or the interest earned; instead, he reinvests it for another year. How much will it be worth at the end of the second year?
   (c) What will the value of Jabu's investment be after five years?

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
2.1 What is beyond 0?	Introduction to negative integers	Pages 19 to 22
2.2 Adding and subtracting with integers	Add and subtract with integers; compare integers; solve problems	Pages 22 to 25
2.3 Multiplying and dividing with integers	Multiply and divide with integers; commutative, associative and distributive properties of operations with integers; mixed calculations with integers	Pages 25 to 30
2.4 Squares, cubes and roots with integers	Squares and cubes of integers; square roots and cube roots of integers	Pages 30 to 31

CAPS time allocation	9 hours
CAPS content specification	Pages 78 to 80

Mathematical background

- The set of **integers** $\boldsymbol{Z}$ = {...; –4; –3; –2; –1; 0; 1; 2; 3; 4; ...} is formed by adding the set {...; –4; –3; –2; –1} to the set of whole numbers $\boldsymbol{N_0}$ = {0; 1; 2; 3; 4; ....}.
- The **focus in this chapter** is on the computational properties of integers and the properties of operations with integers.
- The **computational properties of integers** describe how calculations with integers are done (refer to LB pages 19 to 28) and include the following:
  - Subtracting a bigger number from a smaller number results in a negative number, for example 10 – 20 = (–10) ("negative ten").
  - Any integer and its additive inverse lie the same distance from and on opposite sides of 0 on the number line, therefore the sum of any integer and its additive inverse is equal to 0, for example 12 + (–12) = 0 or (–12) + 12 = 0 ("negative twelve plus twelve equals zero").
  - Adding an integer has the same effect as subtracting its additive inverse, for example 3 + (–10) and 3 – 10 produce the same result.
  - Subtracting an integer has the same effect as adding its additive inverse, for example 3 – (–10) and 3 + 10 produce the same result.
  - The product of a positive and a negative integer is negative, which means that the quotient of a negative and a positive integer is negative, or the quotient of two negative integers is positive.
  - The product of two negative integers is positive, which means that the quotient of a positive and a negative integer is negative.
- The **associative, commutative and distributive properties of operations with integers** are also addressed (refer to LB pages 25 to 29).
- **Number patterns and repeated addition** are used to demonstrate the reasonableness of the computational properties of integers. Learners have to continue number patterns that involve "counting" and operations with integers (refer to LB pages 21, 23 and 24 and question 1 on page 25).
- The two properties "subtracting an integer is the same as adding its additive inverse" and "adding an integer is the same as subtracting its additive inverse" have **absolutely nothing** to do with the property that "a negative number × a positive number = a negative number". Learners should not be allowed to develop the serious misconception that "operations can be done on signs", for example the idea that "a minus times a minus is a plus". Learners should also be protected against the fallacy that subtraction of a negative number is the same thing as multiplying two negative numbers, for example, 10 – (–4) = 10 + 4 because "subtracting an integer is the same as adding its additive inverse", **NOT** because (–1) × (–4) = 4.

2.1 What is beyond 0?

WHY PEOPLE DECIDED TO HAVE NEGATIVE NUMBERS

Background information

In this section the following computational properties of integers are addressed:

- Subtracting a bigger number from a smaller number results in a **negative number**, for example, 10 – 20 = (–10) ("ten minus twenty equals negative ten").
- Any integer and its **additive inverse** lie the same distance from and on opposite sides of 0 on the number line, therefore the sum of any integer and its additive inverse is equal to 0, for example:
  - 12 + (–12) = 0 ("twelve plus negative twelve equals zero") or
  - (–12) + 12 = 0 ("negative twelve plus twelve equals zero").
- **Adding an integer** has the same effect as subtracting its additive inverse. For example: 3 + (–10) and 3 – 10 produce the same result.
- **Subtracting an integer** has the same effect as adding its additive inverse. For example: 3 – (–10) and 3 + 10 produce the same result.

Teaching guidelines

By the end of LB page 19 learners should understand the following computational property of integers:

A negative number describes the result where a number is subtracted from a number smaller than itself.

Misconceptions

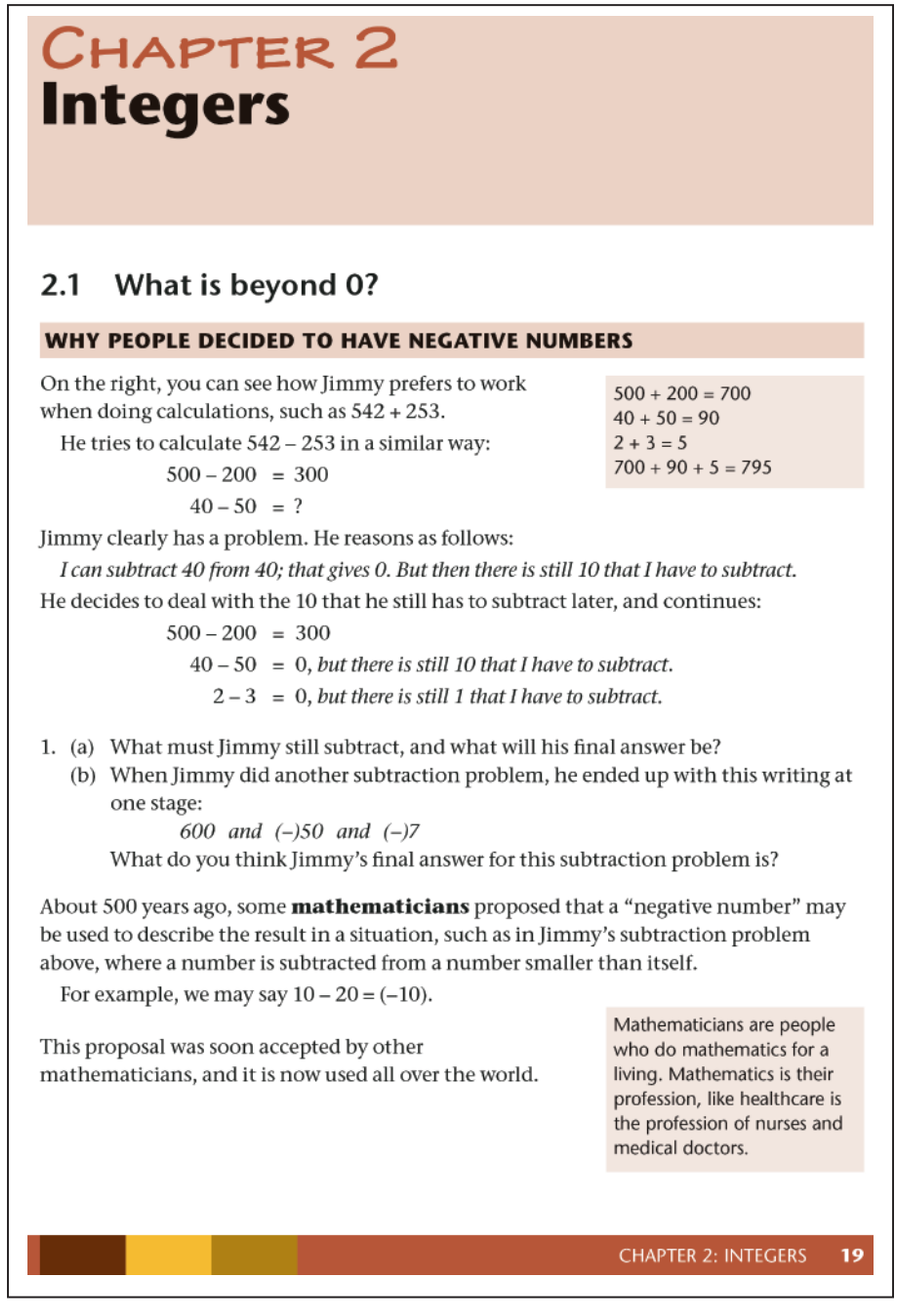
A calculation like 10 – 20 = (–10) should be read as "ten minus (or subtract) twenty equals *negative* ten" and NOT as "ten minus (or subtract) twenty equals *minus* ten".

Note on question 1

This question creates the need for negative numbers.

Answers

1. (a) He must still subtract 11; 300 – 11 = 289
   (b) 543



CHAPTER 2
Integers

2.1 What is beyond 0?

WHY PEOPLE DECIDED TO HAVE NEGATIVE NUMBERS

On the right, you can see how Jimmy prefers to work when doing calculations, such as 542 + 253.

500 + 200 = 700
40 + 50 = 90
2 + 3 = 5
700 + 90 + 5 = 795

He tries to calculate 542 – 253 in a similar way:

500 – 200 = 300
40 – 50 = ?

Jimmy clearly has a problem. He reasons as follows:

I can subtract 40 from 40; that gives 0. But then there is still 10 that I have to subtract.

He decides to deal with the 10 that he still has to subtract later, and continues:

500 – 200 = 300
40 – 50 = 0, *but there is still 10 that I have to subtract.*
2 – 3 = 0, *but there is still 1 that I have to subtract.*

1. (a) What must Jimmy still subtract, and what will his final answer be?
   (b) When Jimmy did another subtraction problem, he ended up with this writing at one stage:
   600 and (–)50 and (–)7
   What do you think Jimmy's final answer for this subtraction problem is?

About 500 years ago, some **mathematicians** proposed that a "negative number" may be used to describe the result in a situation, such as in Jimmy's subtraction problem above, where a number is subtracted from a number smaller than itself.

For example, we may say 10 – 20 = (–10).

This proposal was soon accepted by other mathematicians, and it is now used all over the world.

Mathematicians are people who do mathematics for a living. Mathematics is their profession, like healthcare is the profession of nurses and medical doctors.

CHAPTER 2: INTEGERS 19

Teaching guidelines (continued)

By the end of LB page 20 learners should understand the following computational property of integers:

Adding a negative number has the same effect as subtracting a positive number.

Misconceptions

A calculation like 20 + (–15) = 20 – 15 = 5 should be read as "twenty plus *negative* fifteen equals twenty minus fifteen, which is equal to five" and NOT as "twenty plus *minus* fifteen equals twenty minus fifteen, which is equal to five". The latter creates the impression that operations can be done on signs and has **absolutely nothing** to do with the property that "a positive number × a negative number = a negative number".

Note on introduction to question 4

- $15 - x = 10 \rightarrow$ x is a positive number because the value of 15 has decreased.
- $15 + x = 10 \rightarrow$ x is a negative number because the value of 15 has decreased.

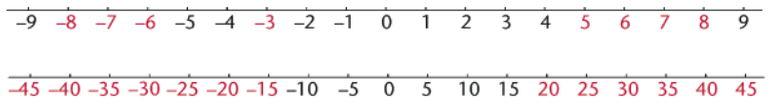
Answers

2. (a) (–4) (b) (–14) (c) (–24)
   (d) (–44) (e) (–184) (f) (–995)
3. See LB page 20 alongside.
4. (a) 200 (b) 40 (c) 200 (d) 40
5. (–60). Subtracting 60 is the same as subtracting 40 and 20.
6. See LB page 21 on the following page.

2. Calculate each of the following:

   (a) 16 – 20 (b) 16 – 30
   (c) 16 – 40 (d) 16 – 60
   (e) 16 – 200 (f) 5 – 1 000

3. Some numbers are shown on the number lines below. Copy the number lines and numbers shown, and fill in the missing numbers.



The following statement is true if the number is 5:

15 – (a certain number) = 10

A few centuries ago, some mathematicians decided they wanted to have numbers that will also make sentences like the following true:

15 + (a certain number) = 10

But to go from 15 to 10 you have to subtract 5.

The numbers 1; 2; 3; 4 etc. are called the **natural numbers**. The natural numbers, 0 and the negative whole numbers together, are called the **integers**.

The number we need to make the sentence *15 + (a certain number) = 10* true must have the following strange property:

If you **add** this number, it should have the **same effect** as to **subtract 5**.

Now the mathematicians of a few centuries ago really wanted to have numbers for which such strange sentences would be true. So they thought:

*Let us decide, and agree amongst ourselves, that the number we call **negative 5** will have the property that if you add it to another number, the effect will be the same as when you subtract the natural number 5.*

This means that the mathematicians agreed that 15 + (–5) is equal to 15 – 5.

Stated differently, instead of adding *negative 5* to a number, you may subtract 5.

Adding a negative number has the same effect as subtracting a natural number.
For example: 20 + (–15) = 20 – 15 = 5

4. Calculate each of the following:

   (a) 500 + (–300) (b) 100 + (–20) + (–40)
   (c) 500 + (–200) + (–100) (d) 100 + (–60)

5. Make a suggestion of what the answer for (–20) + (–40) should be. Give reasons for your suggestion.

Background information (continued)

Number patterns can be used to illustrate computational properties of integers.

Pattern P	Pattern Q	Pattern R	Pattern S
3 + 0 = 3	3 – 0 = 3	6 – 0 = 6	6 + 0 = 6
3 + (–1) = 2	3 – 1 = 2	6 – (–1) = 7	6 + 1 = 7
3 + (–2) = 1	3 – 2 = 1	6 – (–2) = 8	6 + 2 = 8
3 + (–3) = 0	3 – 3 = 0	6 – (–3) = 9	6 + 3 = 9
3 + (–4) = –1	3 – 4 = –1	6 – (–4) = 10	6 + 4 = 10
3 + (–5) = –2	3 – 5 = –2	6 – (–5) = 11	6 + 5 = 11

- Patterns P and Q confirm that adding a negative number has the same effect as subtracting a positive number.
- Patterns R and S confirm that subtracting a negative number has the same effect as adding a positive number.

Teaching guidelines (continued)

At the end of LB page 21 learners should understand the following computational property of integers:

Adding a negative number has the same effect as subtracting a positive number.

Misconceptions

Reading 20 – (–15) = 20 + 15 = 35 as "twenty minus *minus* fifteen equals twenty plus fifteen" creates the impression that operations can be done on signs.

Note on question 6

Number patterns and repeated addition of negative numbers are used to illustrate computational properties of integers, for example:

in question 6(a) –1 is added repeatedly: 10 + (–1) = 9; 9 + (–1) = 8; 8 + (–1) = 7; etc.

Note on introduction to question 7

- $15 - x = 10 \rightarrow x$ is a positive number because the value of 15 has decreased.
- $15 + x = 10 \rightarrow x$ is a negative number because the value of 15 has decreased.

Answers

7. (a) 40 (b) 40 (c) 20 (d) 20
   (e) 60 (f) 60 (g) 0 (h) 0

6. Copy and continue the lists of numbers to complete the following table:

(a)	(b)	(c)	(d)	(e)	(f)	(g)
10	100	3	–3	–20	150	0
9	90	6	–6	–18	125	–5
8	80	9	–9	–16	100	–10
7	70	12	–12	–14	75	–15
6	60	15	–15	–12	50	–20
5	50	18	–18	–10	25	–25
4	40	21	–21	–8	0	–30
3	30	24	–24	–6	–25	–35
2	20	27	–27	–4	–50	–40
1	10	30	–30	–2	–75	–45
0	0	33	–33	0	–100	–50
–1	–10	36	–36	2	–125	–55
–2	–20	39	–39	4	–150	–60
–3	–30	42	–42	6	–175	–65
–4	–40	45	–45	8	–200	–70

The following statement is true if the number is 5:

15 + (a certain number) = 20

What properties should a number have so that it makes the following statement true?

15 – (a certain number) = 20

To go from 15 to 20 you have to add 5. The number we need to make the sentence *15 – (a certain number)* = 20 true, must have the following property:

If you **subtract** this number, it should have the **same effect** as to **add 5**.

Let us agree that 15 – (–5) is equal to 15 + 5.

Stated differently, instead of subtracting *negative 5* from a number, you may add 5.

Subtracting a negative number has the same effect as adding a natural number.

For example: 20 – (–15) = 20 + 15 = 35

7. Calculate each of the following:

(a) 30 – (–10) (b) 30 + 10 (c) 30 + (–10) (d) 30 – 10
(e) 30 – (–30) (f) 30 + 30 (g) 30 + (–30) (h) 30 – 30

You probably agree that:

5 + (–5) = 0 10 + (–10) = 0 and 20 + (–20) = 0

Teaching guidelines (continued)

- At the end of question 9 on LB page 22 learners should understand the following computational property of integers:

  The sum of any integer and its additive inverse is equal to 0.

- Use additive inverses to explain certain computations with integers, for example: 8 + (–5) = 3 + (5 + (–5)) = 3 + 0 = 3 (using the associative property).

Answers

8. (a) –24 (b) 24 (c) 103 (d) –2 348
9. (a) 13 + 30 – 30 = 13 (b) 70 + 80 + (–80) = 70

STATEMENTS THAT ARE TRUE FOR MANY DIFFERENT NUMBERS

Background information

- Some statements are true for a **limited number of pairs of numbers**, for example, the statement (natural number) + (natural number) = 10 has five different solutions:
  1 + 9 = 10; 2 + 8 = 10; 3 + 7 = 10; 4 + 6 = 10; 5 + 5 = 10
- Some statements are true for an **infinite number of pairs of numbers**, for example, the statement (integer) + (integer) = 10 has infinite different solutions – i.e. all those above as well as solutions like 11 + (–1) = 10; 12 + (–2) = 10; 13 + (–3) = 10; 14 + (–4) = 10; 15 + (–5) = 10; etc.

2.2 Adding and subtracting with integers

ADDING CAN MAKE LESS AND SUBTRACTION CAN MAKE MORE

Background information

Natural numbers are defined by the set $N = \{1; 2; 3; 4; 5; ...\}$

Teaching guidelines

Summarise the three computational properties of integers (see shaded text above).

Answers

1. (a) 10 (b) 10 (c) 3 (d) 3
2. (a) 30 (b) 30 (c) 12 (d) 12 (e) 12 (f) 12 (g) 12 (h) 12
   (i) 8 (j) 8 (k) 8 (l) 8 (m) 8 (n) 8 (o) 0

We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (–3), are called **additive inverses**. They wipe each other out when you add them.

When you add any number to its additive inverse, the answer is 0 (the additive property of 0). For example, 120 + (–120) = 0.

What may each of the following be equal to?
(–8) + 5
(–5) + (–8)

8. Write the additive inverse of each of the following numbers:
   (a) 24 (b) –24
   (c) –103 (d) 2 348

The idea of additive inverses may be used to explain why 8 + (–5) is equal to 3:

$$8 + (-5) = 3 + \boxed{5 + (-5)} = 3 + 0 = 3$$

9. Use the idea of additive inverses to explain why each of these statements is true:
   (a) 43 + (–30) = 13 (b) 150 + (–80) = 70

STATEMENTS THAT ARE TRUE FOR MANY DIFFERENT NUMBERS

For how many different pairs of numbers can the following statement be true, if only natural (positive) numbers are allowed?

a number + another number = 10

For how many different pairs of numbers can the statement be true if negative numbers are also allowed?

2.2 Adding and subtracting with integers

ADDING CAN MAKE LESS AND SUBTRACTION CAN MAKE MORE

1. Calculate each of the following:
   (a) 10 + 4 + (–4) (b) 10 + (–4) + 4
   (c) 3 + 8 + (–8) (d) 3 + (–8) + 8

Natural numbers can be arranged in any order to add and subtract them. This is also the case for integers.

The numbers 1; 2; 3; 4; ... that we use to count, are called **natural numbers**.

2. Calculate each of the following:
   (a) 18 + 12 (b) 12 + 18 (c) 2 + 4 + 6
   (d) 6 + 4 + 2 (e) 2 + 6 + 4 (f) 4 + 2 + 6
   (g) 4 + 6 + 2 (h) 6 + 2 + 4 (i) 6 + (–2) + 4
   (j) 4 + 6 + (–2) (k) 4 + (–2) + 6 (l) (–2) + 4 + 6
   (m) 6 + 4 + (–2) (n) (–2) + 6 + 4 (o) (–6) + 4 + 2

Background information (continued)

- A **closed number sentence** is one in which all numbers are known, for example, 8 + 2 = 10.
- An **open number sentence** is one in which some numbers are not known at first, for example 8 – (a number) = 10 or 8 – x = 10.

Teaching guidelines

Discuss the difference between closed and open number sentences.

Note on question 6

- In question 6(a) the answers in the number pattern increase. At the end of the pattern it shows that subtracting a negative integer has the same effect as adding its additive inverse.
- In question 6(b) the answers in the number pattern decrease. At the end of the pattern it shows that adding a negative integer has the same effect as subtracting its additive inverse.

Answers

3. (a) 5 (b) 5 (c) 12 (d) 12 (e) 20
   (f) 10 (g) 0 (h) 2 (i) 4 (j) –6
4. (a) 20 + 30 = 50 (b) 50 + (–30) = 20
   (c) 20 + (–10) = 10 (d) 75 + (–25) = 50
   (e) –25 + (–25) = –50
5. (a) –7 + 50 + (–50) = –7 + 0 = –7
   (b) –25 + 85 + (–85) = –25 + 0 = –25
6. See LB page 23 alongside.
7. (a) 20 (b) 100

3. Calculate each of the following:
   (a) (–5) + 10 (b) 10 + (–5) (c) (–8) + 20
   (d) 20 – 8 (e) 30 + (–10) (f) 30 + (–20)
   (g) 30 + (–30) (h) 10 + (–5) + (–3)
   (i) (–5) + 7 + (–3) + 5 (j) (–5) + 2 + (–7) + 4

4. In each case, find the number that makes the statement true. Give your answer by writing a closed number sentence.
   (a) 20 + (an unknown number) = 50
   (b) 50 + (an unknown number) = 20
   (c) 20 + (an unknown number) = 10
   (d) (an unknown number) + (–25) = 50
   (e) (an unknown number) + (–25) = –50

5. Use the idea of additive inverses to explain why each of the following statements is true:
   (a) 43 + (–50) = –7 (b) 60 + (–85) = –25

Statements like these are also called number sentences.
An incomplete number sentence, where some numbers are not known at first, is sometimes called an **open number sentence**:
8 – (a number) = 10
A **closed number sentence** is where all the numbers are known:
8 + 2 = 10

6. Copy and complete the following table as far as you can:

(a)	(b)	(c)
5 – 8 = –3	5 + 8 = 13	8 – 3 = 5
5 – 7 = –2	5 + 7 = 12	7 – 3 = 4
5 – 6 = –1	5 + 6 = 11	6 – 3 = 3
5 – 5 = 0	5 + 5 = 10	5 – 3 = 2
5 – 4 = 1	5 + 4 = 9	4 – 3 = 1
5 – 3 = 2	5 + 3 = 8	3 – 3 = 0
5 – 2 = 3	5 + 2 = 7	2 – 3 = –1
5 – 1 = 4	5 + 1 = 6	1 – 3 = –2
5 – 0 = 5	5 + 0 = 5	0 – 3 = –3
5 – (–1) = 6	5 + (–1) = 4	(–1) – 3 = –4
5 – (–2) = 7	5 + (–2) = 3	(–2) – 3 = –5
5 – (–3) = 8	5 + (–3) = 2	(–3) – 3 = –6
5 – (–4) = 9	5 + (–4) = 1	(–4) – 3 = –7
5 – (–5) = 10	5 + (–5) = 0	(–5) – 3 = –8
5 – (–6) = 11	5 + (–6) = –1	(–6) – 3 = –9

7. Calculate each of the following:
   (a) 80 + (–60) (b) 500 + (–200) + (–200)

Note on questions 10

Learners who read these expressions properly will clearly distinguish between operation signs ("plus" or "minus") and signs of integers ("positive" or "negative"). For example:

- In question 10(a) the expression 20 – (–10) means "twenty minus negative ten".
- In question 10(e) the expression (–20) – (–10) means "negative twenty minus negative ten".

Note on question 11

Challenge learners to interpret the results in each column, for example:

- The start of column 11(a) shows that subtracting a negative integer has the same effect as adding its additive inverse.
- The end of column 11(b) shows that the sum of two negative integers is negative.
- The start of column 11(c) shows that subtracting a negative integer has the same effect as adding its additive inverse.
- The end of column 11(c) shows that the difference between two negative integers is:
  - positive if the first number is bigger than the second number
  - negative if the first number is smaller than the second number.

Answers

8. (a) Yes. 60 (b) –40
9. (a) 0 (b) 30 (c) 0 (d) –30
10. (a) 30 (b) 200 (c) 10 (d) 0
    (e) –10 (f) 0 (g) –30 (h) –200
11. See LB page 24 alongside.
12. (a) True. $-2 - 3 = -2 + (-3)$ (b) False. $2 + (-3) \neq 2 + 3$
    (c) False. $2 - (-3) \neq 2 - 3$ (d) True. $2 + (-3) = 2 - 3$

8. (a) Is $100 + (-20) + (-20) = 60$, or does it equal to something else?
   (b) What do you think $(-20) + (-20)$ is equal to?
9. Calculate each of the following:
   (a) 20 – 20 (b) 50 – 20 (c) (–20) – (–20) (d) (–50) – (–20)
10. Calculate each of the following:
    (a) 20 – (–10) (b) 100 – (–100) (c) 20 + (–10) (d) 100 + (–100)
    (e) (–20) – (–10) (f) (–100) – (–100) (g) (–20) + (–10) (h) (–100) + (–100)
11. Copy and complete the following table as far as you can:

(a)	(b)	(c)
5 – (–8) = 13	(–5) + 8 = 3	8 – (–3) = 11
5 – (–7) = 12	(–5) + 7 = 2	7 – (–3) = 10
5 – (–6) = 11	(–5) + 6 = 1	6 – (–3) = 9
5 – (–5) = 10	(–5) + 5 = 0	5 – (–3) = 8
5 – (–4) = 9	(–5) + 4 = –1	4 – (–3) = 7
5 – (–3) = 8	(–5) + 3 = –2	3 – (–3) = 6
5 – (–2) = 7	(–5) + 2 = –3	2 – (–3) = 5
5 – (–1) = 6	(–5) + 1 = –4	1 – (–3) = 4
5 – 0 = 5	(–5) + 0 = –5	0 – (–3) = 3
5 – 1 = 4	(–5) + (–1) = –6	(–1) – (–3) = 2
5 – 2 = 3	(–5) + (–2) = –7	(–2) – (–3) = 1
5 – 3 = 2	(–5) + (–3) = –8	(–3) – (–3) = 0
5 – 4 = 1	(–5) + (–4) = –9	(–4) – (–3) = –1
5 – 5 = 0	(–5) + (–5) = –10	(–5) – (–3) = –2

12. In each case, state whether the statement is true or false and give a numerical example to demonstrate your answer:
    (a) Subtracting a positive number from a negative number has the same effect as adding the additive inverse of the positive number.
    (b) Adding a negative number to a positive number has the same effect as adding the additive inverse of the negative number.
    (c) Subtracting a negative number from a positive number has the same effect as subtracting the additive inverse of the negative number.
    (d) Adding a negative number to a positive number has the same effect as subtracting the additive inverse of the negative number.

Answers

12. (e) False. $-2 + 3 \neq -2 + (-3)$ (f) True. $-2 + 3 = -2 - (-3)$
    (g) False. $-2 - 3 \neq -2 - (-3)$ (h) True. $2 - (-3) = 2 + 3$

COMPARING INTEGERS AND SOLVING PROBLEMS

Background information

Number lines can be used to discuss the following **features of integers**:

- The **further** an integer lies **to the right** of 0, the **larger** its value.
- The **further** an integer lies **to the left** of 0, the **smaller** its value.
- Any positive integer lies to the **right** of any negative integer.
- Any negative integer lies to the **left** of any positive integer.
- The larger of **two positive integers** lies to the **right** of the smaller one because the larger one lies **further from 0** than the smaller one.
- The larger of **two negative integers** lies to the **right** of the smaller one because the larger one lies **closer to 0** than the smaller one.

Teaching guidelines

Discuss how a number line can be used to compare two integers, for example:

- 5 lies further from 0 than 3, which shows that $5 > 3$
- 6 lies closer to 0 than 9, which shows that $6 < 9$
- -3 lies closer to 0 than -5, which shows that $-3 > -5$
- -8 lies further from 0 than -2, which shows that $-8 < -2$.

Answers

1. See LB page 25 alongside.
2. $19\ °C - (-5\ °C) = 24\ °C$
3. $-150\ m + 75\ m = -75\ m$
   She is now 75 m below the surface.
4. $2\ 200\ m - 800\ m = 1\ 400\ m$
5. $1\ 200\ m - (-860\ m) = 2\ 060\ m$
6. $-4\ °C + 19\ °C = 15\ °C$

(e) Adding a positive number to a negative number has the same effect as adding the additive inverse of the positive number.
(f) Adding a positive number to a negative number has the same effect as subtracting the additive inverse of the positive number.
(g) Subtracting a positive number from a negative number has the same effect as subtracting the additive inverse of the positive number.
(h) Subtracting a negative number from a positive number has the same effect as adding the additive inverse of the negative number.

COMPARING INTEGERS AND SOLVING PROBLEMS

1. Rewrite and include <, > or = to make the relationship between the numbers true:
   (a) $-103 < -99$ (b) $-699 > -701$
   (c) $30 > -30$ (d) $10 - 7 > -(10 - 7)$
   (e) $-121 > -200$ (f) $12 - 5 > -(12 + 5)$
   (g) $-199 < -110$
2. At 5 a.m. in Bloemfontein the temperature was $-5\ °C$. At 1 p.m., it was $19\ °C$. By how many degrees did the temperature rise?
3. A diver swims 150 m below the surface of the sea. She moves 75 m towards the surface. How far below the surface is she now?
4. One trench in the ocean is 800 m deep and another is 2 200 m deep. What is the difference in their depths?
5. An island has a mountain which is 1 200 m high. The surrounding ocean has a depth of 860 m. What is the difference in height?
6. On a winter's day in Upington the temperature rose by $19\ °C$. If the minimum temperature was $-4\ °C$, what was the maximum temperature?

2.3 Multiplying and dividing with integers

MULTIPLICATION WITH INTEGERS

1. Calculate each of the following:
   (a) $-5 + -5 + -5 + -5 + -5 + -5 + -5 + -5 + -5 + -5$
   (b) $-10 + -10 + -10 + -10 + -10$
   (c) $-6 + -6 + -6 + -6 + -6 + -6 + -6 + -6 + -6 + -6$
   (d) $-8 + -8 + -8 + -8 + -8 + -8$
   (e) $-20 + -20 + -20 + -20 + -20 + -20 + -20$

2.3 Multiplying and dividing with integers

MULTIPLICATION WITH INTEGERS

Background information

Number patterns can be used to investigate multiplication rules for integers.

Pattern P

5 × 2 = 10
5 × 1 = 5
5 × 0 = 0
5 × (–1) = –5
5 × (–2) = –10

Pattern Q

2 × 5 = 10
1 × 5 = 5
0 × 5 = 0
–1 × 5 = –5
–2 × 5 = –10

Pattern R

–5 × 2 = –10
–5 × 1 = –5
–5 × 0 = 0
–5 × (–1) = 5
–5 × (–2) = 10

Teaching guidelines

At the end of LB page 26 learners should have discovered the following:

- With integers, multiplication is commutative.
- *The product of two positive numbers is a positive number* (see pattern P).
- *The product of a positive number and a negative number is a negative number* (see pattern P).
- *The product of a negative number and a positive number is a negative number* (see pattern Q).
- With integers, multiplication distributes over addition and subtraction.

Answers

1. (a) –50 (b) –50 (c) –48 (d) –48 (e) –140
2. (a) ✗ (b) ✓ (c) ✓ (d) ✓
   (e) ✓ (f) ✗ (g) ✗ (h) ✓
3. Yes. e.g. 2 + 3 = 3 + 2; –48 + 3 = 3 + (–48); –48 + (–50) = –50 + (–48)
4. (a) –200 (b) –20 (c) –200 (d) –100 (e) –580 (f) 58
5. (a) 200 (b) 20 (c) 200 (d) –80 (e) –800 (f) –800
6. (a) See LB page 26 alongside. (b) The distributive property
7. (a) Yes
   (b) 14 × (35 + 58) = 14 × 35 + 14 × 58
   14 × (–35 + 58) = 14 × (–35) + 14 × 58
8. See LB page 26 alongside.
9. 10 × (–20) = –200 –500 + 30 = –470 –500 + 300 = –200

2. In each case, show whether you agree (✓) or disagree (✗) with the given statement:
   (a) 10 × (–5) = 50 (b) 8 × (–6) = (–8) × 6
   (c) (–5) × 10 = 5 × (–10) (d) 6 × (–8) = –48
   (e) (–5) × 10 = 10 × (–5) (f) 8 × (–6) = 48
   (g) 4 × 12 = –48 (h) (–4) × 12 = –48

Multiplication of integers is commutative:
(–20) × 5 = 5 × (–20)

3. Is addition of integers commutative? Demonstrate your answer with three different examples.
4. Calculate each of the following:
   (a) 20 × (–10) (b) (–5) × 4 (c) (–20) × 10
   (d) 4 × (–25) (e) 29 × (–20) (f) (–29) × (–2)
5. Calculate each of the following:
   (a) 10 × 50 + 10 × (–30) (b) 50 + (–30) (c) 10 × **(**50 + (–30)**)**
   (d) (–50) + (–30) (e) 10 × (–50) + 10 × (–30) (f) 10 × **(**(–50) + (–30)**)**

The product of two positive numbers is a positive number, for example: 5 × 6 = 30.

The product of a positive number and a negative number is a negative number, for example: 5 × (–6) = –30.

The product of a negative number and a positive number is a negative number, for example: (–5) × 6 = –30.

6. (a) Four numerical expressions are given below. Which expressions do you expect to have the same answers? Do not do the calculations.

   14 × (23 + 58) 23 × (14 + 58) 14 × 23 + 14 × 58 14 × 23 + 58

   (b) What property of operations is demonstrated by the fact that two of the above expressions have the same value?

7. Consider your answers for question 6.
   (a) Does multiplication distribute over addition in the case of integers?
   (b) Illustrate your answer with two examples.
8. Three numerical expressions are given below. Which expressions do you expect to have the same answers? Do not do the calculations.

   10 × **(**(–50) – (–30)**)** 10 × (–50) – (–30) 10 × (–50) – 10 × (–30)

9. Do the three sets of calculations given in question 8.

Teaching guidelines (continued)

At the end of question 11 learners should have discovered the final multiplication rule for integers:

The product of two negative numbers is a positive number
(see pattern R on TG page 31).

Misconceptions

- Learners who do not read mathematical expressions which contain integers properly tend to mix up operations ("plus" and "minus") and signs of integers ("positive" and "negative").
- Learners should also be protected against the fallacy that subtraction of a negative number is the same thing as multiplying two negative numbers, for example $10 - (-4) = 10 + 4$ because "subtracting an integer is the same as adding its additive inverse", **NOT** because $(-1) \times (-4) = 4$.
- Avoid the idea that:
  - "a plus times a plus is a plus"
  - "a plus times a minus is a minus"
  - "a minus times a plus is a minus"
  - "a minus times a minus is a plus".

Note on question 12

To avoid confusion, distinguish between operations ("plus" and "minus") and signs of integers ("positive" and "negative").

Answers

10. $-10 \times 2 = -20$
11. 30
12. (a) 50 (b) -50
    (c) 50 (d) -50
    (e) $200 + 120 = 320$ (f) $-20 \times (-16) = 320$
    (g) $200 - 120 = 80$ (h) $-20 \times -4 = 80$

Your work in questions 5, 8 and 9 demonstrates that multiplication with a positive number distributes over addition and subtraction of integers. For example:

$10 \times \mathbf{(}5 + (-3)\mathbf{)} = 10 \times 2 = \mathbf{20}$ and $10 \times 5 + 10 \times (-3) = 50 + (-30) = \mathbf{20}$

$10 \times \mathbf{(}5 - (-3)\mathbf{)} = 10 \times 8 = \mathbf{80}$ and $10 \times 5 - 10 \times (-3) = 50 - (-30) = \mathbf{80}$

10. Calculate: $(-10) \times \mathbf{(}5 + (-3)\mathbf{)}$

Now consider the question of whether or not multiplication with a negative number distributes over addition and subtraction of integers. For example, would $(-10) \times 5 + (-10) \times (-3)$ also have the answer -20, as does $(-10) \times \mathbf{(}5 + (-3)\mathbf{)}$?

11. What must $(-10) \times (-3)$ be equal to, if we want $(-10) \times 5 + (-10) \times (-3)$ to be equal to -20?

In order to ensure that multiplication distributes over addition and subtraction in the system of integers, we have to agree that

(a negative number) × (a negative number) is a positive number.

For example: $(-10) \times (-3) = 30$.

12. Calculate each of the following:

(a) $(-10) \times (-5)$ (b) $(-10) \times 5$
(c) 10×5 (d) $10 \times (-5)$
(e) $(-20) \times (-10) + (-20) \times (-6)$ (f) $(-20) \times \mathbf{(}(-10) + (-6)\mathbf{)}$
(g) $(-20) \times (-10) - (-20) \times (-6)$ (h) $(-20) \times \mathbf{(}(-10) - (-6)\mathbf{)}$

Here is a summary of the **properties of integers** that make it possible to do calculations with integers:

- When a number is added to its additive inverse, the result is 0, for example $(+12) + (-12) = 0$.
- Adding an integer has the same effect as subtracting its additive inverse. For example, $3 + (-10)$ can be calculated by doing $3 - 10$, and the answer is -7.
- Subtracting an integer has the same effect as adding its additive inverse. For example, $3 - (-10)$ can be calculated by doing $3 + 10$, and the answer is 13.
- The product of a positive and a negative integer is negative, for example $(-15) \times 6 = -90$.
- The product of a negative and a negative integer is positive, for example $(-15) \times (-6) = 90$.

DIVISION WITH INTEGERS

Background information

- Division is the inverse of multiplication.
- If two numbers and the value of their product are known, the answer to two division sums are also known. For example,
  if $3 \times 5 = 15$ then $15 \div 3 = 5$ and $15 \div 5 = 3$.

Teaching guidelines

By the end of LB page 28 learners should be familiar with the following:

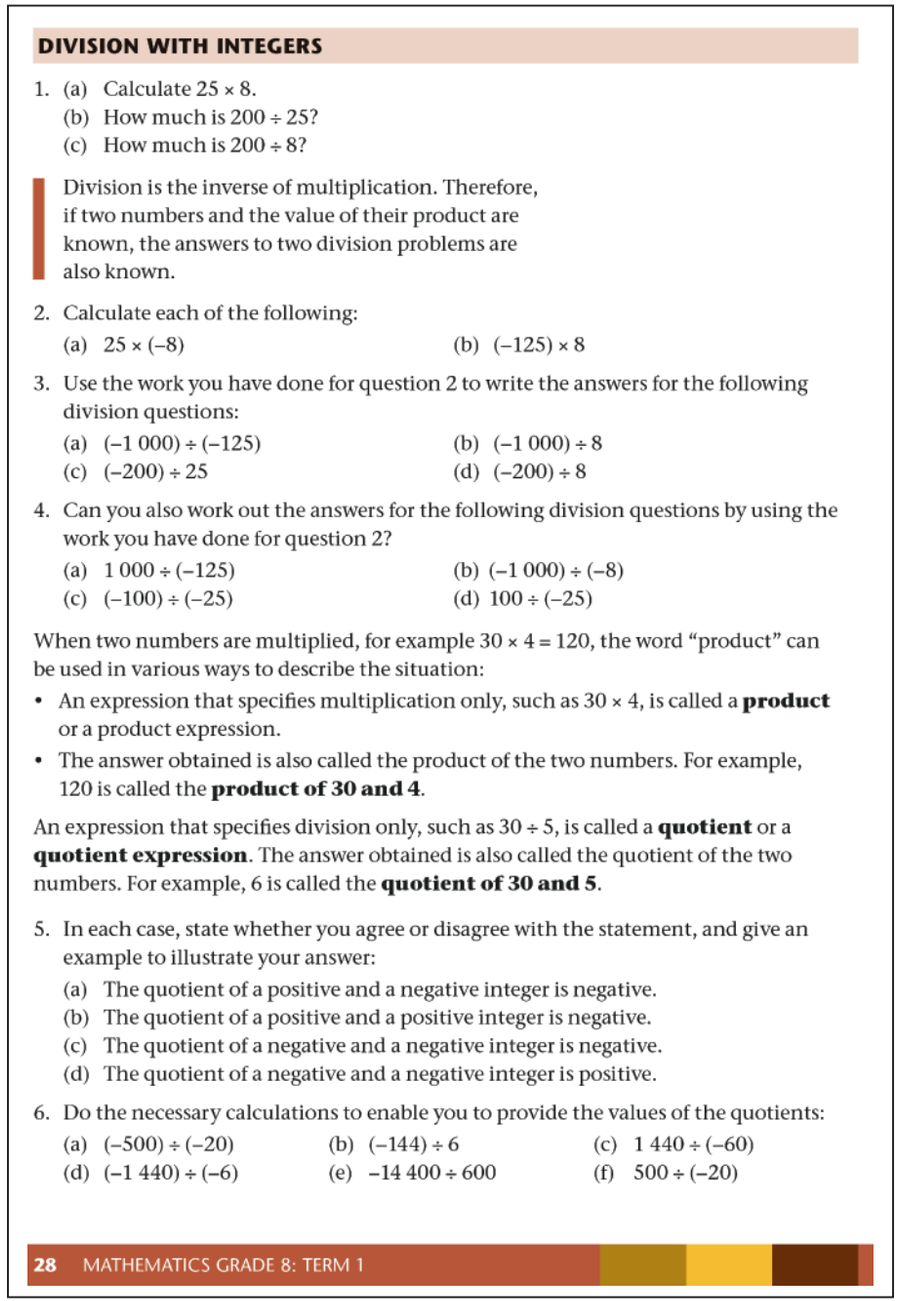
- *A positive number divided by a positive number is a positive number.*
  $3 \times 5 = 15$ so $15 \div 5 = 3$
- *A positive number divided by a negative number is a negative number.*
  $(-3) \times (-5) = 15$ so $15 \div (-5) = (-3)$
- *A negative number divided by a positive number is a negative number.*
  $(-3) \times 5 = (-15)$ so $(-15) \div 5 = (-3)$
- *A negative number divided by a negative number is a positive number.*
  $3 \times (-5) = (-15)$ so $(-15) \div (-5) = 3$

Misconceptions

Learners will develop a **serious misconception** if signs of integers are confused with operations. Avoid the following at all costs: "a plus divided by a plus is a plus"; "a plus divided by a minus is a minus"; "a minus divided by a plus is a minus" and "a minus divided by a minus is a plus".

Answers

1. (a) 200 (b) 8 (c) 25
2. (a) –200 (b) –1 000
3. (a) 8 (b) –125 (c) –8 (d) –25
4. (a) –8 (b) 125 (c) 4 (d) –4
5. (a) Agree. $-100 \div 2 = -50$
   (b) Disagree. $20 \div 2 = 10$, which is positive.
   (c) Disagree. $-20 \div (-2) = 10$, which is positive.
   (d) Agree. $-20 \div (-2) = 10$, which is positive.
6. (a) 25 (b) –24 (c) –24 (d) 240 (e) –24 (f) –25



DIVISION WITH INTEGERS

1. (a) Calculate 25×8.
   (b) How much is $200 \div 25$?
   (c) How much is $200 \div 8$?

Division is the inverse of multiplication. Therefore, if two numbers and the value of their product are known, the answers to two division problems are also known.

2. Calculate each of the following:
   (a) $25 \times (-8)$ (b) $(-125) \times 8$
3. Use the work you have done for question 2 to write the answers for the following division questions:
   (a) $(-1\,000) \div (-125)$ (b) $(-1\,000) \div 8$
   (c) $(-200) \div 25$ (d) $(-200) \div 8$
4. Can you also work out the answers for the following division questions by using the work you have done for question 2?
   (a) $1\,000 \div (-125)$ (b) $(-1\,000) \div (-8)$
   (c) $(-100) \div (-25)$ (d) $100 \div (-25)$

When two numbers are multiplied, for example $30 \times 4 = 120$, the word "product" can be used in various ways to describe the situation:

- An expression that specifies multiplication only, such as 30×4, is called a **product** or a product expression.
- The answer obtained is also called the product of the two numbers. For example, 120 is called the **product of 30 and 4**.

An expression that specifies division only, such as $30 \div 5$, is called a **quotient** or a **quotient expression**. The answer obtained is also called the quotient of the two numbers. For example, 6 is called the **quotient of 30 and 5**.

5. In each case, state whether you agree or disagree with the statement, and give an example to illustrate your answer:
   (a) The quotient of a positive and a negative integer is negative.
   (b) The quotient of a positive and a positive integer is negative.
   (c) The quotient of a negative and a negative integer is negative.
   (d) The quotient of a negative and a negative integer is positive.
6. Do the necessary calculations to enable you to provide the values of the quotients:
   (a) $(-500) \div (-20)$ (b) $(-144) \div 6$ (c) $1\,440 \div (-60)$
   (d) $(-1\,440) \div (-6)$ (e) $-14\,400 \div 600$ (f) $500 \div (-20)$

28 MATHEMATICS GRADE 8: TERM 1

THE ASSOCIATIVE PROPERTIES OF OPERATIONS WITH INTEGERS

Background information (refer to LB page 26)

- Multiplication with integers is commutative.
- Multiplication distributes over addition and subtraction with integers.

Teaching guidelines

At the end of question 5 on LB page 29 learners should be familiar with this statement: "Multiplication with integers is associative."

Answers

1. A. $6 \times 5 = 30$; $30 \times 10 = 300$ B. $10 \times 10 = 100$; $100 \times 3 = 300$
   C. $50 \times 3 = 150$; $150 \times 2 = 300$ D. $15 \times 2 = 30$; $30 \times 10 = 300$
2. (a) Yes
   (b) A. $-6 \times 5 = -30$; $-30 \times (-10) = 300$
   B. $10 \times (-10) = -100$; $-100 \times (-3) = 300$
   C. $-50 \times (-3) = 150$; $150 \times 2 = 300$
   D. $-15 \times 2 = -30$; $-30 \times (-10) = 300$
3. B. $2 \times 5 \times 10 \times 3$ C. $10 \times 5 \times 3 \times 2$ D. $3 \times 5 \times 2 \times 10$
4. $1 + 2 + 3 + (-4) = 2$; $2 + 3 + (-4) + 1 = 2$; $3 + (-4) + 1 + 2 = 2$; $(-4) + 2 + 1 + 3 = 2$
5. (a) 70 (b) 70 (c) -70 (d) 70 (e) -70

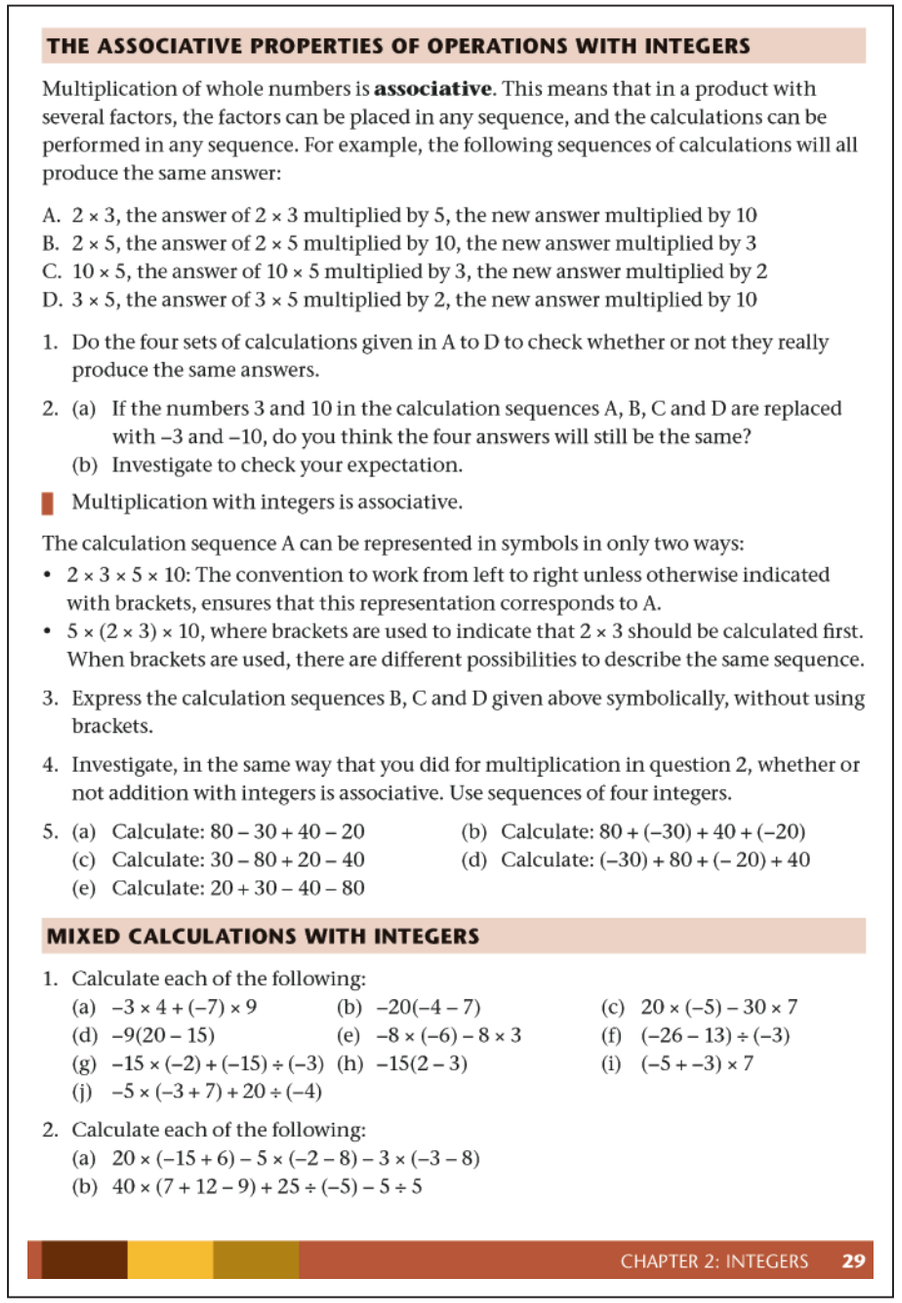
MIXED CALCULATIONS WITH INTEGERS

Teaching guidelines

The distributive property does NOT require that "brackets should be removed" before a product expression is evaluated.

Answers

1. (a) $-12 - 63 = -75$ (b) $-20(-11) = 220$ (c) $-100 - 210 = -310$
   (d) $-9(5) = -45$ (e) $48 - 24 = 24$ (f) $-39 \div (-3) = 13$
   (g) $30 + 5 = 35$ (h) $-15(-1) = 15$ (i) $-8 \times 7 = -56$
   (j) $-5 \times 4 + (-5) = -25$
2. (a) $20 \times (-9) - 5 \times (-10) - 3 \times (-11) = -180 + 50 + 33 = -97$
   (b) $40 \times 10 - 5 - 1 = 394$



THE ASSOCIATIVE PROPERTIES OF OPERATIONS WITH INTEGERS

Multiplication of whole numbers is **associative**. This means that in a product with several factors, the factors can be placed in any sequence, and the calculations can be performed in any sequence. For example, the following sequences of calculations will all produce the same answer:

A. 2×3, the answer of 2×3 multiplied by 5, the new answer multiplied by 10
B. 2×5, the answer of 2×5 multiplied by 10, the new answer multiplied by 3
C. 10×5, the answer of 10×5 multiplied by 3, the new answer multiplied by 2
D. 3×5, the answer of 3×5 multiplied by 2, the new answer multiplied by 10

1. Do the four sets of calculations given in A to D to check whether or not they really produce the same answers.
2. (a) If the numbers 3 and 10 in the calculation sequences A, B, C and D are replaced with -3 and -10, do you think the four answers will still be the same?
   (b) Investigate to check your expectation.

Multiplication with integers is associative.

The calculation sequence A can be represented in symbols in only two ways:

- $2 \times 3 \times 5 \times 10$: The convention to work from left to right unless otherwise indicated with brackets, ensures that this representation corresponds to A.
- $5 \times (2 \times 3) \times 10$, where brackets are used to indicate that 2×3 should be calculated first. When brackets are used, there are different possibilities to describe the same sequence.

3. Express the calculation sequences B, C and D given above symbolically, without using brackets.
4. Investigate, in the same way that you did for multiplication in question 2, whether or not addition with integers is associative. Use sequences of four integers.
5. (a) Calculate: $80 - 30 + 40 - 20$ (b) Calculate: $80 + (-30) + 40 + (-20)$
   (c) Calculate: $30 - 80 + 20 - 40$ (d) Calculate: $(-30) + 80 + (-20) + 40$
   (e) Calculate: $20 + 30 - 40 - 80$

MIXED CALCULATIONS WITH INTEGERS

1. Calculate each of the following:
   (a) $-3 \times 4 + (-7) \times 9$ (b) $-20(-4 - 7)$ (c) $20 \times (-5) - 30 \times 7$
   (d) $-9(20 - 15)$ (e) $-8 \times (-6) - 8 \times 3$ (f) $(-26 - 13) \div (-3)$
   (g) $-15 \times (-2) + (-15) \div (-3)$ (h) $-15(2 - 3)$ (i) $(-5 + -3) \times 7$
   (j) $-5 \times (-3 + 7) + 20 \div (-4)$
2. Calculate each of the following:
   (a) $20 \times (-15 + 6) - 5 \times (-2 - 8) - 3 \times (-3 - 8)$
   (b) $40 \times (7 + 12 - 9) + 25 \div (-5) - 5 \div 5$

Answers

2. (c) $-50(-5) + 30(-3) - 20(-4) = 250 - 90 + 80 = 240$
   (d) $-5 \times 0 = 0$
   (e) $-4(-20) + 7(-30) - 10(-40) = 80 - 210 + 400 = 270$
   (f) $-3(-8)(-6)(-15) = 2\,160$
   (g) $-100 - 30 + 30 - 15 = -115$
   (h) $125 - 100 + 400 - 2\,250 = -1\,825$
   (i) $-110 \div (-11) - 12 \div (-6) = 10 + 2 = 12$

2.4 Squares, cubes and roots with integers

SQUARES AND CUBES OF INTEGERS

Background information

- A **square** is any number which is a product of two identical factors.
- The **square root** of a number is another number which, when squared, will equal the first number.
- A **cube** is any number which is a product of three identical factors.
- The **cube root** of a number is another number which, when cubed, will equal the first number.

Teaching guidelines

Revise the concepts listed above.

Answers

1. (a) 400 (b) −400
2. (a) −400 (b) 400
3. See LB page 30 alongside.
4. (a) if x is negative (b) never
   (c) if x is negative (d) if x is positive and > 1
5. See LB page 30 alongside.
6. (a) 7 (b) No
7. (a) 5 or −5 (b) Yes

(c) $-50(20 - 25) + 30(-10 + 7) - 20(-16 + 12)$
(d) $-5 \times (-3 + 12 - 9)$
(e) $-4 \times (30 - 50) + 7 \times (40 - 70) - 10 \times (60 - 100)$
(f) $-3 \times (-14 + 6) \times (-13 + 7) \times (-20 + 5)$
(g) $20 \times (-5) + 10 \times (-3) + (-5) \times (-6) - (3 \times 5)$
(h) $-5(-20 - 5) + 10(-7 - 3) - 20(-15 - 5) + 30(-40 - 35)$
(i) $(-50 + 15 - 75) \div (-11) + (6 - 30 + 12) \div (-6)$

2.4 Squares, cubes and roots with integers

SQUARES AND CUBES OF INTEGERS

1. Calculate each of the following:
   (a) 20×20 (b) $20 \times (-20)$
2. Write the answers for each of the following:
   (a) $(-20) \times 20$ (b) $(-20) \times (-20)$
3. Copy and complete the following table:

x	1	−1	2	−2	5	−5	10	−10
x^2 which is $x \times x$	1	1	4	4	25	25	100	100
x^3	1	−1	8	−8	125	−125	1 000	−1 000

4. In each case, state for which values of x, in the table in question 3, the given statement is true.
   (a) x^3 is a negative number (b) x^2 is a negative number
   (c) $x^2 > x^3$ (d) $x^2 < x^3$
5. Copy and complete the following table:

x	3	−3	4	−4	6	−6	7	−7
x^2	9	9	16	16	36	36	49	49
x^3	27	−27	64	−64	216	−216	343	−343

6. Ben thinks of a number. He adds 5 to it and his answer is 12.
   (a) What number did he think of?
   (b) Is there another number that would also give 12 when 5 is added to it?
7. Lebo also thinks of a number. She multiplies the number by itself and gets 25.
   (a) What number did she think of?
   (b) Is there more than one number that will give 25 when multiplied by itself?

Background information (continued)

- 10^2 and $(-10)^2$ are both equal to 100 therefore:
  - the positive square root of 100 is 10
  - the negative square root of 100 is (–10).
- $10^3 = 1\,000$ and $(-10)^3 = (-1\,000)$ therefore:
  - the cube root of 1 000 is 10
  - the cube root of (–1 000) is (–10).

Teaching guidelines (continued)

At the end of LB page 31 learners should be able to provide the following:

- The positive and negative square roots of any square number, for example:
  $5^2 = 25 \rightarrow \sqrt{25} = 5$ and $-\sqrt{25} = (-5)$
- The cube root of any cube number, for example:
  $5^3 = 125 \rightarrow \sqrt[3]{125} = 5$
- The cube root of the additive inverse of any cube number, for example:
  $(-5)^3 = (-125) \rightarrow \sqrt[3]{(-125)} = (-5)$

Answers

8. 3
9. (a) 8; –8 (b) 3; –3
10. to 13. See LB page 31 alongside.

8. Mary thinks of a number and calculates (the number) × (the number) × (the number). Her answer is 27. What number did Mary think of?

10^2 is 100 and $(-10)^2$ is also 100.
Both 10 and (–10) are called **square roots** of 100.
10 may be called the **positive square root** of 100,
and (–10) may be called the **negative square root** of 100.

9. Write the positive square root and the negative square root of each number:
   (a) 64 (b) 9

10. Copy and complete the following table:

Number	1	4	9	16	25	36	49	64
Positive square root	1	2	3	4	5	6	7	8
Negative square root	–1	–2	–3	–4	–5	–6	–7	–8

11. Copy and complete the following tables:

(a)

x	1	2	3	4	5	6	7	8
x^3	1	8	27	64	125	216	343	512

(b)

x	–1	–2	–3	–4	–5	–6	–7	–8
x^3	–1	–8	–27	–64	–125	–216	–343	–512

3^3 is 27 and $(-5)^3$ is –125.
3 is called the **cube root** of 27, because $3^3 = 27$.
–5 is called the cube root of –125 because $(-5)^3 = -125$.

12. Copy and complete the following table:

Number	–1	8	–27	–64	–125	–216	1 000
Cube root	–1	2	–3	–4	–5	–6	10

The symbol $\sqrt{\ }$ is used to indicate "root".

$\sqrt[3]{-125}$ represents the **cube root** of –125. That means $\sqrt[3]{-125} = -5$.

$\sqrt[2]{36}$ represents the **positive square root** of 36, and $-\sqrt[2]{36}$ represents the **negative square root**. The "2" that indicates "square" is normally omitted, so $\sqrt{36} = 6$ and $-\sqrt{36} = -6$.

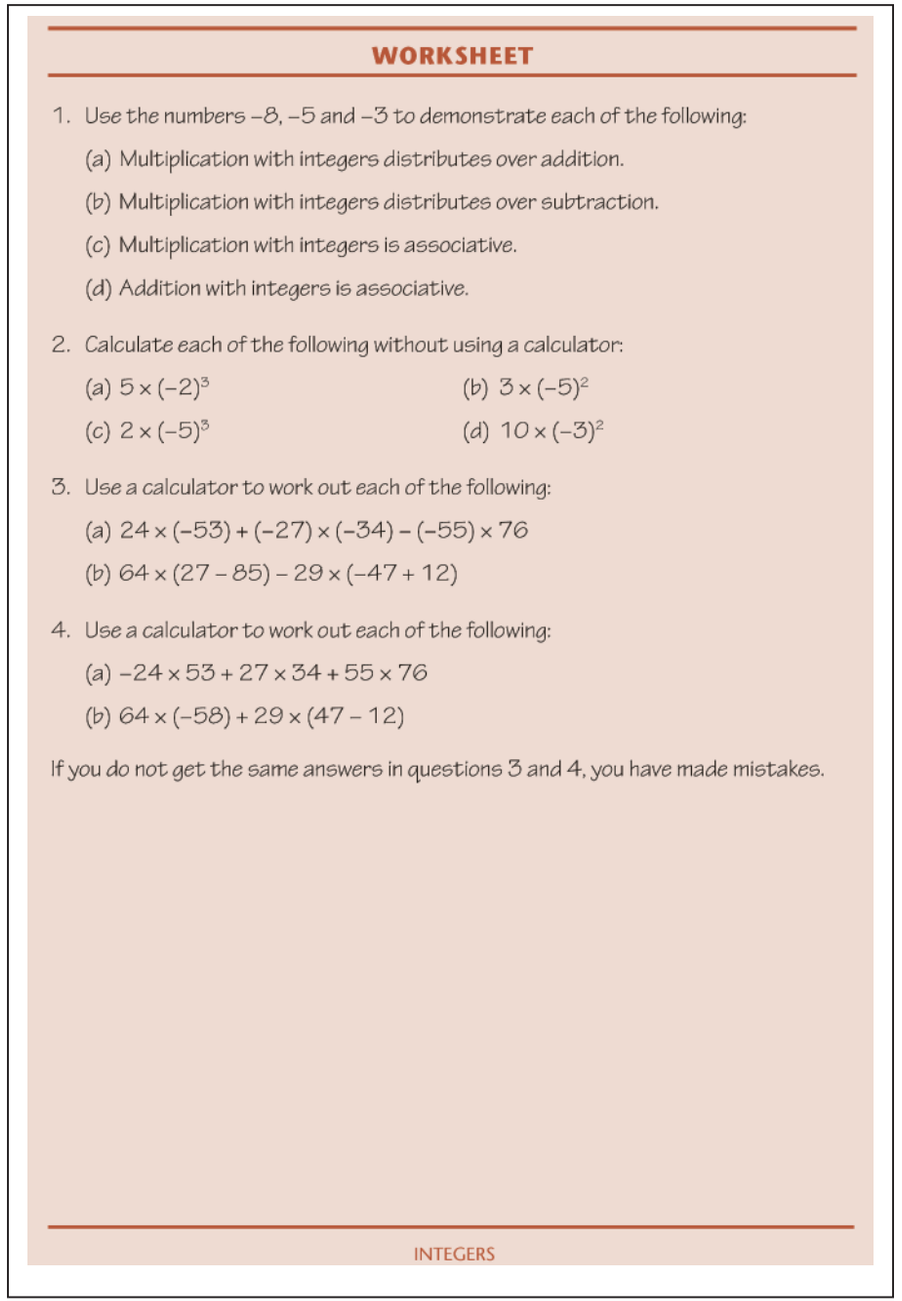
13. Copy and complete the following table:

$\sqrt[3]{-8}$	$\sqrt{121}$	$\sqrt[3]{-64}$	$-\sqrt{64}$	$\sqrt{64}$	$\sqrt[3]{-1}$	$-\sqrt{1}$	$\sqrt[3]{-216}$
–2	11	–4	–8	8	–1	–1	–6

WORKSHEET

Answers

1. (a) $-8(-5 + (-3)) = -8(-5) + (-8)(-3)$
   (b) $-8(-5 - (-3)) = -8(-5) - (-8)(-3)$
   (c) $-8 \times (-3) \times (-5) = -5 \times (-8) \times (-3)$
   (d) $-8 + (-3) + (-5) = -5 + (-8) + (-3)$
2. (a) $5 \times (-8) = -40$
   (b) $3 \times 25 = 75$
   (c) $2 \times (-125) = -250$
   (d) $10 \times 9 = 90$
3. (a) $-1\,272 + 918 + 4\,180 = 3\,826$
   (b) $64(-58) - 29(-35) = -3\,712 + 1\,015 = -2\,697$
4. (a) 3 826
   (b) –2 697



WORKSHEET

1. Use the numbers –8, –5 and –3 to demonstrate each of the following:
   (a) Multiplication with integers distributes over addition.
   (b) Multiplication with integers distributes over subtraction.
   (c) Multiplication with integers is associative.
   (d) Addition with integers is associative.
2. Calculate each of the following without using a calculator:
   (a) $5 \times (-2)^3$ (b) $3 \times (-5)^2$
   (c) $2 \times (-5)^3$ (d) $10 \times (-3)^2$
3. Use a calculator to work out each of the following:
   (a) $24 \times (-53) + (-27) \times (-34) - (-55) \times 76$
   (b) $64 \times (27 - 85) - 29 \times (-47 + 12)$
4. Use a calculator to work out each of the following:
   (a) $-24 \times 53 + 27 \times 34 + 55 \times 76$
   (b) $64 \times (-58) + 29 \times (47 - 12)$

If you do not get the same answers in questions 3 and 4, you have made mistakes.

INTEGERS

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
3.1 Revision	Exponential notation (base; exponent); squares; cubes; square and cube roots	Pages 33 to 36
3.2 Working with integers	Integers in exponential form	Pages 36 to 37
3.3 Laws of exponents	Product of powers; power of a power; power of a product; quotient of powers; power of zero	Pages 37 to 44
3.4 Calculations	Mixed operations	Pages 44 to 45
3.5 Squares, cubes and roots of rational numbers	Square of a fraction; square root of a fraction; cube of a fraction	Pages 45 to 46
3.6 Scientific notation	Very large numbers	Page 47

CAPS time allocation	9 hours
CAPS content specification	Pages 81 to 84

Mathematical background

- The **exponential notation** is a shorthand notation for repeated multiplication with the same number. The exponent, for example the "5" in 2^5, indicates the number of occurrences of the factor 2 in the product $2 \times 2 \times 2 \times 2 \times 2$.
  - Squaring a number resembles the process of finding the area of a square when its side length is given.
  - Square rooting a number resembles the process of finding the side length of a square when its area is given.
  - Squaring and square rooting are inverse operations. For example, the square of 5 is 25 ($5^2 = 25$), therefore the square root of 25 is 5 ($\sqrt{25} = 5$).
  - Cubing a number resembles the process of finding the volume of a cube when its side length is given.
  - Cube rooting a number resembles the process of finding the side length of a cube when its volume is given.
  - Cubing and cube rooting are inverse operations. For example, the cube of 5 is 125 ($5^3 = 125$), therefore the cube root of125 is 5 ($\sqrt[3]{125} = 5$).
- This chapter commences with revision of the squares of natural numbers up to 12 and the cubes of natural numbers up to 6, which should be considered as mental arithmetic.
- Learners have already worked with exponents in Grade 7. The new ideas in this chapter are the following:
  - powers of integers and fractions
  - laws of exponents
  - use of scientific notation.
- Notion of reversal of a process is important in this work. For example, raising to a power and finding a root are inverse processes.

3.1 Revision

EXPONENTIAL NOTATION

Background information

The exponential notation is easily misunderstood. A common error is to confuse it with non-repeated multiplication (repeated addition) and mistakes like $2^3 = 2 \times 3 = 6$ are then made. To avoid this, highlight to learners the similarity and differences between the following two notations:

- the **multiplication notation** for repeated addition of the same number, for example, $7 \times 5 = 5 + 5 + 5 + 5 + 5 + 5 + 5$, and
- the **exponential notation** for repeated multiplication with the same number, e.g. $5^7 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$.

Teaching guidelines

Distinguish between the **shorthand notations** for:

- repeated multiplication with the same number ($4 \times 4 \times 4 = 4^3$)
- repeated addition with the same number ($4 + 4 + 4 = 3 \times 4$)

Discuss of the concepts of the **power**, **base** and **exponent** of a number written in exponential form (refer to LB page 33 alongside).

Answers

1. (a) 8 (b) 64 (c) 27 (d) 729
2. (a) 2^3 (b) 2^6 (c) 3^3 (d) 3^6
3. (a) $5 \times 5 = 25$ (b) $2 \times 2 \times 2 \times 2 \times 2 = 32$ (c) $10 \times 10 = 100$
   (d) $15 \times 15 = 225$ (e) $3 \times 3 \times 3 \times 3 = 81$ (f) $4 \times 4 \times 4 = 64$
   (g) $2 \times 2 \times 2 = 8$ (h) $3 \times 3 = 9$

SQUARES

Background information

Learners should be able to use different ways to express exponential statements in words. For example, 3^2 can be read as "three squared" or as "three to the power two".

A **square** is any number which is a product of two identical factors.

CHAPTER 3
Exponents

3.1 Revision

EXPONENTIAL NOTATION

1. Calculate each of the following:
   (a) $2 \times 2 \times 2$ (b) $2 \times 2 \times 2 \times 2 \times 2 \times 2$
   (c) $3 \times 3 \times 3$ (d) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

Instead of writing $3 \times 3 \times 3 \times 3 \times 3 \times 3$, we can write 3^6.
We read this as "3 to the power of 6". The number 3 is the **base** and 6 is the **exponent**.
When we write $3 \times 3 \times 3 \times 3 \times 3 \times 3$ as 3^6, we are using **exponential notation**.

2. Write each of the following in exponential form:
   (a) $2 \times 2 \times 2$ (b) $2 \times 2 \times 2 \times 2 \times 2 \times 2$
   (c) $3 \times 3 \times 3$ (d) $3 \times 3 \times 3 \times 3 \times 3 \times 3$
3. Calculate each of the following:
   (a) 5^2 (b) 2^5 (c) 10^2 (d) 15^2
   (e) 3^4 (f) 4^3 (g) 2^3 (h) 3^2

SQUARES

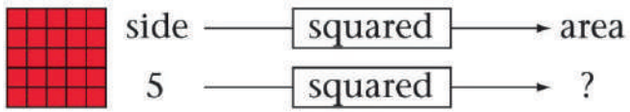
To square a number is to multiply it by itself.
The square of 8 is 64 because 8×8 equals 64.
We write 8×8 as 8^2 in exponential form.
We read 8^2 as **eight squared**.

1. Copy and complete the following table:

	Number	Square the number	Exponential form	Square
(a)	1	1×1	1^2	1
(b)	2	2×2	2^2	4
(c)	3	3×3	3^2	9

Teaching guidelines

To square a number means to multiply it by itself. This resembles finding the area of a square when the length of any side is given.



Answers

1. See the table on LB pages 33 and 34.
2. (a) $9 \times 16 = 144$ (b) $4 \times 9 = 36$ (c) $4 \times 25 = 100$ (d) $4 \times 16 = 64$
3. See the answers on LB p 34 alongside.

CUBES

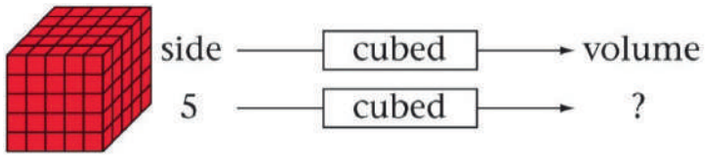
Background information

Learners should be able to use different ways to express exponential statements in words. For example, 3^3 can be read as "three cubed" or as "three to the power three".

A **cube** is any number which is a product of three identical factors.

Teaching guidelines

To cube a number means to multiply it by itself and again by itself. This resembles finding the volume of a cube when the length of any edge is given.



Answers

1. See the table on LB page 34 alongside.
2. (a) $8 \times 27 = 216$ (b) $8 \times 125 = 1\,000$ (c) $8 \times 64 = 512$ (d) $1 \times 729 = 729$
3. (a) True (b) False. $2^3 \times 5^3 = 1\,000$ (which is 10^3)
   (c) True (d) False. $1^3 \times 9^3 = 9^3 = 729$ (and not 10^3 (1 000))

	Number	Square the number	Exponential form	Square
(d)	4	4×4	4^2	16
(e)	5	5×5	5^2	25
(f)	6	6×6	6^2	36
(g)	7	7×7	7^2	49
(h)	8	8×8	8^2	64
(i)	9	9×9	9^2	81
(j)	10	10×10	10^2	100
(k)	11	11×11	11^2	121
(l)	12	12×12	12^2	144

2. Calculate each of the following:
   (a) $3^2 \times 4^2$ (b) $2^2 \times 3^2$ (c) $2^2 \times 5^2$ (d) $2^2 \times 4^2$
3. Copy and complete the following statements to make them true:
   (a) $3^2 \times 4^2 = \ldots 12^2 \ldots$ (b) $2^2 \times 3^2 = \ldots 6^2 \ldots$
   (c) $2^2 \times 5^2 = \ldots 10^2 \ldots$ (d) $2^2 \times 4^2 = \ldots 8^2 \ldots$

CUBES

To cube a number is to multiply it by itself and then by itself again. The cube of 3 is 27 because $3 \times 3 \times 3$ equals 27.
We write $3 \times 3 \times 3$ as 3^3 in exponential form.
We read 3^3 as **three cubed**.

1. Copy and complete the following table:

	Number	Cube the number	Exponential form	Cube
(a)	1	$1 \times 1 \times 1$	1^3	1
(b)	2	$2 \times 2 \times 2$	2^3	8
(c)	3	$3 \times 3 \times 3$	3^3	27
(d)	4	$4 \times 4 \times 4$	4^3	64
(e)	5	$5 \times 5 \times 5$	5^3	125
(f)	6	$6 \times 6 \times 6$	6^3	216
(g)	7	$7 \times 7 \times 7$	7^3	343

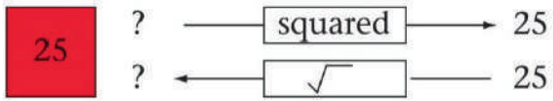
Answers

2. (a) $8 \times 27 = 216$ (b) $8 \times 125 = 1\,000$ (c) $8 \times 64 = 512$ (d) $1 \times 729 = 729$
3. (a) True (b) False. $2^3 \times 5^3 = 1\,000$ (which is 10^3)
   (c) True (d) False. $1^3 \times 9^3 = 9^3 = 729$ (and not 10^3 (1 000))

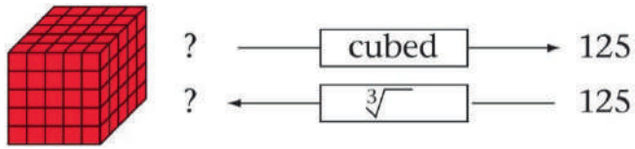
SQUARE AND CUBE ROOTS

Background information

The flow diagram below can be used to determine what number, when squared, equals 25. This resembles finding the side length of a square when its area is given. The answer is 5, because $5 \times 5 = 25$.



The following flow diagram can be used to determine what number, when cubed, equals 125. This resembles finding the side length of a cube when its volume is given. The answer is 5, because $5 \times 5 \times 5 = 125$.



Teaching guidelines

- Refer to the first flow diagram above. Reverse its direction and change its operation to $\sqrt{\ }$ (the symbol for square rooting). Use it to show that the question "What number, when squared, equals 25?" means the same as the question "What is the square root of 25?" This illustrates that square rooting is the inverse operation of squaring.
- Refer to the second flow diagram above. Reverse its direction and change its operation to $\sqrt[3]{\ }$ (the symbol for cube rooting). Use it to show that the question "What number, when cubed, equals 125?" means the same as the question "What is the cube root of 125?" This illustrates that cube rooting is the inverse operation of cubing.

Note on notation for roots

- The symbol for square roots is $\sqrt{\ }$, or $\sqrt[2]{\ }$.
- The symbols for other roots are $\sqrt[3]{\ }$, $\sqrt[4]{\ }$, $\sqrt[5]{\ }$, and so on.

Answers

1. See the table on LB page 35 alongside.

	Number	Cube the number	Exponential form	Cube
(h)	8	$8 \times 8 \times 8$	8^3	512
(i)	9	$9 \times 9 \times 9$	9^3	729
(j)	10	$10 \times 10 \times 10$	10^3	1 000

2. Calculate each of the following:
   (a) $2^3 \times 3^3$ (b) $2^3 \times 5^3$ (c) $2^3 \times 4^3$ (d) $1^3 \times 9^3$
3. Which of the following statements are true? If a statement is false, rewrite it as a true statement.
   (a) $2^3 \times 3^3 = 6^3$ (b) $2^3 \times 5^3 = 7^3$ (c) $2^3 \times 4^3 = 8^3$ (d) $1^3 \times 9^3 = 10^3$

SQUARE AND CUBE ROOTS

To find the square root of a number we ask the question:
Which number was multiplied by itself to get a square?

The square root of 16 is 4 because $4 \times 4 = 16$.

The question: **Which number was multiplied by itself to get 16?** is written mathematically as $\sqrt{16}$.

The answer to this question is written as $\sqrt{16} = 4$.

1. Copy and complete the following table:

	Number	Square of the number	Square root of the square of the number	Reason
(a)	1	1	1	$1 \times 1 = 1$
(b)	2	4	2	$2 \times 2 = 4$
(c)	3	9	3	$3 \times 3 = 9$
(d)	4	16	4	$4 \times 4 = 16$
(e)	5	25	5	$5 \times 5 = 25$
(f)	6	36	6	$6 \times 6 = 36$
(g)	7	49	7	$7 \times 7 = 49$
(h)	8	64	8	$8 \times 8 = 64$
(i)	9	81	9	$9 \times 9 = 81$
(j)	10	100	10	$10 \times 10 = 100$
(k)	11	121	11	$11 \times 11 = 121$
(l)	12	144	12	$12 \times 12 = 144$

Answers

2. (a) 12, because 12 × 12 = 144 (b) 10, because 10 × 10 = 100
   (c) 9, because 9 × 9 = 81 (d) 8, because 8 × 8 = 64
3. See the table on LB page 36 alongside.
4. (a) 6, because 6 × 6 × 6 = 216 (b) 2, because 2 × 2 × 2 = 8
   (c) 5, because 5 × 5 × 5 = 125 (d) 3, because 3 × 3 × 3 = 27
   (e) 4, because 4 × 4 × 4 = 64 (f) 10, because 10 × 10 × 10 = 1 000

3.2 Working with integers

REPRESENTING INTEGERS IN EXPONENTIAL FORM

Background information

Learners should be familiar with the following multiplication rules for integers:

- The product of two positive integers is a positive integer.
- The product of a positive integer and a negative integer is a negative integer.
- The product of a negative integer and a positive integer is a negative integer.
- The product of two negative integers is a positive integer.

Teaching guidelines

Revise the multiplication rules for integers.

Misconceptions

- Learners who do not properly read mathematical expressions which contain integers tend to mix up operations ("plus" and "minus") and signs of integers ("positive" and "negative").
- Learners should also be protected against the fallacy that subtraction of a negative number is the same thing as multiplying two negative numbers, for example 10 – (–4) = 10 + 4 because "subtracting an integer is the same as adding its additive inverse", **NOT** because (–1) × (–4) = 4.
- Avoid the idea that "a plus times a plus is a plus", "a plus times a minus is a minus", "a minus times a plus is a minus" and "a minus times a minus is a plus".

Answers

1. (a) –8 (b) 16 (c) 25 (d) –125 (e) 1 (f) –1

2. Calculate the following. Justify your answer.
   (a) $\sqrt{144}$ (b) $\sqrt{100}$ (c) $\sqrt{81}$ (d) $\sqrt{64}$

To find the cube root of a number we ask the question: Which number was multiplied by itself and again by itself to get a cube?

The cube root of 64 is 4 because 4 × 4 × 4 = 64.
The question: **Which number was multiplied by itself and again by itself (or cubed) to get 64?** is written mathematically as $\sqrt[3]{64}$. The answer to this question is written as $\sqrt[3]{64} = 4$.

3. Copy and complete the following table:

	Number	Cube of the number	Cube root of the cube of the number	Reason
(a)	1	1	1	1 × 1 × 1 = 1
(b)	2	8	2	2 × 2 × 2 = 8
(c)	3	27	3	3 × 3 × 3 = 27
(d)	4	64	4	4 × 4 × 4 = 64
(e)	5	125	5	5 × 5 × 5 = 125
(f)	6	216	6	6 × 6 × 6 = 216
(g)	7	343	7	7 × 7 × 7 = 343
(h)	8	512	8	8 × 8 × 8 = 512
(i)	9	729	9	9 × 9 × 9 = 729
(j)	10	1 000	10	10 × 10 × 10 = 1 000

4. Calculate the following and give reasons for your answers:
   (a) $\sqrt[3]{216}$ (b) $\sqrt[3]{8}$ (c) $\sqrt[3]{125}$
   (d) $\sqrt[3]{27}$ (e) $\sqrt[3]{64}$ (f) $\sqrt[3]{1\,000}$

3.2 Working with integers

REPRESENTING INTEGERS IN EXPONENTIAL FORM

1. Calculate the following, without using a calculator:
   (a) –2 × –2 × –2 (b) –2 × –2 × –2 × –2
   (c) –5 × –5 (d) –5 × –5 × –5
   (e) –1 × –1 × –1 × –1 (f) –1 × –1 × –1

Answers

2. (a) -4 (b) $-2 \times -2 = 4$
   (c) $-5 \times -5 \times -5 = -125$ (d) -125
3. (a) Learners will have different responses to this question.
   (b) Possible answers: For question 2(a) "The calculator calculated -2^2 as $-(2 \times 2) = -4$, which is correct, and not $-2 \times -2 = 4$, which is incorrect." (Some learners make this mistake.) Or: "When I calculated question 2(b), I didn't enter the brackets into the calculator correctly, so the calculator gave an incorrect answer."
4. (a) $(-2)^3$ (b) $(-2)^4$ (c) $(-5)^2$
   (d) $(-5)^3$ (e) $(-1)^4$ (f) $(-1)^3$
5. (a) $-3 \times -3 = 9$ (b) $-3 \times -3 \times -3 = -27$
   (c) $-2 \times -2 \times -2 \times -2 = 16$ (d) $-2 \times -2 \times -2 \times -2 \times -2 \times -2 = 64$
   (e) $-2 \times -2 \times -2 \times -2 \times -2 = -32$ (f) $-3 \times -3 \times -3 \times -3 = 81$
6. (a) Positive. The power is even. (b) Negative. The power is odd.
   (c) Positive. The power is even. (d) Negative. The power is odd.
7. (a) False. $(-3)^2 = -3 \times -3 = 9$ (b) False. $-3^2 = -9$ because $-3^2 = -(3^2)$
   (c) True. Both are equal to -25. (d) True. Both are equal to -1.
   (e) False. $-6 \times -6 \times -6 = -216$ (f) True. Both are equal to 64.

3.3 Laws of exponents

PRODUCT OF POWERS

Background information

The exponential law for the **product of powers** states the following: The product of powers with the same base is equal to the same base, raised to the sum of the exponents of the powers. For example:

- $2^3 \times 2^4 = 2^{3+4} = 2^7$
- $3^6 = 3^{2+4} = 3^2 \times 3^4$ (the law of the product of powers in reverse).

Teaching guidelines

Use expanded notation to find the product of two powers.

Answers

1. 2^{12}

2. Calculate each of the following:
   (a) -2^2 (b) $(-2)^2$
   (c) $(-5)^3$ (d) -5^3
3. Use your calculator to work out the answers to question 2.
   (a) Are your answers to question 2(a) and (b) different or the same as those of the calculator?
   (b) If your answers are different to those of the calculator, try to explain how the calculator did the calculations differently from you.

The calculator "understands" **-5^2** and **$(-5)^2$** as two different numbers.
It understands **-5^2** as **$-5 \times 5 = -25$** and **$(-5)^2$** as **$-5 \times -5 = 25$**.

4. Write the following in exponential form:
   (a) $-2 \times -2 \times -2$ (b) $-2 \times -2 \times -2 \times -2$
   (c) -5×-5 (d) $-5 \times -5 \times -5$
   (e) $-1 \times -1 \times -1 \times -1$ (f) $-1 \times -1 \times -1$
5. Calculate each of the following:
   (a) $(-3)^2$ (b) $(-3)^3$
   (c) $(-2)^4$ (d) $(-2)^6$
   (e) $(-2)^5$ (f) $(-3)^4$
6. Say whether the sign of the answer is negative or positive. Explain why.
   (a) $(-3)^6$ (b) $(-5)^{11}$
   (c) $(-4)^{20}$ (d) $(-7)^5$
7. Say whether the following statements are true or false. If a statement is false, rewrite it as a correct statement.
   (a) $(-3)^2 = -9$ (b) $-3^2 = 9$
   (c) $(-5^2) = -5^2$ (d) $(-1)^3 = -1^3$
   (e) $(-6)^3 = -18$ (f) $(-2)^6 = 2^6$

3.3 Laws of exponents

PRODUCT OF POWERS

1. A product of 2s is given below. Describe it using exponential notation, that is, write it as a power of 2.
   $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Answers

2. (a) $2^3 \times 2^9$
   (b) $2^5 \times 2^5 \times 2^2$
   (c) $2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2$
   (d) $2^3 \times 2^3 \times 2^3 \times 2^3$
   (e) $2^{10} \times 2^2$
3. See the answers on LB page 38 alongside.

Teaching guidelines (continued)

Discuss the concept of **repeated factor**.

Answers

4. See the table on LB page 38 alongside.

2. Express each of the following as a product of the powers of 2, as indicated by the brackets.
   (a) $(2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$
   (b) $(2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2)$
   (c) $(2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$
   (d) $(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
   (e) $(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2)$
3. Copy and complete the following statements so that they are true. You may want to refer to your answers to question 2(a) to (e) to help you.
   (a) $2^3 \times \ldots 2^9 \ldots = 2^{12}$
   (b) $2^5 \times \ldots 2^5 \ldots \times 2^2 = 2^{12}$
   (c) $2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 = \ldots 2^{12} \ldots$
   (d) $2^8 \times \ldots 2^4 \ldots = 2^{12}$
   (e) $2^3 \times 2^3 \times 2^3 \times \ldots 2^3 \ldots = 2^{12}$
   (f) $2^6 \times \ldots 2^6 \ldots = 2^{12}$
   (g) $2^2 \times 2^{10} = \ldots 2^{12} \ldots$

Suppose we are asked to simplify: $3^2 \times 3^4$.
The solution is: $3^2 \times 3^4 = 9 \times 81$
$= 729$
$= 3^6$

The base (3) is a repeated factor. The exponents (2 and 4) tell us the number of times each factor is repeated.

We can explain this solution in the following manner:
$3^2 \times 3^4 = \underbrace{3 \times 3}_{\text{2 factors}} \times \underbrace{3 \times 3 \times 3 \times 3}_{\text{4 factors}} = \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3}_{\text{6 factors}} = 3^6$

4. Copy and complete the following table:

	Product of powers	Repeated factor	Total number of times the factor is repeated	Simplified form
(a)	$2^7 \times 2^3$	2	10	2^{10}
(b)	$5^2 \times 5^4$	5	6	5^6
(c)	$4^1 \times 4^5$	4	6	4^6
(d)	$6^3 \times 6^2$	6	5	6^5
(e)	$2^8 \times 2^2$	2	10	2^{10}
(f)	$5^3 \times 5^3$	5	6	5^6
(g)	$4^2 \times 4^4$	4	6	4^6
(h)	$2^1 \times 2^9$	2	10	2^{10}

Notes on question 5

Determine whether learners understand the following:

- When you **multiply two or more powers that have the same base**, the answer has the same base, but its exponent is equal to the **sum** of the exponents of the powers you are multiplying.
- In **algebraic notation** it means that $a^m \times a^n = a^{m+n}$, where m and n are natural numbers and a is not zero.

Answers

5. (a) $2^{3+4} = 2^7$ (b) $10^{1+2+3} = 10^6$
   (c) $3^{2+3} = 3^5$ (d) $5^{3+2} = 5^5$
6. (a) 10^2 (b) 10^3
   (c) 10^4 (d) $10^{2+3+4} = 10^9$
   (e) $10^2 \times 10^3 \times 10^4 = 10^9$ (f) 10^9
7. (a) x^9 (b) $x^2 \times x^3 \times x^4 = x^{2+3+4} = x^9$
   (c) $x^4 \times x^2 \times x^2 \times x = x^{4+2+2+1} = x^9$ (d) $x^6 \times x^3 = x^{6+3} = x^9$
   (e) $x^3 \times y^3$ (f) $a^2 \times b^2$
8. See the first table on LB page 39 alongside.

RAISING A POWER TO A POWER

Background information

The exponential law for a **power of a power** states the following: A power raised to a power is equal to the same base, raised to the product of the exponents.

- $(2^3)^4 = 2^{3\times4} = 2^{12}$
- $3^8 = 3^{2\times4} = (3^2)^4$ (the law for a power of a power in reverse)

Teaching guidelines

Use expanded notation to raise a power to a power.

Answers

1. See the last table on LB page 39 alongside.

When you multiply two or more powers that have the same base, the answer has the same base, but its exponent is equal to the sum of the exponents of the numbers you are multiplying.

We can express this symbolically as $a^m \times a^n = a^{m+n}$, where m and n are natural numbers and a is not zero.

5. What is wrong with these statements? Correct each one.
   (a) $2^3 \times 2^4 = 2^{12}$ (b) $10 \times 10^2 \times 10^3 = 10^{1\times2\times3} = 10^6$
   (c) $3^2 \times 3^3 = 3^6$ (d) $5^3 \times 5^2 = 15 \times 10$
6. Express each of the following numbers as a single power of 10.
   Example: 1 000 000 as a power of 10 is 10^6.
   (a) 100 (b) 1 000 (c) 10 000
   (d) $10^2 \times 10^3 \times 10^4$ (e) $100 \times 1\,000 \times 10\,000$ (f) 1 000 000 000
7. Write each of the following products in exponential form:
   (a) $x \times x \times x \times x \times x \times x \times x \times x \times x \times x$
   (b) $(x \times x) \times (x \times x \times x) \times (x \times x \times x \times x)$
   (c) $(x \times x \times x \times x \times x) \times (x \times x) \times (x \times x) \times x$
   (d) $(x \times x \times x \times x \times x \times x \times x \times x) \times (x \times x \times x)$
   (e) $(x \times x \times x) \times (y \times y \times y)$
   (f) $(a \times a) \times (b \times b)$
8. Copy and complete the following table:

	Product of powers	Repeated factor	Total number of times the factor is repeated	Simplified form
(a)	$x^7 \times x^3$	x	10	x^{10}
(b)	$x^2 \times x^4$	x	6	x^6
(c)	$x^1 \times x^5$	x	6	x^6
(d)	$x^3 \times x^2$	x	5	x^5
(e)	$x^8 \times x^2$	x	10	x^{10}
(f)	$x^3 \times x^3$	x	6	x^6
(g)	$x^1 \times x^9$	x	10	x^{10}

RAISING A POWER TO A POWER

1. Copy and complete the table of powers of 2:

x	1	2	3	4	5	6	7	8	9	10	11
2^x	2	4	8	16	32	64	128	256	512	1 024	2 048
	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}

Answers

1. See the first table on LB page 40 alongside.
2. See the tables on LB page 40 alongside.
3. See the table on LB page 40 alongside.
4. See the answers on LB page 40 alongside.
5. (a) Yes (b) Yes
   (c) No (d) Yes
   (e) Yes (f) No
   (g) Yes (h) No
6. (a) $8 = 2^3$. So $8^5 = 2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{3+3+3+3+3} = 2^{3\times5} = 2^{15}$
   (b) Yes
   (c) $2^{3\times5}$

x	12	13	14	15	16	17	18
2^x	4 096	8 192	16 384	32 768	65 536	131 072	262 144
	2^{12}	2^{13}	2^{14}	2^{15}	2^{16}	2^{17}	2^{18}

2. Copy and complete the table of powers of 3:

x	1	2	3	4	5	6	7	8	9
3^x	3	9	27	81	243	729	2 187	6 561	19 683
	3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8	3^9

x	10	11	12	13	14
3^x	59 049	177 147	531 441	1 594 323	4 782 969
	3^{10}	3^{11}	3^{12}	3^{13}	3^{14}

3. Copy and complete the table. You can read the values from the tables you made in questions 1 and 2.

Product of powers	Repeated factor	Power of power notation	Total number of repetitions	Simplified form	Value
$2^4 \times 2^4 \times 2^4$	2	$(2^4)^3$	12	2^{12}	4 096
$3^2 \times 3^2 \times 3^2 \times 3^2$	3	$(3^2)^4$	8	3^8	6 561
$2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3$	2	$(2^3)^5$	15	2^{15}	32 768
$3^4 \times 3^4 \times 3^4$	3	$(3^4)^3$	12	3^{12}	531 441
$2^6 \times 2^6 \times 2^6$	2	$(2^6)^3$	18	2^{18}	262 144

4. Copy and complete by using your table of powers of 2 to find the answers for the following:
   (a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = \ldots 2^{12} \ldots = \ldots 4\,096 \ldots$
   (b) $(2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = \ldots 2^{12} \ldots = \ldots 4\,096 \ldots$
   (c) $16^3 = \ldots (2^4)^3 \ldots = \ldots 2^{12} \ldots = \ldots 4\,096 \ldots$
5. Use your table of powers of 2 to find the answers for the following:
   (a) Is $16^3 = 2^{12}$? (b) Is $2^4 \times 2^4 \times 2^4 = 2^{12}$?
   (c) Is $2^4 \times 2^3 = 2^{12}$? (d) Is $(2^4)^3 = 2^4 \times 2^4 \times 2^4$?
   (e) Is $(2^4)^3 = 2^{12}$? (f) Is $(2^4)^3 = 2^{4+3}$?
   (g) Is $(2^4)^3 = 2^{4\times3}$? (h) Is $(2^2)^5 = 2^{2+5}$?
6. (a) Express 8^5 as a power of 2. It may help to first express 8 as a power of 2.
   (b) Can $(2^3) \times (2^3) \times (2^3) \times (2^3) \times (2^3)$ be expressed as $(2^3)^5$?
   (c) Is $(2^3)^5 = 2^{3+5}$ or is $(2^3)^5 = 2^{3\times5}$?

Notes on question 9

Determine whether learners understand the following:

- When you **raise a power to a power** the answer has the same base, but its exponent is equal to the **product** of the two exponents.
- In **algebraic notation** it means that $(a^m)^n = a^{m \times n}$, where m and n are natural numbers and a is not zero.

Answers

7. (a) Learners may argue that $4 \times 4 \times 4 = 64$ and $64 = 2^6$.
   (b) $4 \times 4 \times 4 = 64$ and $64 = 2^6$. Learners may also do it differently.
   (c) Yes
   (d) $2^{2 \times 3}$
8. (a) $3^3 \times 3^3 = 3^{3+3} = 3^6$
   (b) $4^3 \times 4^3 = 4^{3+3} = 4^6$
   (c) $2^4 \times 2^4 = 2^{4+4} = 2^8$
   (d) $9^2 \times 9^2 = 9^{2+2} = 9^4$
   (e) $3^3 \times 3^3 \times 3^3 = 3^{3+3+3} = 3^9$
   (f) $4^3 \times 4^3 \times 4^3 = 4^{3+3+3} = 4^9$
   (g) $5^4 \times 5^4 \times 5^4 = 5^{4+4+4} = 5^{12}$
   (h) $9^2 \times 9^2 \times 9^2 = 9^{2+2+2} = 9^6$
9. (a) 5^{40}
   (b) 10^{20}
   (c) 6^{16}
   (d) 5^{40}
10. For example, $(5^4)^3$; $(5^3)^4$; $(5^2)^6$; $(5^6)^2$
11. See the table on LB page 41 alongside.

7. (a) Express 4^3 as a power of 2.
   (b) Calculate $2^2 \times 2^2 \times 2^2$ and express your answer as a single power of 2.
   (c) Can $(2^2) \times (2^2) \times (2^2)$ be expressed as $(2^2)^3$?
   (d) Is $(2^2)^3 = 2^{2+3}$ or is $(2^2)^3 = 2^{2 \times 3}$?
8. Simplify the following:
   Example: $(10^2)^2 = 10^2 \times 10^2 = 10^{2+2} = 10^4 = 10\,000$
   (a) $(3^3)^2$ (b) $(4^3)^2$ (c) $(2^4)^2$ (d) $(9^2)^2$
   (e) $(3^3)^3$ (f) $(4^3)^3$ (g) $(5^4)^3$ (h) $(9^2)^3$

$(a^m)^n = a^{m \times n}$, where m and n are natural numbers and a is not equal to zero.

9. Simplify:
   (a) $(5^4)^{10}$ (b) $(10^4)^5$ (c) $(6^4)^4$ (d) $(5^4)^{10}$
10. Write 5^{12} as a power of powers of 5 in two different ways.

To simplify $(x^2)^5$ we can write it out as a product of powers or we can use a shortcut.

$(x^2)^5 = x^2 \times x^2 \times x^2 \times x^2 \times x^2$

$= \underbrace{x \times x} \times \underbrace{x \times x} \times \underbrace{x \times x} \times \underbrace{x \times x} \times \underbrace{x \times x} = x^{10}$

2×5 factors = 10 factors

11. Copy and complete the following table:

	Expression	Write as a product of the powers and then simplify	Use the rule $(a^m)^n$ to simplify
(a)	$(a^4)^5$	$a^4 \times a^4 \times a^4 \times a^4 \times a^4$ $= a^{4+4+4+4+4} = a^{20}$	$(a^4)^5 = a^{4 \times 5} = a^{20}$
(b)	$(b^{10})^5$	$b^{10} \times b^{10} \times b^{10} \times b^{10} \times b^{10}$ $= b^{10+10+10+10+10}$ $= b^{50}$	$(b^{10})^5 = b^{10 \times 5}$ $= b^{50}$
(c)	$(x^7)^3$	$x^7 \times x^7 \times x^7$ $= x^{7+7+7}$ $= x^{21}$	$(x^7)^3 = x^{7 \times 3}$ $= x^{21}$
(d)	$(s^6)^4$	$s^6 \times s^6 \times s^6 \times s^6$ $= s^{6+6+6+6}$ $= s^{24}$	$(s^6)^4 = s^{6 \times 4}$ $= s^{24}$
(e)	$(y^3)^7$	$y^3 \times y^3 \times y^3 \times y^3 \times y^3 \times y^3 \times y^3$ $= y^{3+3+3+3+3+3+3}$ $= y^{21}$	$y^{3 \times 7} = y^{21}$

POWER OF A PRODUCT

Background information

The exponential law for the **power of a product** states the following: A power of a product is equal to the product of the factors, each raised to the same power.

- $(2 \times 3)^4 = 2^4 \times 3^4$
- $3^2 \times 4^2 = (3 \times 4)^2$ (the law for a power of a product in reverse)

Teaching guidelines

Use expanded notation to find a power of a product.

Misconceptions

Learners sometimes form misconceptions about exponents that can inhibit their progress in Mathematics considerably. The teacher should especially be on the lookout for the following misconception:

- Multiplying bases and adding/multiplying exponents, for example, $3^4 5^2 = 15^6$ or $3^4 3^2 = 9^6$ or 9^8. Learners who initially learnt about the exponential notation without understanding what it represents may be especially prone to this error. Too much emphasis is often placed, when the exponential notation is introduced, on the terminology **base**, **power** and **exponent** while the meaning, a short way of indicating repeated multiplication, is not prioritised.

Answers

1. See the table on LB page 42 alongside.
2. (a) True. $36 = 4 \times 9$
   (b) True. $216 = 8 \times 27$
   (c) True. $7\,776 = 32 \times 243$
   (d) False. $2^4 \times 3^4 = 6^4$ not 6^8
3. See the table on LB page 42 alongside.

POWER OF A PRODUCT

1. Copy and complete the table. You may use your calculator when you are not sure of a value.

	x	1	2	3	4	5
(a)	2^x	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
(b)	3^x	$3^1 = 3$	$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$
(c)	6^x	$6^1 = 6$	$6^2 = 36$	$6^3 = 216$	$6^4 = 1\,296$	$6^5 = 7\,776$

2. Use the table in question 1 to answer the questions below. Are these statements true or false? If a statement is false, rewrite it as a correct statement.
   (a) $6^2 = 2^2 \times 3^2$ (b) $6^3 = 2^3 \times 3^3$
   (c) $6^5 = 2^5 \times 3^5$ (d) $6^8 = 2^4 \times 3^4$

3. Copy and complete the following table:

	Expression	The bases of the expression are factors of ...	Equivalent expression
(a)	$2^6 \times 5^6$	10	10^6
(b)	$3^2 \times 4^2$	12	12^2
(c)	$4^2 \times 2^2$	8	8^2
(d)	$7^5 \times 8^5$	56	56^5
(e)	$2^3 \times 15^3$	30	30^3
(f)	$3^5 \times x^5$	$3x$	$(3x)^5$
(g)	$7^2 \times z^2$	$7z$	$(7z)^2$
(h)	$4^3 \times y^3$	$4y$	$(4y)^3$
(i)	$2^6 \times m^6$	$2m$	$(2m)^6$
(j)	$2^3 \times m^3$	$2m$	$(2m)^3$
(k)	$2^{10} \times y^{10}$	$2y$	$(2y)^{10}$

12^2 can be written in terms of its factors as $(2 \times 6)^2$ or as $(3 \times 4)^2$.
We already know that $12^2 = 144$.
What this tells us is that both $(2 \times 6)^2$ and $(3 \times 4)^2$ also equal to 144.

Notes on question 4

Determine whether learners understand the following:

- When you **raise a product to a power**, the answer is the **product** of the factors, each raised to the same power.
- In **algebraic notation** it means that $(a \times b)^m = a^m \times b^m$, where m is a natural number and a and b are not zero.

Answers

4. (a) $(3 \times 5)^2 = 15^2$ (b) $(5 \times 2)^3 = 10^3$ (c) $(7 \times 4)^4 = 28^4$
   (d) $(2 \times 6)^3 = 12^3$ (e) $(4 \times 2)^4 = 8^4$ (f) $(5 \times 7)^2 = 35^2$
5. (a) $(2 \times 3)^3 = 2^3 \times 3^3$ (b) $(3 \times 5)^2 = 3^2 \times 5^2$ (c) $(3 \times 7)^4 = 3^4 \times 7^4$
   (d) $(2 \times 3)^5 = 2^5 \times 3^5$ (e) $(2 \times 9)^2 = 2^2 \times 9^2$ or $(3 \times 6)^2 = 3^2 \times 6^2$ (f) s^7t^7
   (g) a^3b^3 (h) $2^2x^2 = 4x^2$ (i) $3^5y^5 = 243y^5$
   (j) $3^2c^2 = 9c^2$ (k) g^4h^4 (l) $4^3x^3 = 64x^3$
6. (a) $243b^5$ (b) $64y^6$ (c) x^2y^2
   (d) $10\,000x^4$ (e) $27x^3$ (f) $25t^2$
   (g) $216m^7$ (h) $144a^2$ (i) n^3p^9

A QUOTIENT OF POWERS

Background information

The exponential law for the **quotient of powers** states the following:

The quotient of powers with the same base is equal to the same base, raised to the difference between the exponents of the powers.

For example:

- $2^6 \div 2^4 = 2^{6-4} = 2^2$
- $3^4 = 3^{9-5} = 3^9 \div 3^5$ (the law for the quotient of powers in reverse)

Teaching guidelines

Use expanded notation to find the quotient of two powers.

Answers

1. (a) 81 (b) 32 (c) 15 625
2. (a) $3^6 \div 3^3 = 729 \div 27 = 27$
   (b) $3^{6-3} = 3^3 = 27$
   (c) Yes, they are both equal to 27.

We write $12^2 = (2 \times 6)^2 = 2^2 \times 6^2 = 4 \times 36 = 144$ or $12^2 = (3 \times 4)^2 = 3^2 \times 4^2 = 9 \times 16 = 144$

A product raised to a power is the product of the factors each raised to the same power.

Using symbols, we write $(a \times b)^m = a^m \times b^m$, where m is a natural number and a and b are not equal to zero.

4. Write each of the following expressions as an expression with one base.
   Example: $3^{10} \times 2^{10} = (3 \times 2)^{10} = 6^{10}$
   (a) $3^2 \times 5^2$ (b) $5^3 \times 2^3$ (c) $7^4 \times 4^4$
   (d) $2^3 \times 6^3$ (e) $4^4 \times 2^4$ (f) $5^2 \times 7^2$
5. Write the following as a product of powers.
   Example: $(3x)^3 = 3^3 \times x^3 = 27x^3$
   (a) 6^3 (b) 15^2 (c) 21^4 (d) 6^5
   (e) 18^2 (f) $(st)^7$ (g) $(ab)^3$ (h) $(2x)^2$
   (i) $(3y)^5$ (j) $(3c)^2$ (k) $(gh)^4$ (l) $(4x)^3$
6. Simplify the following expressions.
   Example: $3^2 \times m^2 = 9 \times m^2 = 9m^2$
   (a) $3^5 \times b^5$ (b) $2^6 \times y^6$ (c) $x^2 \times y^2$
   (d) $10^4 \times x^4$ (e) $3^3 \times x^3$ (f) $5^2 \times t^2$
   (g) $6^3 \times m^7$ (h) $12^2 \times a^2$ (i) $n^3 \times p^9$

A QUOTIENT OF POWERS

Consider the following table:

x	1	2	3	4	5	6
2^x	2	4	8	16	32	64
3^x	3	9	27	81	243	729
5^x	5	25	125	625	3 125	15 625

Answer questions 1 to 4 by referring to the table when needed.

1. Give the value of each of the following:
   (a) 3^4 (b) 2^5 (c) 5^6
2. (a) Calculate $3^6 \div 3^3$. (Read the values of 3^6 and 3^3 from the table and then divide. You may use a calculator where necessary.)
   (b) Calculate 3^{6-3}.
   (c) Is $3^6 \div 3^3$ equal to 3^3? Explain.

Notes on question 5

Determine whether learners understand the following:

- When you **divide two powers that have the same base**, the answer has the same base, but its exponent is equal to the **difference** between the exponents of the powers you are multiplying.
- In **algebraic notation** it means that $a^m \div a^n = a^{m-n}$, where m and n are natural numbers and a is not zero (division by 0 is undefined).

Answers

3. (a) $2^{6-2} = 2^4 = 16$ (b) $2^6 \div 2^2 = 64 \div 4 = 16$ (c) $2^{6\div 2} = 2^3 = 8$
   (d) 8 (e) 16
   (f) Statement A is true because $2^6 \div 2^2 = 64 \div 4 = 16 = 2^4$.
   Statement B is false because $2^6 \div 2^2 = 64 \div 4 = 16$ but $2^3 = 8$.
4. (a) False. $5^6 \div 5^4 = 15\,625 \div 625 = 25$; $5^{6-4} = 5^2$
   (b) True. $3^{4-1} = 3^3 = 27$; $3^4 \div 3 = 81 \div 3 = 27$
   (c) True (d) True
5. (a) $2^{12-10} = 2^2 = 4$ (b) $6^{17-14} = 6^3 = 216$
   (c) $10^{20-14} = 10^6 = 1\,000\,000$ (d) $5^{11-8} = 5^3 = 125$
6. (a) $x^{12-10} = x^2$ (b) $y^{17-14} = y^3$
   (c) $t^{20-14} = t^6$ (d) $n^{11-8} = n^3$

THE POWER OF ZERO

Background information

The exponential rule for a power of zero states the following: Any number raised to the power of zero is always equal to 1.

- $2^6 \div 2^6 = 2^0 = 1$ because 2^6 divided by 2^6 is equal to 1.
- $1 = 4^0 = 4^5 \div 4^5$ (the rule for a power of zero in reverse)

Teaching guidelines

Discuss why any number raised to the power of zero is equal to 1.

Answers

1. (a) $2^{12-12} = 2^0$ (b) $6^{17-17} = 6^0$ (c) $6^{14-14} = 6^0$ (d) $2^{10-10} = 2^0$
2. (a) 1 (b) 1 (c) 1 (d) 1

3. (a) Calculate the value of 2^{6-2}.
   (b) Calculate the value of $2^6 \div 2^2$.
   (c) Calculate the value of $2^{6\div 2}$.
   (d) Read the value of 2^3 from the table.
   (e) Read the value of 2^4 from the table.
   (f) Which of the statements below is true? Give an explanation for your answer.

   A. $2^6 \div 2^2 = 2^{6-2} = 2^4$ B. $2^6 \div 2^2 = 2^{6\div 2} = 2^3$

   To calculate 4^{5-3}, we first do the calculation in the exponent, that is, we subtract 3 from 5. Then we can calculate 4^2 as $4 \times 4 = 16$.

4. Say which of the statements below are true and which are false. Rewrite false statements as correct statements.
   (a) $5^6 \div 5^4 = 5^{6\div 4}$ (b) $3^{4-1} = 3^4 \div 3$
   (c) $5^6 \div 5 = 5^{6-1}$ (d) $2^5 \div 2^3 = 2^2$

$a^m \div a^n = a^{m-n}$
Here, m and n are natural numbers, and m is a number greater than n and a is not zero.

5. Simplify the following. Do not use a calculator.
   Example: $3^{17} \div 3^{12} = 3^{17-12} = 3^5 = 243$
   (a) $2^{12} \div 2^{10}$ (b) $6^{17} \div 6^{14}$ (c) $10^{20} \div 10^{14}$ (d) $5^{11} \div 5^8$
6. Simplify the following:
   (a) $x^{12} \div x^{10}$ (b) $y^{17} \div y^{14}$ (c) $t^{20} \div t^{14}$ (d) $n^{11} \div n^8$

THE POWER OF ZERO

1. Simplify the following:
   (a) $2^{12} \div 2^{12}$ (b) $6^{17} \div 6^{17}$ (c) $6^{14} \div 6^{14}$ (d) $2^{10} \div 2^{10}$

We define $a^0 = 1$.
Any number raised to the power of zero is always equal to 1.

2. Simplify the following:
   (a) 100^0 (b) x^0 (c) $(100x)^0$ (d) $(5x^3)^0$

3.4 Calculations

MIXED OPERATIONS

Simplify the following:

1. $3^3 + \sqrt[3]{-27} \times 2$
2. $5 \times (2+3)^2 + (-1)^0$

3.4 Calculations

MIXED OPERATIONS

Teaching guidelines

Revise the difference between operations ("plus" and "minus") and signs of integers ("positive" and "negative").

Misconceptions

Be on the lookout for the misconception that "exponents distribute", for example, that $(2 + 3)^2 = 2^2 + 3^2$. To help learners who make this mistake, ask them to calculate 5^2 as well as $2^2 + 3^2$.

Answers

1. $27 + (-3) \times 2 = 27 - 6 = 21$
2. $5 \times 5^2 + 1 = 125 + 1 = 126$
3. $9 \times 8 + 5 \times 10 = 72 + 50 = 122$
4. $\frac{10}{10} + 3^2 = 1 + 9 = 10$
5. $4 \times 4 + 6 + 9 \times 10 = 16 + 6 + 90 = 112$
6. $64 \div 8 + 12 = 8 + 12 = 20$

3.5 Squares, cubes and roots of rational numbers

SQUARING A FRACTION

Background information

The **square of a fraction** is equal to the square of the numerator divided by the square of the denominator.

Teaching guidelines

Use expanded notation to find the square of a fraction.

Answers

1. See the table on LB page 45 alongside.
2. (a) $\frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$ (b) $\frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$ (c) $\frac{7}{8} \times \frac{7}{8} = \frac{49}{64}$
3. (a) $(0{,}6)^2 = (\frac{6}{10})^2 = \frac{36}{100} = 0{,}36$ (b) $(0{,}8)^2 = (\frac{8}{10})^2 = \frac{64}{100} = 0{,}64$

3. $3^2 \times 2^3 + 5 \times \sqrt{100}$
4. $\frac{\sqrt[3]{1\,000}}{\sqrt{100}} + (4 - 1)^2$
5. $\sqrt{16} \times \sqrt{16} + \sqrt[3]{216} + 3^2 \times 10$
6. $4^3 \div 2^3 + \sqrt{144}$

3.5 Squares, cubes and roots of rational numbers

SQUARING A FRACTION

Squaring or cubing a fraction or a decimal fraction is no different from squaring or cubing an integer.

1. Copy and complete the following table:

	Fraction	Square the fraction	Value of the square of the fraction
(a)	$\frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
(b)	$\frac{2}{3}$	$\frac{2}{3} \times \frac{2}{3}$	$\frac{4}{9}$
(c)	$\frac{3}{4}$	$\frac{3}{4} \times \frac{3}{4}$	$\frac{9}{16}$
(d)	$\frac{2}{5}$	$\frac{2}{5} \times \frac{2}{5}$	$\frac{4}{25}$
(e)	$\frac{3}{5}$	$\frac{3}{5} \times \frac{3}{5}$	$\frac{9}{25}$
(f)	$\frac{2}{6}$	$\frac{2}{6} \times \frac{2}{6}$	$\frac{4}{36}$
(g)	$\frac{3}{7}$	$\frac{3}{7} \times \frac{3}{7}$	$\frac{9}{49}$
(h)	$\frac{11}{12}$	$\frac{11}{12} \times \frac{11}{12}$	$\frac{121}{144}$

2. Calculate each of the following:
   (a) $\left(\frac{3}{2}\right)^2$ (b) $\left(\frac{4}{5}\right)^2$ (c) $\left(\frac{7}{8}\right)^2$
3. (a) Use the fact that 0,6 can be written as $\frac{6}{10}$ to calculate $(0{,}6)^2$.
   (b) Use the fact that 0,8 can be written as $\frac{8}{10}$ to calculate $(0{,}8)^2$.

FINDING THE SQUARE ROOT OF A FRACTION

Background information

The **square root of a common fraction** can be calculated by the square root of the numerator divided by the square root of the denominator.

Teaching guidelines

Use products of factors to find the square root of a fraction.

Answers

1. See the table on LB page 46 alongside.
2. (a) $\frac{5}{4}$ (b) $\frac{9}{12}$ (c) $\frac{20}{30}$ (d) $\frac{6}{9}$
3. (a) $\sqrt{0{,}01} = \sqrt{\frac{1}{100}} = \frac{1}{10} = 0{,}1$ (b) $\sqrt{0{,}49} = \sqrt{\frac{49}{100}} = \frac{7}{10} = 0{,}7$
4. (a) $\sqrt{\frac{9}{100}} = \frac{3}{10} = 0{,}3$ (b) $\sqrt{\frac{64}{100}} = \frac{8}{10} = 0{,}8$ (c) $\sqrt{\frac{144}{100}} = \frac{12}{10} = 1{,}2$

CUBING A FRACTION

Background information

The **cube of a common fraction** is the cube of the numerator divided by the cube of the denominator.

Teaching guidelines

Use expanded notation to find the cube of a fraction.

Answers

1. (a) $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$ (b) $\frac{5}{10} \times \frac{5}{10} \times \frac{5}{10} = \frac{125}{1\,000}$
   (c) $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$ (d) $\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{64}{125}$
2. (a) $(0{,}6)^3 = (\frac{6}{10})^3 = \frac{216}{1\,000} = 0{,}216$
   (b) $(0{,}8)^3 = (\frac{8}{10})^3 = \frac{512}{1\,000} = 0{,}512$
   (c) $(0{,}7)^3 = (\frac{7}{10})^3 = \frac{343}{1\,000} = 0{,}343$

FINDING THE SQUARE ROOT OF A FRACTION

1. Copy and complete the following table:

	Fraction	Writing the fraction as a product of factors	Square root
(a)	$\frac{81}{121}$	$\frac{9}{11} \times \frac{9}{11}$	$\sqrt{\frac{81}{121}} = \frac{9}{11}$
(b)	$\frac{64}{81}$	$\frac{8}{9} \times \frac{8}{9}$	$\sqrt{\frac{64}{81}} = \frac{8}{9}$
(c)	$\frac{49}{169}$	$\frac{7}{13} \times \frac{7}{13}$	$\sqrt{\frac{49}{169}} = \frac{7}{13}$
(d)	$\frac{100}{225}$	$\frac{10}{15} \times \frac{10}{15}$	$\sqrt{\frac{100}{225}} = \frac{10}{15}$

2. Determine the following:
   (a) $\sqrt{\frac{25}{16}}$ (b) $\sqrt{\frac{81}{144}}$ (c) $\sqrt{\frac{400}{900}}$ (d) $\sqrt{\frac{36}{81}}$
3. (a) Use the fact that 0,01 can be written as $\frac{1}{100}$ to calculate $\sqrt{0{,}01}$.
   (b) Use the fact that 0,49 can be written as $\frac{49}{100}$ to calculate $\sqrt{0{,}49}$.
4. Calculate each of the following:
   (a) $\sqrt{0{,}09}$ (b) $\sqrt{0{,}64}$ (c) $\sqrt{1{,}44}$

CUBING A FRACTION

One half cubed is equal to one eighth.

We write this as $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

1. Calculate each of the following:
   (a) $\left(\frac{2}{3}\right)^3$ (b) $\left(\frac{5}{10}\right)^3$ (c) $\left(\frac{5}{6}\right)^3$ (d) $\left(\frac{4}{5}\right)^3$
2. (a) Use the fact that 0,6 can be written as $\frac{6}{10}$ to find $(0{,}6)^3$.
   (b) Use the fact that 0,8 can be written as $\frac{8}{10}$ to calculate $(0{,}8)^3$.
   (c) Use the fact that 0,7 can be written as $\frac{7}{10}$ to calculate $(0{,}7)^3$.

3.6 Scientific notation

VERY LARGE NUMBERS

Background information

In **scientific notation** a number is expressed as a product of:

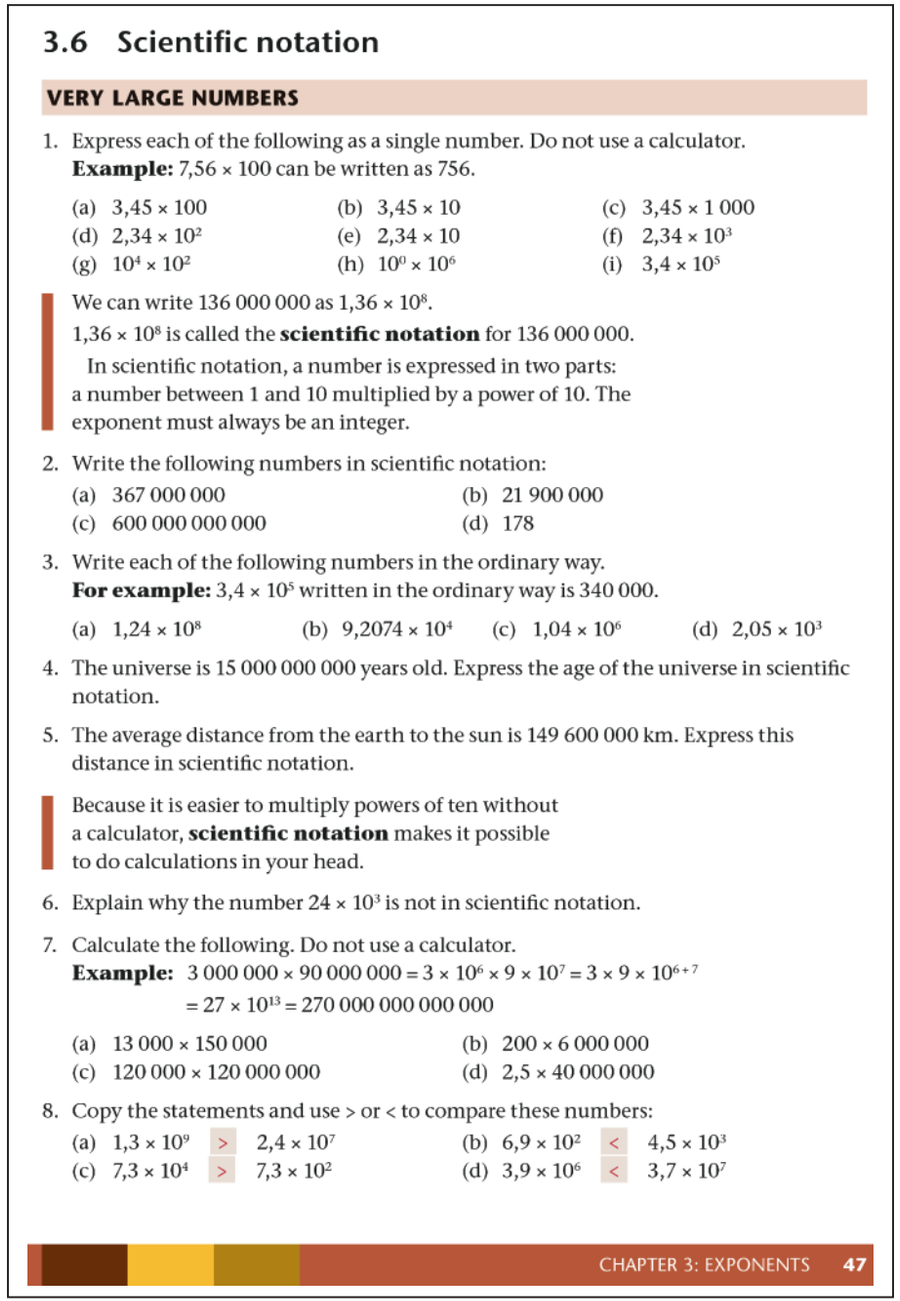
- a number between 1 and 10, and
- a power of 10 with an exponent that is an integer.

Teaching guidelines

- Write an ordinary number in scientific notation:
  $345 = 3{,}45 \times 100 = 3{,}45 \times 10^2$.
- Write a number in scientific notation as an ordinary number:
  $7{,}65 \times 10^3 = 7{,}65 \times 1\,000 = 7\,650$.

Answers

1. (a) 345 (b) 34,5 (c) 3 450
   (d) 234 (e) 23,4 (f) 2 340
   (g) $10^6 = 1\,000\,000$ (h) $1 \times 1\,000\,000 = 1\,000\,000$ (i) 340 000
2. (a) $3{,}67 \times 10^8$ (b) $2{,}19 \times 10^7$ (c) 6×10^{11} (d) $1{,}78 \times 10^2$
3. (a) 124 000 000 (b) 92 074 (c) 1 040 000 (d) 2 050
4. $1{,}5 \times 10^{10}$ years
5. $1{,}496 \times 10^8$ km
6. It is not in scientific notation because 24 is not a number between 1 and 10.
7. (a) $13 \times 10^3 \times 15 \times 10^4 = 13 \times 15 \times 10^3 \times 10^4 = 13 \times 15 \times 10^{3+4} = 195 \times 10^7$
   $= 1\,950\,000\,000$
   (b) $2 \times 10^2 \times 6 \times 10^6 = 2 \times 6 \times 10^2 \times 10^6 = 2 \times 6 \times 10^{2+6} = 12 \times 10^8$
   $= 1\,200\,000\,000$
   (c) $12 \times 10^4 \times 12 \times 10^7 = 12 \times 12 \times 10^4 \times 10^7 = 144 \times 10^{4+7} = 144 \times 10^{11}$
   $= 14\,400\,000\,000\,000$
   (d) $2{,}5 \times 4 \times 10^7 = 10 \times 10^7 = 10^8 = 100\,000\,000$
8. See the answers on LB page 47 alongside.



3.6 Scientific notation

VERY LARGE NUMBERS

1. Express each of the following as a single number. Do not use a calculator.
   Example: 7,56 × 100 can be written as 756.

   (a) $3{,}45 \times 100$ (b) $3{,}45 \times 10$ (c) $3{,}45 \times 1\,000$
   (d) $2{,}34 \times 10^2$ (e) $2{,}34 \times 10$ (f) $2{,}34 \times 10^3$
   (g) $10^4 \times 10^2$ (h) $10^0 \times 10^6$ (i) $3{,}4 \times 10^5$

We can write 136 000 000 as $1{,}36 \times 10^8$.
$1{,}36 \times 10^8$ is called the **scientific notation** for 136 000 000.
In scientific notation, a number is expressed in two parts:
a number between 1 and 10 multiplied by a power of 10. The exponent must always be an integer.

2. Write the following numbers in scientific notation:
   (a) 367 000 000 (b) 21 900 000
   (c) 600 000 000 000 (d) 178
3. Write each of the following numbers in the ordinary way.
   For example: $3{,}4 \times 10^5$ written in the ordinary way is 340 000.

   (a) $1{,}24 \times 10^8$ (b) $9{,}2074 \times 10^4$ (c) $1{,}04 \times 10^6$ (d) $2{,}05 \times 10^3$
4. The universe is 15 000 000 000 years old. Express the age of the universe in scientific notation.
5. The average distance from the earth to the sun is 149 600 000 km. Express this distance in scientific notation.

Because it is easier to multiply powers of ten without a calculator, **scientific notation** makes it possible to do calculations in your head.

6. Explain why the number 24×10^3 is not in scientific notation.
7. Calculate the following. Do not use a calculator.
   Example: $3\,000\,000 \times 90\,000\,000 = 3 \times 10^6 \times 9 \times 10^7 = 3 \times 9 \times 10^{6+7}$
   $= 27 \times 10^{13} = 270\,000\,000\,000\,000$

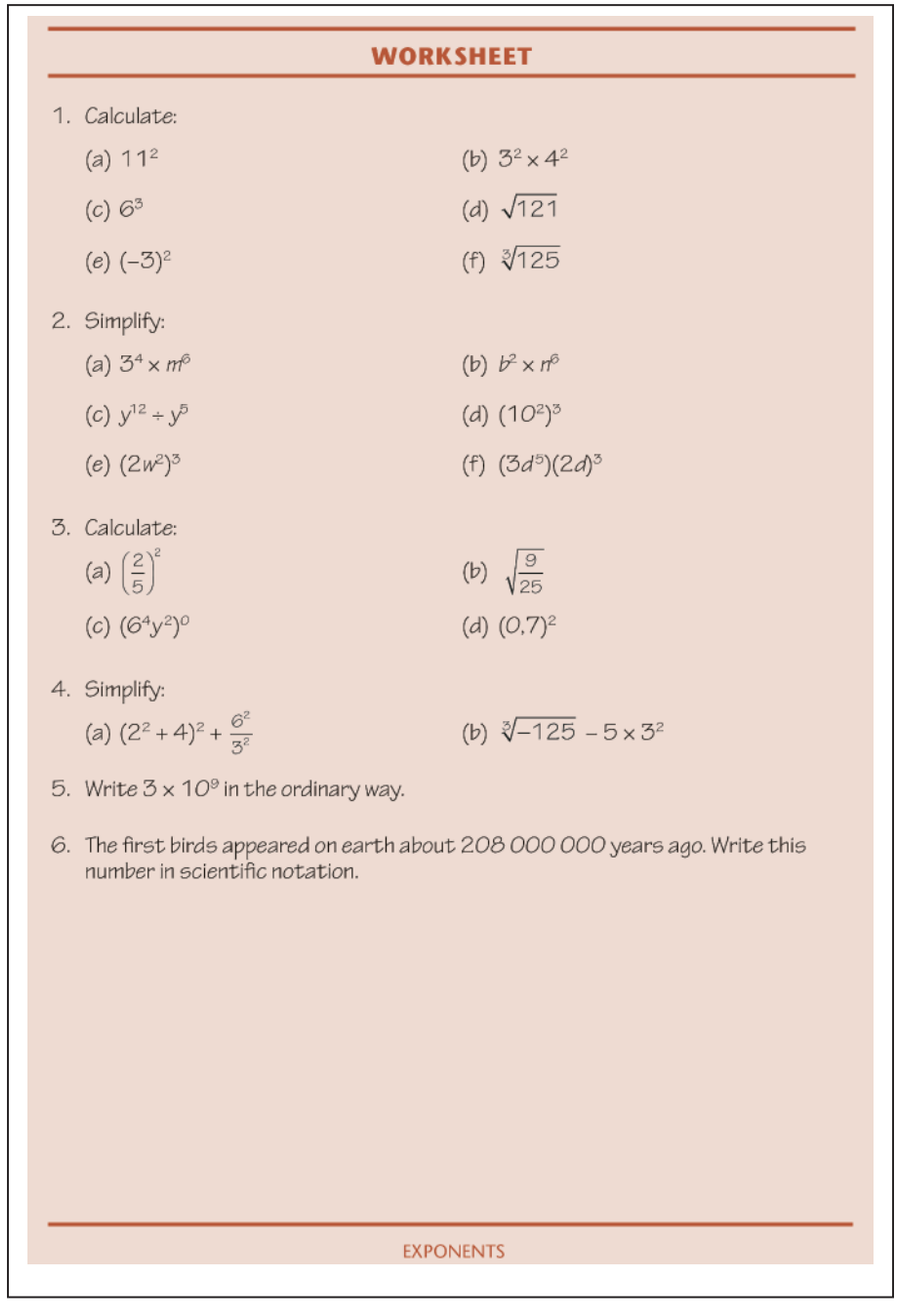
   (a) 13 000 × 150 000 (b) 200 × 6 000 000
   (c) 120 000 × 120 000 000 (d) 2,5 × 40 000 000
8. Copy the statements and use > or < to compare these numbers:
   (a) $1{,}3 \times 10^9 > 2{,}4 \times 10^7$ (b) $6{,}9 \times 10^2 < 4{,}5 \times 10^3$
   (c) $7{,}3 \times 10^4 > 7{,}3 \times 10^2$ (d) $3{,}9 \times 10^6 < 3{,}7 \times 10^7$

CHAPTER 3: EXPONENTS 47

WORKSHEET

Answers

1. (a) $11 \times 11 = 121$
   (b) $9 \times 16 = 144$
   (c) $6 \times 6 \times 6 = 216$
   (d) 11
   (e) $-3 \times -3 = 9$
   (f) 5
2. (a) $81m^6$
   (b) b^2n^6
   (c) $y^{12-5} = y^7$
   (d) 10^6
   (e) $2^3 \times w^6 = 8w^6$
   (f) $3d^5 \times 8d^3 = 24d^8$
3. (a) $\frac{2^2}{5^2} = \frac{4}{25}$
   (b) $\frac{\sqrt{9}}{\sqrt{25}}$
   (c) 1
   (d) $0{,}49$
4. (a) $(4+4)^2 = \frac{2^2 \times 3^2}{3^2} = 8^2 + 2^2 = 64 + 4 = 68$ or
   $(4+4)^2 + \frac{36}{9} = 8^2 + 4 = 64 + 4 = 68$
   (b) $-5 - 5 \times 9 = -5 - 45 = -50$
5. $3\,000\,000\,000$
6. $2{,}08 \times 10^8$



WORKSHEET

1. Calculate:
   (a) 11^2
   (b) $3^2 \times 4^2$
   (c) 6^3
   (d) $\sqrt{121}$
   (e) $(-3)^2$
   (f) $\sqrt[3]{125}$
2. Simplify:
   (a) $3^4 \times m^6$
   (b) $b^2 \times m^6$
   (c) $y^{12} \div y^5$
   (d) $(10^2)^3$
   (e) $(2w^2)^3$
   (f) $(3d^5)(2d)^3$
3. Calculate:
   (a) $\left(\frac{2}{5}\right)^2$
   (b) $\sqrt{\frac{9}{25}}$
   (c) $(6^4y^2)^0$
   (d) $(0{,}7)^2$
4. Simplify:
   (a) $(2^2+4)^2 + \frac{6^2}{3^2}$
   (b) $\sqrt[3]{-125} - 5 \times 3^2$
5. Write 3×10^9 in the ordinary way.
6. The first birds appeared on earth about 208 000 000 years ago. Write this number in scientific notation.

EXPONENTS

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
4.1 The term–term relationship in a sequence	Finding relationships between terms in a sequence to generate more terms: adding or subtracting; multiplying or dividing; using other methods	Pages 49 to 52
4.2 The position–term relationship in a sequence	Finding relationships between the position of a term and the term to generate more terms; developing rules that show the relationship	Pages 52 to 53
4.3 Investigating and extending geometric patterns	Translating geometric patterns to number sequences by developing rules to find the relationship between terms and their position in the sequence	Pages 53 to 56
4.4 Describing patterns in different ways	Investigating different ways to describe the same sequence	Pages 56 to 57

CAPS time allocation	4,5 hours
CAPS content specification	Pages 85 to 88

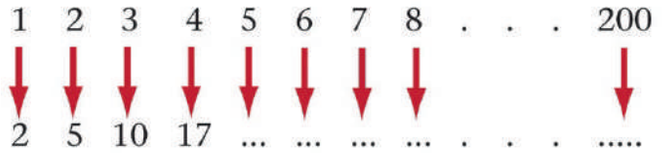
Mathematical background

We can describe a **sequence** by finding the **relationship between successive terms**. For example, in the sequence 3; 12; 48; 192; ... a term can be found by multiplying the previous term by 4. A formula for this action could be written as $T_n = 4T_{n-1}$. Applying the same action to each successive term is called **recursion**, and a formula like the one above is called a **recursive formula**. A recursive formula is used to determine the next term of a sequence using one or more of the preceding terms.

A disadvantage of recursive reasoning is that it would be time consuming, for example, to determine the two hundredth number in a given sequence. In some cases, it is possible to recognise a relationship between two variable quantities as shown below.

The sequence shown here shows a relationship between a term and its position in the sequence.

Each number in the bottom sequence is one more than the square of the corresponding number in the upper sequence that shows its position. This relationship can be expressed by means of the formula: Term value = (term position)^2 + 1 or $T_n = n^2 + 1$.



We can determine any term in the sequence easily if we can establish the relationship between the term and its position in the sequence.

Some sequences grow by adding or subtracting the same number to each successive term or by multiplying or dividing by the same number. For example:

3; 6; 9; 12; ... (add 3) or 1; 3; 9; 27; ... (multiply by 3)

Other sequences grow when the amount added increases from term to term, for example: 1; 3; 6; 10; 15; ... (start by adding two and add one more each time), or when the term number is squared: 1; 4; 9; 16; 25; ... There could be other rules for sequences, such as for a Fibonacci sequence: 1; 1; 2; 3; 5; 8; 13; 21; 35; ... where the first two terms are the same and then each successive term is the sum of the previous two terms.

4.1 The term-term relationship in a sequence

GOING FROM ONE TERM TO THE NEXT

Teaching guidelines

Given a sequence of numbers, learners have to identify a pattern or relationship between consecutive terms in order to extend the pattern.

A number pattern can be analysed by focusing on the relationship between consecutive terms as shown in the examples below:

- 3; 7; 11; 15; 19; ... starting with 3, 4 is added to get the next term each time
- 1; 2; 4; 7; 11; 16; 22; ... starting with 1, add 1 then 2 and then increase the number that is added by 1 each time (so: add 1, 2, 3, 4, ...).

Make sure that learners understand the meaning of the word "consecutive" and that they are comfortable with referring to the consecutive numbers in a sequence as "terms".

Answers

1. 26; 29; 32 – Add 3 each time.
2. 53; 68; 85 – Add consecutive odd numbers: 1 then 3 then 5 and so on.
3. 128; 256; 512 – Multiply by 2 each time.
4. 21; 23; 25 – Add 2 each time or count in odd numbers.
5. 49; 59; 70 – Add consecutive natural numbers: 1 then 2 then 3 and so on.
6. 1 458; 4 374; 13 122 – Multiply by 3 each time.
7. 37; 41; 45 – Add 4 each time.
8. 128; 256; 512 – Multiply by 2 each time.

ADDING OR SUBTRACTING THE SAME NUMBER

Teaching guidelines

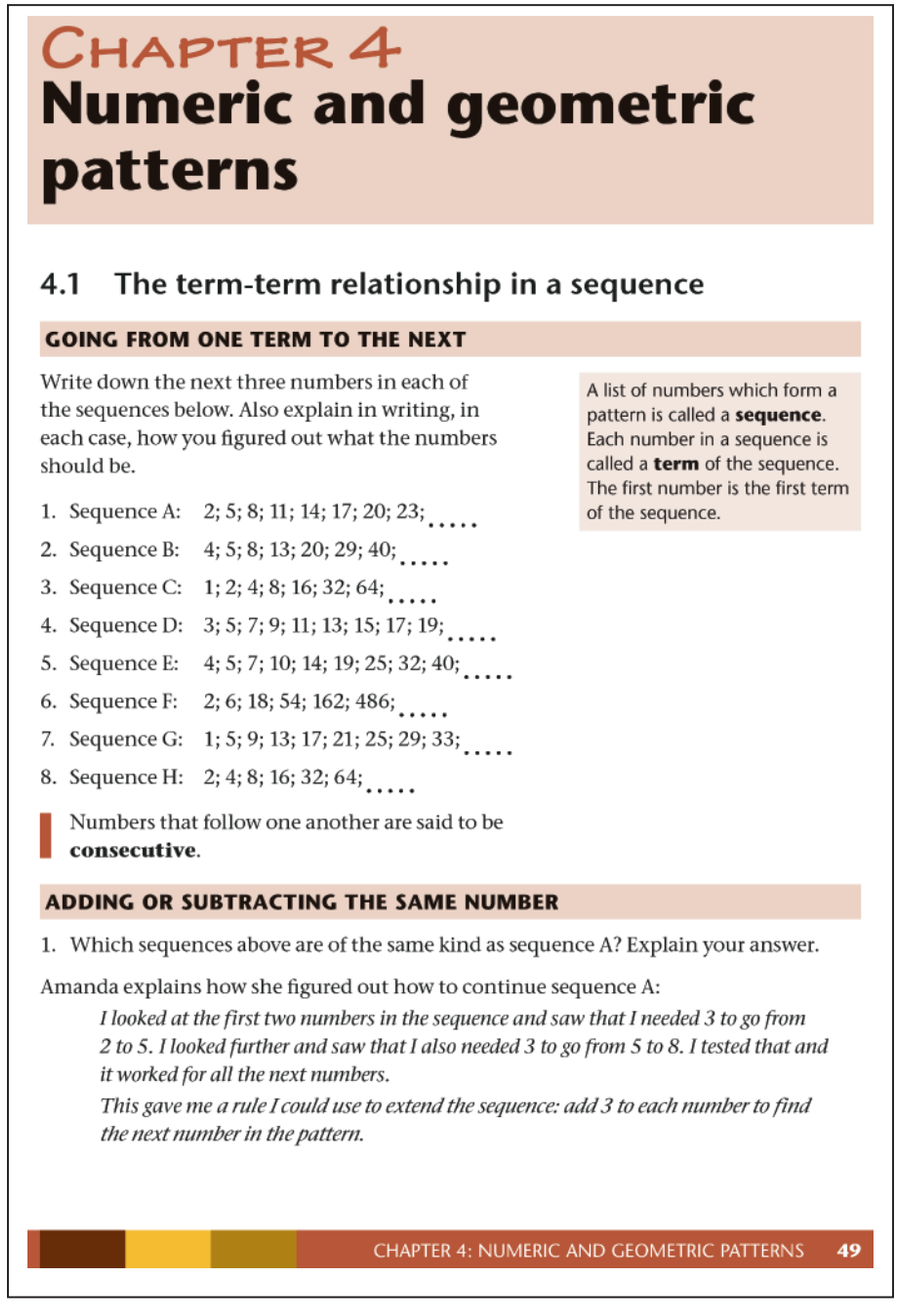
If learners analyse the sequences correctly they should reach the result that the sequences grow by adding a constant value to each term to get the next term.

Let learners also work with sequences where they subtract a number or, in other words, add a negative number to extend the sequence, for example:

49; 46; 43; 40; ... either subtract 3 or add –3.

Answers

1. Sequence D and sequence G. A constant number is added each time.



CHAPTER 4

Numeric and geometric patterns

4.1 The term-term relationship in a sequence

GOING FROM ONE TERM TO THE NEXT

Write down the next three numbers in each of the sequences below. Also explain in writing, in each case, how you figured out what the numbers should be.

A list of numbers which form a pattern is called a **sequence**. Each number in a sequence is called a **term** of the sequence. The first number is the first term of the sequence.

1. Sequence A: 2; 5; 8; 11; 14; 17; 20; 23; .....
2. Sequence B: 4; 5; 8; 13; 20; 29; 40; .....
3. Sequence C: 1; 2; 4; 8; 16; 32; 64; .....
4. Sequence D: 3; 5; 7; 9; 11; 13; 15; 17; 19; .....
5. Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; .....
6. Sequence F: 2; 6; 18; 54; 162; 486; .....
7. Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33; .....
8. Sequence H: 2; 4; 8; 16; 32; 64; .....

Numbers that follow one another are said to be **consecutive**.

ADDING OR SUBTRACTING THE SAME NUMBER

1. Which sequences above are of the same kind as sequence A? Explain your answer.

Amanda explains how she figured out how to continue sequence A:

I looked at the first two numbers in the sequence and saw that I needed 3 to go from 2 to 5. I looked further and saw that I also needed 3 to go from 5 to 8. I tested that and it worked for all the next numbers.

This gave me a rule I could use to extend the sequence: add 3 to each number to find the next number in the pattern.

CHAPTER 4: NUMERIC AND GEOMETRIC PATTERNS 49

Answers

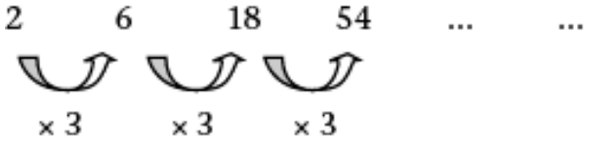
2. See the answers on LB page 50 alongside.
   The rules are:
   (a) add 7 (b) subtract 10 (c) add 1,5
   (d) add 1,8 (e) add $\frac{3}{4}$ (f) subtract 0,11

3. See the answers on LB page 50 alongside.

MULTIPLYING OR DIVIDING WITH THE SAME NUMBER

Teaching guidelines

A number pattern can be analysed by focusing on the relationship between consecutive terms as shown in the example below.



In the above case, each term is three times the previous term and this rule can also be expressed by means of this formula: Term value = 3 × previous term.

Misconceptions

Learners may get confused with two methods of generating the next term in a sequence; the method where each term is found by multiplying a term by a constant value and the method where the position number is multiplied by a constant value. These methods usually do not generate the same set of numbers.

Answers

1. Yes it works; you multiply each number by 2 to find the next number.

Tamara says you can also find the pattern by working backwards and subtracting 3 each time:

14 – 3 = 11; 11 – 3 = 8; 8 – 3 = 5; 5 – 3 = 2

When the **differences** between consecutive terms of a sequence are the same, we say the difference is **constant**.

2. Provide a rule to describe the relationship between the numbers in the sequence. Use this rule to calculate the missing numbers in each sequence.
   (a) 1; 8; 15; 22 ; 29 ; 36 ; 43 ; 50 ; ...
   (b) 10 020; 10 010 ; 10 000 ; 9 990 ; 9 980; 9 970; 9 960 ; 9 950 ; 9 940; 9 930; ...
   (c) 1,5; 3,0; 4,5; 6,0 ; 7,5 ; 9,0 ; 10,5 ; 12,0 ; ...
   (d) 2,2; 4,0; 5,8; 7,6 ; 9,4 ; 11,2 ; 13,0 ; 14,8 ; ...
   (e) $45\frac{3}{4}$; $46\frac{1}{2}$; $47\frac{1}{4}$; 48; $48\frac{3}{4}$; $49\frac{1}{2}$; $50\frac{1}{4}$; 51 ; $51\frac{3}{4}$; ...
   (f) 100,60 ; 100,49; 100,38; 100,27; 100,16 ; 100,05 ; 99,94; 99,83; 99,72; ...

3. Copy and complete the following table:

Input number	1	2	3	4	5	8	12	23	n
Input number + 7	8	9	10	11	12	15	19	30	$n + 7$

MULTIPLYING OR DIVIDING WITH THE SAME NUMBER

Take another look at sequence F on page 49: 2; 6; 18; 54; 162; 486; ...

Piet explains that he figured out how to continue the sequence F:

I looked at the first two terms in the sequence and wrote 2 × ? = 6.
When I multiplied the first number by 3, I got the second number: 2 × 3 = 6.
I then checked to see if I could find the next number if I multiplied 6 by 3: 6 × 3 = 18.
I continued checking in this way: 18 × 3 = 54; 54 × 3 = 162 and so on.
This gave me a rule I can use to extend the sequence and my rule was: multiply each number by 3 to calculate the next number in the sequence.

Zinhle says you can also find the pattern by working backwards and dividing by 3 each time:

54 ÷ 3 = 18; 18 ÷ 3 = 6; 6 ÷ 3 = 2

The number that we multiply with to get the next term in the sequence is called a **ratio**. If the number we multiply with remains the same throughout the sequence, we say it is a **constant ratio**.

1. Check whether Piet's reasoning works for sequence H on page 49: 2; 4; 8; 16; 32; 64; ...
2. Describe, in words, the rule for finding the next number in the sequences on the next page. Write down the next five terms of each sequence.

Answers

2. (a) 10 000; 100 000; 1 000 000; 10 000 000; 100 000 000
   Multiply each number by 10 to calculate the next number.
   (b) 1; 0,5; 0,25; 0,125; 0,0625
   Divide each number by 2 to calculate the next number.
   (c) 567; –1 701; 5 103; –15 309; 45 927
   Multiply each number by –3 to calculate the next number.
   (d) 192; 768; 3 072; 12 288; 49 152
   Multiply each term by 4 to calculate the next number.
   (e) 27; –9; 3; –1; $\frac{1}{3}$
   Divide each number by –3 to calculate the next number.
3. (a) See the answers on LB page 51 alongside.
   (b) See the answers on LB page 51 alongside.

NEITHER ADDING NOR MULTIPLYING BY THE SAME NUMBER

Teaching guidelines

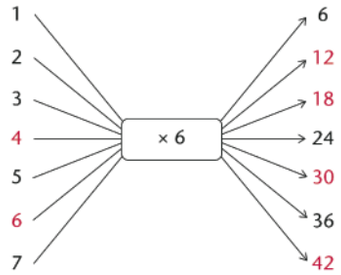
Let learners investigate what type of sequence each of the sequences A to H are in question 1 and separate those that grow by adding the same number or multiplying by the same number. They can write rules for those sequences and find rules for the others that don't grow in the way mentioned above.

Answers

1. (a) Sequence E. Different numbers are added to each term.
   (b) A different calculation is done to find each new term (neither adding the same number nor multiplying by the same number).
2. (a) 65; 82; 101; 122; 145
   (b) 50 + **15** = 65 65 + **17** = 82 82 + **19** = 101
   101 + **21** = 122 122 + **23** = 145

(a) 1; 10; 100; 1 000; ..... (b) 16; 8; 4; 2; ..... (c) 7; –21; 63; –189; .....
(d) 3; 12, 48; ..... (e) 2 187; –729; 243; –81; .....

3. (a) Copy and fill in the missing output and input numbers:



What is the term-to-term rule for the output numbers here, + 6 or × 6?

(b) Copy and complete the following table:

Input numbers	1	2	3	4	5	6	12	x
Output numbers	6	12	18	24	30	36	72	$6x$

NEITHER ADDING NOR MULTIPLYING BY THE SAME NUMBER

1. Consider sequences A to H again and answer the questions that follow:
   Sequence A: 2; 5; 8; 11; 14; 17; 20; 23; ...
   Sequence B: 4; 5; 8; 13; 20; 29; 40; ...
   Sequence C: 1; 2; 4; 8; 16; 32; 64; ...
   Sequence D: 3; 5; 7; 9; 11; 13; 15; 17; 19; ...
   Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; ...
   Sequence F: 2; 6; 18; 54; 162; 486; ...
   Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33; ...
   Sequence H: 2; 4; 8; 16; 32; 64; ....
   (a) Which other sequence(s) is/are of the same kind as sequence B? Explain.
   (b) In what way are sequences B and E different from the other sequences?

There are sequences where there is neither a constant difference nor a constant ratio between consecutive terms and yet a pattern still exists, as in the case of sequences B and E.

2. Consider the sequence: 10; 17; 26; 37; 50; ...
   (a) Write down the next five numbers in the sequence.
   (b) Eric observed that he can calculate the next term in the sequence as follows: 10 + **7** = 17; 17 + **9** = 26; 26 + **11** = 37. Use Eric's method to check whether your numbers in question 2(a) above are correct.

Answers

3. (a) Yes. Test: The difference between 17 and 10 is 7. Increase the difference by 2 to get 9. To get the next term, add 9 to 17 to get 26. Continue this way of reasoning to get the other terms.
   (b) No. Test: The difference between 17 and 10 is 7. Increase the difference by 1 to get 8. To get the next term add 8 to 17. It gives 25 which is not the next term in the sequence.
   (c) Yes. The test is the same as that in question 3(a).
4. (a) 36; 49; 64; 81; 100 The term value is the square of the term number.
   (b) 77; 98; 121; 146; 173 Add consecutive odd numbers: 11; 13; 15; ...
   (c) 104; 139; 179; 224; 274 Add consecutive multiples of 5: 10; 15; 20; ...
   (d) 26; 33; 41; 50; 60 Add consecutive natural numbers: 1; 2; 3; ...

4.2 The position-term relationship in a sequence

USING POSITION TO MAKE PREDICTIONS

Teaching guidelines

Learners have to identify a relationship between the term and its position in the sequence. Let them work through the development of the rule for sequence A on LB page 52.

Learners may try to adapt the recursive relationship to try to come up with a shortcut method without much success. They can find a valid shortcut, however, if they focus on the structure of the values in: 1; 5; 9; 13; 17; ...

- First number in the sequence: $1 = 1$
- Second number in the sequence: $5 = 1 + 4 = 1 + 1 \times 4$
- Third number in the sequence: $9 = 1 + 4 + 4 = 1 + 2 \times 4$
- Fourth number in the sequence: $13 = 1 + 4 + 4 + 4 = 1 + 3 \times 4$

We can recognise the pattern in this structure of values and generalise it as the nth number in the sequence $= 1 + (n - 1) \times 4$.

Misconceptions

Learners mix up rules, for example in sequence H they would want to say multiply the position number by 2, which works for the first two terms only.

Answers

1. Sequence D and sequence G. A constant number is added each time.

3. Which of the statements below can Eric use to describe the relationship between the numbers in the sequence in question 2? Test the rule for the first three terms of the sequence and then simply write "yes" or "no" next to each statement.
   (a) Increase the difference between consecutive terms by two each time.
   (b) Increase the difference between consecutive terms by one each time.
   (c) Add two more than you added to get the previous term.
4. Provide a rule to describe the relationship between the numbers in the sequences below. Use your rule to provide the next five numbers in each sequence.
   (a) 1; 4; 9; 16; 25; .....
   (b) 2; 13; 26; 41; 58; .....
   (c) 4; 14; 29; 49; 74; .....
   (d) 5; 6; 8; 11; 15; 20; .....

4.2 The position-term relationship in a sequence

USING POSITION TO MAKE PREDICTIONS

1. Take another look at sequences A to H. Which sequence(s) are of the same kind as sequence A? Explain.

Sequence A: 2; 5; 8; 11; 14; 17; 20; 23;...
Sequence B: 4; 5; 8; 13; 20; 29; 40; ...
Sequence C: 1; 2; 4; 8; 16; 32; 64; ...
Sequence D: 3; 5; 7; 9; 11; 13; 15; 17; 19; ...
Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; ...
Sequence F: 2; 6; 18; 54; 162; 486; ...
Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33; ...
Sequence H: 2; 4; 8; 16; 32; 64; ...

Sizwe has been thinking about Amanda and Tamara's explanations of how they worked out the rule for sequence A and has drawn up a table. He agrees with them but says that there is another rule that will also work. He explains:

My table shows the terms in the sequence and the difference between consecutive terms:

	1st term	2nd term	3rd term	4th term						
A:	5	8	11	14						
differences		+3	+3	+3	+3	+3	+3	+3	+3	+3

Sizwe reasons that the following rule will also work:

Multiply the position of the number by 3 and add 2 to the answer.
I can write this rule as a number sentence: Position of the number × 3 + 2
*I use my number sentence to check: **1** × 3 + 2 = 5; **2** × 3 + 2 = 8; **3** × 3 + 2 = 11*

2. (a) The term number or the position of the number in the sequence.
   (b) It is the constant difference between the terms.
3. (a) $7 \times 3 + 2 = 21 + 2 = 23$
   (b) $10 \times 3 + 2 = 30 + 2 = 32$
   (c) hundredth term $= 100 \times 3 + 2 = 300 + 2 = 302$
4. (a) Position of the number $\times 2 + 1$ (or $2n + 1$)
   (b) twenty-eighth term $= 28 \times 2 + 1 = 56 + 1 = 57$

MORE PREDICTIONS

Teaching guidelines

Discuss with learners that the words "position in the sequence" could be replaced by a symbol, for example the letter "n".

Answers

1. Position in the sequence $\times 3 + 1$ (or $3n + 1$)
   - tenth term $= 10 \times 3 + 1 = 30 + 1 = 31$
   - fifty-fourth term $= 54 \times 3 + 1 = 162 + 1 = 163$
2. Position in the sequence $\times 5 - 1$ (or $5n - 1$)
   - eighth term $= 8 \times 5 - 1 = 40 - 1 = 39$
   - sixteenth term $= 16 \times 5 - 1 = 80 - 1 = 79$
3. Position in the sequence $\times 12 - 9$ (or $12n - 9$)
   - fourth term $= 4 \times 12 - 9 = 48 - 9 = 39$
   - seventh term $= 7 \times 12 - 9 = 84 - 9 = 75$
   - thirtieth term $= 30 \times 12 - 9 = 360 - 9 = 351$
4. See the answers on LB page 53 alongside.

4.3 Investigating and extending geometric patterns

Background information



We can represent numbers as an arrangement of dots or circles in various shapes, for example triangles, squares, pentagons, hexagons and so on. These numbers are called polygonal numbers. For example, 15 balls can be arranged as a triangle with five balls in a row, four in a row above them, three in a row above those, two in a row above the three and one in the top row. These numbers are sometimes called figurate numbers.

These polygons are two-dimensional but we can extend the representation to three and even to higher dimensions.

2. (a) What do the numbers in bold in Sizwe's number sentence stand for?
   (b) What does the number 3 in Sizwe's number sentence stand for?
3. Consider the sequence 5; 8; 11; 14; ...
   Apply Sizwe's rule to the sequence and determine:
   (a) term number 7 of the sequence
   (b) term number 10 of the sequence
   (c) the hundredth term of the sequence
4. Consider the sequence: 3; 5; 7; 9; 11; 13; 15; 17; 19; ...
   (a) Use Sizwe's explanation to find a rule for this sequence.
   (b) Determine the 28th term of the sequence.

MORE PREDICTIONS

Copy and complete the following tables by calculating the missing terms:

1.

Position in sequence	1	2	3	4	10	54
Term	4	7	10	13	31	163

2.

Position in sequence	1	2	3	4	8	16
Term	4	9	14	19	39	79

3.

Position in sequence	1	2	3	4	7	30
Term	3	15	27	39	75	351

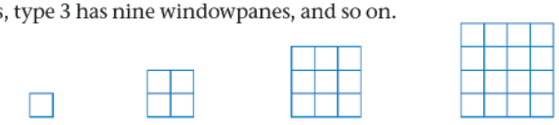
4. Copy the following table. Use the rule **position in the sequence × (position in the sequence + 1)** to complete it.

Position in sequence	1	2	3	4	5	6
Term	2	6	12	20	30	42

4.3 Investigating and extending geometric patterns

SQUARE NUMBERS

A factory makes window frames. Type 1 has one windowpane, type 2 has four windowpanes, type 3 has nine windowpanes, and so on.



The patterns formed by these representations are called geometric patterns, but they represent patterns or sequences of numbers.

SQUARE NUMBERS

Teaching guidelines

Figurate numbers often form sequences. Look at the drawings of the square numbers to see how they grow: 1, 4, 9, 16, ... etc. The pattern in position n has n^2 circles. Discuss with learners why these numbers are called square numbers.

Answers

1. 25
2. 36
3. 49
4. There will be 144 windowpanes. To find the number of windowpanes you square the type number. So for type 12 there are $12 \times 12 = 144$ windowpanes.
5. $15 \times 15 = 225$ $20 \times 20 = 400$

TRIANGULAR NUMBERS

Teaching guidelines

Learners start off with a circle and add two circles to make a triangle.

Answers

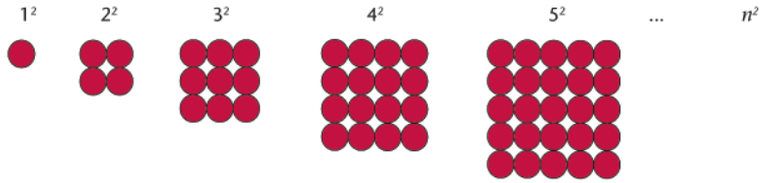
1. (a) 5 (b) 4 (c) 3 (d) 2 (e) 1
   (f) 15 circles: $5 + 4 + 3 + 2 + 1 = 15$ (add the number of circles in each row)
2. $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$ circles
3. $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$ circles

1. How many windowpanes will there be in type 5?
2. How many windowpanes will there be in type 6?
3. How many windowpanes will there be in type 7?
4. How many windowpanes will there be in type 12? Explain.
5. Copy and complete the table. Show your calculations.

Frame type	1	2	3	4	15	20
Number of windowpanes	1	4	9	16	225	400

In algebra we think of a square as a number that is obtained by multiplying a number by itself. So, 1 is also a square because $1 \times 1 = 1$.

The symbol n is used below to represent the *position number* in the expression that gives the rule (n^2) when generalising.



TRIANGULAR NUMBERS

Therese uses circles to form a pattern of triangular shapes:



1. If the pattern is continued, how many circles must Therese have:
   (a) in the bottom row of picture 5?
   (b) in the second row from the bottom of picture 5?
   (c) in the third row from the bottom of picture 5?
   (d) in the second row from the top of picture 5?
   (e) in the top row of picture 5?
   (f) in total in picture 5? Show your calculations.
2. How many circles does Therese need to form triangle picture 7? Show the calculation.
3. How many circles does Therese need to form triangle picture 8?

Answers

4. See the completed table on the LB page 55 alongside.
   - $5 + 4 + 3 + 2 + 1 = 15$
   - $6 + 5 + 4 + 3 + 2 + 1 = 21$
   - $12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 78$
   - $15 + 14 + 13 + 12 + 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 120$

Teaching guidelines

Learners should see the rule to get the sequence of triangular numbers: start with 1; add 2 to get the second number; add 3 to get the next; add 4 and so on. So the number of circles in a picture or pattern number is found by adding consecutive natural numbers. To get the tenth pattern, add ten circles. The problem is that you have to know the total number of circles in the ninth pattern in order to do that. So you will have to start at the beginning and add on until you get to the pattern number that you want the total of. This is not an effective way of calculating.

Work through the explanation leading up to question 5. The original set of triangles has been rearranged to form right-angled triangles, and then the number of circles in each picture of the original sequence has been doubled.

Learners can see that the width of the rectangles consist of the same number of circles as the pattern number and the length has one more circle. Therefore, the number of circles in a pattern is given by the width × (width + 1) which is the same as: pattern number × (pattern number +1).

The number of circles in the original sequence was doubled in each pattern, so to get the number of circles in a particular pattern of the original sequence, we halve the number of circles in the same pattern number of the rectangular sequence.

Notes on the questions

If the pattern number is represented by the symbol n, we can write $n(n + 1)$ to get the number of circles in the rectangular row.

So the number of circles in pattern n of the original sequence will be $\frac{1}{2}n(n + 1)$.

Answers

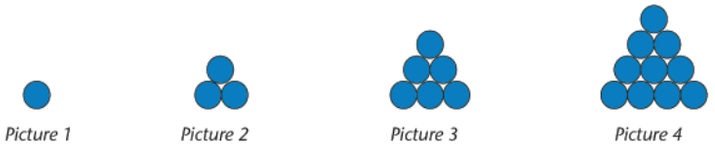
5. See the answers on LB page 55 alongside.

4. Copy and complete the following table. Show all your work.

Picture number	1	2	3	4	5	6	12	15
Number of circles	1	3	6	10	15	21	78	120

More than 2 500 years ago, Greek mathematicians already knew that the numbers 3, 6, 10, 15 and so on could form a triangular pattern. They represented these numbers with dots which they arranged in such a way that they formed equilateral triangles, hence the name **triangular numbers**. Algebraically, we think of them as sums of consecutive natural numbers starting with 1.

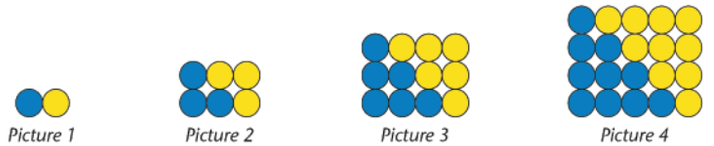
Let us revisit the activity on triangular numbers that we did on the previous page.



Picture 1 *Picture 2* *Picture 3* *Picture 4*

So far, we have determined the number of circles in the pattern by adding consecutive natural numbers. If we were asked to determine the number of circles in picture 200, for example, it would take us a very long time to do so. We need to find a quicker method of finding any triangular number in the sequence.

Consider the arrangement below.



Picture 1 *Picture 2* *Picture 3* *Picture 4*

We have added the yellow circles to the original blue circles and then rearranged the circles in such a way that they are in a rectangular form.

5. Picture 2 is three circles long and two circles wide. Copy and complete the following sentences:
   (a) Picture 3 is 4 circles long and 3 circles wide.
   (b) Picture 1 is 2 circles long and 1 circle wide.
   (c) Picture 4 is 5 circles long and 4 circles wide.
   (d) Picture 5 is 6 circles long and 5 circles wide.

Answers

6. (a) 90
   (b) 42
   (c) 30
   (d) 380

7. (a) Circles in picture 20 $= \frac{21 \times 20}{2} = \frac{420}{2} = 210$

   (b) Circles in picture 35 $= \frac{36 \times 35}{2} = \frac{1\,260}{2} = 630$

4.4 Describing patterns in different ways

T-SHAPED NUMBERS

Teaching guidelines

The pattern can be described using the add-on rule to consecutive terms or through the relationship between the position of the term in the sequence and the value of the term.

Discuss with learners how Plan B is more useful than Plan A – i.e. Plan B allows you to find the number of squares in any pattern number immediately, whereas if you use Plan A, you have to count from the beginning, adding three each time.

Plan B gives a rule to get the number of squares in a pattern:
pattern number × 3 – 2 or $3n - 2$, where n is the pattern number.

Answers

1. (a) 13
   (b) 43
   (c) See the answers on LB page 56 alongside.

6. How many circles will there be in a picture that is:
   (a) ten circles long and nine circles wide?
   (b) seven circles long and six circles wide?
   (c) six circles long and five circles wide?
   (d) 20 circles long and 19 circles wide?

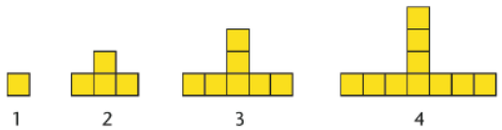
Suppose we want to use a quicker method to determine the number of circles in picture 15. We know that picture 15 is 16 circles long and 15 circles wide. This gives a total of 15 × 16 = 240 circles. But we must compensate for the fact that the yellow circles were originally not there by halving the total number of circles. In other words, the original figure has 240 ÷ 2 = 120 circles.

7. Use the above reasoning to calculate the number of circles in:
   (a) picture 20 (b) picture 35

4.4 Describing patterns in different ways

T-SHAPED NUMBERS

The pattern below is made from squares.



1. (a) How many squares will there be in pattern 5?
   (b) How many squares will there be in pattern 15?
   (c) Copy and complete the following table:

Pattern number	1	2	3	4	5	6	20
Number of squares	1	4	7	10	13	16	58

You can use the following three plans (or methods) to calculate the number of squares for pattern 20. Study each one carefully.

Plan A:

To get from one square to four squares, you have to add three squares. To get from four squares to seven squares, you have to add three squares. To get from seven squares to ten squares, you have to add three squares. Continue to add three squares for each pattern until pattern 20.

Plan B:

Multiply the pattern number by three and subtract two. Pattern 20 will therefore have 20 × 3 – 2 squares.

Answers

2. (a) Both A and B. Both plans work if you try them out.
   (b) Learners' give their own explanations.
   (c) Yes. See the answers on LB page 57 alongside.

... AND SOME OTHER SHAPES

Teaching guidelines

Discuss the two methods: adding on three each time or finding and using a rule to continue the sequence.

Then develop a rule: pattern number × 3 = 2 or $3n + 2$, where n is the position of the pattern in the sequence, or the pattern number.

Learners should see by now that finding a rule (if possible) makes calculations easier when finding values of terms in the sequence.

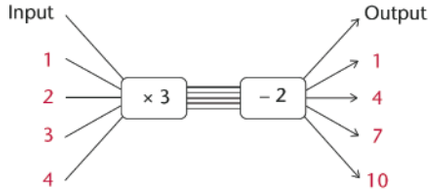
Answers

1. See the completed pattern 4 on LB page 57 alongside.
2. (a) Three tiles are added to make each figure: one tile across and two tiles on the right-hand side downwards. To make the seventeenth figure, three tiles must be added thirteen times (17 – 4 = 13) to the 14 tiles of the fourth figure. So there will be 14 + 13 × 3 = 53 tiles.
   (b) 3 × 23 + 2 = 69 + 2 = 71 tiles
   (c) See the answer on the LB page 57 alongside.
   (d) 50 × 3 + 2 = 150 + 2 = 152 tiles

Plan C:

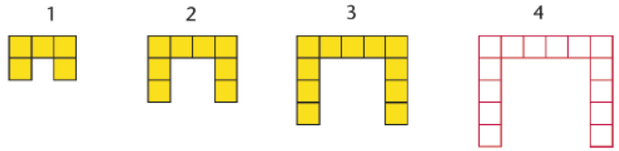
The number of squares in pattern 5 is 13. Pattern 20 will therefore have 13 × 4 = 52 squares because 20 = 5 × 4.

2. (a) Which method or plan (A, B or C) will give the right answer? Explain why.
   (b) Which of the above plans did you use? Explain why?
   (c) Can this flow diagram be used to calculate the number of squares?

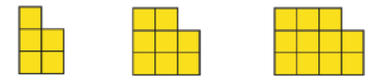


... AND SOME OTHER SHAPES

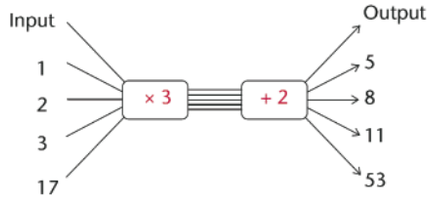
1. Three figures are given below. Draw the next figure in the tile pattern.



2. (a) If the pattern is continued, how many tiles will there be in the 17th figure? Answer this question by analysing what happens.
   (b) Thato decides that it easier for him to see the pattern when the tiles are rearranged as shown on the right:



   Use Thato's method to determine the number of tiles in the 23rd figure.
   (c) Copy and complete the following flow diagram by writing the appropriate operators so that it can be used to calculate the number of tiles in any figure of the pattern.



   (d) How many tiles will there be in the 50th figure if the pattern is continued?

WORKSHEET

Teaching guidelines

You can use the worksheet as a test for summative assessment or for the purpose of formative assessment.

Answers

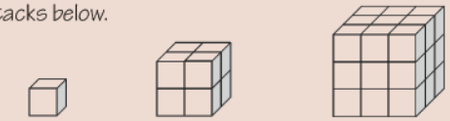
1. (a) 58; 74; 92; 112
   Add consecutive even numbers: 2 then 4 then 6 and so on.
   (b) 486; 1 458; 4 374; 13 122
   Multiply each number by 3 to find the next term.
   (c) 31; 37; 43; 49
   Add 6 to find the next term.
2. (a) See the completed table on LB page 58 alongside.
   (b) Multiply the position number by 7 and subtract 4.
3. (a) 125
   (b) See the completed table on LB page 58 alongside.
   (c) Cube the stack number, i.e. (stack number)^3.

WORKSHEET

1. Write down the next four terms in each sequence. Also explain, in each case, how you figured out what the terms are.
   (a) 2; 4; 8; 14; 22; 32; 44; .....
   (b) 2; 6; 18; 54; 162; .....
   (c) 1; 7; 13; 19; 25; .....
2. (a) Copy and complete the following table by calculating the missing terms:

Position in sequence	1	2	3	4	5	7	10
Term	3	10	17	24	31	45	66

   (b) Write the rule to calculate the term from the position in the sequence in words.
3. Consider the stacks below.



   (a) How many cubes will there be in stack 5?
   (b) Copy and complete the following table:

Stack number	1	2	3	4	5	6	10
Number of cubes	1	8	27	64	125	216	1 000

   (c) Write down the rule to calculate the number of cubes for any stack number.

NUMERIC AND GEOMETRIC PATTERNS

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
5.1 Constant and variable quantities	Understanding dependence of one variable on another; using and completing flow diagrams; using tables and words to describe what happens in a flow diagram	Pages 59 to 62
5.2 Different ways to describe relationships	Describing relationships in terms of flow diagrams; tables, words and symbolic formulae	Pages 63 to 65
5.3 Algebraic symbols for variables and relationships	Using algebraic symbols to write formulae that describe relationships between two quantities	Pages 65 to 66

CAPS time allocation	3 hours
CAPS content specification	Pages 88 to 89

Mathematical background

A **functional relationship** shows the connection between a set of input values and a set of output values. The input value is changed in a calculation to give the output value. For example, if apples are sold at R16,00 per kilogram then the more kilograms we buy, the higher the cost for the apples.

This is an example of a situation in which one variable quantity (in this case the cost of the apples) depends on another variable quantity (in this case the number of kilograms bought), and is uniquely determined by it. In Mathematics, we refer to the relationship between the corresponding values of such related variables as a "function". The function concept is at the heart of school algebra and calculus.

- Input variable = independent variable, usually denoted by a symbol, like x
- Output variable = dependent variable, usually denoted by a symbol, like y.

A functional relationship is usually represented by a **function rule** where the dependent variable, y, is expressed in terms of the independent variable, x. In the example above the cost, y, for x kilograms of apples will be given by $y = 16{,}00x$. The cost, y, depends on the number of kilograms, x, of apples bought.

A function can be represented in more than one way:

- in words
- in a table with each input value matching an output value
- in a flow diagram
- symbolically in a formula
- in a graph.

A function could describe a more complicated situation, for example: a carpet shampoo machine is hired for a flat rate of R250 plus R20 per day. The equation that gives the cost to hire the machine for x days is given by $y = 20x + 250$. The flat rate is constant. The rule consists of a variable term and a constant term.

When the output number is given, the formula becomes an equation, for example: $18 = 5x + 3$ or $5x + 3 = 18$.

5.1 Constant and variable quantities

LOOKING FOR CONNECTIONS BETWEEN QUANTITIES

Teaching guidelines

In this section learners learn about dependence of one variable on another. The idea of a function is built on the idea of a variable. Consequently, to empower learners to attain the understanding indicated by the CAPS, it is critical that learners acquire the concept of a variable, as opposed to a constant. It is useful for learners to think of situations where the value of one quantity depends on the value of another and changes as the other changes.

The situation in question 7 broadens out to the concepts covered in questions 8 and 9. Question 7 recaps number patterns touched on in the previous chapter.

Misconceptions

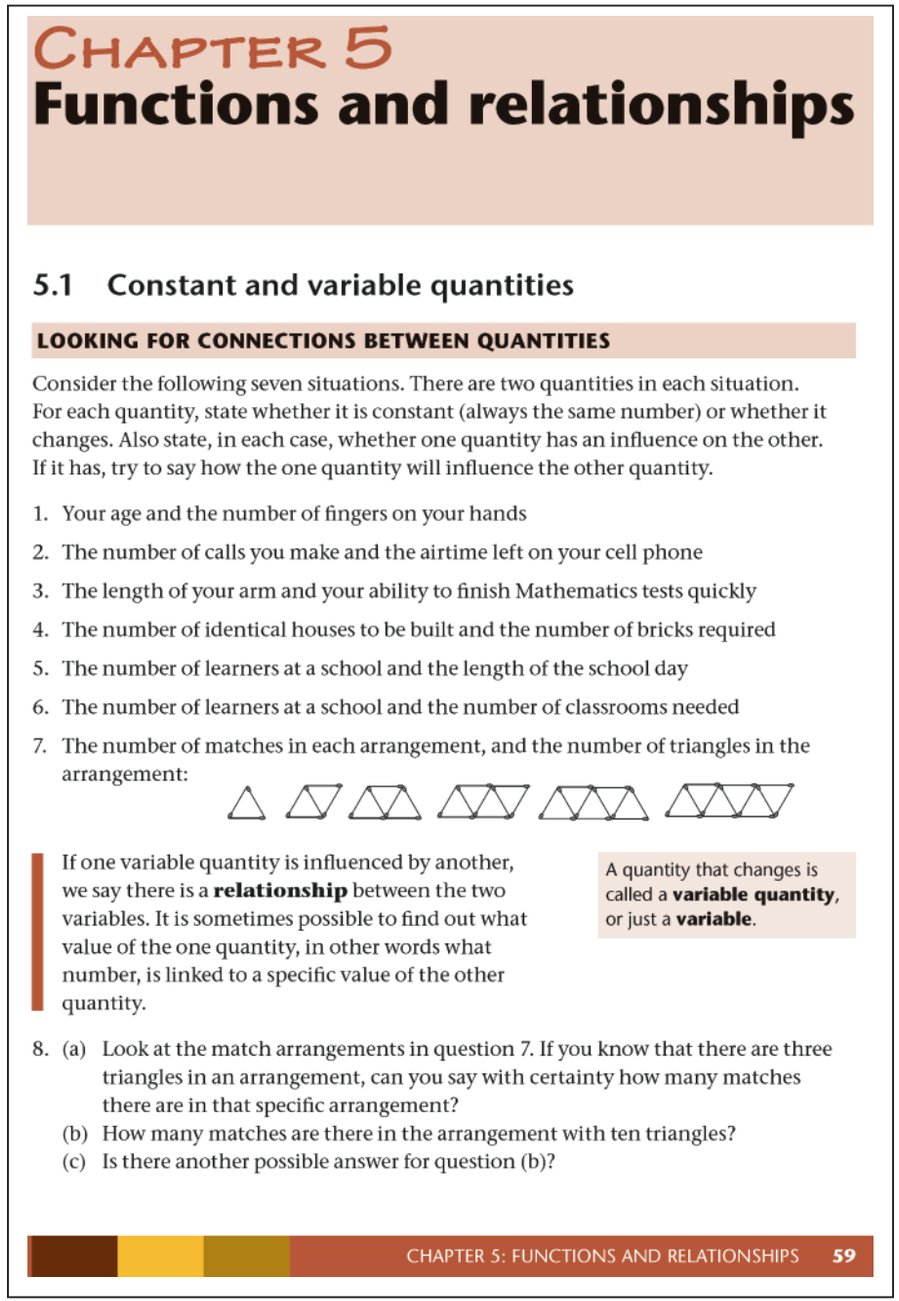
Learners misunderstand the concept of variable and of dependence.

Notes on the questions

The questions have been carefully chosen so that learners can understand: some quantities are constant and that there is no dependence in such situations; or that a quantity may depend on the value of another quantity.

Answers

1. Age – changes.
   Number of fingers on hands – constant.
   No influence.
2. Number of calls – changes.
   Airtime left on cellphone – changes.
   The more calls you make, the less airtime will be left on your cellphone.
3. Length of your arm – constant.
   How quickly you can finish Mathematics tests – changes.
   No influence.
4. Number of houses – changes.
   Number of bricks – changes.
   The more houses, the more bricks required.
5. Number of learners at a school – changes.
   Length of the school day – constant.
   No influence.
6. Number of learners – changes.
   Number of classrooms – changes.
   The more learners, the more classrooms needed.
7. Number of triangles – changes.
   Number of matches – changes.
   As more triangles are made, more matches are needed.
8. (a) Yes, seven matches.
   (b) 21. For every triangle added to the already existing ones, you have to add two matches.
   (c) No



CHAPTER 5

Functions and relationships

5.1 Constant and variable quantities

LOOKING FOR CONNECTIONS BETWEEN QUANTITIES

Consider the following seven situations. There are two quantities in each situation. For each quantity, state whether it is constant (always the same number) or whether it changes. Also state, in each case, whether one quantity has an influence on the other. If it has, try to say how the one quantity will influence the other quantity.

1. Your age and the number of fingers on your hands
2. The number of calls you make and the airtime left on your cell phone
3. The length of your arm and your ability to finish Mathematics tests quickly
4. The number of identical houses to be built and the number of bricks required
5. The number of learners at a school and the length of the school day
6. The number of learners at a school and the number of classrooms needed
7. The number of matches in each arrangement, and the number of triangles in the arrangement:

If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. It is sometimes possible to find out what value of the one quantity, in other words what number, is linked to a specific value of the other quantity.

A quantity that changes is called a **variable quantity**, or just a **variable**.

8. (a) Look at the match arrangements in question 7. If you know that there are three triangles in an arrangement, can you say with certainty how many matches there are in that specific arrangement?
   (b) How many matches are there in the arrangement with ten triangles?
   (c) Is there another possible answer for question (b)?

CHAPTER 5: FUNCTIONS AND RELATIONSHIPS 59

Answers

9. Yes, it shows the connection between the number of triangles in an arrangement (input) and how many matches are needed (output). If you add two matches, another triangle is added to the arrangement. The reason you have to multiply by 2 and add 1 is that to complete the first triangle you need two matches + one match.

COMPLETING SOME FLOW DIAGRAMS

Teaching guidelines

What the input numbers will be depends on the context. For example, in questions 7 to 9 on pages 59 to 60 of the LB, the input numbers can only be whole numbers. It is unlikely that half a match will be used and negative numbers as an input value will have no meaning.

If there is no context, for example as in question 1 on LB page 60, the input numbers could be any real number. Unless the set of numbers from which the input numbers are taken is specified, the input numbers could be any real number. If the set of numbers is specified, it would be said, for example, "input numbers are whole numbers or integers", etc.

The operator(s) is the calculation instruction: what to do with the input numbers to get the output numbers.

Misconceptions

Learners may think that the only possible input numbers are those shown in the flow diagram.

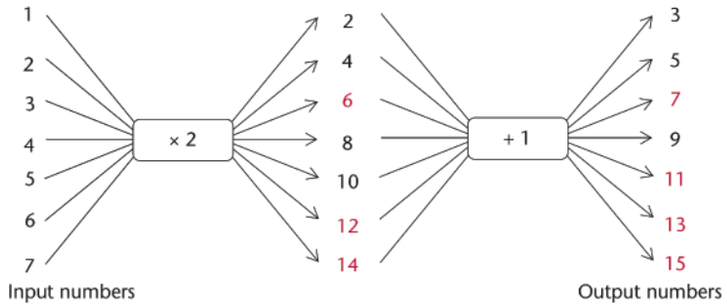
Teaching guidelines

Flow diagrams are useful as they help learners to:

- reinforce the concept of dependence: the value of the output numbers depends on the value of the input numbers
- understand more complicated rules, for example two-step rules such as multiply by 5 and add 14
- understand the order of the operations and learn how to insert brackets where needed, for example: × 5 first and then + 14 can be written as: input × 5 + 14, but + 14 multiply by 5 has to be written as: (input + 14) × 5

  understand how the calculations have been done on the input numbers, which makes it easier to grasp the inverse operations when they solve equations.

9. Copy and complete the following flow diagrams by filling in all the missing numbers. Do you see any connections between the situation in question 7 on page 59 and this flow diagram? If so, describe the connections.

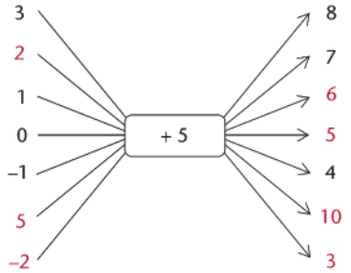


COMPLETING SOME FLOW DIAGRAMS

A relationship between two quantities can be shown with a flow diagram, such as those below. Unfortunately, only some of the numbers can be shown on a flow diagram.

1. Copy the following flow diagram. Calculate the output numbers. Some input numbers are missing. Choose and insert your own input numbers.

(a)



Each input number in a flow diagram has a corresponding **output number**. The first (top) **input number** corresponds to the first output number. The second input number corresponds to the second output number and so on. We call + 5 the **operator**.

(b) What type of numbers are the given input numbers?

(c) In the above flow diagram, the output number 8 corresponds to the input number 3. Copy and complete the following sentences:

In the relationship shown in the above flow diagram, the output number ....4.... *corresponds to the input number –1.*

The input number ....2.... *corresponds to the output number 7.*

If more places are added to the flow diagram, the input number ....26.... *will correspond to the output number 31.*

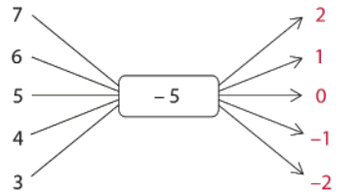
Teaching guidelines

In these questions the **function concept** is laid down: variable input; a rule applied to input gives variable output. Do not rush through this section – it is important that learners get enough experience working with these functional relationships.

Answers

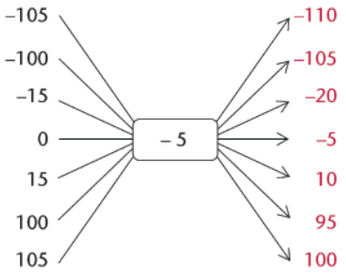
1. (a) See the completed flow diagram on LB page 60 on the previous page.
   (b) Integers
   (c) See the answers on LB page 60 on the previous page.
2. (a) See the completed flow diagram on LB page 61 alongside.
   (b) Input and output are switched around. This operation is the additive inverse of +5 in question 1.
3. (a) See the completed flow diagram on LB page 61 alongside.
   (b) See the completed flow diagram on LB page 61 alongside.
   (c) –5; subtracting 5 is the same as adding (–5).
   (d) –68; subtracting (–68) is the same as adding 68.
4. See the completed flow diagram on LB page 61 alongside.

2. (a) Copy and complete this flow diagram.
   (b) Compare this flow diagram to the flow diagram in question 1. What link do you find between the two?

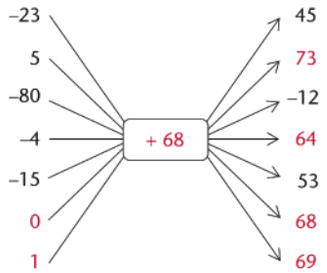


3. Copy and complete the following flow diagrams. You have to find out what the **operator** for (b) is, and fill it in yourself.

(a)

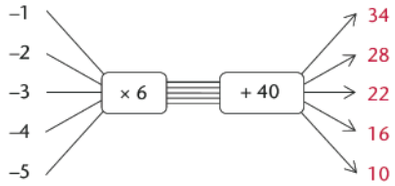


(b)



   (c) What number can you add in (a), instead of subtracting 5, that will produce the same output numbers?
   (d) What number can you subtract in (b), instead of adding a number, that will produce the same output numbers?

4. Copy and complete the following flow diagram:



A completed flow diagram shows two kinds of information:

- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

The flow diagram that you completed in question 4 shows the following information:

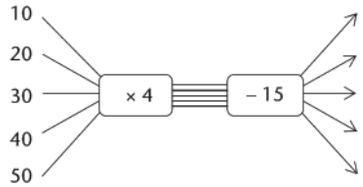
- Each input number is multiplied by 6, then 40 is added to produce the output numbers.
- The input and output numbers are connected, as shown in the table on page 62.

Answers

5. (a) Multiply the input number by 4 and then subtract 15 to get the output number.
   (b) See the table on LB page 62 alongside.
6. (a) See the flow diagram on LB page 62 alongside.
   (b) See the flow diagram on LB page 62 alongside.
   (c) The input numbers 100 square metres; apply the input number to the flow diagram in (a); 100 × 4 000 + 180 000 = R580 000.

Input numbers	–1	–2	–3	–4	–5
Output numbers	34	28	22	16	10

5. (a) Describe, in words, how the following output numbers can be calculated:



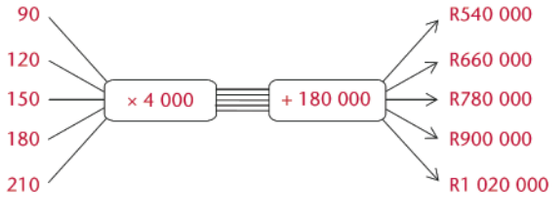
(b) Copy the following table and use it to show which output numbers are connected to which input numbers in the above flow diagram.

Input number	10	20	30	40	50
Output number	25	65	105	145	185

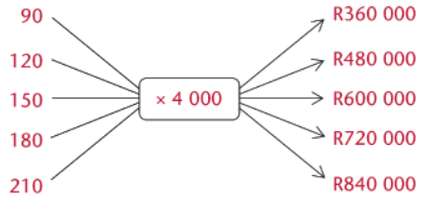
6. The following information is available about the floor space and cost of houses in a new development. The cost of an empty stand is R180 000.

Floor space in square metres (m^2)	90	120	150	180	210
Cost of house and stand	540 000	660 000	780 000	900 000	1 020 000

(a) Represent the above information in the following flow diagram:



(b) Show what the houses only will cost, if you get the stand for free.



(c) Try to figure out what the cost of a house and stand will be, if there are exactly one hundred 1 m by 1 m sections of floor space in the house.

5.2 Different ways to describe relationships

A RELATIONSHIP BETWEEN RED DOTS AND BLUE DOTS

Teaching guidelines

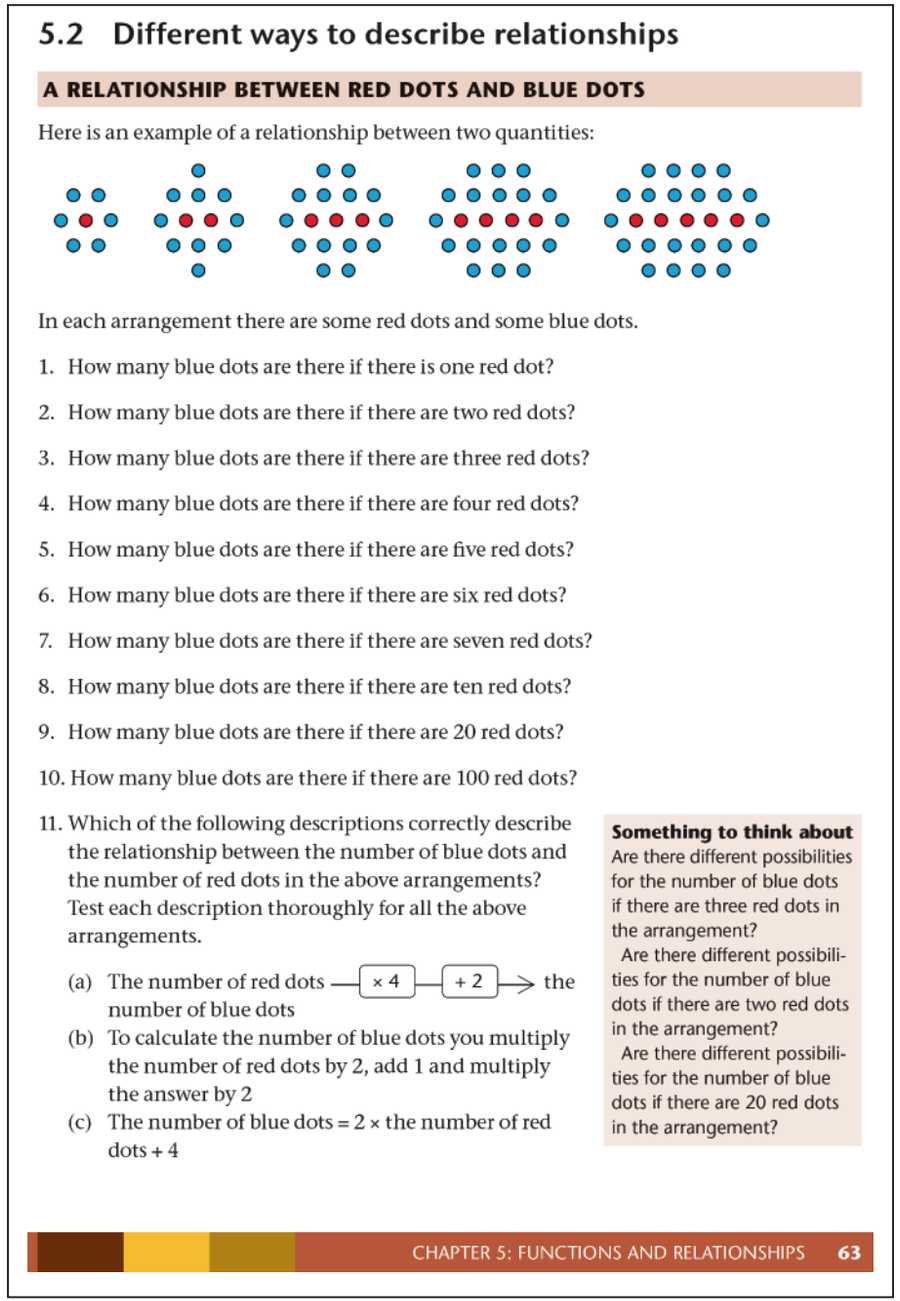
Learners use a number pattern. This is work which they should be familiar with from the previous chapter.

This type of numeric pattern develops an understanding of functional relationships, in which there is a dependent variable (output), and an independent variable (input), and where there is a unique output for any given input value.

The terms "dependent" and "independent" variable, and indeed the term "variable" itself, are powerful descriptors of the elements of functional relationships. You may want to build on the experiences learners had with patterns in the previous chapter, by using some of the patterns they worked with there and applying the terms "dependent" and "independent variable" to these patterns; using the different ways of describing the relationships.

Answers

1. 6
2. 10
3. 14
4. 18
5. 22
6. 26
7. 30
8. 42
9. 82
10. 402
11. (a); (b); (d); (e); (f); (g); (h)



5.2 Different ways to describe relationships

A RELATIONSHIP BETWEEN RED DOTS AND BLUE DOTS

Here is an example of a relationship between two quantities:

In each arrangement there are some red dots and some blue dots.

1. How many blue dots are there if there is one red dot?
2. How many blue dots are there if there are two red dots?
3. How many blue dots are there if there are three red dots?
4. How many blue dots are there if there are four red dots?
5. How many blue dots are there if there are five red dots?
6. How many blue dots are there if there are six red dots?
7. How many blue dots are there if there are seven red dots?
8. How many blue dots are there if there are ten red dots?
9. How many blue dots are there if there are 20 red dots?
10. How many blue dots are there if there are 100 red dots?
11. Which of the following descriptions correctly describe the relationship between the number of blue dots and the number of red dots in the above arrangements? Test each description thoroughly for all the above arrangements.

    (a) The number of red dots → × 4 → + 2 → the number of blue dots
    (b) To calculate the number of blue dots you multiply the number of red dots by 2, add 1 and multiply the answer by 2
    (c) The number of blue dots = 2 × the number of red dots + 4

Something to think about
Are there different possibilities for the number of blue dots if there are three red dots in the arrangement?
Are there different possibilities for the number of blue dots if there are two red dots in the arrangement?
Are there different possibilities for the number of blue dots if there are 20 red dots in the arrangement?

CHAPTER 5: FUNCTIONS AND RELATIONSHIPS 63

TRANSLATING BETWEEN DIFFERENT LANGUAGES OF DESCRIPTION

Teaching guidelines

The different ways in which the same functional relationship can be represented is prominent in this grade, and hence in the work in this section.

Show learners how a functional relationship can be represented in:

- A table of corresponding values of the dependent and independent variables, for example in a relationship between green and yellow dots:

Number of green dots	1	2	3	4	5
Number of yellow dots	8	13	18	23	28

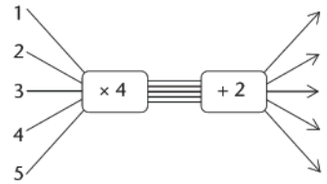
  A table shows which output numbers correspond to certain input numbers.
- A verbal description that gives the rule, for example: to calculate the number of yellow dots, you multiply the number of green dots by five and add three to the answer.
- A flow diagram that gives the rule, for example: 

- A word formula, for example: the number of yellow dots = five multiplied by the number of green dots plus three. The rule is given in the formula.

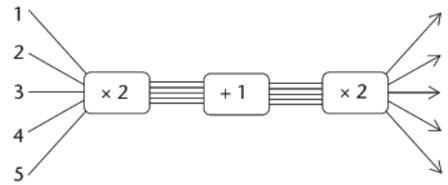
(d)

Number of red dots	1	2	3	4	5	6
Number of blue dots	6	10	14	18	22	26

(e)



(f)



(g) The number of blue dots = 4 × the number of red dots + 2
(h) The number of blue dots = 2 × (2 × the number of red dots + 1)
(Remember that the calculations inside the brackets are done first.)

The descriptions in (c), (g) and (h) above are called **word formulae**.

TRANSLATING BETWEEN DIFFERENT LANGUAGES OF DESCRIPTION

A relationship between two quantities can be described in different ways, including:

- a table of values of the two quantities
- a flow diagram
- a word formula
- a symbol formula (or symbolic formula).

You will learn about symbolic formulae in Section 5.3.

1. The relationship between two quantities is described as follows:
   The second quantity is always three times the first quantity plus 8.
   The first quantity varies between 1 and 5, and it is always a whole number.

Answers

1. (a) See the flow diagram on LB page 65 alongside.
   (b) See the table on LB page 65 alongside.
   (c) output number = input number × 3 + 8
2. (a) See the table on LB page 65 alongside.
   (b) output number = (input number + 5) × 3

5.3 Algebraic symbols for variables and relationships

DESCRIBING PROCEDURES IN DIFFERENT WAYS

Teaching guidelines

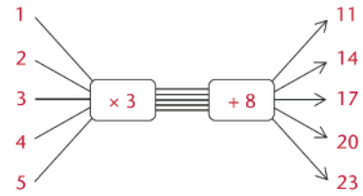
Learners progress from describing the relationship in a word formula to using symbols in the formula. The symbols replace the words "input number" and "output number".

Explain that we usually use x and y to indicate variables, but that the symbol we use is not as important as the meaning we attach to the symbol. In other words, x and y represent numbers that change, known as variables.

Answers

1. (a) See the completed table on LB page 65 alongside.
   output number = input number × 10 + 15
   input numbers: multiples of 5 starting from 5.
   output numbers: Start with 65 and add 50 to get the next term.

(a) Describe this relationship using the flow diagram.



(b) Describe the relationship between the two quantities using this table:

Input number	1	2	3	4	5
Output number	11	14	17	20	23

(c) Describe the relationship between the two quantities using a word formula.

2. The relationship between two quantities is described as follows:
   The input numbers are the first five odd numbers.

value of the one quantity — + 5 — × 3 → *the corresponding value of the other quantity*

(a) Describe this relationship using a table.

Input number	1	3	5	7	9
Output number	18	24	30	36	42

(b) Describe the relationship using a word formula.

5.3 Algebraic symbols for variables and relationships

DESCRIBING PROCEDURES IN DIFFERENT WAYS

1. In each case, do four things:
   - Complete the table.
   - Describe the relationship with a word formula.
   - Describe the input numbers in words.
   - Describe the output numbers in words.

(a) input number — × 10 — + 15 → output number

Input number	5	10	15	20	25	30
Output number	65	115	165	215	265	315

output number = ...

Answers

1. (b) See the completed table on LB page 66 alongside.
   output number = (input number + 15) × 10
   input numbers: multiples of 5 starting from 5.
   output numbers: multiples of 50 starting from 200.

   (c) See the completed table on LB page 66 alongside.
   output number = (input number × 2 + 3) × 5
   input numbers: multiples of 5 starting from 5.
   output numbers: Start with 65 and add 50 to get the next term.
2. $y = 10(x + 15)$ $y = 5(2x + 3)$
3. (a) output number = input number × 7 + 10
   (b) output number = (input number + 10) × 7
   (c) output number = (input number × 2 + 10) × 7

WRITING SYMBOLIC FORMULAE

Teaching guidelines

Give learners plenty of practice writing symbolic formulae by replacing the input variable by the letter x and the output variable by the letter y, and using flow diagrams if needed.

Misconceptions

Learners could think in terms of only one value for the variable, or not take into account that the variables could be negative numbers or fractions.

Answers

1. $y = 4x - 7$
2. $y = 5(x - 7)$
3. $y = 5(x - 7) + 3$

(b) input number → [+ 15] → [× 10] → output number

Input number	5	10	15	20	25	30
Output number	200	250	300	350	400	450

(c) input number → [× 2] → [+ 3] → [× 5] → output number

Input number	5	10	15	20	25	30
Output number	65	115	165	215	265	315

Formulae with symbols

Instead of writing "input number" and "output number" in formulae, you may just write a single letter symbol as an abbreviation.

Many years ago, mathematicians adopted the convention of using the letter symbol x as an abbreviation for the "input number", and the letter symbol y as an abbreviation for the "output number".

Letter symbols other than x and y are also used to indicate variable quantities.

The word formula you wrote for question 1(a) can be written more shortly as:

$y = 10 \times x + 15$

Mathematicians have also agreed that one may leave the ×-sign (multiplication sign) out when writing **symbolic formulae**.

Note that it is not at all wrong to use the multiplication sign in symbolic formulae.

So, instead of $y = 10 \times x + 15$, we may write $y = 10x + 15$.

2. Rewrite your word formulae in questions 1(b) and 1(c) as symbolic formulae.
3. Write a word formula for each of the following relationships:
   (a) $y = 7x + 10$ (b) $y = 7(x + 10)$ (c) $y = 7(2x + 10)$

WRITING SYMBOLIC FORMULAE

Describe each of the following relationships with a symbolic formula:

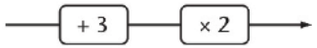
1. To calculate the output number, the input number is multiplied by 4 and 7 is subtracted from the answer.
2. To calculate the output number, 7 is subtracted from the input number and the answer is multiplied by 5.
3. To calculate the output number, 7 is subtracted from the input number, the answer is multiplied by 5 and 3 is added to this answer.

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
6.1 Algebraic language	Understanding algebraic language; interpreting expressions as calculations and instructions; how to simplify and evaluate expressions; introducing conventions of algebraic language	Pages 67 to 70
6.2 Add and subtract like terms	Knowing how to use properties of operations to rearrange terms in order to add or subtract like terms; understanding the usefulness of simplifying expressions; testing the correctness of simplifications using substitution	Pages 70 to 72

CAPS time allocation	4,5 hours
CAPS content specification	Page 90

Mathematical background

An algebraic expression is a description of certain calculations that have to be done in a certain order. An algebraic expression is just one way to describe a sequence of calculations; verbal descriptions and flow diagrams are two other ways of doing this. For example, a flow diagram explicitly shows the order in which the calculations must be done.

Words	Flow diagram	Expression
Add three to a number and then multiply the answer by two.		$2(x + 3)$

Important note: It is important to correctly understand the relationships between algebraic expressions, formulae and equations.

- An **algebraic expression** is a symbolic description of a set of calculations that can be performed on different values of a variable. For example, the expression $3x + 5$ means "*multiply the value of x by three and add five to the answer*".
- A **formula** is a description of how the values of a variable, that depends on one or more other variables, can be calculated. The formula $y = 3x + 5$ means "*to calculate the value of y that corresponds to a specific value of x, multiply the value of x by three and add five*". A symbolic formula "contains" an algebraic expression; in fact, the formula is actually an algebraic expression.
- An **equation** is a statement about an unknown number (a constant, not a variable). An equation states what result (answer) is obtained if certain calculations are performed on an unknown constant. For example, the equation $2x + 3 = 15$ states that "*when a certain unknown number is multiplied by two and three is added to the result, the answer is fifteen*". To "solve an equation" means to find out what the unknown number is.

Substitution is used to find out when expressions are equal in value. Algebraic expressions that have the same numerical value for the same values of x, but look different, are called **equivalent expressions**.

6.1 Algebraic language

WORDS, DIAGRAMS AND SYMBOLS

Teaching guidelines

Stress the fact that an expression is a set of calculation instructions. Remind learners that an expression can be given in words, as a flow diagram and as an algebraic expression using symbols.

Let learners discuss the strengths of each representation. For example, ask in what way the flow diagram is helpful.

Teach learners the conventions we use in algebraic language, for example, how to use brackets and why they are necessary (the operation in brackets is to be done first); a multiplication sign between two letter symbols or between a letter symbol and a constant can be omitted ($4 \times x = 4x$ and $a \times b = ab$); we usually write the terms in an expression so that the constant term is written last ($4x + 3y - 5$).

Misconceptions

Learners have difficulty with using brackets in expressions such as 1(a) on LB page 67 where addition has to be performed before multiplication.

Some learners forget the order of operations in an expression such as 1(d) on LB page 67 and would like to perform the operations in linear order: subtract 5 from 10 and then multiply by x.

Notes on the questions

Learners have opportunities to translate between the three forms of representation. In doing so, they recognise and interpret rules or relationships represented in symbolic form. Each representation highlights certain aspects.

When learners engage with an activity such as in question 1, they engage with algebraic conventions and language.

Answers

1. See the completed table on LB page 67 alongside.
2. (a) $-2a + b$
   (b) $2a$

CHAPTER 6

Algebraic expressions 1

6.1 Algebraic language

WORDS, DIAGRAMS AND SYMBOLS

1. Copy and complete the following table:

	Words	Flow diagram	Expression
	Multiply a number by two and add six to the answer.	× 2 → + 6 →	$2 \times x + 6$
(a)	Add three to a number and then multiply the answer by two.	+ 3 → × 2 →	$2 \times (x + 3)$
(b)	Multiply a number by five and then subtract one from the answer.	× 5 → − 1 →	$5 \times x - 1$
(c)	Multiply a number by four and then add seven to the answer.	× 4 → + 7 →	$7 + 4 \times x$
(d)	Multiply a number by negative five and then add ten to the answer.	× −5 → + 10 →	$10 - 5 \times x$

An **algebraic expression** indicates **a sequence of calculations** that can also be described in words or by means of a flow diagram.

The flow diagram illustrates the **order** in which the calculations must be done.

In algebraic language, the **multiplication sign is usually omitted**. So, we write $2x$ instead of $2 \times x$. We also write $x \times 2$ as $2x$.

2. Write the following expressions in "normal" algebraic language:
   (a) $-2 \times a + b$ (b) $a2$

CHAPTER 6: ALGEBRAIC EXPRESSIONS 1 67

LOOKING DIFFERENT BUT YET THE SAME

Teaching guidelines

Learners must see that some expressions have the same values for only some values of the variable and others have the same values for all values of the variables. Discuss with learners how expressions like $2x + 3x$ and $5x$ are the same.

Use the correct language:– i.e. use "equivalent" to describe expressions that have the same value for all values of the variable.

Misconceptions

Learners want to "close" an expression, for example, $2 + 3 = 5$, so learners incorrectly want to write $2x + 3$ as $5x$. Tables such as in question 1 on LB page 68 could help learners to understand the correct way to add expressions.

Answers

1. See the completed table on LB page 68 alongside.
2. (a) same answer (15) (b) same answer (50)
3. (a) different answers (9 and 15) (b) different answers (23 and 50)
4. $2x + 3x$ and $5x$

   They are both equal to 5 for $x = 1$.

   They are both equal to 15 for $x = 3$.

   They are both equal to 35 for $x = 7$.

   They are both equal to 50 for $x = 10$.
5. (a) True. The two expressions have the same numerical values for the same values of x.

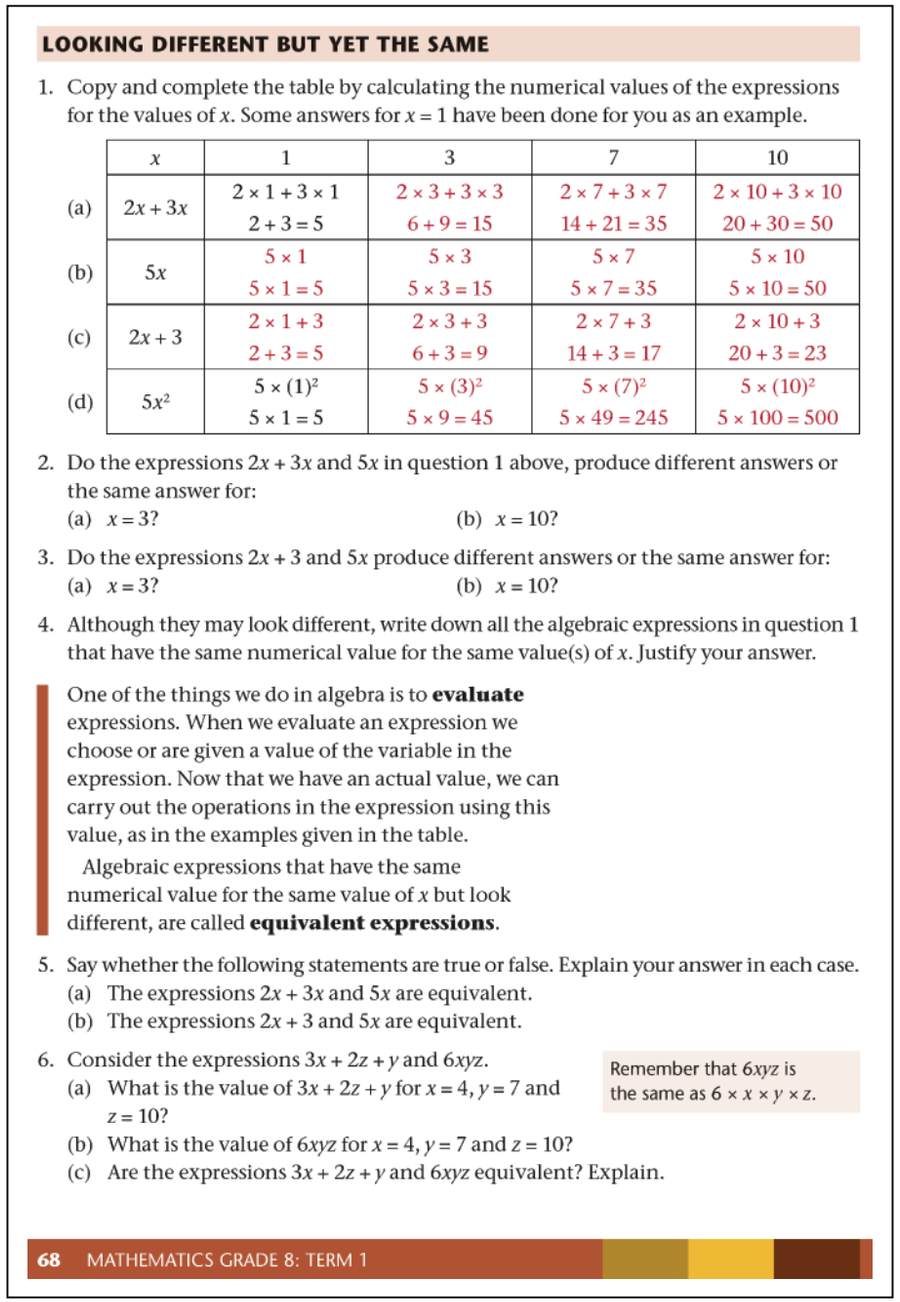
   (b) False. The two expressions have different values for the same values of x, except for $x = 1$.
6. (a) The value of $3x + 2z + y$ for $x = 4$, $y = 7$ and $z = 10$

   $= 3 \times 4 + 2 \times 10 + 7 = 12 + 20 + 7 = 39$

   (b) $6xyz$ for $x = 4$, $y = 7$ and $z = 10$

   $= 6 \times 4 \times 7 \times 10 = 1\,680$

   (c) They are not because they have different numerical values for the same values of x, y and z.



LOOKING DIFFERENT BUT YET THE SAME

1. Copy and complete the table by calculating the numerical values of the expressions for the values of x. Some answers for $x = 1$ have been done for you as an example.

	x	1	3	7	10
(a)	$2x + 3x$	$2 \times 1 + 3 \times 1$ $2 + 3 = 5$	$2 \times 3 + 3 \times 3$ $6 + 9 = 15$	$2 \times 7 + 3 \times 7$ $14 + 21 = 35$	$2 \times 10 + 3 \times 10$ $20 + 30 = 50$
(b)	$5x$	5×1 $5 \times 1 = 5$	5×3 $5 \times 3 = 15$	5×7 $5 \times 7 = 35$	5×10 $5 \times 10 = 50$
(c)	$2x + 3$	$2 \times 1 + 3$ $2 + 3 = 5$	$2 \times 3 + 3$ $6 + 3 = 9$	$2 \times 7 + 3$ $14 + 3 = 17$	$2 \times 10 + 3$ $20 + 3 = 23$
(d)	$5x^2$	$5 \times (1)^2$ $5 \times 1 = 5$	$5 \times (3)^2$ $5 \times 9 = 45$	$5 \times (7)^2$ $5 \times 49 = 245$	$5 \times (10)^2$ $5 \times 100 = 500$

2. Do the expressions $2x + 3x$ and $5x$ in question 1 above, produce different answers or the same answer for:
   (a) $x = 3$? (b) $x = 10$?
3. Do the expressions $2x + 3$ and $5x$ produce different answers or the same answer for:
   (a) $x = 3$? (b) $x = 10$?
4. Although they may look different, write down all the algebraic expressions in question 1 that have the same numerical value for the same value(s) of x. Justify your answer.

One of the things we do in algebra is to **evaluate** expressions. When we evaluate an expression we choose or are given a value of the variable in the expression. Now that we have an actual value, we can carry out the operations in the expression using this value, as in the examples given in the table.

Algebraic expressions that have the same numerical value for the same value of x but look different, are called **equivalent expressions**.

5. Say whether the following statements are true or false. Explain your answer in each case.
   (a) The expressions $2x + 3x$ and $5x$ are equivalent.
   (b) The expressions $2x + 3$ and $5x$ are equivalent.
6. Consider the expressions $3x + 2z + y$ and $6xyz$.
   (a) What is the value of $3x + 2z + y$ for $x = 4$, $y = 7$ and $z = 10$?
   (b) What is the value of $6xyz$ for $x = 4$, $y = 7$ and $z = 10$?
   (c) Are the expressions $3x + 2z + y$ and $6xyz$ equivalent? Explain.

Remember that $6xyz$ is the same as $6 \times x \times y \times z$.

Teaching guidelines

Use and explain the correct language such as: product, coefficient, term, like terms and unlike terms. Explain that:

- Products are made up of factors (the factors are separated by multiplication signs).
- Terms are separated by plus or minus signs. Terms can contain products.
- Like terms can simplify to one term, for example $3x + 7x = 10x$.

Misconceptions

Learners often find it difficult to simplify an expression like $3x^2 + 5x^2$ and want to write $8x^4$. It is useful to let them write the expressions out in full:

$$(x^2 + x^2 + x^2) + (x^2 + x^2 + x^2 + x^2 + x^2) = 8x^2$$

Answers

7. (a) $6x$ (b) $13x$ (c) $12b$ (d) $19m$ (e) $6x^2$ (f) $13x^2$
8. $3x^2 + 8x^2 + 2x^2 = 13x^2$ The coefficient of x^2 is 13.
9. (a) See answers on LB page 69 alongside.
   (b) $12 \times 3 \times 2 = 72$
   (c) The two expressions are not equivalent because they produce different results for the same values of x and y.
   (d) They are unlike terms. They have different variables.
10. (a) Expressions A, B and D
   (b)

	$x = 10$	$x = 17$	$x = 54$
A.	$6 \times 10 + 4 \times 10$	$6 \times 17 + 4 \times 17$	$6 \times 54 + 4 \times 54$
	$= 60 + 40 = 100$	$= 102 + 68 = 170$	$= 324 + 216 = 540$
B.	$10 \times 10 = 100$	$10 \times 17 = 170$	$10 \times 54 = 540$
D.	$9 \times 10 + 10$	$9 \times 17 + 17$	$9 \times 54 + 54$
	$= 90 + 10 = 100$	$= 153 + 17 = 170$	$= 486 + 54 = 540$

   (c) They are like terms because they have the same variable raised to the same power. The power of x is 1 for both terms.
   (d) They are unlike terms because even though they have the same variable (x), it is not raised to the same power. For the term $10x$, the power of x is 1 and for the term $10x^2$, the power of x is 2.

To show that the two expressions in question 5(a) are equivalent, we write $2x + 3x = 5x$.

We can explain this as:

$2x + 3x = (x + x) + (x + x + x) = 5x$

We say the expression $2x + 3x$ **simplifies** to $5x$.

The term $3x$ is a product. The number 3 is called the **coefficient** of x.

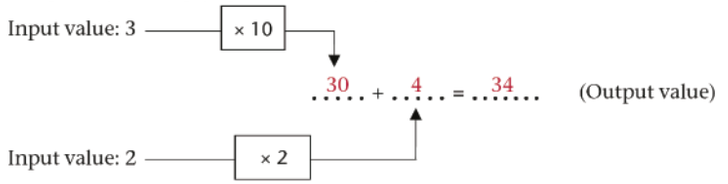
7. In each case below, write down an expression equivalent to the one given:
   (a) $3x + 3x$ (b) $3x + 8x + 2x$
   (c) $8b + 2b + 2b$ (d) $7m + 2m + 10m$
   (e) $3x^2 + 3x^2$ (f) $3x^2 + 8x^2 + 2x^2$
8. What is the coefficient of x^2 for the expression equivalent to $3x^2 + 8x^2 + 2x^2$?

In an expression that can be written as a sum, the different parts of the expression are called the **terms of the expression**. For example, $3x$, $2z$ and y are the terms of the expression $3x + 2z + y$.

An expression can have **like terms** or **unlike terms**, or both.

Like terms are terms that have the **same variable(s) raised to the same power**. The terms $2x$ and $3x$ are examples of like terms.

9. (a) Copy the diagram. Calculate the numerical value of $10x + 2y$ for $x = 3$ and $y = 2$ by completing the empty spaces in the diagram.



   (b) What is the output value for the expression $12xy$ for $x = 3$ and $y = 2$?
   (c) Are the expressions $10x + 2y$ and $12xy$ equivalent? Explain.
   (d) Are the terms $10x$ and $2y$ like or unlike terms? Explain.
10. (a) Which of the following algebraic expressions do you think will give the same results?
   A. $6x + 4x$ B. $10x$ C. $10x^2$ D. $9x + x$
   (b) Test the algebraic expressions you have identified for the following values of x:
   $x = 10$ $x = 17$ $x = 54$
   (c) Are the terms $6x$ and $4x$ like or unlike terms? Explain.
   (d) Are the terms $10x$ and $10x^2$ like or unlike terms? Explain.

Answers

11. When deciding whether terms are like terms or not, we do not concentrate on the order in which the variables appear in the terms. Both terms contain the variable x raised to the power 2 and the variable y raised to the power 3. So they are like terms. (Note: Some learners may use the commutative property of numbers to explain that $x^2 \times y^3 = y^3 \times x^2$.)
12. All three terms have exactly the same variables, raised to the same power.

6.2 Add and subtract like terms

REARRANGE TERMS AND THEN COMBINE LIKE TERMS

Teaching guidelines

If expressions are equivalent, they will have the same value for all values of the variable.

After learners have completed the table in question 1, discuss the different values with them, paying special attention to the equivalent expressions. You could ask questions such as: "Which expression would you rather use if you have to substitute values for the variable?", the answer we hope for is: "The expression with the fewest terms." One reason is that the possibility of making mistakes is smaller with fewer substitutions.

Notes on the questions

The questions have been designed for learners to experience that adding unlike terms does not give the same values as the original expression. For example, substituting the same values into $35x + 100$ and $135x$ does not give the same answers, therefore $35x + 100$ and $135x$ are not equivalent. The question could be asked: Could $35x + 100$ be simplified, or is it as simple as it can be?

Answers

1. See the completed table on LB page 70 alongside.
2. $30x + 80 + 5x + 20$ and $35x + 100$
3. For the two expressions to be equivalent, they must have the same numerical values when the same values of x are substituted in both expressions. The expressions are not equal for values of x other than $x = 1$, therefore they are not equivalent.

11. Ashraf and Hendrik have a disagreement about whether the terms $7x^2y^3$ and $301y^3x^2$ are like terms or not. Hendrik thinks they are not, because in the first term the x^2 comes before the y^3, whereas in the second term the y^3 comes before the x^2.
    Explain to Hendrik why his argument is not correct.
12. Explain why the terms $5abc$, $10acb$ and $15cba$ are like terms.

6.2 Add and subtract like terms

REARRANGE TERMS AND THEN COMBINE LIKE TERMS

1. Copy and complete the table by evaluating the expressions for the given values of x:

x	1	2	10
$30x + 80$	$30 \times 1 + 80$ $= 30 + 80 = 110$	$30 \times 2 + 80$ $= 60 + 80 = 140$	$30 \times 10 + 80$ $= 300 + 80 = 380$
$5x + 20$	$5 \times 1 + 20$ $= 5 + 20 = 25$	$5 \times 2 + 20$ $= 10 + 20 = 30$	$5 \times 10 + 20$ $= 50 + 20 = 70$
$30x + 80 + 5x + 20$	$30 \times 1 + 80 +$ $5 \times 1 + 20$ $= 30 + 80 + 5 + 20$ $= 135$	$30 \times 2 + 80 +$ $5 \times 2 + 20$ $= 60 + 80 + 10 + 20$ $= 170$	$30 \times 10 + 80 +$ $5 \times 10 + 20$ $= 300 + 80 + 50 + 20$ $= 450$
$35x + 100$	$35 \times 1 + 100$ $= 35 + 100 = 135$	$35 \times 2 + 100$ $= 70 + 100 = 170$	$35 \times 10 + 100$ $= 350 + 100 = 450$
$135x$	135×1 $= 135$	135×2 $= 270$	135×10 $= 1\,350$

2. Write down all the expressions in the table that are equivalent.
3. Tim thinks that the expressions $135x$ and $35x + 100$ are equivalent because for $x = 1$, they both have the same numerical value 135. Explain to Tim why the two expressions are not equivalent.

We have already come across the commutative and associative properties of operations. We will now use these properties to help us form equivalent algebraic expressions.

Commutative property
The order in which we add or multiply numbers does not change the answer: $a + b = b + a$ and $ab = ba$

Associative property
The way in which we group three or more numbers when adding or multiplying does not change the answer: $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$

Teaching guidelines

Explain what we mean by the term "simplify"– i.e. changing an expression into a form that is easier to work with, usually because the simplified expression involves fewer calculations.

Explain that the commutative and associative properties of operations are useful when we want to simplify expressions because they allow us to rearrange and combine the terms.

Use questions 6 to 8 to make learners aware of the usefulness of substitution as a checking device to prove the equivalence of the simplified and the original expressions.

Misconceptions

Be aware that learners who do not grasp the principles may want to add unlike terms: $35x + 100$ to get $135x$.

Notes on the questions

Even though learners will progress to adding terms wherever they are positioned in an expression, it is useful to have them physically apply the commutative and associative principles so that while they rearrange terms, they are learning how to combine like terms.

The questions in this section and the next are designed to help learners realise that manipulation of algebraic expressions is useful, because it leads to different but equivalent expressions that are convenient to work with.

Answers

4. (a) $13x + 6x + 7 - 2 = 19x + 5$
   (b) $21x + 7x - 8 + 15 = 28x + 7$
   (c) $18c + 5c - 7c - 12d = 16c - 12d$
   (d) $3abc + 7abc + 4 - 6 = 10abc - 2$
   (e) $12x^2 - 2x^2 + 2x + 8x = 10x^2 + 10x$
   (f) $7m^3 + 9m^3 + 7m^2 + 1 = 16m^3 + 7m^2 + 1$
5. (a) $15x + 7y + 25x + 3 + 2y$
   $= 15x + 25x + 7y + 2y + 3$
   $= 40x + 9y + 3$
   (b) $12mn + 8mn$
   $= 20mn$
6. D. Because $5x^2 + x + 7 + x - 9 = 5x^2 + x + x + 7 - 9 = 5x^2 + 2x - 2$
7. B. Because $6x^2 - x + 4 + x^2 - 5 = 6x^2 + x^2 - x + 4 - 5 = 7x^2 - x - 1$
8. C. Because $5x^2 + 2x + 4 + 3x^2 - 5x - 1 = 5x^2 + 3x^2 + 2x - 5x + 4 - 1$
   $= 8x^2 - 3x + 3$

We can find an equivalent expression by **rearranging** and **combining like terms**, as shown below:

$30x + 80 + 5x + 20$
Hence $30x + (80 + 5x) + 20$
Hence $30x + (5x + 80) + 20$
$= (30x + 5x) + (80 + 20)$
$= 35x + 100$

The terms 80 and 20 are called **constants**. The numbers 30 and 5 are called **coefficients**.

Brackets are used in the expression on the left to show how the like terms have been rearranged.

Like terms are combined to form a single term.

The terms $30x$ and $5x$ are combined to get the new term $35x$, and the terms 80 and 20 are combined to form the new term: 100. We say that the **expression** $30x + 80 + 5x + 20$ is **simplified** to a new expression $35x + 100$.

4. Simplify the following expressions:
   (a) $13x + 7 + 6x - 2$ (b) $21x - 8 + 7x + 15$ (c) $18c - 12d + 5c - 7c$
   (d) $3abc + 4 + 7abc - 6$ (e) $12x^2 + 2x - 2x^2 + 8x$ (f) $7m^3 + 7m^2 + 9m^3 + 1$

When you are not sure about whether or not you correctly simplified an expression, it is always advisable to check your work by evaluating the original expression and the simplified expression for some values.

When we use a value of the variable in the expression, we call it a **substitution**.

5. Make a simpler expression that is equivalent to the given expression. Test your answer for three different values of x, and redo your work until you get it right.
   (a) Simplify $(15x + 7y) + (25x + 3 + 2y)$ (b) Simplify $12mn + 8mn$

In questions 6 to 8 below, write down the letter representing the correct answer. Explain why you think your answer is correct.

6. The sum of $5x^2 + x + 7$ and $x - 9$ is:
   A. $x^2 - 2$ B. $5x^2 + 2x + 16$ C. $5x^2 + 16$ D. $5x^2 + 2x - 2$
7. The sum of $6x^2 - x + 4$ and $x^2 - 5$ is equivalent to:
   A. $7x^2 - x + 9$ B. $7x^2 - x - 1$ C. $6x^4 - x - 9$ D. $7x^4 - x - 1$
8. The sum of $5x^2 + 2x + 4$ and $3x^2 - 5x - 1$ can be expressed as:
   A. $8x^2 + 3x + 3$ B. $8x^2 + 3x - 3$ C. $8x^2 - 3x + 3$ D. $8x^2 - 3x - 3$

Combining like terms is a useful algebraic habit. It allows us to replace an expression with another expression that may be convenient to work with.

Do the questions on page 72 to get a sense of what we are talking about.

CONVENIENT REPLACEMENTS

Teaching guidelines

Point out to learners that it is much quicker to evaluate $10x$ than $x + x + x + x + x + x + x + x + x + x$.

Answers

1. $x + x + x + x + x + x + x + x + x + x = 10x$
   (a) $10 \times 2 = 20$ (b) $10 \times 50 = 500$
2. $x + x + x + z + z + y = 3x + 2z + y$
   (a) $3 \times 4 + 2 \times 10 + 7 = 12 + 20 + 7 = 39$
   (b) $3 \times 0 + 2 \times 22 + 8 = 0 + 44 + 8 = 52$
3. Yes. $3x$ and $7x$ are like terms and when we add them, we get a simplified expression, $10x$. We can replace the expression $3x + 7x$ with the expression $10x$ and then evaluate the simplified expression ($10x$) instead of the original expression.

   Evaluating $10x$ for $x = 20$ gives $10 \times 20 = 200$, which is equivalent to evaluating $3 \times 20 + 7 \times 20 = (3 + 7) \times 20 = 10 \times 20$.
4. Yes. $5x$ and $3x$ are like terms and when we add them, we get a simplified expression, $8x$. We can replace the expression $5x + 3x$ with the expression $8x$ and then evaluate the simplified expression ($8x$) instead of the original expression.

   Evaluating 8x for x = 8 gives $8 \times 8 = 64$, which is equivalent to evaluating $5 \times 8 + 3 \times 8 = (5 + 3) \times 8 = 8 \times 8$.
5. No. The terms $7x$ and 5 are unlike terms and cannot be combined by adding. We can only evaluate the expression by substituting the given value of x into the expression $7x$ and then add 5 to the answer.
   $7 \times 10 + 5 = 70 + 5 = 75$ and not 120.
6. No. $5x$ and 3 are unlike terms; they cannot be added to get $8x$.
7. $12x + 25x + 75x + 8x = 120x$ when $x = 6$: $120 \times 6 = 720$
8. $3x^2 + 7 + 2x^2 + 3 = 3x^2 + 2x^2 + 7 + 3 = 5x^2 + 10$
   when $x = 5$: $5 \times (5)^2 + 10 = 5 \times 25 + 10 = 125 + 10 = 135$
9. Zama subtracted unlike terms $2n$ and -1 and got n instead of adding $2n$ and $6n$ to get $8n$ and then simplifying the given expression to get $8n - 1$. She was supposed to evaluate the expression $8n - 1$ for $n = 4$ by doing the calculation $8 \times 4 - 1 = 32 - 1 = 31$.

   (Note: Learners may also comment on the fact that Zama calculated $(4)^2$ as $4 \times 2 = 8$ instead of as $4 \times 4 = 16$.)

CONVENIENT REPLACEMENTS

1. Consider the expression $x + x + x + x + x + x + x + x + x + x + x$. What is the value of the expression in each of the following cases?
   (a) $x = 2$ (b) $x = 50$
2. Consider the expression $x + x + x + z + z + y$. What is the value of the expression in each of the following cases?
   (a) $x = 4, y = 7, z = 10$ (b) $x = 0, y = 8, z = 22$
3. Suppose you have to evaluate $3x + 7x$ for $x = 20$. Will calculating 10×20 give the correct answer? Explain.

Suppose we evaluate the expression $3x + 7x$ for $x = 20$ without first combining the like terms. We will have to do **three** calculations, namely 3×20, then 7×20 and then find the sum of the two: $3 \times 20 + 7 \times 20 = 60 + 140 = 200$.

But if we first combine the like terms $3x$ and $7x$ into one term $10x$, we only have to do **one calculation**: $10 \times 20 = 200$. This is one way of thinking about the convenience or usefulness of simplifying an algebraic expression.

4. The expression $5x + 3x$ is given and you are required to evaluate it for $x = 8$. Will calculating 8×8 give the correct answer? Explain.
5. Suppose you have to evaluate $7x + 5$ for $x = 10$. Will calculating 12×10 give the correct answer? Explain.
6. The expression $5x + 3$ is given and you have to evaluate it for $x = 8$. Will calculating 8×8 give the correct answer? Explain.

Samantha was asked to evaluate the expression $12x^2 + 2x - 2x^2 + 8x$ for $x = 12$. She thought to herself that just substituting the value of x directly into the terms would require a lot of work. She first combined the like terms as shown below:

$12x^2 - \mathbf{2x^2 + 2x} + 8x$
$= 10x^2 + 10x$

The terms $+ 2x$ and $- 2x^2$ change positions by the commutative property of operations.

Then for $x = 10$, Samantha found the value of $10x^2 + 10x$ by calculating:

$10 \times 10^2 + 10 \times 10$
$= 1\,000 + 100$
$= 1\,100$

Use Samatha's way of thinking for questions 7 to 9.

7. What is the value of $12x + 25x + 75x + 8x$ when $x = 6$?
8. Evaluate $3x^2 + 7 + 2x^2 + 3$ for $x = 5$.
9. When Zama was asked to evaluate the expression $2n - 1 + 6n$ for $n = 4$, she wrote down the following:
   $2n - 1 + 6n = n + 6n = 6n^2$
   Hence for $n = 4$: $6 \times (4)^2 = 6 \times 8 = 48$. Explain where Zama went wrong and why.

WORKSHEET

Teaching guidelines

This worksheet can be used for formative assessment or as a test for summative assessment.

Answers

1. See the completed table shown on LB page 73 alongside.
2. A and C. In A, the variable is y for both terms and is raised to the same power. In C, the variables are y and z; y is raised to the power 2 and z to the power 1.
3. (a) $2c + 3d$
   (b) $7def$
4. (a) the coefficient of x^2
   (b) -5
   (c) a constant
5. All three terms have exactly the same variables, raised to the same power.
6. (a) $7 + 8 = 15$
   (b) $9 \times 7 = 63$
   (c) $7 - 7 = 0$
7. (a) $18c + 5c - 7c + 12d = 16c + 12d$
   (b) $3def + 7def + 4 - 6 = 10def - 2$
8. (a) $2 \times (3)^2 + 3 \times (-1)$
   $= 2 \times 9 + (-3)$
   $= 18 - 3 = 15$
   (b) $(2 \times 3)^2 + 3 \times (-1)$
   $= (6)^2 - 3$
   $= 36 - 3 = 33$
9. (a) $20y$ (b) $10c$ (c) $20b - 2$
   (d) $m + 3n + 2$ (e) $3h^2 + 20$ (f) $7e^2f + 7ef + 2$
10. (a) $3y + 3y + 3y + 3y + 3y + 3y = 18y$
    $18 \times 18 = 324$
    (b) $13y + 14 - 3y + 6 = 10y + 20$
    $10 \times 200 + 20 = 2\,000 + 20 = 2\,020$
    (c) $20 - y^2 + 101y^2 + 80 = 100y^2 + 100$
    $100 \times (1)^2 + 100 = 100 + 100 = 200$
    (d) $12y^2 + 3yz + 18y^2 + 2yz = 30y^2 + 5yz$
    $30 \times (3)^2 + 5 \times 3 \times 2 = 30 \times 9 + 30 = 270 + 30 = 300$

WORKSHEET

1. Copy and complete the following table:

	Words	Flow diagram	Expression
(a)	Multiply a number by three and add two to the answer	$\times 3$ → $+ 2$ →	$3x + 2$
(b)	Multiply a number by nine and subtract six from the answer	$\times 9$ → $- 6$ →	$9x - 6$
(c)	Multiply a number by seven and subtract three from the answer	$\times 7$ → $- 3$ →	$7x - 3$

2. Which of the following pairs consist of like terms? Explain.
   A. $3y; -7y$ B. $14e^2; 5e$ C. $3y^2z; 17y^2z$ D. $-bcd; 5bd$
3. Write the following in the "normal" algebraic way:
   (a) $c2 + d3$ (b) $7 \times d \times e \times f$
4. Consider the expression $12x^2 - 5x + 3$.
   (a) What is the number 12 called? (b) Write down the coefficient of x.
   (c) What name is given to the number 3?
5. Explain why the terms $5pqr$; $-10prq$ and $15qrp$ are like terms.
6. If $y = 7$, what is the value of each of the following?
   (a) $y + 8$ (b) $9y$ (c) $7 - y$
7. Simplify the following expressions:
   (a) $18c + 12d + 5c - 7c$ (b) $3def + 4 + 7def - 6$
8. Evaluate the following expressions for $y = 3$, $z = -1$:
   (a) $2y^2 + 3z$ (b) $(2y)^2 + 3z$
9. Write each of the following algebraic expressions in the simplest form:
   (a) $5y + 15y$ (b) $5c + 6c - 3c + 2c$ (c) $4b + 3 + 16b - 5$
   (d) $7m + 3n + 2 - 6m$ (e) $5h^2 + 17 - 2h^2 + 3$ (f) $7e^2f + 3ef + 2 + 4ef$
10. Evaluate each of the following expressions:
    (a) $3y + 3y + 3y + 3y + 3y + 3y$ for $y = 18$
    (b) $13y + 14 - 3y + 6$ for $y = 200$
    (c) $20 - y^2 + 101y^2 + 80$ for $y = 1$
    (d) $12y^2 + 3yz + 18y^2 + 2yz$ for $y = 3$ and $z = 2$

ALGEBRAIC EXPRESSIONS 1

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
7.1 Setting up equations	Finding numbers, called solutions, that make the equations true	Page 74
7.2 Solving equations by inspection	Using tables to recognise equivalent expressions (with the same solution)	Pages 75 to 76
7.3 More examples	Finding solutions by inspection and testing them	Page 76

CAPS time allocation	3 hours
CAPS content specification	Page 91

Mathematical background

An **equation** states what result (answer) is obtained if certain calculations are performed on an unknown number (a constant, not a number). To "solve an equation" means to find out what the unknown number is.

The focus in this chapter is on solving equations by inspection only. To clarify what "solution by inspection" may be taken to mean, it is helpful to consider different examples of equations:

- Equation 1: $12 + x = 20$ is quite easy to solve. The question to answer is "12 plus what equals 20?" Basic number knowledge tells us that $12 + 8 = 20$ and therefore $x = 8$, which makes this equation true.
- Equation 2: $3x + 8 = 20$ can be solved in almost the same way as the first equation, but it requires an extra step. We know that $12 + 8 = 20$, therefore $3x = 12$. We also know that $3 \times 4 = 12$, so it follows that $x = 4$.

These two examples demonstrate one form of "solving by inspection". Solution by inspection promotes understanding of what is meant by "the solution of an equation", namely *the numerical value of the unknown that makes the equation true*. It is therefore helpful for learners to spend some time doing it.

- Equation 3: $5x + 4 = 3x + 10$ cannot be solved as above. One way to try to solve it is to assign different values to x and compare the left-hand and right-hand sides of the equation. For example, if x is 1: $5x + 4 = 9$ and $3x + 10 = 13$; $x = 1$ is not the solution. If $x = 2$: $5x + 4 = 14$ and $3x + 10 = 16$. If $x = 3$: $5x + 4 = 19$ and $3x + 10 = 19$; so 3 is the solution. This method can also be called "solving by inspection", but it clearly differs from the method used for equations 1 and 2 above. We literally "searched" for the solution to Equation 3 by trying different values for x. It is a "trial and improve" or a "numerical search" method.
- Equation 4: $3x + 5 = 61 - 5x$: A numerical search for the solution can be described as follows: choose $x = 1$: $3x + 5 = 8$ and $61 - 5x = 56$, i.e. $3x + 5$ is 48 less than $61 - 5x$; choose $x = 10$: $3x + 5 = 35$ and $61 - 5x = 11$, i.e. $3x + 5$ is 24 more than $61 - 5x$. The solution is between 1 and 10. It may be useful to try $x = 5$.
  It may be useful to show the results of the numerical search in a table as shown here.
  The solution is the value of x for which there is no difference.

x	1	10	5	6	7
$3x + 5$	8	35	20	23	26
$61 - 5x$	56	11	36	31	26
Difference	–48	24	–16	–8	0

7.1 Setting up equations

Teaching guidelines

Write an expression on the board, for example $x + 13$ and ask learners what the value of x should be. The answer could be any value. Then write $x + 13 = 19$ and ask the same question. Now there is only one possible value that x can have: 6. It is the value that makes the equation true. Remind learners of the difference between an equation and an expression.

Learners practise writing equations in question 2.

Notes on the questions/tasks

Up to and including Grade 7, the term "number sentences" is used, so the term "equation" may be new for Grade 8 learners.

Answers

1. (a) False. $-3 - 3 = -6$ (b) False. $-2 \times -2 \times -2 = -8$
   (c) False. $3 \times -3 = -9$ (d) False. $3 \times 1 = 3$
   (e) True. $6 \times 7 + 5 = 42 + 5 = 47$
2. (a) 8. Because $8 \times 10 = 80$ (or because $80 \div 10 = 8$)
   (b) 17. Because $17 + 83 = 100$ (or because $100 - 83 = 17$)
   (c) 20. Because $20 \div 5 = 4$ (or because $4 \times 5 = 20$)
   (d) 5. Because $5 \times 4 = 20$ (or because $20 \div 4 = 5$)
   (e) 50. Because $50 \times 2 = 100$ (or because $100 \div 2 = 50$)
   (f) 2. Because $2 \times 2 \times 2 \times 2 \times 2 = 32$
   (g) Doesn't exist. A negative number raised to the power four (an even power) is always positive.
   (h) 6. Because $6 \times 15 = 90$ (or because $90 \div 15 = 6$)
   (i) -96. Because $-96 + 93 = -3$ (or because $-3 - 93 = -96$)
   (j) 30. Because $30 \div 2 = 15$ (or because $15 \times 2 = 30$)
3. (a) Eight times a number equals seventy-two. The number is 9.
   (b) Two fifths of a number equals two. The number is 5.
   (c) Two times a number plus five equals twenty-one. The number is 8.
   (d) Twelve plus nine times a number equals thirty. The number is 2.
   (e) Thirty minus two times a number equals forty. The number is -5.
   (f) Five times a number plus four equals three times that same number plus ten. The number is 3.

CHAPTER 7
Algebraic equations 1

7.1 Setting up equations

An **equation** is a mathematical sentence that is true for some numbers but false for other numbers. The following are examples of equations:

$x + 3 = 11$ and $2x = 8$

$x + 3 = 11$ is true if $x = 8$, but false if $x = 3$.

When we look for a number or numbers that make an equation true, we say that we are **solving the equation**. For example, $x = 4$ is the **solution** of $2x = 8$ because it makes $2x = 8$ true. (Check: $2 \times 4 = 8$)

LOOKING FOR NUMBERS TO MAKE STATEMENTS TRUE

1. Are the following statements true or false? Justify your answer.
   (a) $x - 3 = 0$, if $x = -3$ (b) $x^3 = 8$, if $x = -2$ (c) $3x = -6$, if $x = -3$
   (d) $3x = 1$, if $x = 1$ (e) $6x + 5 = 47$, if $x = 7$
2. Find the original number. Show your reasoning.
   (a) A number multiplied by 10 is 80.
   (b) Add 83 to a number and the answer is 100.
   (c) Divide a number by 5 and the answer is 4.
   (d) Multiply a number by 4 and the answer is 20.
   (e) Twice a number is 100. (f) A number raised to the power 5 is 32.
   (g) A number raised to the power 4 is –81. (h) Fifteen times a number is 90.
   (i) 93 added to a number is –3. (j) Half a number is 15.
3. Write the equations below in words using "a number" in place of the letter symbol x. Then write what you think "the number" is in each case.
   Example: $4 + x = 23$. *Four plus a number equals twenty-three. The number is 19.*
   (a) $8x = 72$ (b) $\frac{2x}{5} = 2$ (c) $2x + 5 = 21$
   (d) $12 + 9x = 30$ (e) $30 - 2x = 40$ (f) $5x + 4 = 3x + 10$

74 MATHEMATICS GRADE 8: TERM 1

7.2 Solving equations by inspection

Teaching guidelines

The concept of one value of the variable that makes an equation true is reinforced in this section. Let learners compare $2x + 3 = x + 4$ with $2x + 3 = 9 - x$, for example, which are two different equations with different solutions.

Let learners try to find the solution to the equation $3x - 2 = 2x + 3$ by extending the table or the two expressions.

Notes on the questions/tasks

Note that when learners solve equations by numerical search as demonstrated here, they would simultaneously practise other items in the CAPS, namely "determine the numerical value of an expression by substitution" and "use substitution in equations to generate tables of ordered pairs". In the table, the rows for x and $2x + 3$ for example, is a set of ordered pairs, as well as the rows for x and $x + 4$, and so on.

Answers

1. (a) $x = 0$ (b) $x = 1$ (c) $x = 1$
   (d) $x = 2$ (e) $x = 3$ (f) $x = 2$
   (g) $x = 2$
2. Equations (b) and (c): the solution for both equations is $x = 1$.
   Equations (d), (f) and (g): the solution for all three equations is $x = 2$.
3. See the completed table on LB page 75 alongside.
   (a) No
   (b) The value of both expressions becomes bigger as x increases.
   (c) Yes, between $x = 10$ and $x = 15$.
   (d) $x = 13$

7.2 Solving equations by inspection

THE ANSWER IS IN PLAIN SIGHT

1. Seven equations are given in the following table. Use the table to find out for which of the given values of x it will be true that the left-hand side of the equation is equal to the right-hand side.

You can read the solutions of an equation from a table.

x	−3	−2	−1	0	1	2	3	4
$2x + 3$	−3	−1	1	3	5	7	9	11
$x + 4$	1	2	3	4	5	6	7	8
$9 - x$	12	11	10	9	8	7	6	5
$3x - 2$	−11	−8	−5	−2	1	4	7	10
$10x - 7$	−37	−27	−17	−7	3	13	23	33
$5x + 3$	−12	−7	−2	3	8	13	18	23
$10 - 3x$	19	16	13	10	7	4	1	−2

(a) $2x + 3 = 5x + 3$ (b) $5x + 3 = 9 - x$ (c) $2x + 3 = x + 4$
(d) $10x - 7 = 5x + 3$ (e) $3x - 2 = x + 4$ (f) $9 - x = 2x + 3$
(g) $10 - 3x = 3x - 2$

Two or more equations can have the same solution. For example, $5x = 10$ and $x + 2 = 4$ have the same solution; $x = 2$ is the solution for both equations.

Two equations are called **equivalent** if they have the same solution.

2. Which of the equations in question 1 have the same solutions? Explain.

3. Copy and complete the following table. Then answer the questions that follow.

You can also do a search by narrowing down the possible solution to an equation.

x	0	5	10	15	20	25	30	35	40
$2x + 3$	3	13	23	33	43	53	63	73	83
$3x - 10$	−10	5	20	35	50	65	80	95	110

(a) Can you find a solution for $2x + 3 = 3x - 10$ in the table?
(b) What happens to the values of $2x + 3$ and $3x - 10$ as x increases? Do they become bigger or smaller?
(c) Is there a point where the value of $3x - 10$ becomes bigger or smaller than the value of $2x + 3$ as the value of x increases? If so, between which x-values does this happen?

This point where the two expressions are equal is called the **break-even point**.

7.3 More examples

Teaching guidelines

Learners should understand what we mean by "finding a solution by inspection". It is "seeing" the answer because one sees the relationship between numbers; for example in $3x = 21$ we have $x = 7$ because $3 \times 7 = 21$.

Answers

1. (a) $x = 7$ (b) $x = 7$ (c) $x = 7$ (d) $x = 7$
2. (a) 7 Because $12 \times 7 = 84$
   (b) 7 Because $\frac{84}{7} = 12$
   (c) 5 Because $8 \times 5 + 8 = 40 + 8 = 48$
   (d) 2 Because $19 - 8 \times 2 = 19 - 16 = 3$
   (e) 4 Because $6 \times 4 - 4 = 24 - 4 = 20$
   (f) -4 Because $-4 \times -4 \times -4 = 16 \times -4 = -64$
   (g) 3 Because $5 \times 5 \times 5 = 25 \times 5 = 125$
   (h) 3 Because $2 \times 2 \times 2 = 8$
   (i) 3 Because $3 \times 3 = 9$
3. (a) $m = 92$ $92 + 8 = 100$ (b) $x = 20$ $20 + 60 = 80$
   (c) $k = 26$ $26 - 26 = 0$ (d) $y = 0$ $105 \times 0 = 0$
   (e) $k = 1$ $1 \times 10 = 10$ (f) $x = 20$ $5 \times 20 = 100$
   (g) $t = 3$ $\frac{15}{3} = 5$ (h) $t = 15$ $\frac{15}{5} = 3$
4. (a) $12x = 36$
   $x = 3$
   $12 \times 3 + 14 = 36 + 14 = 50$
   (b) $75 = 15m$
   $m = 5$
   $15 \times 5 + 25 = 75 + 25 = 100$
   (c) $100 = 20x$
   $x = 5$
   $\frac{100}{5} = 20$
   (d) $7m = 35$
   $m = 5$
   $7 \times 5 + 5 = 35 + 5 = 40$
   (e) $2x = 2$
   $x = 1$
   (f) $3x = 21$
   $x = 7$
   (g) $2x = -10$
   $x = -5$
   (h) $\frac{x}{7} = 3$
   $x = 21$
   (i) $36 = 9x$
   $x = 4$
   (j) $x = 4 \times \frac{6}{2}$
   $x = 12$

(d) Now that you narrowed down where the possible solution can be, try other possible values for x until you find out for what value of x the statement $2x + 3 = 3x - 10$ is true.

"Searching" for the solution of an equation by using tables or by narrowing down to the possible solution is called **solution by inspection**.

7.3 More examples

LOOKING FOR AND CHECKING SOLUTIONS

1. What is the solution for the equations shown below?
   (a) $x - 3 = 4$ (b) $x + 2 = 9$ (c) $3x = 21$ (d) $3x + 1 = 22$

When a certain number is the solution of an equation we say that the number **satisfies** the equation. For example, $x = 4$ satisfies the equation $3x = 12$ because $3 \times 4 = 12$.

2. Choose the number in brackets that satisfies the equation. Explain your choice.
   (a) $12x = 84$ {5; 7; 10; 12}
   (b) $\frac{84}{x} = 12$ {−7; 0; 7; 10}
   (c) $48 = 8k + 8$ {−5; 0; 5; 10}
   (d) $19 - 8m = 3$ {−2; −1; 0; 1; 2}
   (e) $20 = 6y - 4$ {3; 4; 5; 6}
   (f) $x^3 = -64$ {−8; −4; 4; 8}
   (g) $5^x = 125$ {−3; −1; 1; 3}
   (h) $2^x = 8$ {1; 2; 3; 4}
   (i) $x^2 = 9$ {1; 2; 3; 4}
3. What makes the following equations true? Check your answers.
   (a) $m + 8 = 100$ (b) $80 = x + 60$ (c) $26 - k = 0$ (d) $105 \times y = 0$
   (e) $k \times 10 = 10$ (f) $5x = 100$ (g) $\frac{15}{t} = 5$ (h) $3 = \frac{t}{5}$
4. Solve the following equations by inspection. Check your answers.
   (a) $12x + 14 = 50$ (b) $100 = 15m + 25$
   (c) $\frac{100}{x} = 20$ (d) $7m + 5 = 40$
   (e) $2x + 8 = 10$ (f) $3x + 10 = 31$
   (g) $-1 + 2x = -11$ (h) $2 + \frac{x}{7} = 5$
   (i) $100 = 64 + 9x$ (j) $\frac{2x}{6} = 4$

Term 2

Chapter 8: Algebraic expressions 2 93
8.1 Expanding algebraic expressions 94
8.2 Simplifying algebraic expressions 97
8.3 Simplifying quotient expressions 99
8.4 Squares, cubes and roots of expressions 102

Chapter 9: Algebraic equations 2 105
9.1 Thinking forwards and backwards 106
9.2 Solving equations using the additive and multiplicative inverses 108
9.3 Solving equations involving powers 109

Chapter 10: Construction of geometric figures 111
10.1 Bisecting lines 112
10.2 Constructing perpendicular lines 114
10.3 Bisecting angles 116
10.4 Constructing special angles without a protractor 118
10.5 Constructing triangles 120
10.6 Properties of triangles 122
10.7 Properties of quadrilaterals 123
10.8 Constructing quadrilaterals 125

Chapter 11: Geometry of 2D shapes 129
11.1 Types of triangles 130
11.2 Unknown angles and sides of triangles 132
11.3 Types of quadrilaterals and their properties 137
11.4 Unknown angles and sides of quadrilaterals 140
11.5 Congruency 141
11.6 Similarity 142

Chapter 12: Geometry of straight lines 147
12.1 Angles on a straight line 148
12.2 Vertically opposite angles 151
12.3 Lines intersected by a transversal 153
12.4 Parallel lines intersected by a transversal 156
12.5 Finding unknown angles on parallel lines 159
12.6 Solving more geometric problems 161

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
8.1 Expanding algebraic expressions	Using the distributive property to produce a sum of terms (sum expressions) by expansion of products (product expressions)	Pages 77 to 80
8.2 Simplifying algebraic expressions	Using rearrangements of terms (commutative and associative properties) to add like terms	Pages 80 to 82
8.3 Simplifying quotient expressions	Learning how to simplify algebraic expressions	Pages 82 to 85
8.4 Squares, cubes and roots of expressions	Simplifying squares and cubes and finding square roots and cube roots of expressions	Pages 85 to 87

CAPS time allocation	9 hours
CAPS content specification	Pages 92 to 93

Mathematical background

Algebraic expressions can be written in many different but equivalent ways, for example: $2(x - 15) = 2x - 30$.

An algebraic expression describes calculations that have to be done in a certain order. A sequence of calculations can also be described using words or flow diagrams.

A **flow diagram** explicitly shows the order in which the calculations must be done. For example:



- The first flow diagram translates into words as "multiply a number by 3 and add 2 to the result". The algebraic expression is $3x + 2$.
- The second flow diagram translates into words as "add 3 to a number and multiply the result by 2". The algebraic expression is $2(x + 3)$. We could apply the distributive property of operations and remove the brackets, thereby expanding the product to a sum, $2(x + 3) = 2x + 6$.
- We use the **commutative and associative properties** of operations (where they apply) to rearrange terms. Addition and multiplication are commutative and associative, but subtraction and division are not ($4 + 2 = 2 + 4$, but $4 - 2 \neq 2 - 4$ and $3 \times 2 = 2 \times 3$, but $6 \div 3 \neq 3 \div 2$).
- We rearrange terms in order to combine like terms.

All the above we do to **simplify expressions**.

We substitute values into expressions. Algebraic expressions that have the same numerical value for the same values of x, but look different, are called **equivalent expressions**.

When we simplify expressions in order to find their square or cube roots, we have to end up with a single term (**product expression**).

8.1 Expanding algebraic expressions

MULTIPLY OFTEN OR MULTIPLY ONCE: IT IS YOUR CHOICE

Teaching guidelines

Using the distributive property reduces the number of calculations needed. Compare the number of calculations for the following expressions:

$ab + ac$ ... three calculations; two multiplications and an addition

$a(b + c)$ ... two calculations; an addition and a multiplication

For example, let $x = 3$ in the following calculations:

$3x + 3 \times 2 = 3 \times 3 + 3 \times 2 = 9 + 6 = 15$ ... three calculations

$3(x + 2) = 3(3 + 2) = 3 \times 5 = 15$ ... two calculations

Misconceptions

Learners may want to conjoin (add incorrectly), for example $x + 2$ being expressed as $2x$. Filling in the table should prevent this as the teacher could point out to learners which expressions give the same results for the same values of x. If there are learners who still persist in conjoining unlike terms, use tables to show them it is incorrect:

$x + 2$ for $x = 1; 2; 3; 4$, etc. gives 3; 4; 5; 6; etc. and

$2x$ for $x = 1; 2; 3; 4$, etc., the same values, gives 2; 4; 6; 8; etc.

Notes on the questions

By doing the questions learners may learn to remove brackets correctly and see the use of the distributive property as a way to limit the number of calculations they have to do.

Answers

1. (a) $5 \times 13 = \mathbf{65}$ $5 \times 87 = \mathbf{435}$ $\mathbf{65} + \mathbf{435} = 500$
   (b) $13 + 87 = \mathbf{100}$ $\mathbf{100} \times 5 = 500$
2. (a) $10 \times 56 = 10 \times 50 + 10 \times 6 = 500 + 60 = 560$
   (b) $10 \times 16 = \mathbf{160}$; $10 \times 40 = \mathbf{400}$; $\mathbf{160} + \mathbf{400} = 560$
3. (a) Sample answer: $12 = x$, $2 = y$. $6(x + y) = 6 \times (12 + 2) = 6 \times 14 = 84$
   (b) Sample answer: $6 \times x = 6 \times 12 = 72$; $6 \times y = 6 \times 2 = 12$; $72 + 12 = 84$
4. (a) and (b) See the completed table on LB page 77 alongside.

CHAPTER 8
Algebraic expressions 2

8.1 Expanding algebraic expressions

MULTIPLY OFTEN OR MULTIPLY ONCE: IT IS YOUR CHOICE

1. (a) Calculate 5×13 and 5×87 and add the two answers.
   (b) Add 13 and 87, and then multiply the answer by 5.
   (c) If you do not get the same answer for questions 1(a) and 1(b), you have made a mistake. Redo your work until you get it right.

If you work correctly, you get the same answer for questions 1(a) and 1(b); this is an example of a certain property of addition and multiplication called the **distributive property**. You use this property each time you multiply a number in parts. For example, you may calculate 3×24 by calculating 3×20 and 3×4, and then add the two answers:

$3 \times 24 = 3 \times 20 + 3 \times 4$

What you saw in question 1 was that $5 \times 100 = 5 \times 13 + 5 \times 87$. This can also be expressed by writing $5(13 + 87)$.

The word *distribute* means "to spread out". The distributive properties may be described as follows:
$a(b + c) = ab + ac$ and
$a(b - c) = ab - ac$,
where a, b and c can be any numbers.

2. (a) Calculate 10×56. (b) Calculate $10 \times 16 + 10 \times 40$.
3. Write down any two numbers smaller than 100. Let us call them x and y.
   (a) Add your two numbers and multiply the answer by 6.
   (b) Calculate $6 \times x$ and $6 \times y$ and add the two answers.
   (c) If you do not get the same answers for (a) and (b), you have made a mistake somewhere. Correct your work.
4. Copy and complete the following table:

(a)

x	1	2	3	4	5
$3(x + 2)$	9	12	15	18	21
$3x + 6$	9	12	15	18	21
$3x + 2$	5	8	11	14	17
$3(x - 2)$	–3	0	3	6	9
$3x - 6$	–3	0	3	6	9
$3x - 2$	1	4	7	10	13

Teaching guidelines

Remind learners that where brackets or algebraic symbols are multiplied, we need not write the multiplication sign. For example, $3 \times x = 3x$ and $3 \times (x + 4)$ can be written as $3(x + 4)$, which makes the expression easier to read.

Remind learners of the order of operations – i.e. the calculations in brackets are done first, if they can be done. For example:

$3(2x + 5 + x) = 3(3x + 5)$ but we cannot simplify inside the brackets any further.

Answers

4. (c) $3(x + 2)$ and $3x + 6$ are equivalent.

   $3(x - 2)$ and $3x - 6$ are equivalent.

   Each pair of expressions has the same numerical values for the same values of x.

   (d) None.

   (e) Learners should find that there are no such numbers.

5. (a) $5 \times 6 + 15 = 30 + 15 = 45$

   (b) $5 \times (6 + 3) = 5 \times 9 = 45$

   (c) Yes we can, because using the distributive property, we can express $5(x + 3)$ as $5x + 15$.

   The two expressions are equivalent expressions and therefore have the same numerical values for the same values of x.

6. The completed flow diagrams are shown on LB page 78 alongside and LB page 79 on the following page.

(b) If you do not get the same answers for the expressions $3(x + 2)$ and $3x + 6$, and for $3(x - 2)$ and $3x - 6$, you have made a mistake somewhere. Correct your work.

In algebra we normally write $3(x + 2)$ instead of $3 \times (x + 2)$. The expression $3 \times (x + 2)$ does not mean that you should first multiply by 3 when you evaluate the expression for a certain value of x. The brackets tell you that the first thing you should do is add the value(s) of x to 2 and then multiply the answer by 3.

However, instead of first adding the values within the brackets and then multiplying the answer by 3, we may just do the calculation $3 \times x + 3 \times 2 = 3x + 6$ as shown in the table.

(c) Which expressions amongst those given in the table on page 77 are equivalent? Explain.

(d) For what value(s) of x is $3(x + 2) = 3x + 2$?

(e) Try to find a value of x such that $3(x + 2) \neq 3x + 6$.

If multiplication is the last step in evaluating an algebraic expression, then the expression is called a **product expression** or, briefly, a **product**. The way you evaluated the expression $3(x + 2)$ in the table is an example of a product expression.

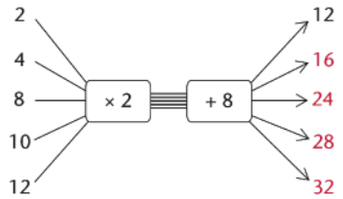
5. (a) Determine the value of $5x + 15$ if $x = 6$.

   (b) Determine the value of $5(x + 3)$ if $x = 6$.

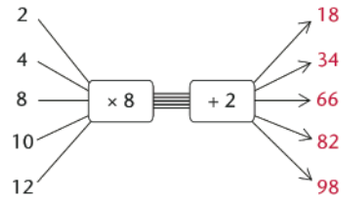
   (c) Can we use the expression $5x + 15$ to calculate the value of $5(x + 3)$ for any values of x? Explain.

6. Copy and complete the following flow diagrams:

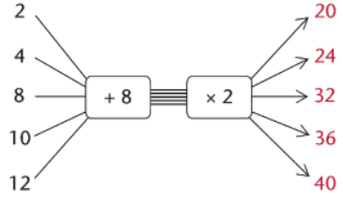
(a)



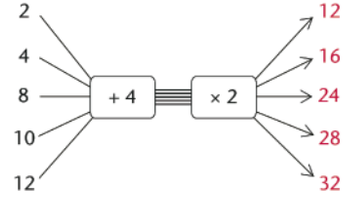
(b)



(c)



(d)



Answers

7. (a) (a) and (d) produce the same output numbers and so do (c) and (f).
   (b) diagram (a): $(x \times 2) + 8$ or $2x + 8$
   diagram (b): $(x \times 8) + 2$ or $8x + 2$
   diagram (c): $(x + 8) \times 2$ or $2(x + 8)$
   diagram (d): $(x + 4) \times 2$ or $2(x + 4)$
   diagram (e): $(x + 1) \times 8$ or $8(x + 1)$
   diagram (f): $(x \times 2) + 16$ or $2x + 16$

PRODUCT EXPRESSIONS AND SUM EXPRESSIONS

Teaching guidelines

The repeated additions in questions 1(a) and 2(a) are simplified using the abbreviated way of writing repeated addition (see notes below), which leads to product expressions.

In questions 1(b) and 2(b) the terms are rearranged using the commutative property and then the short way of writing repeated addition is applied to the like terms, which leads to sum expressions.

Show learners that the answers in the (a) part of each question are equivalent to the answers in the (b) part – i.e. the (b) parts are the expansions of the (a) parts.

Misconceptions

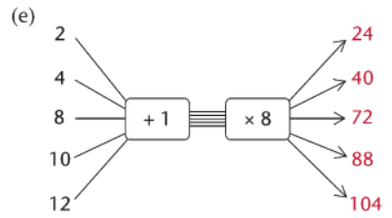
Check that learners do not try to write $5(9x)$ in question 2.

Notes on the questions

Repeated addition, for example: $x + x + x + x + x + x +$ for n **terms**, can be abbreviated to $n \times x$ or simply nx, whereas repeated multiplication is abbreviated using the index notation, for example $x \times x \times x \times$ for n **factors** is written as x^n.

Answers

1. See the completed expressions on LB page 79 alongside.
2. See the completed expressions on LB page 79 alongside.
3. (a) $3x + 21$ (b) $20x + 10$
   (c) $4x^2 + 6x$ (d) $6p + 3q$
   (e) $t^2 + 9t$ (f) $xy + xz$
   (g) $2b^2 + 2ab - 8b$ (h) $k^3 - k^2m$



7. (a) Which of the flow diagrams in question 6(a) to (f) produce the same output numbers?
   (b) Write an algebraic expression for each of the flow diagrams in question 6.

PRODUCT EXPRESSIONS AND SUM EXPRESSIONS

1. Copy and complete the following:
   (a) $(3 + 6) + (3 + 6) + (3 + 6) + (3 + 6) + (3 + 6)$
   $= 5 \times (3 + 6)$
   (b) $(3 + 6) + (3 + 6) + (3 + 6) + (3 + 6) + (3 + 6)$
   $= (3 + 3 + 3 + 3 + 3) + (6 + 6 + 6 + 6 + 6)$
   $= (5 \times 3) + (5 \times 6)$

2. Copy and complete the following:
   (a) $(3x + 6) + (3x + 6) + (3x + 6) + (3x + 6) + (3x + 6)$
   $= 5(3x + 6)$
   (b) $(3x + 6) + (3x + 6) + (3x + 6) + (3x + 6) + (3x + 6)$
   $= (3x + 3x + 3x + 3x + 3x) + (6 + 6 + 6 + 6 + 6)$
   $= (5 \times 3x) + (5 \times 6)$

3. In each case, write an expression without brackets that will give the same results as the given expression.
   (a) $3(x + 7)$ (b) $10(2x + 1)$
   (c) $x(4x + 6)$ (d) $3(2p + q)$
   (e) $t(t + 9)$ (f) $x(y + z)$
   (g) $2b(b + a - 4)$ (h) $k^2(k - m)$

The process of writing product expressions as sum expressions is called **expansion**. It is sometimes also referred to as **multiplication of algebraic expressions**.

Teaching guidelines

Remind learners how to correctly multiply by negative numbers.

Answers

4. (a) See the completed table on LB page 80 alongside.
   Learners practise substitution using the table.
   (b) From the table it is clear the $3(x + 2y + 4z) = 3x + 6y + 12z$

5. (a) $2x^2 + 2x + 2$
   (b) $pq + pr + ps$
   (c) $-3x - 6y - 9z$
   (d) $2x^3 + x^2 + 7x$
   (e) $48x - 12x^2$
   (f) $48x - 12x^2$
   (g) $24x^2 - 15x - 24x^2 + 20x = 5x$
   (h) $10x(5x) = 50x^2$

8.2 Simplifying algebraic expressions

EXPAND, REARRANGE AND THEN COMBINE LIKE TERMS

Teaching guidelines

Learners should be able to apply the distributive principle (remove brackets) correctly by now.

Take care when explaining how to simplify expressions with negative numbers, that learners preserve the rules of multiplication with negative numbers.

Remind learners that $(5 + x)^2$ means $(5 + x)(5 + x)$. They may have difficulty removing these brackets. You could explain it, using the distributive property, as follows:

$(5 + x)(5 + x) = 5(5 + x) + x(5 + x) = 25 + 5x + 5x + x^2$ and simplify to $25 + 10x + x^2$ by combining like terms.

Answers

1. (a) $3x + 6$
   (b) $20x + 5x + 15 = 25x + 15$
   (c) $11x + 28$
   (d) $x^2 + 10x + 25$
   (e) $6x + 9$
   (f) $x^2 + 2$

4. (a) Copy and complete the following table for the given values of x, y and z.

	$3(x + 2y + 4z)$	$3x + 6y + 12z$	$3x + 2y + 4z$
$x = 1$ $y = 2$ $z = 3$	$3(1 + 4 + 12)$ $= 3 \times 17$ $= 51$	$3 + 12 + 36$ $= 51$	$3 + 4 + 12$ $= 19$
$x = 10$ $y = 20$ $z = 30$	$3(10 + 40 + 120)$ $= 3 \times 170$ $= 510$	$30 + 120 + 360$ $= 510$	$30 + 40 + 120$ $= 190$
$x = 23$ $y = 60$ $z = 100$	$3(23 + 120 + 400)$ $= 3 \times 543$ $= 1\,629$	$69 + 360 + 1\,200$ $= 1\,629$	$69 + 120 + 400$ $= 589$
$x = 14$ $y = 0$ $z = 1$	$3(14 + 0 + 4)$ $= 3 \times 18$ $= 54$	$42 + 0 + 12$ $= 54$	$42 + 0 + 4$ $= 46$
$x = 5$ $y = 9$ $z = 32$	$3(5 + 18 + 128)$ $= 3 \times 151$ $= 453$	$15 + 54 + 384$ $= 453$	$15 + 18 + 128$ $= 161$

   (b) Which sum expression and product expression are equivalent?

5. For each expression, write an equivalent expression without brackets.
   (a) $2(x^2 + x + 1)$
   (b) $p(q + r + s)$
   (c) $-3(x + 2y + 3z)$
   (d) $x(2x^2 + x + 7)$
   (e) $6x(8 - 2x)$
   (f) $12x(4 - x)$
   (g) $3x(8x - 5) - 4x(6x - 5)$
   (h) $10x(3x(8x - 5) - 4x(6x - 5))$

8.2 Simplifying algebraic expressions

EXPAND, REARRANGE AND THEN COMBINE LIKE TERMS

1. Write the shortest possible equivalent expression without brackets.
   (a) $x + 2(x + 3)$
   (b) $5(4x + 3) + 5x$
   (c) $5(x + 5) + 3(2x + 1)$
   (d) $(5 + x)^2$
   (e) $-3(x^2 + 2x - 3) + 3(x^2 + 4x)$
   (f) $x(x - 1) + x + 2$

When you are not sure if you have simplified an expression correctly, you should always check your work by evaluating the original expression and the simplified expression for some values of the variables.

Teaching guidelines

Remind learners to be vigilant when they remove brackets after a negative number, for example $-2(x + 1) = -2 \times x + (-2) \times 1 = -2x - 2$. This is an area where learners often make mistakes.

Another mistake learners often make is that they do not multiply all the terms inside a bracket by the coefficient in front of the bracket when they remove brackets, for example for $3(x - 2z)$ they write $3x - 2z$ instead of $3x - 6z$.

Answers

2. (a) $120 + (5 \times 100) - 20 = 120 + 500 - 20 = 600$
   (b) $6 \times 102 = 6 \times 100 = 600$
   (c) Yes, because when you simplify $x(x + 2) + 5x^2 - 2x$ you get $6x^2$. The two expressions are therefore equivalent expressions. Equivalent expressions always have the same numerical values for the same values of x.
3. (a) $3x + 6 = 3(-5) + 6 = -15 + 6 = -9$
   (b) $25x + 15 = 25(-5) + 15 = -125 + 15 = -110$
   (c) $11x + 28 = 11(-5) + 28 = -55 + 28 = -27$
   (d) $x^2 + 10x + 25 = (-5)^2 + 10(-5) + 25 = 25 - 50 + 25 = 0$
   (e) $6x + 9 = 6(-5) + 9 = -30 + 9 = -21$
   (f) $x^2 + 2 = (-5)^2 + 2 = 25 + 2 = 27$
4. See the completed table on LB page 81 alongside.
5. (a) True (b) False
6. (a) $3x - 2y - z$ (b) $-x + 3y - 6z$

2. (a) Evaluate $x(x + 2) + 5x^2 - 2x$ for $x = 10$.
   (b) Evaluate $6x^2$ for $x = 10$.
   (c) Can we use the expression $6x^2$ to calculate the values of the expression $x(x + 2) + 5x^2 - 2x$ for any given value of x? Explain.

This is how a sum expression for $x(x + 2) + 5x^2 - 2x$ can be made:

$$\begin{aligned} x(x + 2) + 5x^2 - 2x &= x \times x + x \times 2 + 5x^2 - 2x \\ &= x^2 + 2x + 5x^2 - 2x \\ &= x^2 + 5x^2 + 2x - 2x \quad \text{[Rearrange and combine like terms.]} \\ &= 6x^2 + 0 \\ &= 6x^2 \end{aligned}$$

3. Evaluate the following expressions for $x = -5$:
   (a) $x + 2(x + 3)$ (b) $5(4x + 3) + 5x$
   (c) $5(x + 5) + 3(2x + 1)$ (d) $(5 + x)^2$
   (e) $-3(x^2 + 2x - 3) + 3(x^2 + 4x)$ (f) $x(x - 1) + x + 2$
4. Copy and complete the following table for the given values of x, y and z.

x	100	80	10	20	30
y	50	40	5	5	20
z	20	30	2	15	10
$x + (y - z)$	130	90	13	10	40
$x - (y - z)$	70	70	7	30	20
$x - y - z$	30	10	3	0	0
$x - (y + z)$	30	10	3	0	0
$x + y - z$	130	90	13	10	40
$x - y + z$	70	70	7	30	20

5. Say whether the following statements are true or false. Refer to the table in question 4. For any values of x, y and z:
   (a) $x + (y - z) = x + y - z$ (b) $x - (y - z) = x - y - z$
6. Write the expressions without brackets. Do not simplify.
   (a) $3x - (2y + z)$ (b) $-x + 3(y - 2z)$

We can simplify algebraic expressions by using properties of operations as shown:

$(5x + 3) - 2(x + 1)$

Hence $5x + 3 - 2x - 2$

Hence $5x - 2x + 3 - 2$

Hence $3x + 1$

$x - (y + z) = x - y - z$

Addition is both associative and commutative.

Answers

7. (a) $22x + 13x - 5 = 35x - 5$ (b) $22x - 13x + 5 = 9x + 5$
   (c) $22x - 13x - 5 = 9x - 5$ (d) $4x - 15 + 6x = 10x - 15$
8. (a) $2x^2 + 2 - x - 2 = 2x^2 - x$ (b) $-3x^2 - 6x + 9 + 3x^2 = -6x + 9$

8.3 Simplifying quotient expressions

FROM QUOTIENT EXPRESSIONS TO SUM EXPRESSIONS

Teaching guidelines

Remind learners to be careful when they work with negative numbers.

Let learners know that we may not divide by 0, as it is not defined.

Misconceptions

Learners often simplify an expression like $\frac{7x^2 + 5x}{x}$ incorrectly by dividing only the first term in the bracket, $(7x^2 + 5x)$, writing $7x + 5x$ as the answer instead of $7x + 5$.

Answers

1. See the completed table on LB page 82 alongside.
2. (a) $7 \times 0 + 5 = 0 + 5 = 5$
   (b) Division by 0 is not allowed, as it is undefined.

7. Write an equivalent expression without brackets for each of the following expressions and then simplify:
   (a) $22x + (13x - 5)$ (b) $22x - (13x - 5)$
   (c) $22x - (13x + 5)$ (d) $4x - (15 - 6x)$
8. Simplify:
   (a) $2(x^2 + 1) - x - 2$ (b) $-3(x^2 + 2x - 3) + 3x^2$

Some of the techniques we have used so far to form equivalent expressions include:

- remove brackets
- rearrange terms
- combine like terms.

8.3 Simplifying quotient expressions

FROM QUOTIENT EXPRESSIONS TO SUM EXPRESSIONS

1. Copy and complete the table for the given values of x.

x	1	7	−3	−10
$7x^2 + 5x$	$7 + 5$ $= 12$	$(7 \times 49) + (5 \times 7)$ $= 343 + 35$ $= 378$	$(7 \times 9) + (5 \times -3)$ $= 63 - 15$ $= 48$	$700 - 50$ $= 650$
$\frac{7x^2 + 5x}{x}$	$\frac{7+5}{1}$ $= 12$	$\frac{(7 \times 49) + (5 \times 7)}{7}$ $= \frac{343 + 35}{7} = \frac{378}{7}$ $= 54$	$\frac{(7 \times 9) - 15}{-3}$ $= \frac{63 - 15}{-3} = \frac{48}{-3}$ $= -16$	$\frac{700 - 50}{-10}$ $= \frac{650}{-10}$ $= -65$
$7x + 5$	$7 + 5$ $= 12$	$49 + 5$ $= 54$	$-21 + 5$ $= -16$	$-70 + 5$ $= -65$
$7x + 5x$	$7 + 5$ $= 12$	$49 + 35$ $= 84$	$-21 - 15$ $= -36$	$-70 - 50$ $= -120$
$7x^2 + 5$	$7 + 5$ $= 12$	$(7 \times 49) + 5$ $= 343 + 5$ $= 348$	$63 + 5$ $= 68$	$700 + 5$ $= 705$

2. (a) What is the value of $7x + 5$ for $x = 0$?
   (b) What is the value of $\frac{7x^2 + 5x}{x}$ for $x = 0$?

Teaching guidelines

Dividing an expression by a single term is applying the distributive property of operations. The explanation is: we can write $\frac{7x^2+5x}{x}$ as $\frac{1}{x}(7x^2+5x)$ because division by a number is the same as multiplying by the reciprocal of the number. The second expression above can then be written as $\frac{1}{x}(7x^2+5x) = \frac{7x^2}{x}+\frac{5x}{x}$, removing the brackets.

Define the term **quotient expression** as an algebraic fraction.

Answers

2. (c) $7x + 5$. It requires only two calculations: one multiplication and one addition.

   $\frac{7x^2+5x}{x}$ requires five calculations: three multiplications, one addition and one division.

   (d) Yes, because they have the same numerical values for the permissible values of x.

   (e) No, because no other expressions have the same numerical values for the same values of x, except for where $x = 1$.

3. See the completed table on LB page 83 alongside.

   (a) $2 - 0 = 2$

   (b) $5x = 0$. Division by 0 is undefined. The expression has no value.

   (c) Yes, they are equivalent (have the same numerical values) for all values of x, except $x = 0$. Division by 0 is not admissible, so the second expression has no value for $x = 0$.

   (d) $2 - x$. It requires only one calculation. $\frac{10x-5x^2}{5x}$ requires six calculations: four multiplications, one subtraction and one division.

(c) Which of the two expressions, $7x + 5$ or $\frac{7x^2+5x}{x}$, requires fewer calculations? Explain.

(d) Are the expressions $7x + 5$ and $\frac{7x^2+5x}{x}$ equivalent, $x = 0$ excluded? Explain.

(e) Are there any other expressions that are equivalent to $\frac{7x^2+5x}{x}$ from those given in the table? Explain.

If division is the last step in evaluating an algebraic expression, then the expression is called a **quotient expression** or an **algebraic fraction**.

The expression $\frac{7x^2+5x}{x}$ is an example of a quotient expression or algebraic fraction.

3. Copy and complete the following table for the given values of x:

x	5	10	−5	−10
$10x - 5x^2$	$50 - 125$ $= -75$	$100 - 500$ $= -400$	$-50 - 125$ $= -175$	$-100 - 500$ $= -600$
$5x$	5×5 $= 25$	5×10 $= 50$	5×-5 $= -25$	5×-10 $= -50$
$\frac{10x-5x^2}{5x}$	$\frac{50-125}{25}$ $= \frac{-75}{25}$ $= -3$	$\frac{100-500}{50}$ $= \frac{-400}{50}$ $= -8$	$\frac{-50-125}{-25}$ $= \frac{-175}{-25}$ $= 7$	$\frac{-100-500}{-50}$ $= \frac{-600}{-50}$ $= 12$
$2 - x$	$2 - 5$ $= -3$	$2 - 10$ $= -8$	$2 - (-5)$ $= 7$	$2 - (-10)$ $= 12$

(a) What is the value of $2 - x$ for $x = 0$?

(b) What is the value of $\frac{10x-5x^2}{5x}$ for $x = 0$?

(c) Are the expressions $2 - x$ and $\frac{10x-5x^2}{5x}$ equivalent, $x = 0$ excluded? Explain.

(d) Which of the two expressions $2 - x$ or $\frac{10x-5x^2}{5x}$ requires fewer calculations? Explain.

We have found that quotient expressions, such as $\frac{10x-5x^2}{5x}$ can sometimes be manipulated to give equivalent expressions, such as $2 - x$.

The value of this is that these equivalent expressions require fewer calculations.

Teaching guidelines

This is probably a good time to let learners understand that whenever they work with an expression like $\frac{10x-5x^2}{5x}$ they should keep in mind that x may not take the value 0. They could even write $\frac{10x-5x^2}{5x}$ $(x \neq 0)$ which will be helpful when they solve equations in future work.

Work through the process of dividing by using the distributive property.

Answers

4. (a) $4x + 5z + 3$
   (b) $5x^2 + 4x$
   (c) $9x + 1$
   (d) $3b - 2a$

5. (a) $\frac{5x^2+3x}{x^2} = 5 + \frac{3}{x} = 5 + \frac{3}{-1} = 5 + (-3) = 2$
   (b) $x = 0$, because $x^2 = 0$ and division by 0 is undefined and therefore not allowed.

Notes on the questions

To **simplify** means different things in different contexts. It depends on what you have to do.

Sometimes reducing an expression to one term is called **simplifying it**, for example $3x + 9 + 5x + 15$ has to be simplified to one term:

- rearrange: $3x + 5x + 9 + 15$
- combine like terms: $8x + 24$
- apply the distributive property: $8(x + 3)$ one term.

Simplifying the following $3(x + 2) + 5(x + 3)$ can only be done if the brackets are removed so that like terms can be added.

- $3(x + 2) + 5(x + 3) = 3x + 6 + 5x + 15 = 8x + 21$ this is the simplest form of the expression.

The expressions $\frac{10x-5x^2}{5x}$ and $2 - x$ are not quite equivalent because for $x = 0$, the value of $2 - x$ can be calculated, while the first expression has no value.

However, we can say that the two expressions are equivalent if they have the same values for all values of x admissible for both expressions.

How is it possible that $\frac{7x^2+5x}{x} = 7x + 5$ and $\frac{10x-5x^2}{5x} = 2 - x$ for all admissible values of x? We say $x = 0$ is not an admissible value of x because division by 0 is not allowed.

One of the methods for finding equivalent expressions for algebraic fractions is by means of division:

$$\frac{7x^2+5x}{x} = \frac{1}{x}(7x^2 + 5x) \quad \text{[just as } \frac{3}{5} = 3 \times \frac{1}{5}\text{]}$$
$$= (\frac{1}{x} \times 7x^2) + (\frac{1}{x} \times 5x) \quad \text{[distributive property]}$$
$$= \frac{7x^2}{x} + \frac{5x}{x}$$
$$= 7x + 5 \quad \text{[provided } x \neq 0\text{]}$$

4. Use the method shown on the previous page to simplify each fraction below:
   (a) $\frac{8x+10z+6}{2}$ (b) $\frac{20x^2+16x}{4}$
   (c) $\frac{9x^2y+xy}{xy}$ (d) $\frac{21ab-14a^2}{7a}$

Simplifying a quotient expression can sometimes lead to a result which still contains quotients, as you can see in the following example:

$$\frac{5x^2+3x}{x^2}$$
$$= \frac{5x^2}{x^2} + \frac{3x}{x^2}$$
$$= 5 + \frac{3}{x}$$

5. (a) Evaluate $\frac{5x^2+3x}{x^2}$ for $x = -1$.
   (b) For the expression $\frac{5x^2+3x}{x^2}$ to be equivalent to $5 + \frac{3}{x}$, which value of x must be excluded? Why?

Teaching guidelines

Let learners write down next to the answers for question 6 which values of x are excluded.

You could ask learners, regarding question 9, whose method they think is better and to give a reason for their choice. (Natasha's method has fewer steps, is therefore faster and leaves less room for mistakes.)

Answers

6. (a) $4x + \frac{2}{x} + 1$ (b) $4 + \frac{1}{n}$
7. (a) $4(2) + \frac{2}{2} + 1 = 10$ (b) $4 + \frac{1}{n} = 4\frac{1}{4}$
8. (a) $3x^3 - 6x^2 + \frac{1}{x}$ (b) $-n^3 - \frac{2}{3}$
9. Natasha first simplified the expression $\frac{x^2+2x+1}{x}$ to $x + 2 + \frac{1}{x}$ and then evaluated the simplified expression. Lebogang substituted $x = 10$ in the original expression.

8.4 Squares, cubes and roots of expressions

SIMPLIFYING SQUARES AND CUBES

Teaching guidelines

You could show learners that $(3x)^2 = 3x \times 3x = \ldots = 3 \times 3 \times x \times x = (3)^2(x)^2 = 9x^2$. This will reinforce the laws of exponents they learnt in the first term.

Let learners use the third law of exponents when they get to question 1(c) $(-3y)^2 = (-3)^2y^2 = 9y^2$. They should be careful when they work with negative numbers.

Misconceptions

Learners often make mistakes with calculations like $(2x^2)^2$ where they do not square the second factor. Making them write out the factors, as in the example $(3x \times 3x)$ in the LB, could help them avoid this.

Answers

1. (a) $4x^2$ (b) $4x^4$ (c) $9y^2$
2. (a) $9x$ (b) $8y$ (c) $15a$
3. (a) $81x^2$ (b) $64y^2$ (c) $225a^2$

6. Simplify the following expressions:
   (a) $\frac{8x^2 + 2x + 4}{2x}$ (b) $\frac{4n + 1}{n}$
7. Evaluate:
   (a) $\frac{8x^2 + 2x + 4}{2x}$ for $x = 2$ (b) $\frac{4n + 1}{n}$ for $n = 4$
8. Simplify:
   (a) $\frac{6x^4 - 12x^3 + 2}{2x}$ (b) $\frac{-6n^4 - 4n}{6n}$
9. When Natasha and Lebogang were asked to evaluate the expression $\frac{x^2 + 2x + 1}{x}$ for $x = 10$, they did it in different ways.

   Natasha's calculation:
   $10 + 2 + \frac{1}{10}$
   $= 12\frac{1}{10}$

   Lebogang's calculation:
   $\frac{100 + 20 + 1}{10}$
   $= \frac{121}{10}$
   $= 12\frac{1}{10}$

   Explain how each of them thought about evaluating the given expression.

8.4 Squares, cubes and roots of expressions

SIMPLIFYING SQUARES AND CUBES

Study the following example:

$(3x)^2 = 3x \times 3x$	Meaning of squaring
$= 3 \times x \times 3 \times x$	
$= 3 \times 3 \times x \times x$	Multiplication is commutative: $a \times b = b \times a$
$= 9x^2$	We say that $(3x)^2$ simplifies to $9x^2$

1. Simplify the following expressions:
   (a) $(2x)^2$ (b) $(2x^2)^2$ (c) $(-3y)^2$
2. Simplify the following expressions:
   (a) $25x - 16x$ (b) $4y + y + 3y$ (c) $a + 17a - 3a$
3. Simplify:
   (a) $(25x - 16x)^2$ (b) $(4y + y + 3y)^2$ (c) $(a + 17a - 3a)^2$

Answers

4. (a) $8x^3$ (b) $-x^3$ (c) $125a^3$
   (d) $343y^6$ (e) $-27m^3$ (f) $8x^9$
5. (a) $3a$ (b) $10x$ (c) $5b$
6. (a) $(3a)^3 = 27a^3$ (b) $(10x)^3 = 1\,000x^3$ (c) $(5b)^3 = 125b^3$
   (d) $(7x)^3 = 343x^3$ (e) $(20x)^3 = 8\,000x^3$ (f) $(6y)^3 = 216y^3$

SQUARE AND CUBE ROOTS OF EXPRESSIONS

Teaching guidelines

Learners should be familiar with the concept of a square root (and a cube root) from previous work, for example $\sqrt{144} = \sqrt{12 \times 12} = 12$, the square root of a number is the number that, multiplied by itself, gives the number of which it is the square root. Finding the square root of a product, as in question 2, should not be too difficult for learners if they grasped the concept.

Misconceptions

When learners do question 4(c) $\sqrt{25a^2 - 16a^2}$, ensure that they do not attempt to find the square roots of the terms separately – i.e. $5a - 4a$. If they do try it this way, substitution of a few values of the variable could help them see the error. For example, use $a = 2$ in the original expression, in the correct answer and then in the wrong answer.

- Original expression: $\sqrt{25a^2 - 16a^2} = \sqrt{25 \times 4 - 16 \times 4} = \sqrt{36} = 6$
- Correct simplified expression: $\sqrt{9a^2} = \sqrt{9 \times 4} = 6$
- Incorrect simplified answer: $5a - 4a = a = 2$. Therefore, we cannot take the square roots of terms, we have to add or subtract them first. If they cannot be added, it means the square root cannot be taken. For example, the square root of $\sqrt{25a^2 - 16y^2}$ cannot be taken unless the values of x and y are known.

Answers

1. Learners should realise that Vuyiswa's method is unnecessarily complicated.
2. (a) True. The number that is squared is $6x$.
   (b) True. The number that is squared is $5x^2$.
3. (a) y^{12} (b) $169x^2$
4. (a) $\sqrt{y^6 \times y^6} = y^6$ (b) $\sqrt{169x^2} = \sqrt{13x \times 13x} = 13x$
   (c) $\sqrt{9a^2} = \sqrt{3a \times 3a} = 3a$ (d) $\sqrt{11y \times 11y} = 11y$

Study the following example:

$(3x)^3 = 3x \times 3x \times 3x$ — Meaning of cubing

$= 3 \times x \times 3 \times x \times 3 \times x$

$= 3 \times 3 \times 3 \times x \times x \times x$ — Multiplication is commutative: $a \times b = b \times a$

$= 27x^3$ — We say that $(3x)^3$ simplifies to $27x^3$.

4. Simplify:
   (a) $(2x)^3$ (b) $(-x)^3$ (c) $(5a)^3$
   (d) $(7y^2)^3$ (e) $(-3m)^3$ (f) $(2x^3)^3$
5. Simplify:
   (a) $5a - 2a$ (b) $7x + 3x$ (c) $4b + b$
6. Simplify:
   (a) $(5a - 2a)^3$ (b) $(7x + 3x)^3$ (c) $(4b + b)^3$
   (d) $(13x - 6x)^3$ (e) $(17x + 3x)^3$ (f) $(20y - 14y)^3$

Always remember to test if the simplified expression is equivalent to the given expression for at least three different values of the given variable.

SQUARE AND CUBE ROOTS OF EXPRESSIONS

1. Thabang and his friend Vuyiswa were asked to simplify $\sqrt{2a^2 \times 2a^2}$.
   Thabang reasoned as follows:
   To find the square root of a number is the same as asking yourself the question: "Which number was multiplied by itself?" The number that is multiplied by itself is $2a^2$ and therefore $\sqrt{2a^2 \times 2a^2} = 2a^2$.
   Vuyiswa reasoned as follows:
   I should first simplify $2a^2 \times 2a^2$ to get $4a^4$ and then calculate $\sqrt{4a^4} = 2a^2$.
   Which of the two methods do you prefer? Explain why.
2. Say whether each of the following is true or false. Give a reason for your answer.
   (a) $\sqrt{6x \times 6x} = 6x$ (b) $\sqrt{5x^2 \times 5x^2} = 5x^2$
3. Simplify:
   (a) $y^6 \times y^6$ (b) $125x^2 + 44x^2$
4. Simplify:
   (a) $\sqrt{y^{12}}$ (b) $\sqrt{125x^2 + 44x^2}$
   (c) $\sqrt{25a^2 - 16a^2}$ (d) $\sqrt{121y^2}$

Teaching guidelines

Make sure that learners combine the terms first before taking the square root.

Remind learners that the order of a root is shown by a number in front of the sign, for example the "3" in front of the cube root sign, for a fourth root there will be a "4". If we mean to find a square root, there need not be a "2" in front of the sign.

Answers

4. (e) $\sqrt{25a^2} = \sqrt{5a \times 5a} = 5a$ (f) $\sqrt{16a^2} = \sqrt{4a \times 4a} = 4a$
5. It means to find out what expression, raised to the power of 3, equals $8x^3$.
6. (a) $8a^3$ (b) $1\,000b^9$ (c) $27x^9$ (d) $-27x^9$
7. (a) $10b^3$ (b) $2a$ (c) $3x$ (d) $-3x$
8. (a) $8x^3$ (b) $-8m^3$
9. (a) $2x$ (b) $-2m$ (c) $5y$ (d) $6a$

WORKSHEET

Teaching guidelines

You can use this worksheet as a test for summative assessment or for formative assessment.

Answers

1. (a) $6b + 6$ (b) $4 - 5e$
   (c) $110mn$ (d) $13pqr + 3$
2. (a) 320 (b) 600
3. (a) $2b + 3$ (b) $\frac{4b+6}{2} = 2b + 3 = 200 + 3 = 203$
4. (a) $4g \times 4g = 16g^2$ (b) $6y \times 6y \times 6y = 216y^3$ (c) $100s^2$
5. (a) $11b$ (b) $4y$ (c) $9d$

(e) $\sqrt{16a^2 + 9a^2}$ (f) $\sqrt{25a^2 - 9a^2}$

5. What does it mean to find the cube root of $8x^3$ written as $\sqrt[3]{8x^3}$?
6. Simplify:
   (a) $2a \times 2a \times 2a$ (b) $10b^3 \times 10b^3 \times 10b^3$
   (c) $3x^3 \times 3x^3 \times 3x^3$ (d) $-3x^3 \times -3x^3 \times -3x^3$
7. Determine the following:
   (a) $\sqrt[3]{1000b^9}$ (b) $\sqrt[3]{2a \times 2a \times 2a}$ (c) $\sqrt[3]{27x^3}$ (d) $\sqrt[3]{-27x^3}$
8. Simplify each of the following expressions:
   (a) $6x^3 + 2x^3$ (b) $-m^3 - 3m^3 - 4m^3$
9. Determine the following:
   (a) $\sqrt[3]{6x^3 + 2x^3}$ (b) $\sqrt[3]{-8m^3}$ (c) $\sqrt[3]{125y^3}$ (d) $\sqrt[3]{93a^3 + 123a^3}$

WORKSHEET

1. Simplify:
   (a) $2(3b + 1) + 4$ (b) $6 - (2 + 5e)$
   (c) $18mn + 22mn + 70mn$ (d) $4pqr + 3 + 9pqr$
2. Evaluate each of the following expressions for $m = 10$:
   (a) $3m^2 + m + 10$ (b) $5(m^2 - 5) + m^2 + 25$
3. (a) Simplify: $\frac{4b+6}{2}$
   (b) Evaluate the expression $\frac{4b+6}{2}$ for $b = 100$.
4. Simplify:
   (a) $(4g)^2$ (b) $(6y)^3$ (c) $(7s + 3s)^2$
5. Determine the following:
   (a) $\sqrt{121b^2}$ (b) $\sqrt[3]{64y^3}$ (c) $\sqrt{63d^2 + 18d^2}$

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
9.1 Thinking forwards and backwards	Using flow diagrams to introduce the use of inverse operations and addition and multiplication properties of equations	Pages 88 to 90
9.2 Solving equations using the additive and multiplicative inverses	Solving equations using additive and multiplicative inverses and addition of like terms to isolate the unknown	Pages 90 to 91
9.3 Solving equations involving powers	Solving equations where the unknowns are exponents	Page 91

CAPS time allocation	3 hours
CAPS content specification	Page 94

Mathematical background

An **equation** states that two expressions are equal.

A **linear equation** in one variable, x, can be written in the form $ax = b$, where a and b are constants and $a \neq 0$.

The value of x that makes the equation true is called a solution of the equation.

In order **to solve an equation** we need to isolate the variable on one side of the equation. Depending on the equation, for example: $5(3 - x) + 2(3 - x) = 14$, we could:

- use the distributive property to remove brackets that group terms: $15 - 5x + 6 - 2x = 14$
- simplify the expression on each side of the equation by combining (adding) like terms: $21 - 7x = 14$
- change the equation to isolate the variable terms on one side of the equation and the constant terms on the other side: $-7x = 14 - 21$; $-7x = -7$
- solve the equation and check the solution $x = -7 \div -7 = 1$; test LHS of the original equation.

To change an equation to the simplest form $ax = b$, we could use inverse operations to produce equivalent equations, which are equations that have the same solution(s). Inverse operations undo each other. We use the inverse operations together with the following properties:

- the addition (or subtraction) property of equality – adding (or subtracting) the same number to each side of an equation gives an equivalent equation
- the multiplication (or division) property of equality – multiplying (or dividing) each side of an equation by the same non-zero number gives an equivalent equation.

To solve an equation with powers, the side that does not contain the unknown should be changed to the exponential form with the same base as the other side. The reasoning is that the equality means the two sides of the equation have the same value. Since the bases are the same and the powers are same (by the = sign), the exponents have to be the same.

9.1 Thinking forwards and backwards

DOING AND UNDOING WHAT HAS BEEN DONE

Teaching guidelines

Learners have solved equations before by inspection (seeing the answer or using tables), now they will use flow diagrams to see what was done to an input value or variable and then undo it from the known output value working backwards to find what the input was. For example, if $4x + 2 = 22$:

- we started with x then multiplied by 4 and added 2
- so start with 22 and undo the addition, subtract 2 to get 20
- then undo the multiplication by dividing by 4, which gives 5
- you get to the unknown $x = 5$.

Learners learn to do the backward calculations using flow diagrams and tables.

Answers

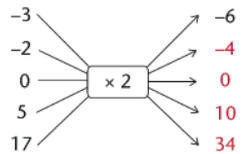
1. See the answers on LB page 88 alongside.
2. See the completed table on LB page 88 alongside.
3. (a) $4x = -28$ (b) $4x = 40$ (c) $4x = 0$
4. (a) See the completed flow diagram on LB page 88 alongside.
   (b) $x = -17$ $-68 \div 4 = -17$
   (c) I divided the output numbers by 4. I did the opposite of the calculation in the flow diagram.
5. (a) See the completed table on LB page 88 alongside.
   (b) See the completed flow diagrams on LB page 88 alongside.
   (c) I divided each output number by 5 to find the input number.

CHAPTER 9
Algebraic equations 2

9.1 Thinking forwards and backwards

DOING AND UNDOING WHAT HAS BEEN DONE

1. Copy and complete the flow diagram on the right by finding the output values.

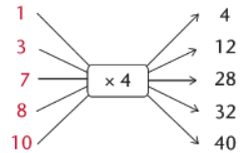


2. Copy and complete the following table:

x	–3	–2	0	5	17
$2x$	–6	–4	0	10	34

3. Evaluate $4x$ if:
   (a) $x = -7$ (b) $x = 10$ (c) $x = 0$

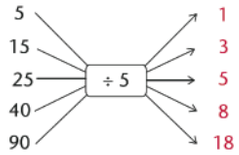
4. (a) Copy and complete the flow diagram on the right by finding the input values.
   (b) Puleng put another integer into the flow diagram and got –68 as an answer. Which integer did she put in? Show your calculation.
   (c) Explain how you worked to find the input numbers when you did question 4(a).

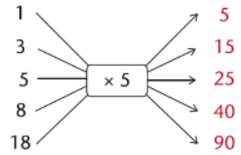


5. (a) Copy and complete the following table:

x	1	3	5	8	18
$5x$	5	15	25	40	90

   (b) Copy and complete the following flow diagrams:





   (c) Explain how you completed the table.

Teaching guidelines

Evaluating an expression by replacing the variable with an input number is called **substitution**. Starting from the output and working backwards to find what the input was is called **solving the equation**.

Learners work with the pairs of flow diagrams to get the concept of **inverse operations**.

Let learners write equations for questions 11 to 14 using x for the unknown number.

Answers

6. Question 1: Doing process, input to output
   Question 2: Doing process, input to output
   Question 3: Doing process, input to output
   Question 4: Undoing process, output to input
   Question 5(a): Undoing process, output to input
   Question 5(b): Doing process, input to output
7. to 9. See the answers on LB page 89 alongside.
10. (a) See the answer on LB page 89 alongside.
    (b) I will first subtract 12 from 20 and then multiply the answer by 8.
11. $10x = 150$. The number is 15 (because $150 \div 10 = 15$).
12. $x \div 5 = 1$. The number is 5 (because $5 \times 1 = 5$).
13. $x + 23 = 107$. The number is 84 (because $107 - 23 = 84$).

One of the things we do in algebra is to **evaluate** expressions. When we evaluate expressions we replace a variable in the expression with an **input number** to obtain the value of the expression called the **output number**. We think of this process as a **doing process**.

However, in other cases we may need to undo what was done. When we know what output number was obtained but do not know what input number was used, we have to **undo** what was done in evaluating the expression. In such a case we say we are **solving an equation**.

6. Look again at questions 1 to 5. For each question, say whether the question required a doing or an undoing process. Give an explanation for your answer (for example: input to output).

7. (a) Copy and complete the following flow diagrams:

   1 —[+ 6]→ 7 7 —[− 6]→ 1

   (b) What do you observe?

8. (a) Copy and complete the following flow diagrams:

   2 —[× 5]→ 10 10 —[÷ 5]→ 2

   (b) What do you observe?

9. (a) Copy and complete the following flow diagrams:

   20 —[× 5]—[+ 5]→ 105 105 —[− 5]—[÷ 5]→ 20

   (b) What do you observe?

10. (a) Copy and complete the following flow diagram:

    64 —[÷ 8]—[+ 12]→ 20

    (b) What calculations will you do to determine what the input number was when the output number is 20?

Solve the following problems by undoing what was done to get the answer:

11. When a certain number is multiplied by 10 the answer is 150. What is the number?
12. When a certain number is divided by 5 the answer is 1. What is the number?
13. When 23 is added to a certain number the answer is 107. What is the original number?

Answers

14. $5x - 2 = 13$. The number is 3 (because $\frac{13+2}{5} = 3$).

9.2 Solving equations using the additive and multiplicative inverses

FINDING THE UNKNOWN

Teaching guidelines

Solving equations working backwards involves using inverse operations.

Explain that the additive inverse of a number undoes what the number did, if a was added, we can undo that by adding $-a$, because $a + (-a) = 0$. For example, if we have $2x + 3 = 7$ and wanted to change the left-hand side to $2x$, we would write $2x + 3 - 3$ but would have to do the same to the right-hand side as well to get $2x + 3 - 3 = 7 - 3$, this would give $2x = 4$.

In the same way, the multiplicative inverse of a number undoes multiplication. If we multiplied by b we can undo it by multiplying by $\frac{1}{b}$ because $b \times \frac{1}{b} = 1$.

We use the inverse operations together with the following properties:

- addition (or subtraction) property of equality by which we add (or subtract) the same number to each side of an equation to get an equivalent equation.
- multiplication (or division) property of equality by which we multiply (or divide) each side of an equation by the same non-zero number to get an equivalent equation.

Misconceptions

Learners do not apply the additive inverse or multiplicative inverse to both sides of the equation when they solve equations, for example they have $3x + 3 = 36$ and write $3x = 36$; or even $3x = 39$, and get the wrong answer.

14. When a certain number is multiplied by 5 and 2 is subtracted from the answer, the final answer is 13. What is the original number?

Moving from the output value to the input value is called **solving the equation for the unknown**.

9.2 Solving equations using the additive and multiplicative inverses

FINDING THE UNKNOWN

Consider the equation $3x + 2 = 23$.

We can represent the equation $3x + 2 = 23$ in a flow diagram, where x represents an unknown number:



When you reverse the process in the flow diagram, you start with the output number 23, then subtract 2 and then divide the answer by 3:



We can write all of the above reverse process as follows:

Subtract 2 from both sides of the equation:

$$3x + 2 - 2 = 23 - 2$$
$$3x = 21$$

Divide both sides by 3:

$$\frac{3x}{3} = \frac{21}{3}$$
$$x = 7$$

We say $x = 7$ is the solution of $3x + 2 = 23$, because $3 \times 7 + 2 = 23$. We say that $x = 7$ makes the equation $3x + 2 = 23$ true.

The numbers + 2 and – 2 are **additive inverses** of each other. When we add a number and its additive inverse we always get 0.

The numbers 3 and $\frac{1}{3}$ are **multiplicative inverses** of each other. When we multiply a number and its multiplicative inverse we always get 1, so $3 \times \frac{1}{3} = 1$.

The additive and multiplicative inverses help us to isolate the unknown value or the input value.

Teaching guidelines

When solving equations, learners have to decide how they will isolate the unknown. They may have to use additive inverses first and then multiplicative inverses. Sometimes they may have to add like terms first, for example, $4x + 6x = 20$; $10x = 20$.

In other equations there may be terms containing the unknown on both sides, as in $5x = 40 + 3x$. Explain that they need to have one term only that contains the unknown. So they must subtract $3x$ on both sides: $5x - 3x = 40 + 3x - 3x$; $2x = 40$.

Answers

1. $x + 10 = 0$; $x = -10$; Checking: $-10 + 10 = 0$
2. $49x + 2 = 100$; $49x = 100 - 2 = 98$; $x = 2$; Checking: $49 \times 2 + 2 = 98 + 2 = 100$
3. $2x = 1$; $x = \frac{1}{2}$; Checking: $2 \times \frac{1}{2} = 1$
4. $20 = 11 - 9x$; $20 - 11 = -9x$; $9 = -9x$; $x = -1$;
   Checking: $-9 \times -1 + 11 = 9 + 11 = 20$
5. $4x + 6x = 20$; $10x = 20$; $x = 2$; Checking: $10 \times 2 = 20$
6. $5x - 3x = 40 + 3x - 3x$; $2x = 40 + 0$; $x = 20$; Checking: $2 \times 20 = 40$
7. $3x + 1 - x = 0$; $2x = -1$; $x = \frac{-1}{2}$; Checking: $2 \times \frac{-1}{2} = -1$
8. $x + 20 + 4x = -55$; $5x = -75$; $x = -15$; Checking: $5 \times -15 + 20 = -75 + 20 = -55$

9.3 Solving equations involving powers

Teaching guidelines

The reasoning behind this method of solving for an unknown is that the equality means the two sides of the equation have the same value. Therefore, the side that does not contain the unknown should be changed to the exponential form with the same base as the other side. Since the bases are the same and the powers are the same (by the = sign), the exponents have to be the same.

Answers

1. See the completed table on LB page 91 alongside.
2. See the completed table on LB page 91 alongside.
3. (a) $2^x = 32 = 2^5$; $x = 5$ (b) $4^x = 16 = 4^2$; $x = 2$
   (c) $6^x = 216 = 6^3$; $x = 3$ (d) $5^{x+1} = 125 = 5^3$; $x + 1 = 3 \therefore x = 2$

Solve the equations below by using the additive and multiplicative inverses. Check your answers.

1. $x + 10 = 0$
2. $49x + 2 = 100$
3. $2x = 1$
4. $20 = 11 - 9x$

In some cases you need to collect like terms before you can solve the equations using additive and multiplicative inverses, as in the example below:

Example: Solve for x: $7x + 3x = 10$

$$10x = 10$$
$$\frac{10x}{10} = \frac{10}{10}$$
$$x = 1$$

5. $4x + 6x = 20$
6. $5x = 40 + 3x$
7. $3x + 1 - x = 0$
8. $x + 20 + 4x = -55$

Also remember:
- **The multiplicative property of 1:** the product of any number and 1 is that number.
- **The additive property of 0:** the sum of any number and 0 is that number.

$7x$ and $3x$ are like terms and can be replaced with one equivalent expression: $(7 + 3)x = 10x$.

9.3 Solving equations involving powers

Solving an exponential equation is the same as asking the question: **To what exponent must the base be raised in order to make the equation true?**

1. Copy and complete the following table:

x	1	3	5	7
2^x	2	8	32	128

2. Copy and complete the following table:

x	0	2	3	5
3^x	1	9	27	243

Karina solved the equation $3^x = 27$ as follows:

$3^x = 27$
Hence $3^x = 3^3$
Hence $x = 3$

The number 27 can be expressed as 3^3 because $3^3 = 27$.

3. Now use Karina's method and solve for x in each of the following:
   (a) $2^x = 32$ (b) $4^x = 16$ (c) $6^x = 216$ (d) $5^{x+1} = 125$

CHAPTER 9: ALGEBRAIC EQUATIONS 2 91

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
10.1 Bisecting lines	Bisecting a line using a ruler; bisecting a line using a compass	Pages 92 to 93
10.2 Constructing perpendicular lines	Constructing a line perpendicular to another line from a point not on the line; constructing a line perpendicular to another line from a point on the line	Pages 94 to 95
10.3 Bisecting angles	Measuring and classifying angles; bisecting angles using a protractor	Pages 96 to 97
10.4 Constructing special angles without a protractor	Constructing angles of 60°, 30° and 120° as well as 90° and 45°	Pages 98 to 99
10.5 Constructing triangles	Constructing triangles when three sides are given, and when certain angles and sides are given; definition of congruent triangles	Pages 99 to 102
10.6 Properties of triangles	Properties of equilateral triangles and isosceles triangles; the sum of the angles in a triangle	Pages 102 to 103
10.7 Properties of quadrilaterals	Properties (angles and side lengths) of quadrilaterals; the sum of the angles in a quadrilateral	Pages 103 to 105
10.8 Constructing quadrilaterals	Using the properties, parallel lines and perpendicular lines, to construct quadrilaterals	Pages 105 to 106

CAPS time allocation	8 hours
CAPS content specification	Page 95

Mathematical background

In order to perform constructions of geometrical figures we must be able to:

- bisect line segments using compasses and rulers
- construct perpendicular lines to another line from points on the line and points not on the line
- bisect angles and construct special angles
- construct triangles when at least three measurements are given, for example two sides and an included angle, or three sides, etc.
- construct quadrilaterals using their properties of parallel lines and equal angles, etc.

We need to know how to classify triangles according to their sides (for example, isosceles, equilateral, scalene, etc.) and according to their angle properties (for example, obtuse-angled, acute-angled and right-angled). Furthermore, equilateral triangles have all their angles equal and isosceles triangles have the two angles opposite the equal sides equal.

10.1 Bisecting lines

BISECTING A LINE SEGMENT WITH A RULER

Teaching guidelines

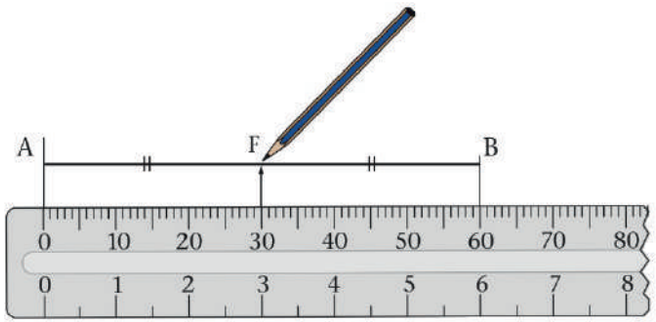
Explain that to bisect a line means to divide the line into two equal parts, so we are looking at finding the midpoint of the line.

If we draw a line through the midpoint of the line, the new line is called a bisector of the original line.

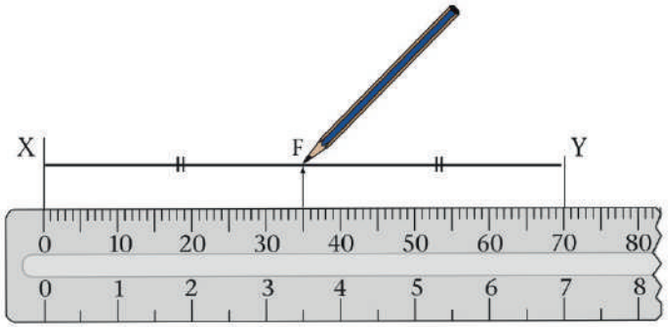
Use a ruler to measure off half of the given line. Mark the midpoint at the halfway mark on the line

Answers

2. Bisecting line AB = 6 cm. Midpoint is at 3 cm.



Bisecting line XY = 7 cm. Midpoint is at 3,5 cm.



CHAPTER 10
Construction of geometric figures

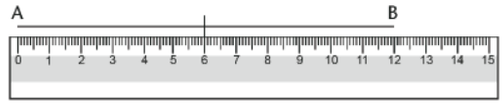
10.1 Bisecting lines

When we construct, or draw geometric figures, we often need to bisect lines or angles. To **bisect** means to cut something into two equal parts. There are different ways to bisect a line segment.

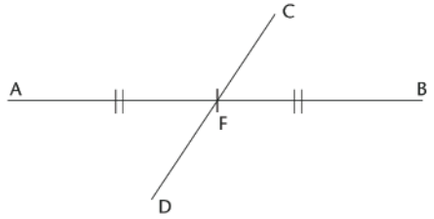
BISECTING A LINE SEGMENT WITH A RULER

1. Read through the following steps:

Step 1: Draw line segment AB and determine its midpoint.



Step 2: Draw any line segment through the midpoint.



The small marks on AF and FB show that AF and FB are equal.

CD is called a **bisector** because it bisects AB. AF = FB.

2. Use a ruler to draw and bisect the following line segments: AB = 6 cm and XY = 7 cm.

BISECTING A LINE SEGMENT WITH A COMPASS AND RULER

Teaching guidelines

Remind learners that their pencils should be sharp. A blunt pencil leads to thick lines and arcs that make it difficult to find the exact point of intersection.

Explain to learners that they should adjust the compass so that its width is slightly longer than half the length of the line segment so that the arcs can intersect. Demonstrate on the board that taking the width too short will lead to an unsuccessful construction.

With this method you not only bisect the line, but you can also draw a perpendicular line through the midpoint. This line is called the perpendicular bisector of the original line.

Remind learners of the meaning of the words **intersect** and **perpendicular**.

To construct a perpendicular bisector using only a ruler and a protractor:

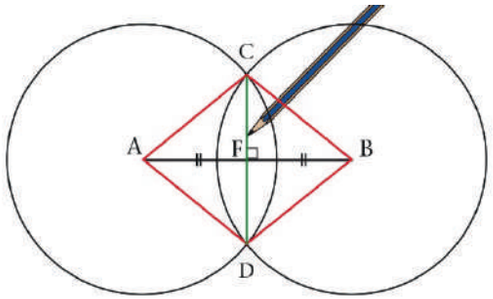
- find the midpoint F of the line with a ruler (see LB page 92)
- at F, construct an angle of 90° using a protractor.

Misconceptions

Learners do not realise why this method works and are therefore not meticulous about keeping the radius of the circles the same, so the constructions fails.

Notes on the questions

Refer to the drawing on the right. The construction works because the circles have the same radius AC = CB = BD = DA, which forms a rhombus and the diagonals of a rhombus bisect each other at right angles.



Answers

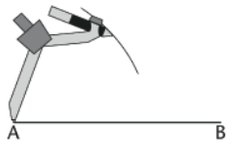
2. Learners' own work.

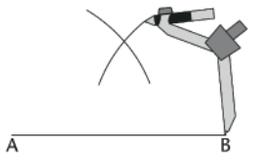
In Grade 6, you learnt how to use a compass to draw circles and parts of circles called arcs. We can use arcs to bisect a line segment.

BISECTING A LINE SEGMENT WITH A COMPASS AND RULER

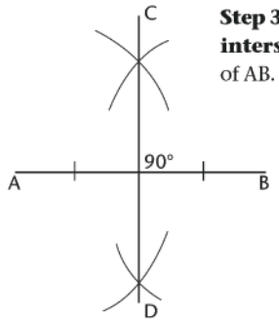
1. Read through the following steps:

Step 1: Place the compass on one endpoint of the line segment (point A). Draw an arc above and below the line. (Notice that all the points on the arc above and below the line are the same distance from point A.)





Step 2: Without changing the compass width, place the compass on point B. Draw an arc above and below the line so that the arcs cross the first two. (The two points where the arcs cross are the same distance away from point A and from point B.)



Step 3: Use a ruler to join the points where the arcs **intersect**. This line segment (CD) is the bisector of AB.

Intersect means to cross or meet.
A **perpendicular** is a line that meets another line at an angle of 90°.

Notice that CD is also **perpendicular** to AB. Therefore, CD is a **perpendicular bisector**.

2. Use a compass and a ruler to practise drawing perpendicular bisectors on line segments.

Try this!
Use only a protractor and ruler to draw a perpendicular bisector on a line segment. (Remember that we use a protractor to measure angles.)

10.2 Constructing perpendicular lines

A line perpendicular to another line can be constructed from:

- a given point that is not on the line
- a given point on the line.

A PERPENDICULAR LINE FROM A GIVEN POINT

Teaching guidelines

The given point is not on the line. Learners start off with a line and a point outside the line.

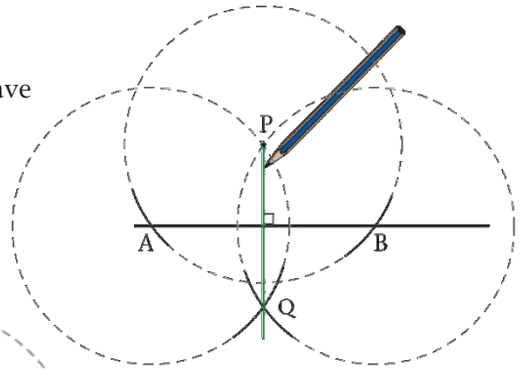
Draw the arcs of a circle so that they intersect the line at A and B.

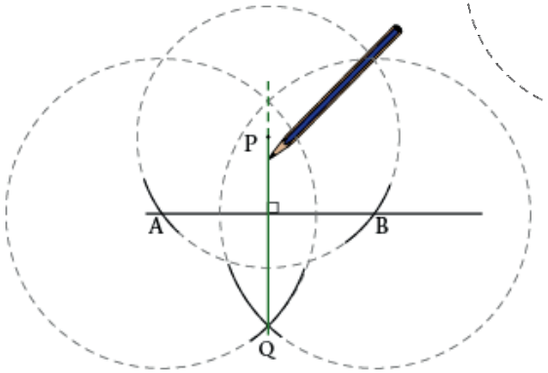
From each of the points A and B, draw another arc so that the two new arcs intersect above or below the line. The width of the compass need not be the same as for the first arcs.

Notes on the questions

The drawings below explain why this construction works.

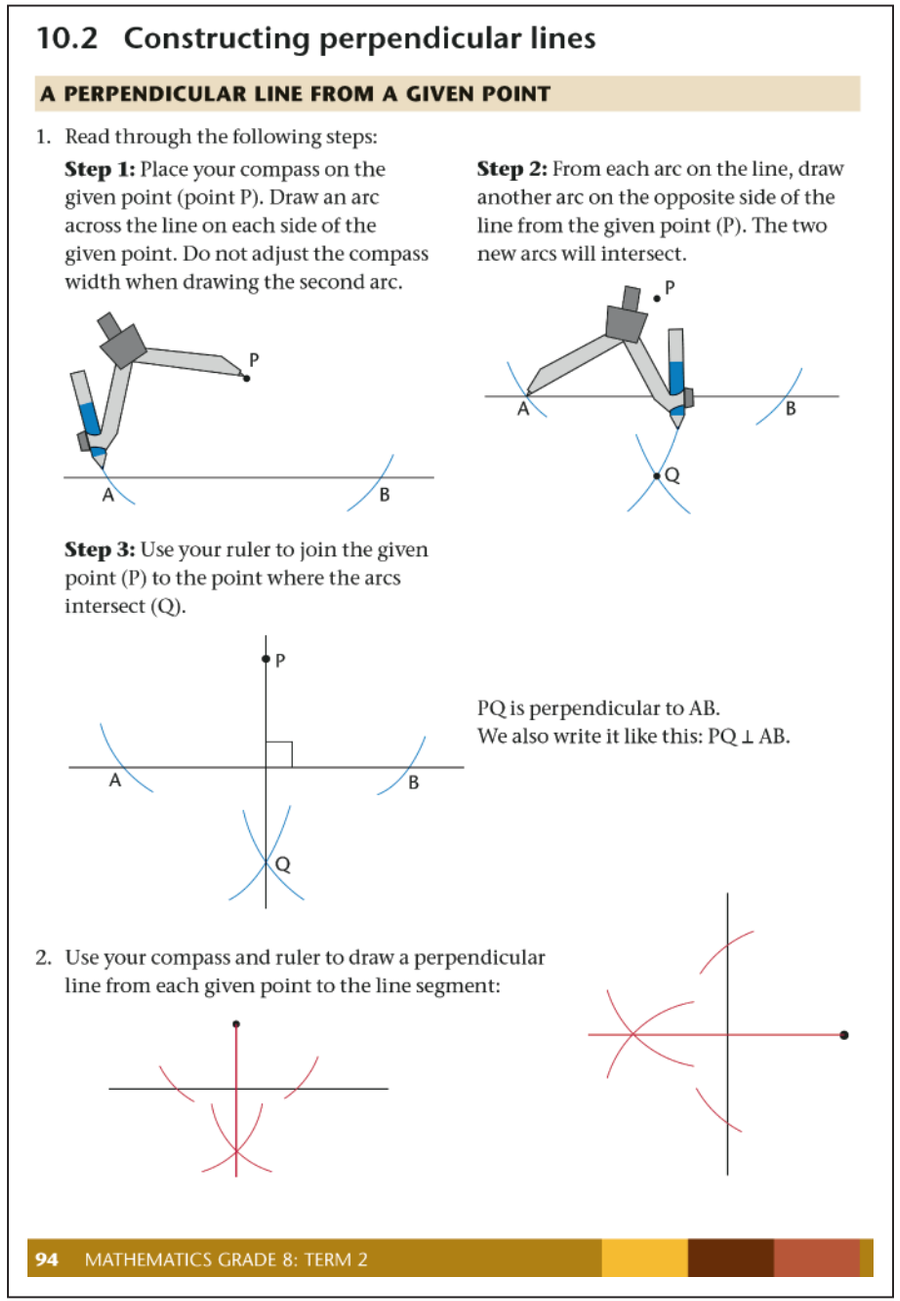
In the first drawing the circles all have the same radius and in the second drawing the circles at A and B have slightly larger radii than the circle drawn at P.





Answers

2. Learners' own work.



10.2 Constructing perpendicular lines

A PERPENDICULAR LINE FROM A GIVEN POINT

1. Read through the following steps:

Step 1: Place your compass on the given point (point P). Draw an arc across the line on each side of the given point. Do not adjust the compass width when drawing the second arc.

P
A
B

Step 2: From each arc on the line, draw another arc on the opposite side of the line from the given point (P). The two new arcs will intersect.

P
A
B
Q

Step 3: Use your ruler to join the given point (P) to the point where the arcs intersect (Q).

P
A
B
Q

PQ is perpendicular to AB.
We also write it like this: PQ ⊥ AB.

2. Use your compass and ruler to draw a perpendicular line from each given point to the line segment:

94 MATHEMATICS GRADE 8: TERM 2

A PERPENDICULAR LINE FROM A GIVEN POINT ON A LINE

Teaching guidelines

Remind learners to always work with a sharp pencil.

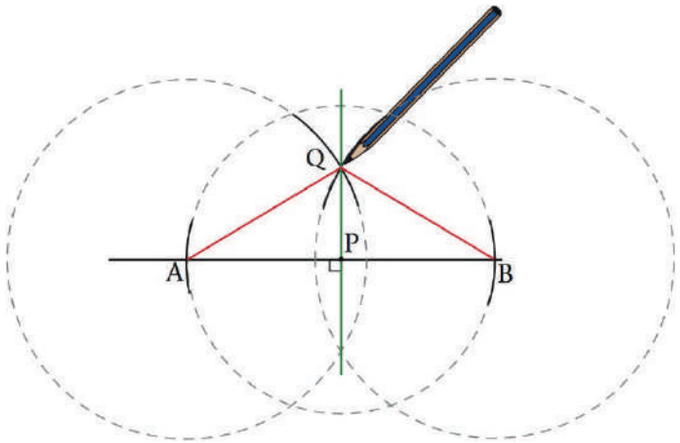
The radius of the circles drawn at the points A and B must be larger than the circle drawn at P.

Notes on the questions/tasks

The drawings below explain why this construction works.

The distance AQ = BQ because the circles A and B have the same radius.

The line PQ bisects the line segment AB perpendicularly.



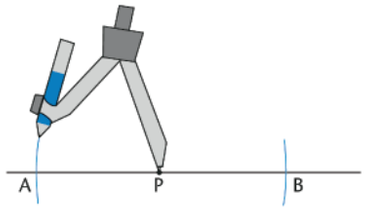
Answers

2. Learners' own work. See the answers on LB page 95 alongside.

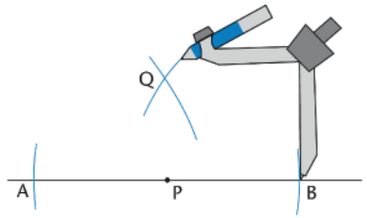
A PERPENDICULAR LINE AT A GIVEN POINT ON A LINE

1. Read through the following steps:

Step 1: Place your compass on the given point (P). Draw an arc across the line on each side of the given point. Do not adjust the compass width when drawing the second arc.

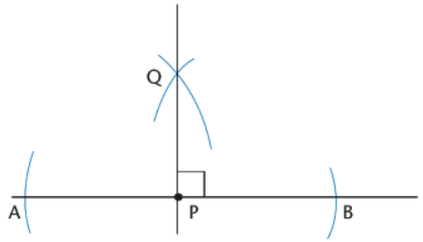


Step 2: Open your compass so that it is wider than the distance from one of the arcs to point P. Place the compass on each arc and draw an arc above or below the point P. The two new arcs will intersect.

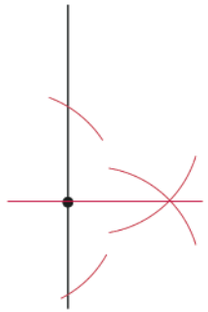


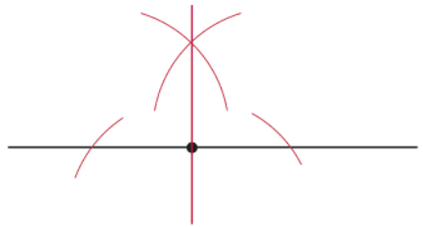
Step 3: Use your ruler to join the given point (P) and the point where the arcs intersect (Q).

$PQ \perp AB$



2. Copy the following diagrams. Use your compass and ruler to draw a perpendicular at the given point on each line:





10.3 Bisecting angles

MEASURING AND CLASSIFYING ANGLES

Teaching guidelines

Remind learners how to measure the magnitude of an angle by doing an example on the board.

Misconceptions

Learners read the wrong scale on the protractor.

Answers

1. See the answers on LB page 96 alongside.
2. See the answers on LB page 96 alongside.
3. ∠1: obtuse
   ∠1 + ∠2: straight
   ∠1 + ∠4: straight
   ∠2 + ∠3: straight
   ∠3 + ∠4: straight
   ∠1 + ∠2 + ∠4: reflex
   ∠1 + ∠2 + ∠3 + ∠4: revolution
   ∠6: right
   ∠7 + ∠8; right
   ∠6 + ∠7 + ∠8: straight
   ∠5 + ∠6 + ∠7: straight
   ∠5 + ∠6: obtuse
   ∠5 + ∠6 + ∠7 + ∠8: reflex
   ∠5 + ∠6 + ∠7 + ∠8 + ∠9: revolution

10.3 Bisecting angles

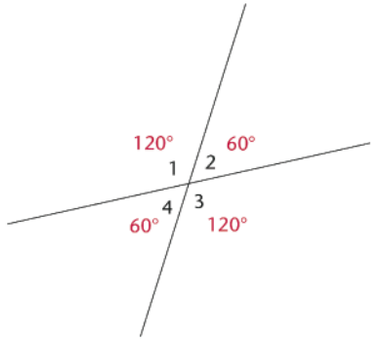
Angles are formed when any two lines meet. We use degrees (°) to measure angles.

MEASURING AND CLASSIFYING ANGLES

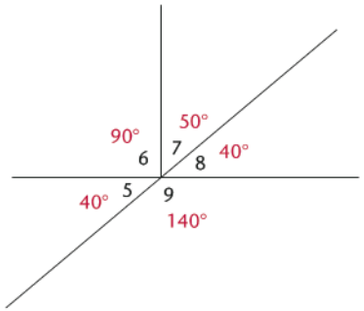
In the figures below, each angle has a number from 1 to 9.

1. Use a protractor to measure the sizes of all the angles in each figure. Write your answers for each figure.

(a)



(b)



2. Copy the statements below and use your answers to write the angle sizes:

$\hat{1} =$ 120°
$\hat{1} + \hat{2} =$ 180°
$\hat{1} + \hat{4} =$ 180°
$\hat{2} + \hat{3} =$ 180°
$\hat{3} + \hat{4} =$ 180°
$\hat{1} + \hat{2} + \hat{4} =$ 240°
$\hat{1} + \hat{2} + \hat{3} + \hat{4} =$ 360°
$\hat{6} =$ 90°
$\hat{7} + \hat{8} =$ 90°
$\hat{6} + \hat{7} + \hat{8} =$ 180°
$\hat{5} + \hat{6} + \hat{7} =$ 180°
$\hat{6} + \hat{5} =$ 130°
$\hat{5} + \hat{6} + \hat{7} + \hat{8} =$ 220°
$\hat{5} + \hat{6} + \hat{7} + \hat{8} + \hat{9} =$ 360°

3. Next to each of your answers above, write down what type of angle it is, namely acute, obtuse, right, straight, reflex or revolution.

BISECTING ANGLES WITHOUT A PROTRACTOR

Teaching guidelines

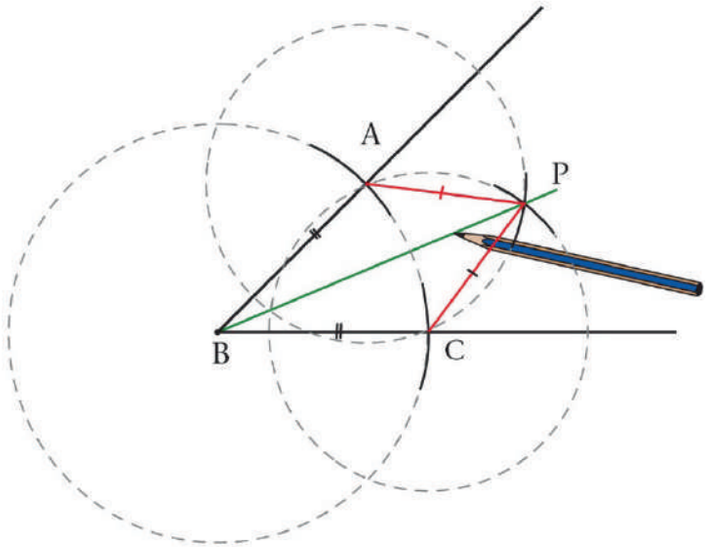
Learners can use any radius to draw the first circle and may change the radius of the next two circles, or they can keep it the same as that of the first circle, but then the drawing may become very large.

Allow learners to experiment with this construction until they can do it smoothly.

Notes on the questions/tasks

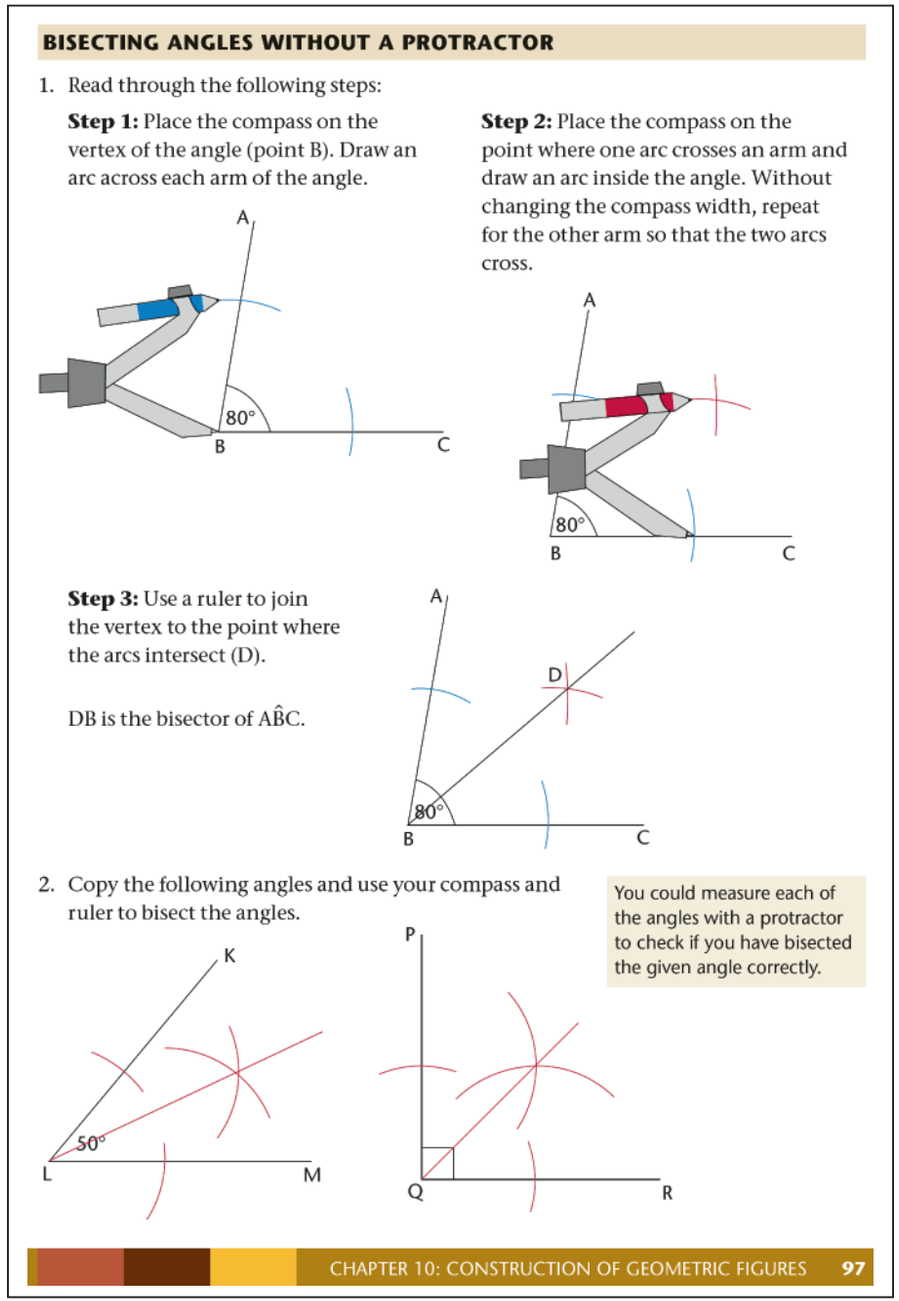
The drawings below explain why this construction works:

- AB = BC = radii of circle B
- AP = PC = radii of circle A and circle C that have the same radius
- So triangle ABP and triangle CBP have all the sides the same length, the angles are the same as well.
- So ∠ABP = ∠CBP.



Answers

2. Learners' own work. See the answers on LB page 97 alongside.

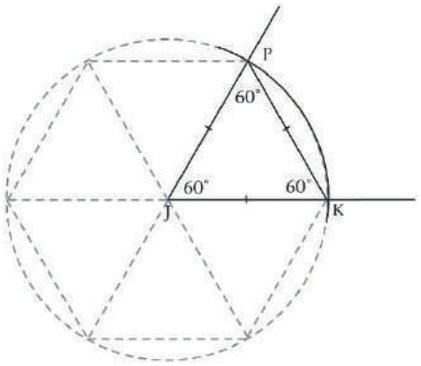


10.4 Constructing special angles without a protractor

CONSTRUCTING ANGLES OF 60°, 30° AND 120°

Teaching guidelines

You could show learners the following construction on the board.



- Draw the arc from J and use the same radius to draw another arc from the point K to intersect the first arc at P.
- Now JK and KP have the same length and JP will have the same length because P lies on the first arc that has radius JK.
- The triangle formed is an equilateral triangle of which all the angles are 60°.
- Produce KJ and show learners that the adjacent angle is 120° as a straight angle is 180°.

Answers

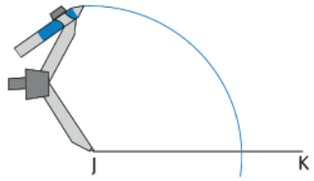
2. (a) See the answers at the top of LB page 99 on the next page.
   (b) See the answers at the top of LB page 99 on the next page.
   (c) Learners should be able to deduce that an angle of 60° that is bisected gives two angles of 30° each.
   (d) 120°
   (e) 180°

10.4 Constructing special angles without a protractor

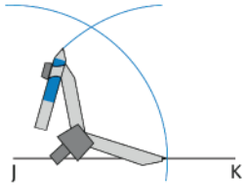
CONSTRUCTING ANGLES OF 60°, 30° AND 120°

1. Read through the following steps:

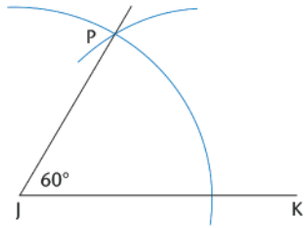
Step 1: Draw a line segment (JK). With the compass on point J, draw an arc across JK and up over above point J.



Step 2: Without changing the compass width, move the compass to the point where the arc crosses JK, and draw an arc that crosses the first one.



Step 3: Join point J to the point where the two arcs meet (point P).
$P\hat{J}K = 60°$



2. (a) Copy the drawing at the top of page 99. Construct an angle of 60° at point B.
   (b) Bisect the angle you constructed.
   (c) Do you notice that the bisected angle consists of two 30° angles?
   (d) Extend line segment BC to A. Then measure the angle adjacent to the 60° angle. What is its size?
   (e) What do the 60° angle and its adjacent angle add up to?

When you learn more about the properties of triangles later, you will understand why the method above creates a 60° angle. Or can you already work this out now? (Hint: What do you know about equilateral triangles?)

Adjacent means "next to".

CONSTRUCTING ANGLES OF 90° AND 45°

Teaching guidelines

If learners could manage the previous construction, they should be able to do this one.

- Construct a right angle as shown in section 10.2.
- Bisect the angle as shown in section 10.2 to get 45°.

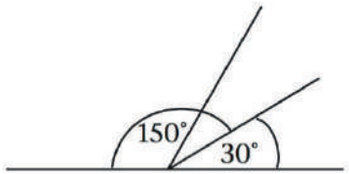
Answers

1. See the answers on LB page 99 alongside.
2. See the answers on LB page 99 alongside.

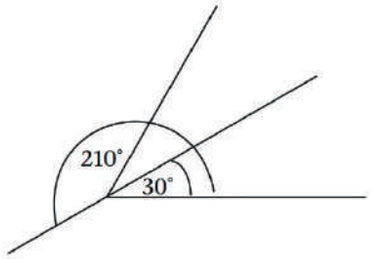
Challenge

Have learners construct the following angles, using their newly gained knowledge of constructing angles.

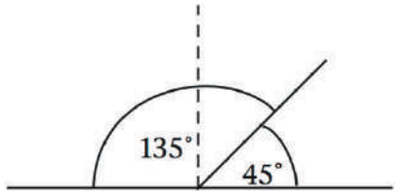
1. 150°
   - 150° = 180° – 30°.
   - Construct a 30° angle.
   - Produce the side.
   - The outer angle is 150°.

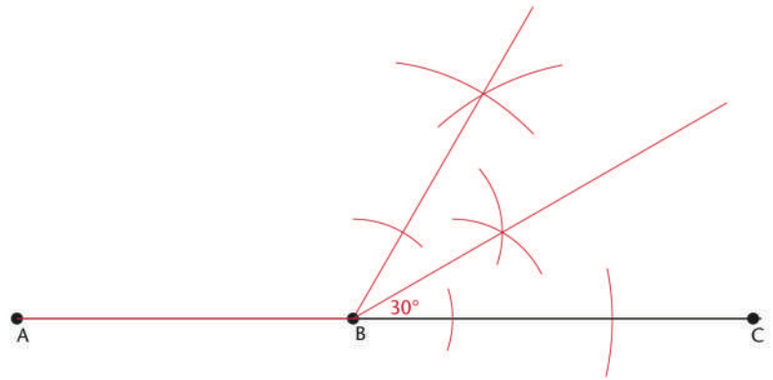


2. 210°
   - 210° = 180° + 30°.
   - Construct a 30° angle.
   - Produce the upper side.
   - The whole angle is 210°.



3. 135°
   - Construct an angle of 45°.
   - Produce the side and take the outer angle.

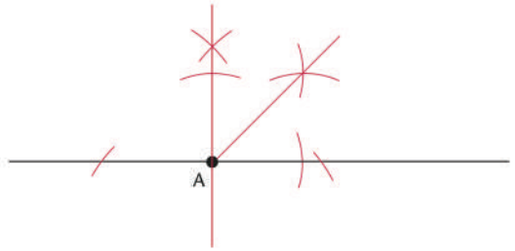




CONSTRUCTING ANGLES OF 90° AND 45°

1. Construct an angle of 90° at point A. Go back to Section 10.2 if you need help.
2. Bisect the 90° angle to create an angle of 45°. Go back to Section 10.3 if you need help.

Challenge
Try to construct the following angles without using a protractor: 150°, 210° and 135°.



10.5 Constructing triangles

In this section, you will learn how to construct triangles. You will need a pencil, a protractor, a ruler and a compass.

A triangle has three sides and three angles. We can construct a triangle when we know some of its measurements, that is, its sides, its angles, or some of its sides and angles.

CHAPTER 10: CONSTRUCTION OF GEOMETRIC FIGURES 99

10.5 Constructing triangles

CONSTRUCTING TRIANGLES

Teaching guidelines

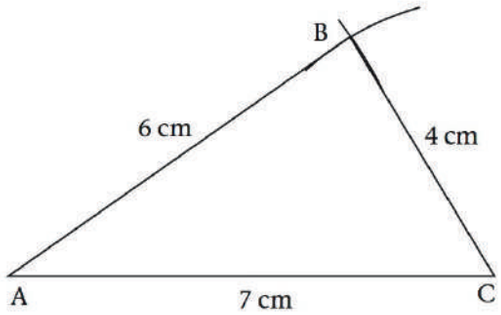
Encourage learners to always make a rough drawing on which they write all the information when they start planning the construction. A rough drawing is a tidy drawing, but need not be accurate. A construction should be as accurate as possible.

Follow the instructions on LB page 100 alongside to construct triangles if three sides are given.

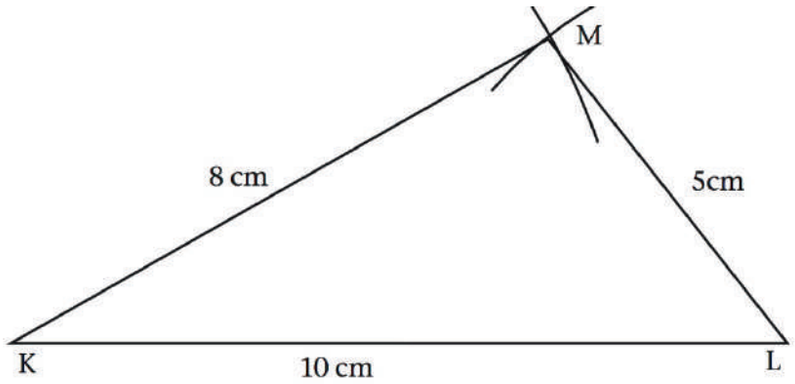
We recommend that you use the longest side as the base of the triangle.

Answers

2. (a)



(b)



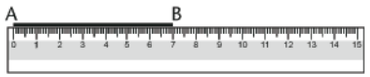
(c) Construct ΔPQR in the same way as shown above.

CONSTRUCTING TRIANGLES

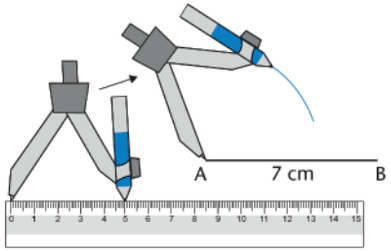
Constructing triangles when three sides are given

1. Read through the following steps. They describe how to construct ΔABC with side lengths of 3 cm, 5 cm and 7 cm.

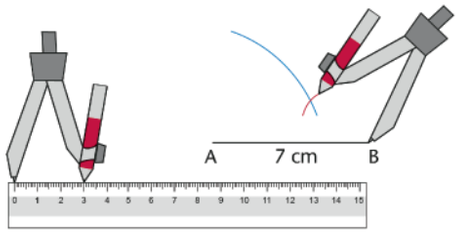
Step 1: Draw one side of the triangle using a ruler. It is often easier to start with the longest side.



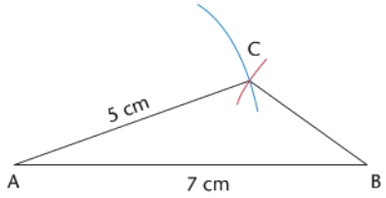
Step 2: Set the compass width to 5 cm. Draw an arc 5 cm away from point A. The third vertex of the triangle will be somewhere along this arc.



Step 3: Set the compass width to 3 cm. Draw an arc from point B. Note where this arc crosses the first arc. This will be the third vertex of the triangle.



Step 4: Use your ruler to join points A and B to the point where the arcs intersect (C).



2. Follow the steps above to construct the following triangles:
   (a) ΔABC with sides 6 cm, 7 cm and 4 cm
   (b) ΔKLM with sides 10 cm, 5 cm and 8 cm
   (c) ΔPQR with sides 5 cm, 9 cm and 11 cm

Teaching guidelines

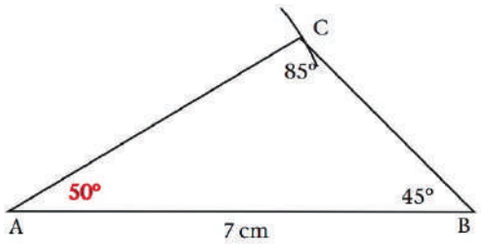
When learners have to construct triangles using a combination of angles and sides, it is imperative that they do a rough sketch. Doing a rough sketch will allow them to see what information they do not have and to plan how the construction will be performed.

Remind learners to work with a sharp pencil.

Answers

3. (a) Two angles and a side between the two angles are given.
See the rough drawing and answers on LB page 101 alongside. Start with the given facts. Draw line AB 7 cm long. At A, construct an angle of 50° and at B, an angle of 45°. Where the upper sides of these angles intersect is angle C. Measure the angle and the sides AC and BC.

In the exercise, two angles and the side between them are given. If the information was two angles and any other side (not between the angles), the construction is slightly more complicated. Let's say the angles B = 45° and C = 85° and AB = 7 cm were given. Learners can first work out the third angle and proceed as described in question 3(a). This implies that they have to know that the sum of the interior angles of a triangle add up to 180°.



(b) In this example the angle between the two known sides is given. The construction is simple: draw KL 9 cm long; at K construct a 35° angle and draw its side KM 8 cm long; join L and M to give the third side. See the rough drawing and answers on LB page 101 alongside.

In the exercise the given angle is between the two sides. If the angle was not between the two sides, the construction would be a little different. First, the 35° angle is drawn and one of its sides, KL, is made 9 cm. At L, with the compass set at 8 cm, an arc is drawn to cut the other side from K at M. All the unknown sides and angles are measured.

(c) Construct the 90° angle and then the other sides or angles. See the rough drawing and answers on LB page 101 alongside.

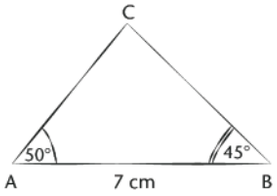
4. See the answers on LB page 101 alongside.

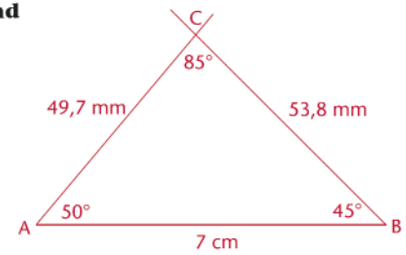
5. Learners' own work.

Constructing triangles when certain angles and sides are given

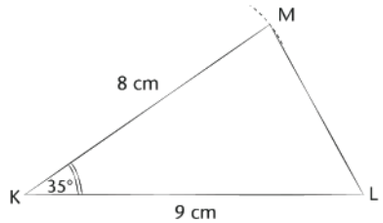
3. Use the rough sketches in (a) to (c) below to construct accurate triangles, using a ruler, compass and protractor. Make sure that:
   - the dotted lines show where you have to use a compass to measure the length of a side
   - you use a protractor to measure the size of the given angles.

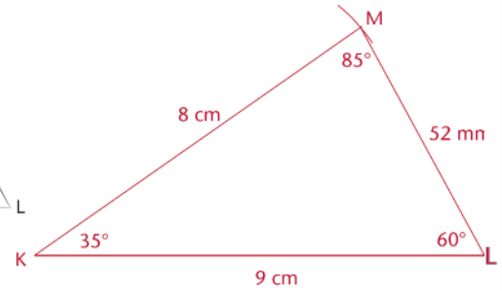
(a) Construct ΔABC, with **two angles and one side given**.



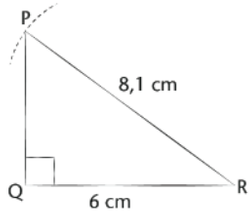


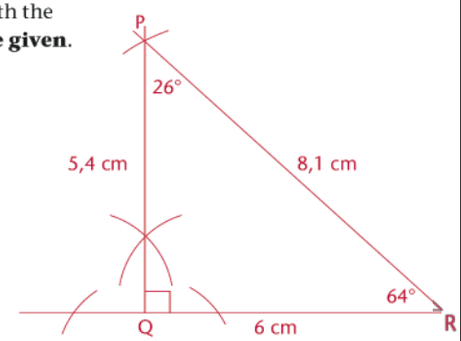
(b) Construct a ΔKLM, with **two sides and an angle given**.





(c) Construct a right-angled ΔPQR, with the **hypotenuse and one other side given**.





Challenge

1. (a) If no sides are given, learners have to choose a length for the sides and proceed as in 3(a).
   (b) Draw XY = 8 cm; at X construct and angle of 50°; at Y draw an arc with radius 7 cm to cut the side drawn from X at Z.
2. For (a) there are many solutions.
   For (b) there is one triangle. All the triangles drawn should be congruent.

10.6 Properties of triangles

PROPERTIES OF EQUILATERAL TRIANGLES

Teaching guidelines

Point out that all equilateral triangles have the same angle sizes.

Answers

1. (a) Learners' own work.
   (b) See the answers on LB page 102 alongside.
2. See the answers on LB page 102 alongside.
3. Learners' own work.

4. Measure the missing angles and sides of each triangle in 3(a) to (c) on the previous page. Write the measurements next to your completed constructions.
5. Compare each of your constructed triangles in question 3(a) to (c) with a classmate's triangles. Are the triangles exactly the same?

If triangles are exactly the same, we say they are **congruent**.

Challenge

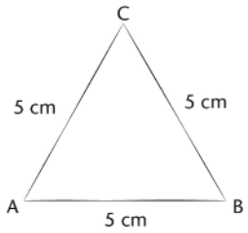
1. Construct these triangles:
   (a) ΔSTU, with three angles given: $\hat{S} = 45°$, $\hat{T} = 70°$ and $\hat{U} = 65°$.
   (b) ΔXYZ, with two sides and the angle opposite one of the sides given: $\hat{X} = 50°$, XY = 8 cm and XZ = 7 cm.
2. Can you find more than one solution for each triangle above? Explain your findings to a classmate.

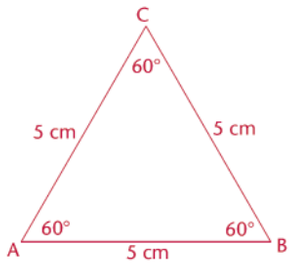
10.6 Properties of triangles

The angles of a triangle can be the same size or different sizes. The sides of a triangle can be the same length or different lengths.

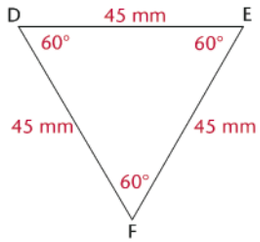
PROPERTIES OF EQUILATERAL TRIANGLES

1. (a) Construct ΔABC.
   (b) Measure and label the sizes of all its sides and angles.





2. Measure and write down the sizes of the sides and angles of ΔDEF on the right.
3. Both triangles in questions 1 and 2 are called **equilateral triangles**. Discuss with a classmate if the following is true for an equilateral triangle:
   - All the sides are equal.
   - All the angles are equal to 60°.



PROPERTIES OF ISOSCELES TRIANGLES

Teaching guidelines

Isosceles triangles have two sides equal and the angles opposite those sides are equal.

Misconceptions

Be aware that learners sometimes confuse isosceles and equilateral triangles.

Answers

1. (a) In ΔDEF DE = DF and in ΔJKL ∠J = ∠L = 70°
   (b) Learners' own work.
2. Learners' own work.

THE SUM OF THE ANGLES IN A TRIANGLE

Teaching guidelines

The conclusion reached by doing question 2 is an important fact about triangles. Learners must be sure of this fact concerning triangles as they will use it often in their future mathematical studies.

Answers

1. 180°
2. (a) and (b) Learners' own work.
   (c) 180°

10.7 Properties of quadrilaterals

Background information

In order to classify shapes we need to know what their properties are.

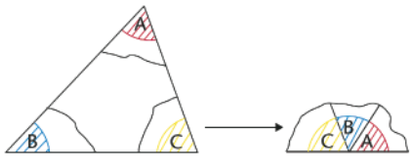
It may be that the shape has more than the minimum required properties, as in the case of the square being a rectangle. It has to have all four angles equal and two pairs of opposite sides equal. It has the extra property that all its sides are also equal, which makes it a special rectangle.

PROPERTIES OF ISOSCELES TRIANGLES

1. (a) Construct ΔDEF with EF = 7 cm, $\hat{E} = 50°$ and $\hat{F} = 50°$.
   Also construct ΔJKL with JK = 6 cm, KL = 6 cm and $\hat{J} = 70°$.
   (b) Measure and label all the sides and angles of each triangle.
2. Both triangles above are called **isosceles triangles**. Discuss with a classmate whether the following is true for an isosceles triangle:
   - Only two sides are equal.
   - Only two angles are equal.
   - The two equal angles are opposite the two equal sides.

THE SUM OF THE ANGLES IN A TRIANGLE

1. Look at your constructed triangles ΔABC, ΔDEF and ΔJKL above and on the previous page. What is the sum of the three angles each time?
2. Did you find that the sum of the interior angles of each triangle is 180°? Do the following to check if this is true for other triangles.



   (a) On a clean sheet of paper, construct any triangle. Label the angles A, B and C and cut out the triangle.
   (b) Neatly tear the angles off the triangle and fit them next to one another.
   (c) Notice that $\hat{A}$, $\hat{B}$ and $\hat{C}$ form a straight angle. Complete: $\hat{A} + \hat{B} + \hat{C} =$ 180°

We can conclude that the interior angles of a triangle always add up to 180°.

10.7 Properties of quadrilaterals

A quadrilateral is any closed shape with four straight sides. We classify quadrilaterals according to their sides and angles. We note which sides are parallel, perpendicular or equal. We also note which angles are equal.

PROPERTIES OF QUADRILATERALS

Teaching guidelines

The quadrilaterals can be subdivided into groups according to their properties. For example, they can be parallelograms, kites, squares, rectangles and trapeziums.

Ask questions to make learners think about the connections between the groups of shapes, for example:

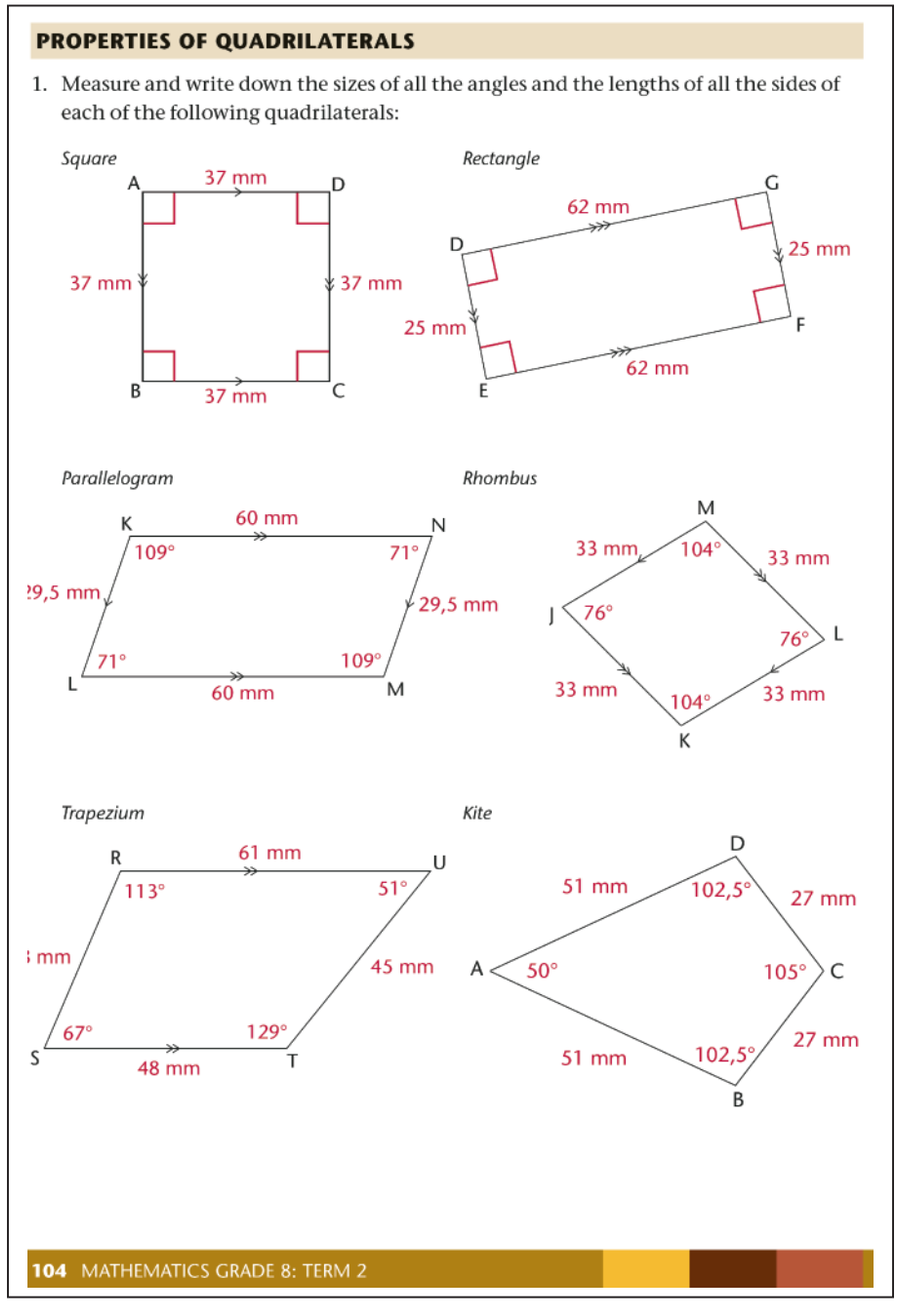
- Could a rectangle be considered a parallelogram?
- Could a square be considered a rhombus, or a rectangle?
- Could a rhombus be considered a kite?

In order to answer these questions we have to determine what the minimum properties are that a shape should have to belong to a group. For example, "What does a square need to have to be considered a rhombus?" (All four sides have to be the same length.)

Sorting quadrilaterals according to their properties is an important activity which helps learners to develop the ability to think analytically and make deductions and connections. They need to gather all the available information and compare the properties of the different shapes in order to classify them.

Answers

1. See the answers on LB page 104 alongside.



PROPERTIES OF QUADRILATERALS

1. Measure and write down the sizes of all the angles and the lengths of all the sides of each of the following quadrilaterals:

Square

Rectangle

Parallelogram

Rhombus

Trapezium

Kite

Answers

2. See the completed table on LB page 105 alongside.

Teaching guidelines

Completing the table of properties of quadrilaterals is an important activity and should not be hurried over.

Allow learners to complete the table and discuss it with each other in small groups or pairs.

In a class discussion, help learners to see that a rectangle, a rhombus and a square are also parallelograms, because they have the required properties of a parallelogram. They have more properties than needed, but that is why they also belong to subgroups, for example, a square is also a rectangle, and a rhombus, but a rhombus is not a rectangle. A kite and a trapezium each belong to a group of their own within the quadrilaterals.

Let learners verbalise these properties and classifications to each other.

SUM OF THE ANGLES IN A QUADRILATERAL

Teaching guidelines

Another way of seeing that the sum of the interior angles of a quadrilateral is 360°, is to draw a few quadrilaterals and draw a diagonal in each. The diagonal divides the quadrilateral into two triangles. We have found that the sum of the interior angles of a triangle is 180°. Therefore, the sum of the interior angles of the two triangles is $2 \times 180° = 360°$.

Answer

1. It is always 360°.
2. It forms a circle, which is 360°.

10.8 Constructing quadrilaterals

Teaching guidelines

Let learners construct a few of the quadrilaterals shown on LB page 104 on the previous page.

2. Use your answers in question 1. Copy the following table and place a ✓ in the correct box to show which property is correct for each shape.

Properties	Parallelogram	Rectangle	Rhombus	Square	Kite	Trapezium
Only one pair of sides are parallel						✓
Opposite sides are parallel	✓	✓	✓	✓		
Opposite sides are equal	✓	✓	✓	✓		
All sides are equal			✓	✓		
Two pairs of adjacent sides are equal			✓	✓	✓	
Opposite angles are equal	✓	✓	✓	✓		
All angles are equal		✓		✓		

SUM OF THE ANGLES IN A QUADRILATERAL

1. Add up the four angles of each quadrilateral on the previous page. What do you notice about the sum of the angles of each quadrilateral?
2. Did you find that the sum of the interior angles of each quadrilateral equals 360°? Do the following to check if this is true for other quadrilaterals:
   (a) On a clean sheet of paper, use a ruler to construct any quadrilateral.
   (b) Label the angles A, B, C and D. Cut out the quadrilateral.
   (c) Neatly tear the angles off the quadrilateral and fit them next to one another.
   (d) What do you notice?

We can conclude that the interior angles of a quadrilateral always add up to 360°.

10.8 Constructing quadrilaterals

You learnt how to construct perpendicular lines in Section 10.2. If you know how to construct parallel lines, you should be able to construct any quadrilateral accurately.

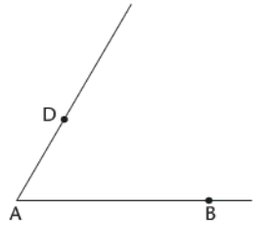
Teaching guidelines

Allow learners to carefully read through and follow the four steps given on LB page 106 for the construction of parallel lines. Then ask learners to go back to reconstruct the quadrilaterals on LB page 104. The exercises on LB page 106 are for practising ideas and skills they have developed.

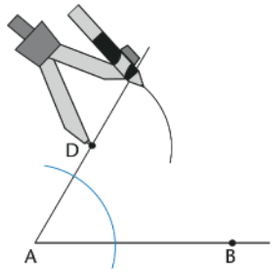
CONSTRUCTING PARALLEL LINES TO DRAW QUADRILATERALS

1. Read through the following steps:

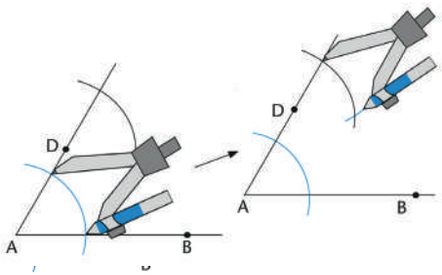
Step 1: From line segment AB, mark a point D. This point D will be on the line that will be parallel to AB. Draw a line from A through D.



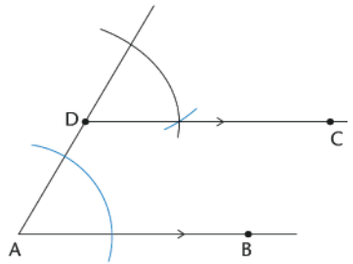
Step 2: Draw an arc from A that crosses AD and AB. Keep the same compass width and draw an arc from point D as shown.



Step 3: Set the compass width to the distance between the two points where the first arc crosses AD and AB. From the point where the second arc crosses AD, draw a third arc to cross the second arc.



Step 4: Draw a line from D through the point where the two arcs meet. DC is parallel to AB.



2. Practise drawing a parallelogram, square and rhombus.
3. Use a protractor to draw quadrilaterals with at least one set of parallel lines.

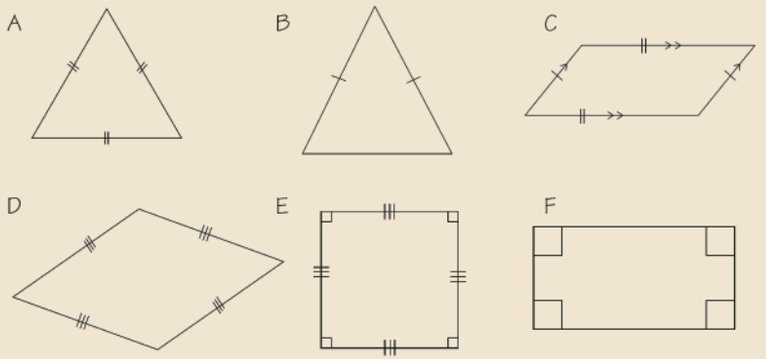
WORKSHEET

Answer

1. Learners' own work.
2. A equilateral triangle
   B isosceles triangle
   C parallelogram
   D rhombus
   E square
   F rectangle
3. (a) square
   (b) rhombus, square, kite
   (c) trapezium
   (d) parallelogram, rhombus, square, rectangle
   (e) square, rectangle
   (f) rhombus, square

WORKSHEET

1. Do the following construction:
   (a) Use a compass and ruler to construct equilateral ΔABC with sides of 9 cm.
   (b) Without using a protractor, bisect $\hat{B}$. Let the bisector intersect AC at point D.
   (c) Use a protractor to measure $A\hat{D}B$. Write the measurement on the drawing.
2. Name the following types of triangles and quadrilaterals:



3. Which of the following quadrilaterals matches each description below? (There may be more than one answer for each.)

parallelogram; rectangle; rhombus; square; kite; trapezium

   (a) All sides are equal and all angles are equal.
   (b) Two pairs of adjacent sides are equal.
   (c) One pair of sides is parallel.
   (d) Opposite sides are parallel.
   (e) Opposite sides are parallel and all angles are equal.
   (f) All sides are equal.

CONSTRUCTION OF GEOMETRIC FIGURES

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
11.1 Types of triangles	Name triangles according to their sides and their angles; looking at their sides; summarising the properties of triangles	Pages 108 to 110
11.2 Unknown angles and sides of triangles	Working out unknown angles and sides using conventions for writing reasons for answers	Pages 110 to 114
11.3 Types of quadrilaterals and their properties	Naming different types of quadrilaterals and investigating their properties	Pages 115 to 118
11.4 Unknown angles and sides of quadrilaterals	Using the properties to find unknown angles and sides of quadrilaterals	Pages 118 to 119
11.5 Congruency	Definition of congruency; notation used to indicate congruent triangles; identifying congruent angles and sides	Pages 119 to 120
11.6 Similarity	Definition of similarity; using definition to check for similarity; recognising enlargements as similar to the original shape; using the properties of similar and congruent figures	Pages 120 to 122

CAPS time allocation	8 hours
CAPS content specification	Pages 96 to 98

Mathematical background

Triangles can be classified according to the properties of their sides or their angles, for example:

Sides		Angles	
Triangle type	**Property**	**Triangle type**	**Property**
Scalene	All three sides have different lengths	Obtuse triangle	One angle is greater than 90°
Isosceles	Two sides have the same length	Acute triangle	All angles are less than 90°
Equilateral	All three sides have the same length	Right-angled triangle	One angle is equal to 90°

Triangles can be classified using both sides and angles, for example: right-angled, isosceles, etc.

Quadrilaterals are classified as closed shapes with four straight sides. The question learners should learn to ask is: What are the properties that define a particular shape? For example, rectangles need four right angles, and therefore a square can be called a rectangle.

Similar shapes have all their sides in the same ratio and corresponding angles are equal.

Shapes are congruent if their corresponding sides are equal (in the ratio 1 : 1) and their corresponding angles are equal. This means that all congruent shapes are also similar.

11.1 Types of triangles

NAMING TRIANGLES ACCORDING TO THEIR SIDES

Teaching guidelines

Learners should know by now how to name triangles according to their sides. Point out that as far as the sides of a triangle are concerned there are only three possibilities:

- all the sides have the same length – such a triangle is called equilateral
- two of the sides have the same length – such a triangle is called isosceles
- all the sides have different lengths – such a triangle is called scalene.

You may want to draw a number of these triangles on the board with the appropriate markings and let learners name them.

Misconceptions

Learners often do not read the information in a drawing, for example the marks that indicate equal sides or angles. Instead, they interpret a drawing visually. If it looks equilateral, they think it is equilateral and will say so.

Answers

1. See the answers on LB page 108 alongside.
2. ΔPQR Isosceles ΔXYZ Equilateral ΔGHI Scalene

NAMING TRIANGLES ACCORDING TO THEIR ANGLES

Teaching guidelines

Point out to learners that we can also refer to triangles by their angles. A triangle can have:

- all three angles acute – an acute-angled triangle (or acute triangle)
- one angle a right angle – a right-angled triangle
- one angle obtuse – an obtuse-angled triangle (or obtuse triangle).

CHAPTER 11
Geometry of 2D shapes

11.1 Types of triangles

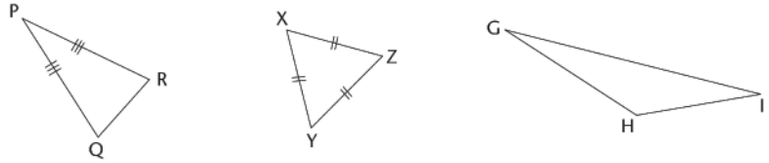
By now, you know that a triangle is a closed 2D shape with three straight sides. We can classify or name different types of triangles according to the lengths of their sides and according to the sizes of their angles.

NAMING TRIANGLES ACCORDING TO THEIR SIDES

1. Copy the table and match the name of each type of triangle with its correct description:

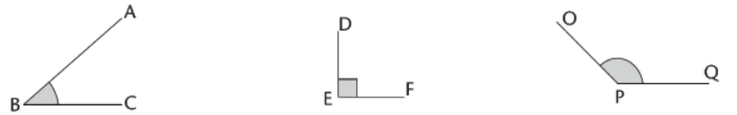
Name of triangle	Description of triangle
Isosceles triangle	All the sides of a triangle are equal.
Scalene triangle	None of the sides of a triangle are equal.
Equilateral triangle	Two sides of a triangle are equal.

2. Name each type of triangle by looking at its sides:



NAMING TRIANGLES ACCORDING TO THEIR ANGLES

Remember the following types of angles:



Acute angle (< 90°) *Right angle (= 90°)* *Obtuse angle (between 90° and 180°)*

Answers

1. No
2. obtuse-angled (or an obtuse triangle)
3. acute-angled (or an acute triangle)
4. 90°

INVESTIGATING THE ANGLES AND SIDES OF TRIANGLES

Teaching guidelines

Remind learners of the fact that the sum of the angles of a triangle is 180°.

Point out the link between the sides and angles of a triangle:

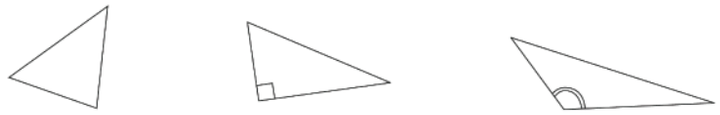
- The largest angle in a triangle is always opposite the longest side.
- If two sides have the same length, the angles opposite these sides are also equal.
- If all three sides are equal, the angles are also all equal.

Let learners summarise the properties of triangles as on LB page 110. They could make posters of these properties.

Answers

1. (a) 180°
   (b) No. Two right angles are 180°.
   (c) No, because the sum of the angles would be greater than 180°.
2. (a) They are equal (60 degrees).
   (b) The two angles opposite the equal sides are equal.
   (c) Yes
   (d) Yes

Study the following triangles and then answer the questions:



Acute triangle *Right-angled triangle* *Obtuse triangle*

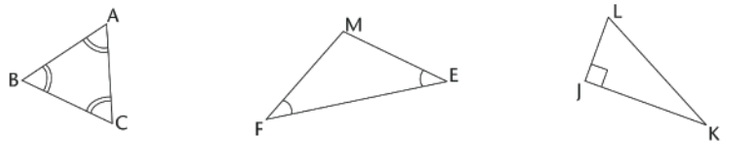
1. Are all the angles of a triangle always equal?
2. When a triangle has an obtuse angle, what kind of triangle is it called?
3. When a triangle has only acute angles, what kind of triangle is it called?
4. What size must one of the angles of a triangle be for it to be called a right-angled triangle?

INVESTIGATING THE ANGLES AND SIDES OF TRIANGLES

1. (a) What is the sum of the interior angles of a triangle?
   (b) Can a triangle have two right angles? Explain your answer.
   (c) Can a triangle have more than one obtuse angle? Explain your answer.

If you cannot work out the answers in question 1(b) and (c), try to construct the triangles to find the answers.

2. Look at the triangles below. The arcs show which angles are equal.



Equilateral triangle *Isosceles triangle* *Right-angled triangle*

(a) ΔABC is an equilateral triangle. What do you notice about its angles?
(b) ΔFEM is an isosceles triangle. What do you notice about its angles?
(c) ΔJKL is a right-angled triangle. Is its longest side opposite the 90° angle?
(d) Construct any three right-angled triangles on a sheet of paper. Is the longest side always opposite the 90° angle?

11.2 Unknown angles and sides of triangles

WORKING OUT UNKNOWN ANGLES AND SIDES

Teaching guidelines

Encourage learners to use the properties of triangles (discussed in section 11.1) to work out answers when they have to find unknown angles and sides of triangles. They will often be required to give reasons for their answers.

Show learners how to write the reasons they will use in shorter form, for example in ΔJKL on LB page 110:

$$\angle J + \angle K + \angle L = 180° \text{ (Interior } \angle s \text{ of a } \Delta)$$
$$\angle J + 40° + 50° = 180°$$
$$\angle J = 180° - 40° - 85°$$
$$= 55°$$

Point out that we create an equation with the information we have, where the unknown is an angle or a side. The normal procedures to solve equations apply here as well; we add or subtract angles to get the unknown angle or side on one side of the equation and then do the same on the other side of the equation.

Properties of triangles:

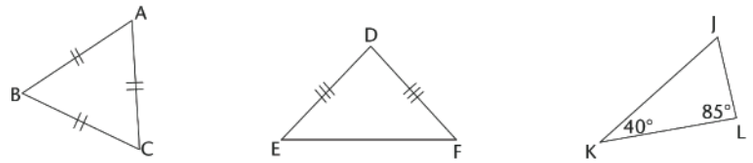
- The **sum of the interior angles** of a triangle is 180°.
- An **equilateral triangle** has all sides equal and each interior angle is equal to 60°.
- An **isosceles triangle** has two equal sides and the angles opposite the equal sides are equal.
- A **scalene triangle** has no sides equal.
- A **right-angled triangle** has a right angle (90°).
- An **obtuse triangle** has one obtuse angle (between 90° and 180°).
- An **acute triangle** has three acute angles (< 90°).

Interior angles are the angles inside a closed shape; not the angles outside of it.

11.2 Unknown angles and sides of triangles

You can use what you know about triangles to obtain other information. When you work out new information, you must always give reasons for the statements you make.

Look at the following examples of working out unknown angles and sides when certain information is given. The reason for each statement is written in square brackets.



$\hat{A} = \hat{B} = \hat{C} = 60°$	[Angles in an equilateral Δ = 60°]
DE = DF	[Given]
$\hat{E} = \hat{F}$	[Angles opposite the equal sides of an isosceles Δ are equal]
$\hat{J} = 55°$	[The sum of the interior angles of a Δ = 180°; so $\hat{J} = 180° - 40° - 85°$]

You can shorten the following reasons in the ways shown:

- Sum of interior angles (∠s) of a triangle (Δ) = 180°: **Interior ∠s of Δ**
- Isosceles triangle has two sides and two angles equal: **Isosceles Δ**
- Equilateral triangle has three sides and three angles equal: **Equilateral Δ**
- Angles forming a straight line = 180°: **Straight line**

Teaching guidelines

Point out to learners how the information that is given in the drawing is used to make an equation that can be used to find the unknown values.

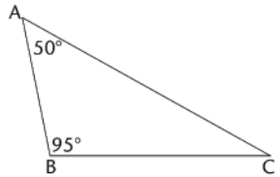
Answers

1. to 4. See the answers on LB page 111 alongside.

WORKING OUT UNKNOWN ANGLES AND SIDES

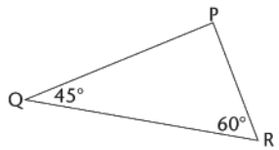
Find the sizes of unknown angles and sides in the following triangles. Always give reasons for every statement.

1. Find $\hat{C}$.



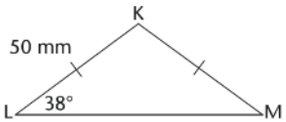
$\hat{A} + \hat{B} + \hat{C} = 180°$ [Interior ∠s of a Δ]
$50° + 95° + \hat{C} = 180°$
$145° + \hat{C} = 180°$
$\hat{C} = 180° - 145°$
$\hat{C} = 35°$

2. Find $\hat{P}$.



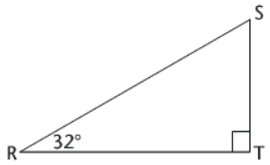
$\hat{Q} + \hat{P} + \hat{R} = 180°$ [Interior ∠s of a Δ]
$45° + \hat{P} + 60° = 180°$
$\hat{P} = 180° - 45° - 60°$
$\hat{P} = 75°$

3. (a) Find KM.
   (b) Find $\hat{K}$.



(a) KM = KL = 50 mm
(b) $\hat{L} = \hat{M} = 38°$ [Isosceles Δ]
$\hat{K} + \hat{L} + \hat{M} = 180°$ [Interior ∠s of a Δ]
$\hat{K} + 38° + 38° = 180°$
$\hat{K} = 180° - 38° - 38°$
$\hat{K} = 104°$

4. What is the size of $\hat{S}$?



$\hat{S} + \hat{T} + \hat{R} = 180°$ [Interior ∠s of a Δ]
$\hat{S} + 90° + 32° = 180°$
$\hat{S} = 180° - 90° - 32°$
$= 58°$

Answers

5. to 6. See the answers on LB page 112 alongside.

WORKING OUT MORE UNKNOWN ANGLES AND SIDES

Teaching guidelines

Learners often overlook information that they need to make an equation. Let them work in pairs or small groups on a few of these problems so that they can share their insights.

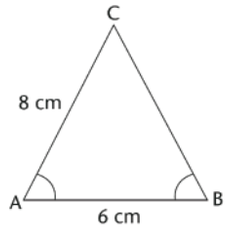
Point out that we usually make use of at least two facts about the triangles.

The answers have been worked out fully on LB page 112 alongside. The communication of the answers is very important. Let learners follow the answers by having them available for learners to study (either on the board or pinned up in the classroom). They should follow all the steps and the reasoning.

Answers

1. See the answers on LB page 112 alongside.

5. (a) Find CB.
(b) Find $\hat{C}$ if $\hat{A} = 50°$.



(a) $CB = CA = 8$ cm [Isosceles Δ]

(b) $\hat{B} = \hat{A} = 50°$

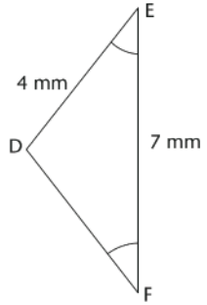
$\hat{C} + \hat{B} + \hat{A} = 180°$ [Interior ∠s of a Δ]

$\hat{C} + 50° + 50° = 180°$

$\hat{C} = 180° - 50° - 50°$

$\hat{C} = 80°$

6. (a) Find DF.
(b) Find $\hat{E}$ if $\hat{D} = 100°$.



(a) $DF = DE = 4$ mm [Isosceles Δ]

(b) $\hat{D} + \hat{E} + \hat{F} = 180°$ [Interior ∠s of a Δ]

$100° + \hat{E} + \hat{E} = 180°$

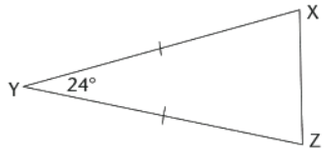
$2\hat{E} = 180° - 100°$

$\hat{E} = 80° \div 2$

$\hat{E} = 40°$

WORKING OUT MORE UNKNOWN ANGLES AND SIDES

1. Calculate the size of $\hat{X}$ and $\hat{Z}$.



$\hat{X} = \hat{Z}$ [Isosceles Δ]

$24° + \hat{X} + \hat{Z} = 180°$ [Interior ∠s of a Δ]

$\hat{X} + \hat{X} = 180° - 24°$

$2\hat{X} = 156°$

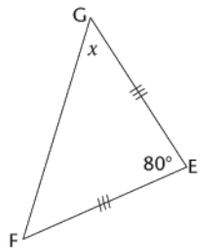
$\hat{X} = 78°$

$\hat{Z} = 78°$

Answers

2. to 5. See the answers on LB page 113 alongside.

2. Calculate the size of x.



$\hat{F} = \hat{G} = x$ [Isosceles Δ]

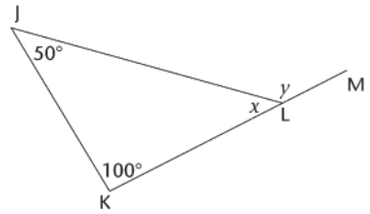
$80° + x + x = 180°$ [Interior ∠s of a Δ]

$2x = 180° - 80°$

$2x = 100°$

$x = 50°$

3. KLM is a straight line. Calculate the size of x and y.



$100° + 50° + x = 180°$ [Interior ∠s of a Δ]

$x = 180° - 100° - 50°$

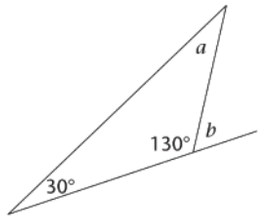
$x = 30°$

$30° + y = 180°$ [Straight line]

$y = 180° - 30°$

$y = 150°$

4. Angle b and an angle with a size of 130° form a straight angle. Calculate the size of a and b.



$130° + b = 180°$ [Straight line]

$b = 180° - 130°$

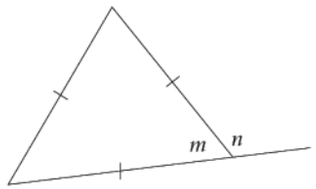
$b = 50°$

$130° + 30° + a = 180°$ [Interior ∠s of a Δ]

$a = 180° - 130° - 30°$

$a = 20°$

5. Angle m and n form a straight angle. Calculate the size of m and n.



$m = 60°$ [Equilateral Δ]

$m + n = 180°$ [Straight line]

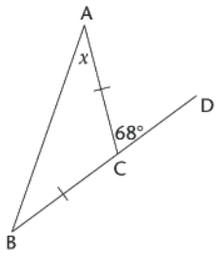
$n = 180° - 60°$

$n = 120°$

Answers

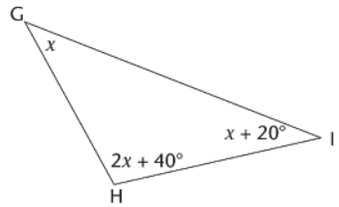
6. to 9. See the answers on LB page 114 alongside.

6. BCD is a straight line segment. Calculate the size of x.



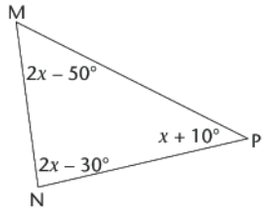
$\hat{B} = \hat{A} = x$ [Equilateral Δ]
$A\hat{C}B = 180° - 68°$ [Straight line]
$A\hat{C}B = 112°$
$112° + x + x = 180°$ [Interior ∠s of a Δ]
$2x = 180° - 112°$
$2x = 68°$
$x = 34°$

7. Calculate the size of x and then the size of $\hat{H}$.



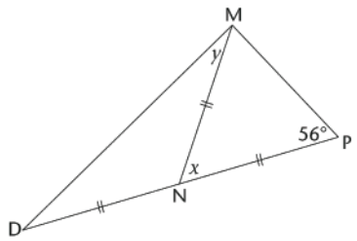
$x + (2x + 40°) + (x + 20°) = 180°$
[Interior ∠s of a Δ]
$4x = 180° - 40° - 20°$
$4x = 120°$
$x = 30°$
$\hat{H} = 2x + 40°$
$\hat{H} = 2(30°) + 40°$
$\hat{H} = 100°$

8. Calculate the size of $\hat{N}$.



$(x + 10°) + (2x - 30°) + (2x - 50°) = 180°$
[Interior ∠s of a Δ]
$5x = 180° + 30° + 50° - 10°$
$5x = 250°$
$x = 50°$
$\hat{N} = 2(50°) - 30°$
$\hat{N} = 70°$

9. DNP is a straight line. Calculate the size of x and y.



$N\hat{M}P = M\hat{P}N = 56°$ [Isosceles Δ]
$x + 56° + 56° = 180°$ [Interior ∠s of a Δ]
$x = 68°$
$M\hat{N}D = 180° - 68°$ [Straight line]
$M\hat{N}D = 112°$
$N\hat{M}D = N\hat{D}M = y$ [Isosceles Δ]
$y + y + 112° = 180°$ [Interior ∠s of a Δ]
$y = 34°$

11.3 Types of quadrilaterals and their properties

THE PROPERTIES OF DIFFERENT TYPES OF QUADRILATERALS

Teaching guidelines

In the previous chapter (10.7) learners investigated some of the properties of quadrilaterals. Let learners use those properties to name the shapes and find new properties.

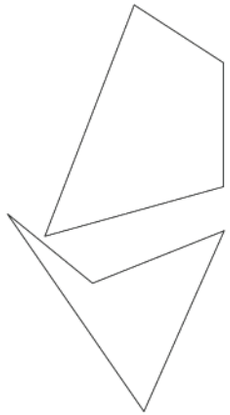
Learners will have to measure the sides and angles of the shapes as no further information is given on the drawings.

Answers

1. (a) Trapeziums. They have one pair of opposite sides parallel. The other sides or angles do not have any special properties in these examples.

11.3 Types of quadrilaterals and their properties

A quadrilateral is a figure with four straight sides which meet at four vertices. In many quadrilaterals all the sides are of different lengths and all the angles are of different sizes.



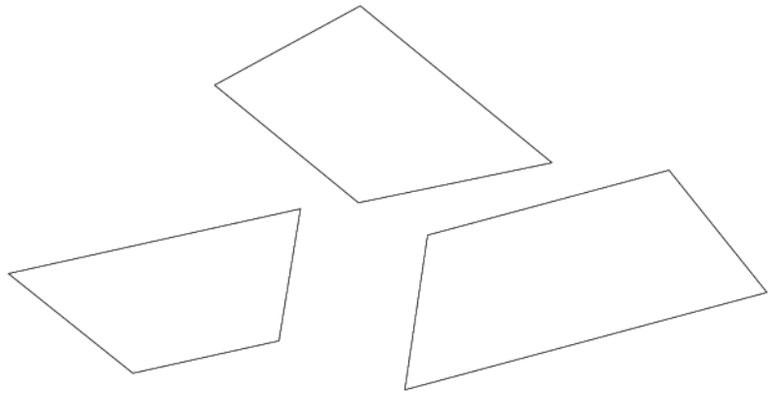
You have previously worked with these types of quadrilaterals, in which some sides have the same lengths, and some angles may be of the same size:

parallelograms
rectangles
kites
rhombuses
squares
trapeziums

THE PROPERTIES OF DIFFERENT TYPES OF QUADRILATERALS

1. In each of the following questions, different examples of a certain type of quadrilateral are given. In each case identify which kind of quadrilateral it is. Describe the properties of each type by making statements about the lengths and directions of the sides and the sizes of the angles of each type. You may have to take some measurements to be able to do this.

(a)



CHAPTER 11: GEOMETRY OF 2D SHAPES 115

Teaching guidelines

Explain that:

- a convex polygon is a polygon in which all the interior angles are smaller than 180°.
- a concave polygon has one or more interior angles that are greater than 180°, for example an arrowhead (the third shape in question 1(c)).

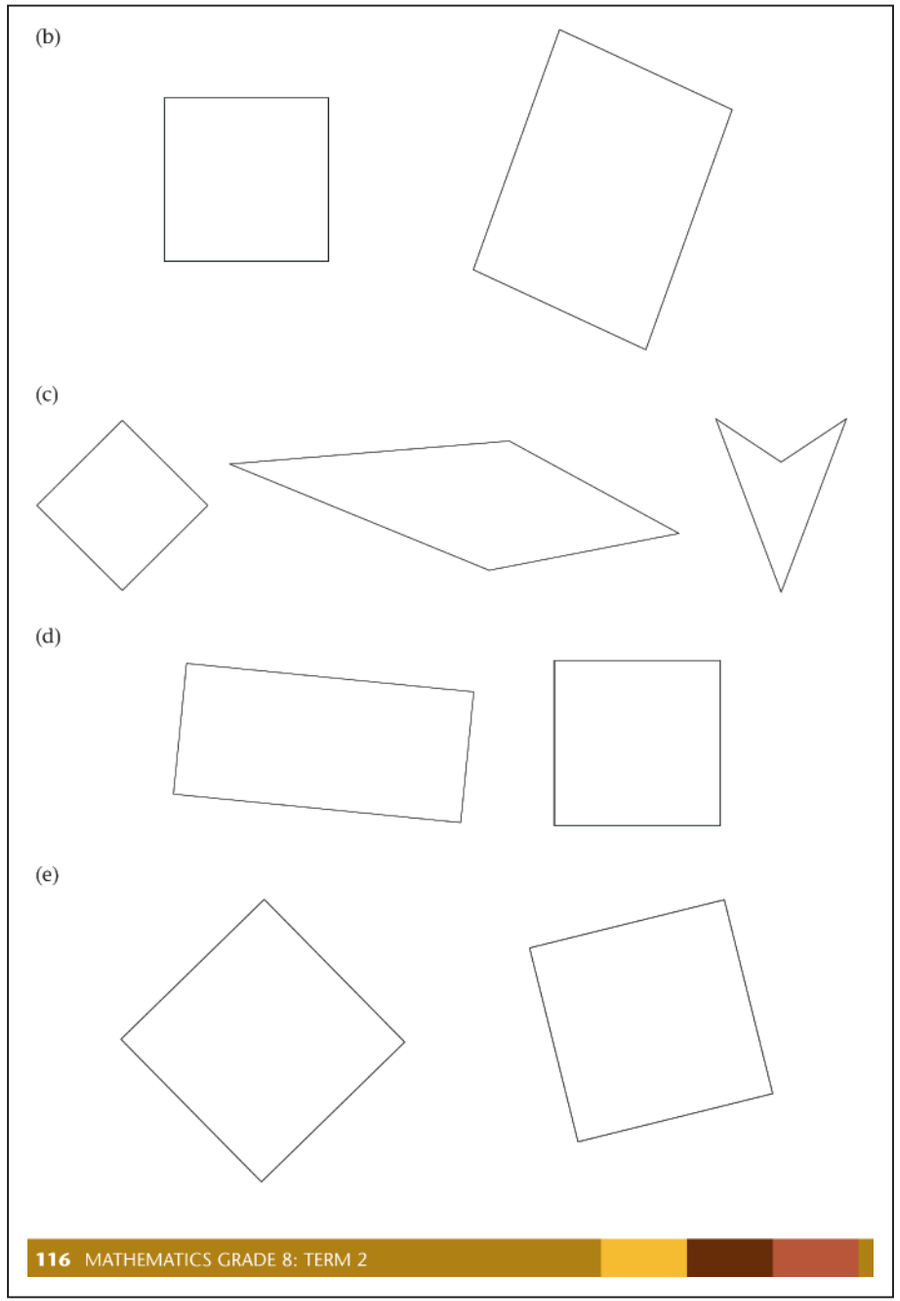
Answers

1. (b) These two shapes are parallelograms because they have opposite sides equal and opposite sides parallel. Both are a special kind of parallelogram (i.e. a rectangle) because all their angles are right angles.

   (c) These shapes are all kites, because they have adjacent sides equal. The first shape is a square, which is a special kind of kite that has additional properties of having all angles right angles and all sides equal. The third shape is a concave polygon, sometimes called an arrowhead.

   (d) These are rectangles, because the opposite sides are equal and parallel, and all the angles are right angles. The second shape is a square, a special kind of rectangle, which also has all sides equal.

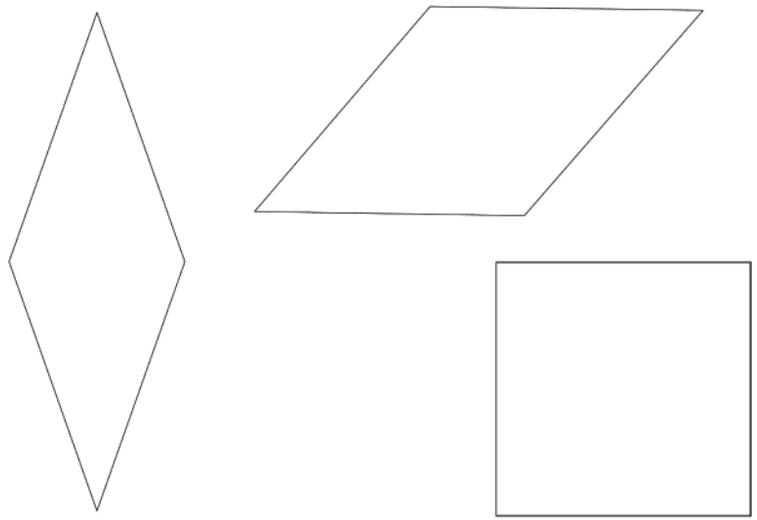
   (e) These shapes are squares, as they have four equal sides and all four angles equal to 90°.



Answers

1. (f) These shapes are all rhombi. All the sides are equal and the opposite sides are parallel. One of the rhombi also has angles that are right angles, which means it is a square (a special kind of rhombus).
2. (a) True
   (b) True
   (c) True
   (d) False
   (e) False
   (f) True
   (g) True
   (h) True
   (i) True
   (j) False
   (k) False

(f)



2. Use your completed lists and the drawings in question 1 to determine if the following statements are true or false:

   (a) A rectangle is a parallelogram.
   (b) A square is a parallelogram.
   (c) A rhombus is a parallelogram.
   (d) A kite is a parallelogram.
   (e) A trapezium is a parallelogram.
   (f) A square is a rhombus.
   (g) A square is a rectangle.
   (h) A square is a kite.
   (i) A rhombus is a kite.
   (j) A rectangle is a rhombus.
   (k) A rectangle is a square.

If a quadrilateral has *all* the properties of another quadrilateral, you can define it in terms of the other quadrilateral, as you have found above.

A **convention** is something (such as a definition or method) that most people agree on, accept and follow.

3. Here are some conventional definitions of quadrilaterals:
   - A **parallelogram** is a quadrilateral with two opposite sides parallel.
   - A **rectangle** is a parallelogram that has all four angles equal to 90°.
   - A **rhombus** is a parallelogram with all four sides equal.
   - A **square** is a rectangle with all four sides equal.
   - A **trapezium** is a quadrilateral with one pair of opposite sides parallel.
   - A **kite** is a quadrilateral with two pairs of adjacent sides equal.

3. There is a summary of the definitions at the bottom of LB page 117 on the previous page.
   (a) A quadrilateral with both pairs of opposite sides equal and all angles equal to 90°.
   (b) A rhombus with all angles equal to 90°.
   (c) A quadrilateral with both pairs of opposites sides equal and both pairs of opposite angles equal.
   (d) A quadrilateral with one pair of opposite angles equal and (two pairs of) adjacent sides equal.
   (e) A quadrilateral with only one pair of sides parallel. (There is no other definition that works.)

11.4 Unknown angles and sides of quadrilaterals

FINDING UNKNOWN ANGLES AND SIDES

Teaching guidelines

As in the case where learners worked with triangles, they make use of the properties of quadrilaterals that they already know.

Do an example on the board so that learners can see how to communicate their answers. Encourage them to write the necessary steps clearly. Their explanations should be clear enough for anyone to follow. They should not leave out any necessary steps.

Answers

1. to 4. See the answers on LB page 118 alongside.

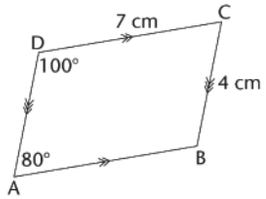
Write down other definitions that work for the following quadrilaterals:
(a) Rectangle (b) Square (c) Rhombus
(d) Kite (e) Trapezium

11.4 Unknown angles and sides of quadrilaterals

FINDING UNKNOWN ANGLES AND SIDES

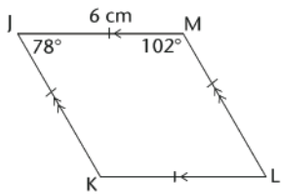
Find the length of all the **unknown sides** and **angles** in the following quadrilaterals. Give reasons to justify your statements. (Also recall that the sum of the angles of a quadrilateral is 360°.)

1.



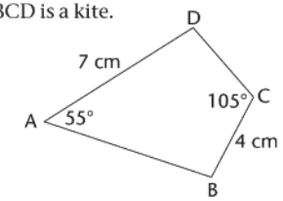
ABCD is a parallelogram [opposite sides ||]
DA = CB = 4 cm [parallelogram]
AB = DC = 7 cm [parallelogram]
$\hat{B} = \hat{D} = 100°$ [parallelogram]
$\hat{A} = \hat{C} = 80°$ [parallelogram]

2.



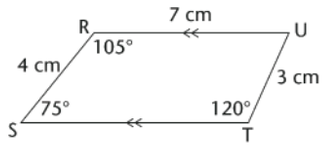
JKLM is a rhombus [opposite sides || and all sides equal]
JK = LM = KL = JM = 6 cm [rhombus]
$\hat{K} = \hat{M} = 102°$ [rhombus]
$\hat{J} = \hat{L} = 78°$ [rhombus]

3. ABCD is a kite.



$\hat{D} = \hat{B}$ [kite]
$\hat{D} + \hat{B} + 55° + 105° = 360°$ [quadrilateral]
$2\hat{D} = 360° - 105° - 55°$
$\hat{D} = 100° = \hat{B}$
AB = AD = 7 cm [kite]
BC = CD = 4 cm [kite]

4. The perimeter of RSTU is 23 cm.



4 + 7 + 3 + ST = 23 cm
ST = 9 cm
$75° + 105° + 120° + \hat{U} = 360°$ [quadrilateral]
$\hat{U} = 60°$

Answers

5. See the answers on LB page 119 alongside.

11.5 Congruency

WHAT IS CONGRUENCY?

Teaching guidelines

Learners have already come across the term congruent. They are familiar with reflections, rotations and translations under which only the position of the shape, and not the shape itself, is changed. Therefore, a shape and its reflection (rotation or translated shape) are congruent.

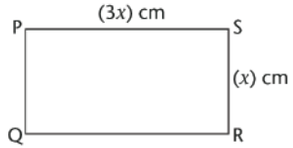
Use congruent shapes to illustrate the principle. Rotate a shape about the origin or reflect it horizontally or vertically so that learners can see that the original shape and its transformation are the same in size and area, and are therefore congruent.

Point out that in congruent shapes the corresponding side lengths are the same and the corresponding angles are equal.

Answers

1. Yes, by the nature of a reflection.
2. Yes, by the nature of a rotation.
3. Yes, by the nature of a translation.

5. PQRS is a rectangle and has a perimeter of 40 cm.

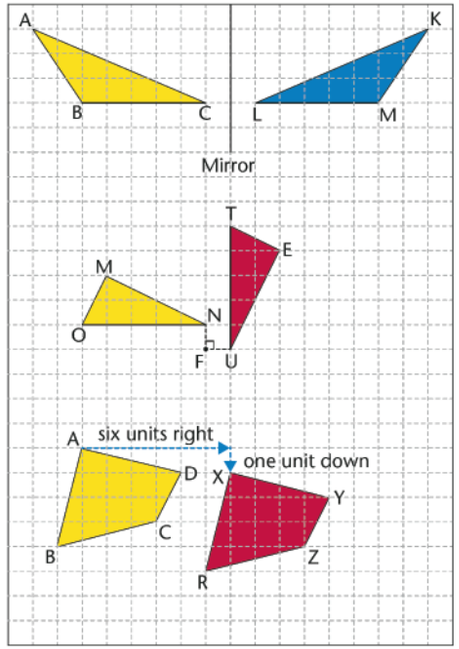


$2(3x + x) = 40$ cm
$8x = 40$ cm
$x = 5$ cm
PS = 3(5) = 15 cm
QR = PS [rectangle] = 15 cm
PQ = SR [rectangle] = x = 5 cm

11.5 Congruency

WHAT IS CONGRUENCY?

1. ΔABC is reflected in the vertical line (mirror) to give ΔKLM.
   Are the sizes and shapes of the two triangles exactly the same?



2. ΔMON is rotated 90° around point F to give you ΔTUE.
   Are the sizes and shapes of ΔMON and ΔTUE exactly the same?

3. Quadrilateral ABCD is translated six units to the right and one unit down to give quadrilateral XRZY.
   Are ABCD and XRZY exactly the same?

In the previous activity, each of the figures was transformed (reflected, rotated or translated) to produce a second figure. The second figure in each pair has **the same angles**, **side lengths**, **size and area** as the first figure. The second figure is therefore an **accurate copy** of the first figure.

Teaching guidelines

Learners usually go wrong when they have to name congruent and similar shapes. Therefore, take time to explain in detail how to name two congruent triangles so that the order in which the angles are mentioned is the order in which they are equal, for example if in ΔABC and ΔKLM ∠A = ∠K; ∠B = ∠M and ∠C = ∠L, we would say ΔABC is congruent to ΔKML and we write ΔABC ≡ ΔKML.

IDENTIFYING CONGRUENT ANGLES AND SIDES

Teaching guidelines

Show learners again that if ΔPQR ≡ ΔXYZ, it means that PQ = XY; QR = YZ, PR = XZ, ∠P = ∠X, ∠Q = ∠Y, ∠R = ∠Z.

Misconceptions

Learners do not understand the notation and give the incorrect information.

Answers

1. to 4. See the answers on LB page 120 alongside.

11.6 Similarity

CHECKING FOR SIMILARITY

Teaching guidelines

Learners worked with similar shapes in previous grades. Shapes that are similar have the same shape and the same corresponding angles, but their sizes differ.

Their corresponding sides are in proportion.

When one figure is an image of another figure, we say that the two figures are **congruent**.
The symbol for congruency is: ≡

The word **congruent** comes from the Latin word *congruere*, which means "to agree". Figures are congruent if they match up perfectly when laid on top of each other.

Notation of congruent figures

When we name shapes that are congruent, we name them so that the matching, or corresponding, angles are in the same order. For example, in **ΔABC** and **ΔKLM** on the previous page:

$\hat{A}$ is congruent to (matches and is equal to) $\hat{K}$.
$\hat{B}$ is congruent to $\hat{M}$.
$\hat{C}$ is congruent to $\hat{L}$.

We therefore use this notation: **ΔABC ≡ ΔKML**.

We cannot assume that, when the angles of polygons are equal, the polygons are congruent. You will learn about the conditions of congruence in Grade 9.

Similarly, for the other pairs of figures on the previous page:
ΔMON ≡ ΔTUE and **ABCD ≡ XRZY**.

The notation of congruent figures also shows which sides of the two figures correspond and are equal. For example, ΔABC ≡ ΔKML shows that:
AB = KM, BC = ML and AC = KL.

The incorrect notation ΔABC ≡ ΔKML would show the following incorrect information:
$\hat{B} = \hat{L}$, $\hat{C} = \hat{M}$, AB = KL and AC = KM.

IDENTIFYING CONGRUENT ANGLES AND SIDES

Copy the following table and write down which angles and sides are equal between each pair of congruent figures:

1. ΔPQR ≡ ΔUCT $\hat{P} = \hat{U}$, $\hat{Q} = \hat{C}$, $\hat{R} = \hat{T}$ PQ = UC, QR = CT, PR = UT	2. ΔKLM ≡ ΔUWC $\hat{K} = \hat{U}$, $\hat{L} = \hat{W}$, $\hat{M} = \hat{C}$ KL = UW, LM = WC, KM = UC
3. ΔGHI ≡ ΔQRT $\hat{G} = \hat{Q}$, $\hat{H} = \hat{R}$, $\hat{I} = \hat{T}$ GH = QR, HI = RT, GI = QT	4. ΔKJL ≡ ΔPOQ $\hat{K} = \hat{P}$, $\hat{J} = \hat{O}$, $\hat{L} = \hat{Q}$ KJ = PO, JL = OQ, KL = PQ

11.6 Similarity

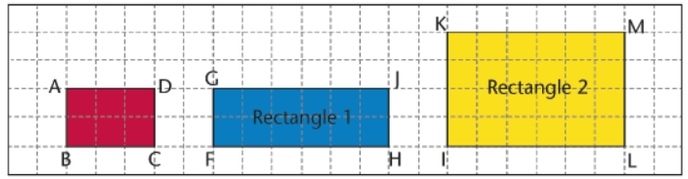
In Grade 7, you learnt that two figures are **similar** when they have the **same shape** (their angles are equal) but they may be **different sizes**. The sides of one figure are proportionally longer or shorter than the sides of the other figure; that is, the length of each side is multiplied or divided by the same number. We say that one figure is an enlargement or a reduction of the other figure.

Answers

1. (a) 2; 1 (they are the same)
   (b) 2; 2
   (c) Rectangle 2. The corresponding sides are enlarged in the same proportion.
2. (a) FC longer than BC? 2
   HF longer than AB? 2
   HG longer than AC? 2
   IK shorter than BC? 2
   JI shorter than AB? 2
   JK shorter than AC? 2
   (b) Yes. The corresponding sides are enlarged in the same proportion.
   (c) Yes. The corresponding sides are reduced in the same proportion.

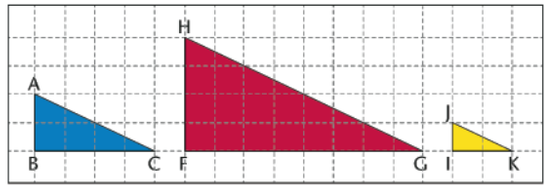
CHECKING FOR SIMILARITY

1. Look at the rectangles below and answer the questions that follow:



   (a) Look at rectangle 1 and ABCD. How many times is FH longer than BC? How many times is GF longer than AB?
   (b) Look at rectangle 2 and ABCD. How many times is IL longer than BC? How many times is LM longer than CD?
   (c) Is rectangle 1 or rectangle 2 an enlargement of rectangle ABCD? Explain your answer.

2. Look at the triangles below and answer the questions that follow:



   (a) How many times is:
   - FG longer than BC?
   - HG longer than AC?
   - JI shorter than AB?
   - HF longer than AB?
   - IK shorter than BC?
   - JK shorter than AC?

   (b) Is ΔHFG an enlargement of ΔABC? Explain your answer.
   (c) Is ΔJIK a reduction of ΔABC? Explain your answer.

In the previous activity, rectangle KILM is an enlargement of rectangle ABCD. Therefore, ABCD is similar to KILM. The symbol for "is similar to" is: |||. So we write: ABCD ||| KILM.

The triangles on the previous page are also similar. ΔHFG is an enlargement of ΔABC, and ΔJIK is a reduction of ΔABC.

In ΔABC and ΔHFG, $\hat{A} = \hat{H}$, $\hat{B} = \hat{F}$ and $\hat{C} = \hat{G}$. We therefore write it like this: ΔABC ||| ΔHFG.

In the same way, ΔABC ||| ΔJIK.

When you enlarge or reduce a polygon, you need to enlarge or reduce all its sides proportionally, or by the same ratio. This means that you multiply or divide each length by the same number.

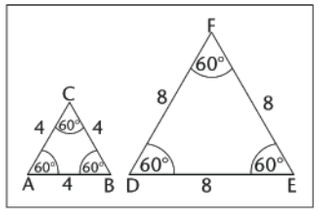
Answers

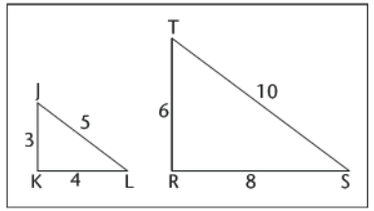
1. Yes. The angles are the same size (equilateral Δ).
   Yes. The corresponding sides are enlarged in the same proportion.
2. No. While RT × 4 = EF, TU is enlarged less than 2 times to FG.
3. ZY is 2 × QR. So XZ = 2PR = 16 cm and XY = 2PQ = 10 cm
4. (a) True, the corresponding sides are in the same proportion (equal, in fact).
   (b) False, the corresponding sides are not necessarily of equal length.
   (c) False, the breadth can be enlarged by one proportion and the length by another.
   (d) True, corresponding sides are in the same proportion as length = breadth.

Similar figures are figures that have the same angles (same shape) but are not necessarily the same size.

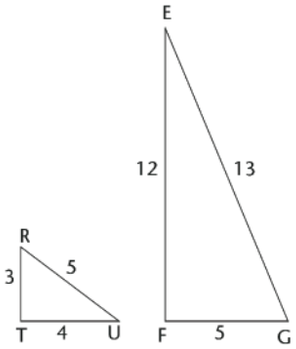
USING PROPERTIES OF SIMILAR AND CONGRUENT FIGURES

1. Are the triangles in each pair similar? Give a reason for each answer.

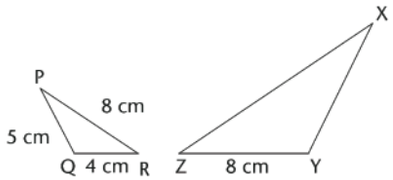




2. Is ΔRTU ||| ΔEFG? Give a reason for your answer.



3. ΔPQR ||| ΔXYZ. Determine the length of XZ and XY.



4. Are the following statements true or false? Explain your answers.
   (a) Figures that are congruent are similar.
   (b) Figures that are similar are congruent.
   (c) All rectangles are similar.
   (d) All squares are similar.

WORKSHEET

Answers

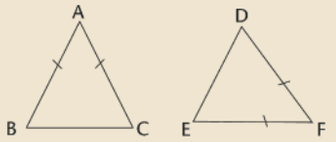
1. (a) See the answer on LB page 123 alongside.
   (b) 40 mm
   (c) $\hat{B} = \hat{C} = 80°$; $\hat{A} + 80° + 80° = 180°$ $\hat{A} = 20°$
   (d) AB = AC = EF = DF
   (e) BC
   (f) $\hat{F} = \hat{A} = 40°$ [Congruent] $\hat{B} = \hat{C}$ [Isosceles Δ]
   $\hat{B} + \hat{B} + 40° = 180°$ [Interior ∠s of a △] $\hat{B} = 70°$
2. (a) Parallelogram. The opposite sides are parallel.
   (b) Yes. Corresponding sides are enlarged in the same proportion.
   (c) 115° [Opposite angles of parallelogram]
   (d) $\hat{M} = 65°$ [Opposite angles of parallelogram]
   $\hat{S} = \hat{M} = 65°$ [Corresponding angles of similar figures]
   (e) KL = 6 cm [Opposite sides of parallelogram]

WORKSHEET

1. Study the triangles below and answer the following questions:

   (a) Choose the correct answer and write it down.
   ΔABC is:

   - ☐ acute and equilateral
   - ☐ obtuse and scalene
   - ☑ acute and isosceles
   - ☐ right-angled and isosceles



   (b) If AB = 40 mm, what is the length of AC?

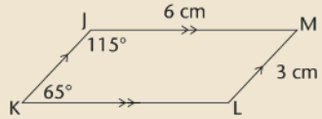
   (c) If $\hat{B} = 80°$, what is the size of $\hat{C}$ and of $\hat{A}$?

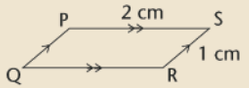
   (d) ΔABC ≡ ΔFDE. Name all the sides in the two triangles that are equal to AB.

   (e) Name the side that is equal to DE.

   (f) If $\hat{F}$ is 40°, what is the size of $\hat{B}$?

2. Look at figures JKLM and PQRS. (Give reasons for your answers below.)





   (a) What type of quadrilateral is JKLM?

   (b) Is JKLM ||| PQRS?

   (c) What is the size of $\hat{L}$?

   (d) What is the size of $\hat{S}$?

   (e) What is the length of KL?

GEOMETRY OF 2D SHAPES

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
12.1 Angles on a straight line	Working with the sum of supplementary angles and using the knowledge to calculate unknown angles on a straight line	Pages 124 to 127
12.2 Vertically opposite angles	Definition of vertically opposite angles; finding unknown vertically opposite angles; making equations using vertically opposite angles	Pages 127 to 130
12.3 Lines intersected by a transversal	Pairs of angles formed by a transversal; identifying angles formed by a transversal	Pages 130 to 132
12.4 Parallel lines intersected by a transversal	Investigating angle sizes; identifying angles on parallel lines	Pages 132 to 134
12.5 Finding unknown angles on parallel lines	Working out unknown angles using known facts about parallel lines; sum of angles in a quadrilateral	Pages 135 to 137
12.6 Solving more geometric problems	Using angle relationships on parallel lines; investigating properties of triangles and quadrilaterals	Pages 137 to 139

CAPS time allocation	9 hours
CAPS content specification	Page 98

Mathematical background

Supplementary angles are angles that add up to 180°. **Complementary angles** add up to 90°.

Angles that share a vertex and a side are called **adjacent angles**.

Adjacent angles on a straight line are **supplementary**.

Vertically opposite angles are formed when two lines intersect and share a vertex but do not have a common side.

Two lines intersected by a transversal form pairs of angles:

- Angles that are in the same position, one at either point of intersection, are called **corresponding angles**, for example an angle above the line and to the left of the transversal.
- Angles of which the positions at the points of intersection are switched are called **alternate angles**. If the one angle is below the line on the right side of the transversal, its alternate angle is above the line on the right side of the transversal.
- Angles that lie on the same side of the transversal and between the two lines are called **co-interior angles**.

When parallel lines are intersected by a transversal, corresponding angle pairs and alternate angle pairs are equal, and co-interior angles are supplementary.

The facts about angles on a straight line, vertically opposite angles, angles formed when parallel lines are cut by a transversal, and the properties of triangles and quadrilaterals can all be used to find unknown angles in geometric figures.

12.1 Angles on a straight line

SUM OF ANGLES ON A STRAIGHT LINE

Teaching guidelines

In this section learners are introduced to concepts such as:

- supplementary and complementary angles
- adjacent angles.

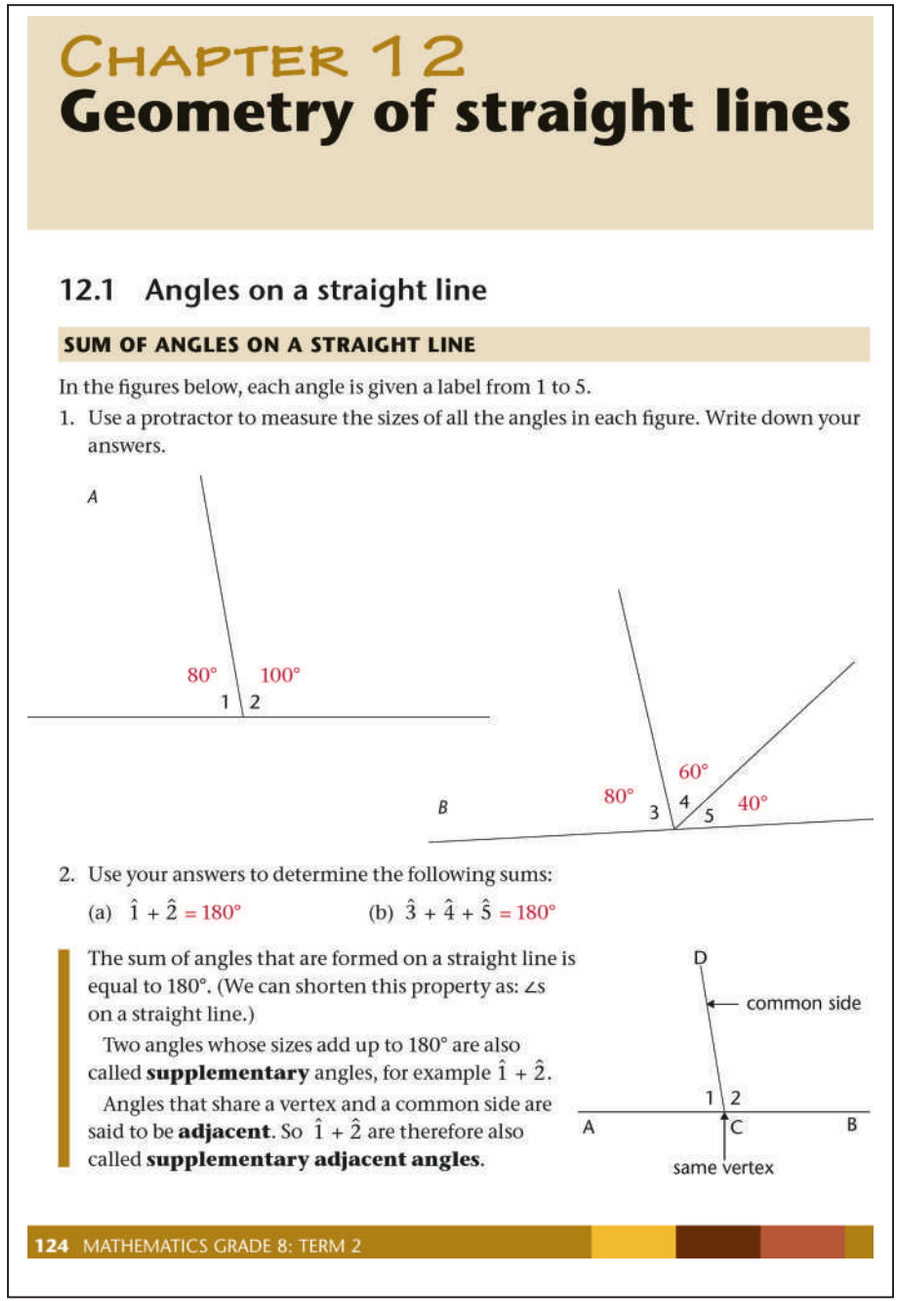
Show learners that adjacent angles share a side and a vertex.

Misconceptions

Learners think that complementary and supplementary angles are always adjacent. This is not true. As long as they add up to 90° or 180° they are complementary and supplementary respectively.

Answers

1. See the answers on LB page 124 alongside.
2. See the answers on LB page 124 alongside.



CHAPTER 12

Geometry of straight lines

12.1 Angles on a straight line

SUM OF ANGLES ON A STRAIGHT LINE

In the figures below, each angle is given a label from 1 to 5.

1. Use a protractor to measure the sizes of all the angles in each figure. Write down your answers.

2. Use your answers to determine the following sums:

   (a) $\hat{1} + \hat{2} = 180°$ (b) $\hat{3} + \hat{4} + \hat{5} = 180°$

The sum of angles that are formed on a straight line is equal to 180°. (We can shorten this property as: ∠s on a straight line.)

Two angles whose sizes add up to 180° are also called **supplementary** angles, for example $\hat{1} + \hat{2}$.

Angles that share a vertex and a common side are said to be **adjacent**. So $\hat{1} + \hat{2}$ are therefore also called **supplementary adjacent angles**.

FINDING UNKNOWN ANGLES ON STRAIGHT LINES

Teaching guidelines

Learners should follow the normal steps to solve an equation after they have made an equation to find an unknown angle.

Do an example on the board so that learners can see how to communicate their answers. Insist on answers that have all the steps written out in full, not just a single answer, for example $x = 117°$. Learners can often see the solution, but do not want to write out the full train of thought and the reason.

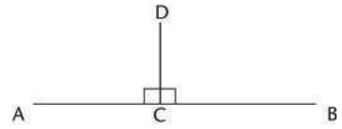
Sometimes they "see" what the answer is without consciously thinking why this is so. Reasoning in geometry is a deductive process which learners have to learn. If we expect answers that are set out in full, with reasons, we are helping learners to develop the process of deductive reasoning.

Answers

1. to 3. See the full answers on LB page 125 alongside.

When two lines are perpendicular, their adjacent supplementary angles are each equal to 90°.

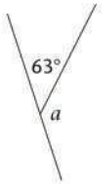
In the drawing alongside, $D\hat{C}A$ and $D\hat{C}B$ are adjacent supplementary angles because they are next to each other (adjacent) and they add up to 180° (supplementary).



FINDING UNKNOWN ANGLES ON STRAIGHT LINES

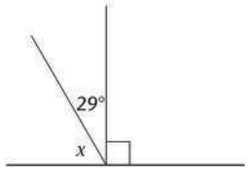
Work out the sizes of the unknown angles below. Build an equation each time as you solve these geometric problems. Always give a reason for every statement you make.

1. Calculate the size of a.



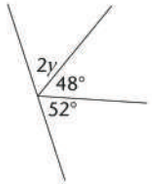
$a + 63° = 180°$ [∠s on a straight line]
$a = 180° - 63°$
$= 117°$

2. Calculate the size of x.



$x + 29° + 90° = 180°$ [∠s on a straight line]
$x = 180° - 29° - 90°$
$x = 61°$

3. Calculate the size of y.



$2y + 48° + 52° = 180°$ [∠s on a straight line]
$2y = 180° - 52° - 48°$
$2y = 80°$
$y = 40°$

FINDING MORE UNKNOWN ANGLES ON STRAIGHT LINES

Teaching guidelines

Learners have to think about the way to answer more than one question about the information given in the drawing.

Each situation is given on a straight line, so the angles are all supplementary.

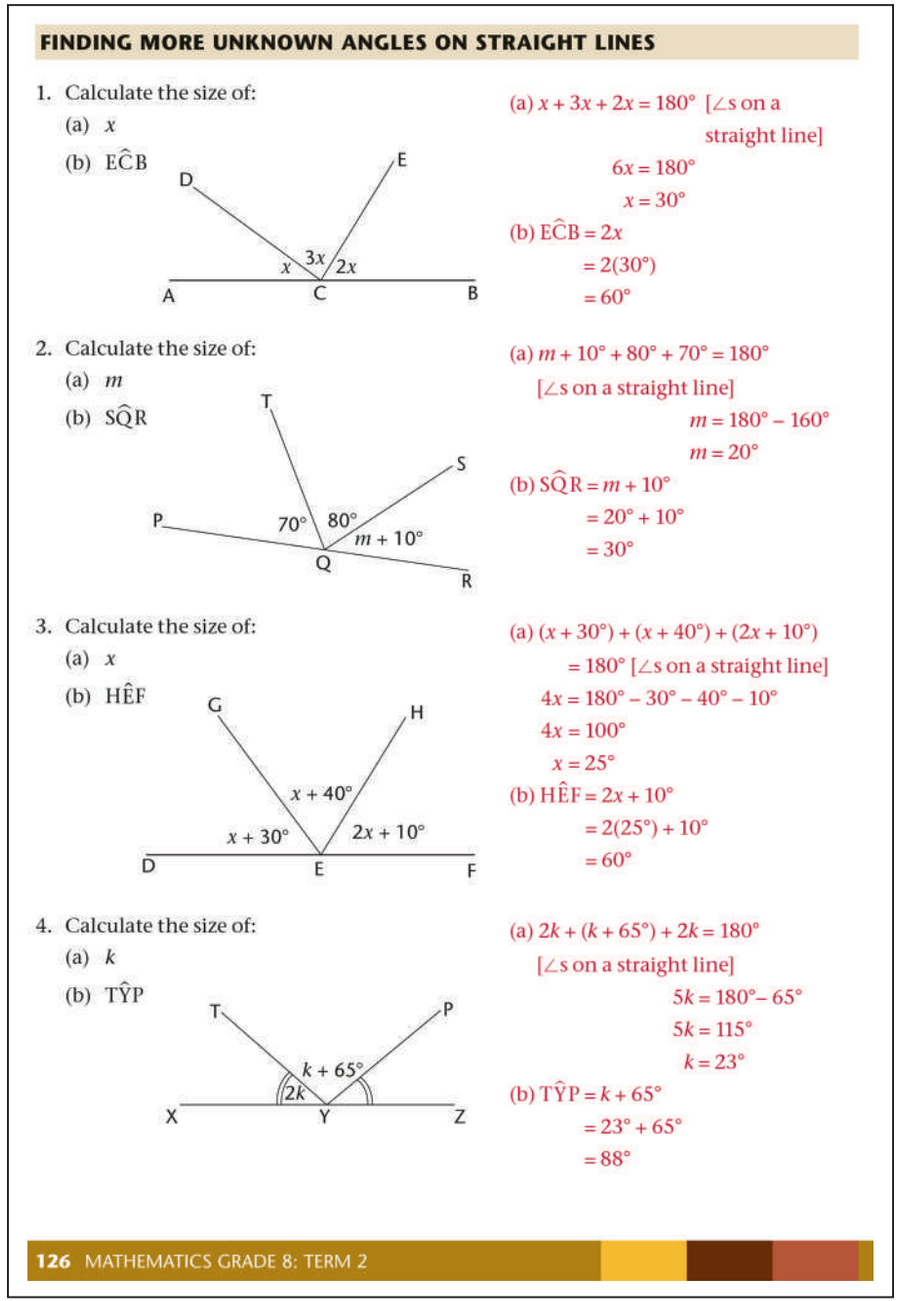
Misconceptions

Sometimes learners guess the size of angles. For example, in question 3 they would say "$x + 40°$ looks as if it could be 90°" and calculate x from there. Show learners how to use the information about straight lines to solve this one, namely that the three angles add up to 180° and they have to make an equation using this fact.

In a situation where all the angles are unknown, learners hesitate as they forget the relationship with the straight line. Impress on them that to make an equation you need to set an unknown value equal to a known value, in this case it is the fact that the sum of the angles on a straight line is 180°.

Answers

1. to 4. See the answers on LB page 126 alongside.



FINDING MORE UNKNOWN ANGLES ON STRAIGHT LINES

1. Calculate the size of:
   (a) x
   (b) $E\hat{C}B$

(a) $x + 3x + 2x = 180°$ [∠s on a straight line]
$6x = 180°$
$x = 30°$
(b) $E\hat{C}B = 2x$
$= 2(30°)$
$= 60°$

2. Calculate the size of:
   (a) m
   (b) $S\hat{Q}R$

(a) $m + 10° + 80° + 70° = 180°$ [∠s on a straight line]
$m = 180° - 160°$
$m = 20°$
(b) $S\hat{Q}R = m + 10°$
$= 20° + 10°$
$= 30°$

3. Calculate the size of:
   (a) x
   (b) $H\hat{E}F$

(a) $(x + 30°) + (x + 40°) + (2x + 10°)$
$= 180°$ [∠s on a straight line]
$4x = 180° - 30° - 40° - 10°$
$4x = 100°$
$x = 25°$
(b) $H\hat{E}F = 2x + 10°$
$= 2(25°) + 10°$
$= 60°$

4. Calculate the size of:
   (a) k
   (b) $T\hat{Y}P$

(a) $2k + (k + 65°) + 2k = 180°$ [∠s on a straight line]
$5k = 180° - 65°$
$5k = 115°$
$k = 23°$
(b) $T\hat{Y}P = k + 65°$
$= 23° + 65°$
$= 88°$

Answers

5. See answers on LB page 127 alongside.

12.2 Vertically opposite angles

WHAT ARE VERTICALLY OPPOSITE ANGLES?

Teaching guidelines

Use reasoning to show learners that vertically opposite angles are equal.

On LB page 127 $\angle a + \angle b = \angle b + \angle c$; and therefore $\angle a = \angle c$; vertically opposite angles are equal.

Misconceptions

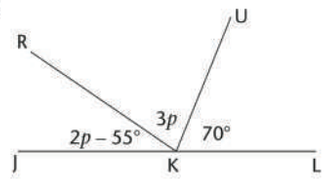
Sometimes learners confuse vertically opposite angles and opposite angles.

Show learners that the opposite angles in a parallelogram, for example, do not share a vertex, while vertically opposite angles share a vertex.

Answers

1. See the answers on LB page 127 alongside.
2. $a = c$ and $b = d$, they are vertically opposite angles.

5. Calculate the size of:
   (a) p
   (b) $J\hat{K}R$

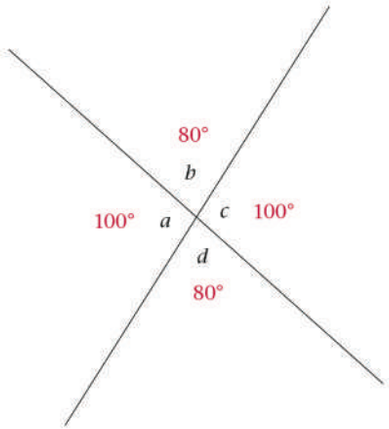


(a) $3p + 70° + (2p - 55°)$
$= 180°$ [$\angle$s on a straight line]
$5p = 180° - 70° + 55°$
$5p = 165°$
$p = 33°$
(b) $J\hat{K}R = 2p - 55°$
$= 2(33°) - 55°$
$= 11°$

12.2 Vertically opposite angles

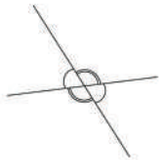
WHAT ARE VERTICALLY OPPOSITE ANGLES?

1. Use a protractor to measure the sizes of all the angles in the figure. Write down your answers.



2. Notice which angles are equal and how these equal angles are formed.

Vertically opposite angles (**vert. opp. $\angle$s**) are the angles opposite each other when two lines intersect.
Vertically opposite angles are **always equal**.



FINDING UNKNOWN ANGLES

Teaching guidelines

Point out to learners how they can use what they have already learnt (for example about angles on a straight line), combined with the new fact about vertically opposite angles, when they provide answers to these questions.

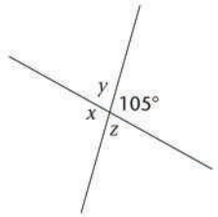
Answers

1. to 4. See the answers on LB page 128 alongside.

FINDING UNKNOWN ANGLES

Calculate the sizes of the unknown angles in the following figures. Always give a reason for every statement you make.

1. Calculate x, y and z.



$x = 105°$ [vert. opp. ∠s]

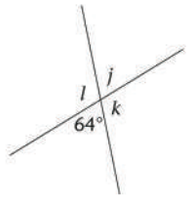
$y + 105° = 180°$ [∠s on a straight line]

$y = 180° - 105°$

$= 75°$

$z = 75°$ [vert. opp. ∠s]

2. Calculate j, k and l.



$64° + k = 180°$ [∠s on a straight line]

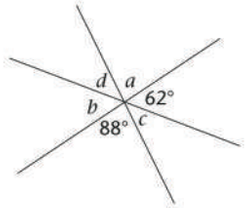
$k = 180° - 64°$

$k = 116°$

$j = 64°$ [vert. opp. ∠s]

$l = 116°$ [vert. opp. ∠s]

3. Calculate a, b, c and d.



$62° + 88° + c = 180°$ [∠s on a straight line]

$c = 180° - 62° - 88°$

$c = 30°$

$a = 88°$ [vert. opp. ∠s]

$b = 62°$ [vert. opp. ∠s]

$d = 30°$ [vert. opp. ∠s]

EQUATIONS USING VERTICALLY OPPOSITE ANGLES

Teaching guidelines

Learners should not have a problem with these questions.

When they write down answers they should think of it as communicating their thought processes. This communication should be clear and precise so that a reader can follow the argument; therefore reasons have to be given where needed. The first sentence with the reason is the most important one, thereafter it is a process of solving an equation.

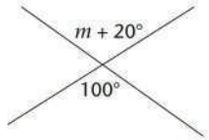
Answers

1. to 4. See the answers on LB page 129 alongside.

EQUATIONS USING VERTICALLY OPPOSITE ANGLES

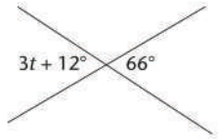
Vertically opposite angles are always equal. We can use this property to build an equation. Then we solve the equation to find the value of the unknown variable.

1. Calculate the value of m.



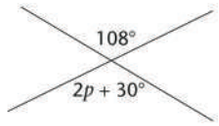
$m + 20° = 100°$ [vert. opp. ∠s]
$m = 100° - 20°$
$= 80°$

2. Calculate the value of t.



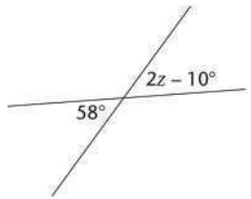
$3t + 12° = 66°$ [vert. opp. ∠s]
$3t = 54°$
$t = 18°$

3. Calculate the value of p.



$108° = 2p + 30°$ [vert. opp. ∠s]
$78° = 2p$
$p = 39°$

4. Calculate the value of z.



$58° = 2z - 10°$ [vert. opp. ∠s]
$68° = 2z$
$z = 34°$

CHAPTER 12: GEOMETRY OF STRAIGHT LINES 129

Answers

5. to 6. See the answers on LB page 130 alongside.

12.3 Lines intersected by a transversal

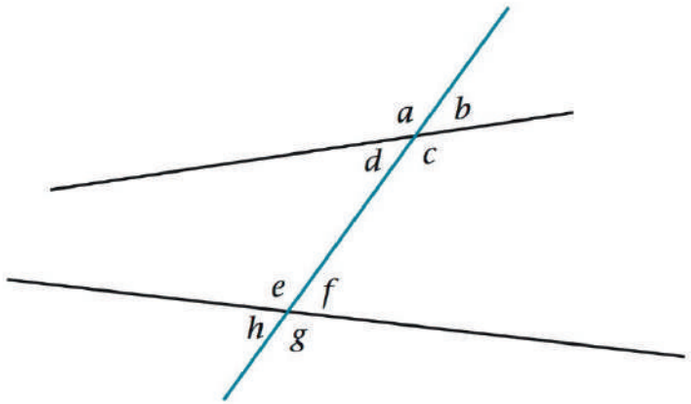
PAIRS OF ANGLES FORMED BY A TRANSVERSAL

Teaching guidelines

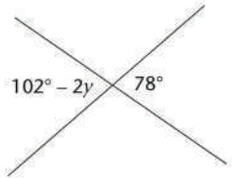
Define a transversal, i.e. a line that intersects at least two other lines.

Discuss the pairs of angles formed, one at each point of intersection, with regard to the positions of the angles. Refer to the drawing below for the following:

- Angles that are in the same position, one at either point of intersection, are called **corresponding angles**. For example, an angle above the line and to the left of the transversal, like *a* and *e* below. Other examples are *d* and *h*, *b* and *f*, and *c* and *g*.
- Angles of which the positions at the points of intersection are switched are called alternate angles. If the one angle is below the line on the right side of the transversal, like *c* below, its alternate angle, *e*, is above the line on the right side of the transversal. These are **alternate interior angles**. Another pair is *d* and *f*. Exterior alternate angle pairs are *g* and *a*; *b* and *h*.
- Angles that lie on the same side of the transversal and between the two lines are called **co-interior angles**. Examples of these are *c* and *f*, and *d* and *e* below.



5. Calculate the value of y.

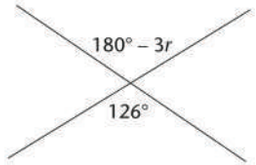


$102° - 2y = 78°$ [vert. opp. ∠s]

$-2y = -24°$

$y = 12°$

6. Calculate the value of r.



$126° = 180° - 3r$ [vert. opp. ∠s]

$3r = 54°$

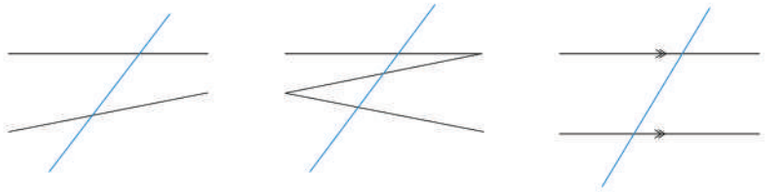
$r = 18°$

12.3 Lines intersected by a transversal

PAIRS OF ANGLES FORMED BY A TRANSVERSAL

A **transversal** is a line that crosses at least two other lines.

The blue line is the transversal.



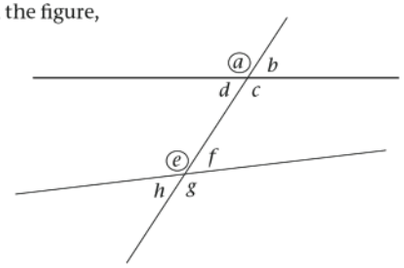
When a transversal intersects two lines, we can compare the sets of angles on the two lines by looking at their positions.

Answers

1. *d* and *h*: Left of the transversal and below a line.
   c and *g*: Right of the transversal and below a line.
2. *d* and *f*: Respectively left of the transversal and below a line, and right of the transversal and above a line.
   c and *e*: Respectively right of the transversal and below a line, and left of the transversal and above a line.
   a and *g*: Respectively left of the transversal and above a line, and right of the transversal and below a line.
   b and *h*: Respectively right of the transversal and above a line, and left of the transversal and below a line.
3. *d* and *e*: Respectively left of the transversal below the top line, and left of the transversal and above the bottom line.
   c and *f*: Respectively right of the transversal below the top line, and right of the transversal and above the bottom line.

The angles that lie on the same side of the transversal and are in matching positions are called **corresponding angles** (**corr. ∠s**). In the figure, these are corresponding angles:

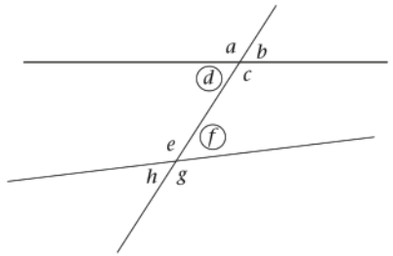
- *a* and *e*
- *b* and *f*
- *d* and *h*
- *c* and *g*.



1. In the figure, *a* and *e* are both left of the transversal and above a line.
   Write down the location of the following corresponding angles. The first one has been done for you.
   b and *f*: Right of the transversal and above the lines.
   d and *h* *c* and *g*

Alternate angles (**alt. ∠s**) lie on opposite sides of the transversal, but are not adjacent or vertically opposite. When the alternate angles lie between the two lines, they are called **alternate interior angles**. In the figure, these are alternate interior angles:

- *d* and *f*
- *c* and *e*.



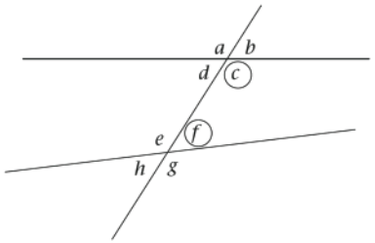
When the alternate angles lie outside of the two lines, they are called **alternate exterior angles**. In the figure, these are alternate exterior angles:

- *a* and *g*
- *b* and *h*.

2. Write down the location of the following alternate angles:
   d and *f* *c* and *e* *a* and *g* *b* and *h*

Co-interior angles (**co-int. ∠s**) lie on the same side of the transversal and between the two lines. In the figure, these are co-interior angles:

- *c* and *f*
- *d* and *e*.



3. Write down the location of the following co-interior angles:
   d and *e* *c* and *f*

IDENTIFYING TYPES OF ANGLES

Teaching guidelines

Note that we are not talking about the size of the angles, simply their position in relation to the transversal and the points of intersection of the transversal with the lines. Learners must become fluent in recognising corresponding angle pairs, alternate angle pairs and co-interior angle pairs.

Answers

1. a and e, c and g, b and f, d and h
2. b and h, c and e
3. a and g, d and f
4. b and e, c and h
5. a and c, h and f, d and b, e and g

12.4 Parallel lines intersected by a transversal

INVESTIGATING ANGLE SIZES

Teaching guidelines

Let learners measure the angles in both figures (i.e. where the lines are parallel and where they are not parallel).

Let them work in small groups and discuss the measurements they took of the angles.

Make sure that learners understand that the alternate angles and corresponding angles are equal, and that co-interior angles are supplementary only if the two lines cut by a transversal are parallel.

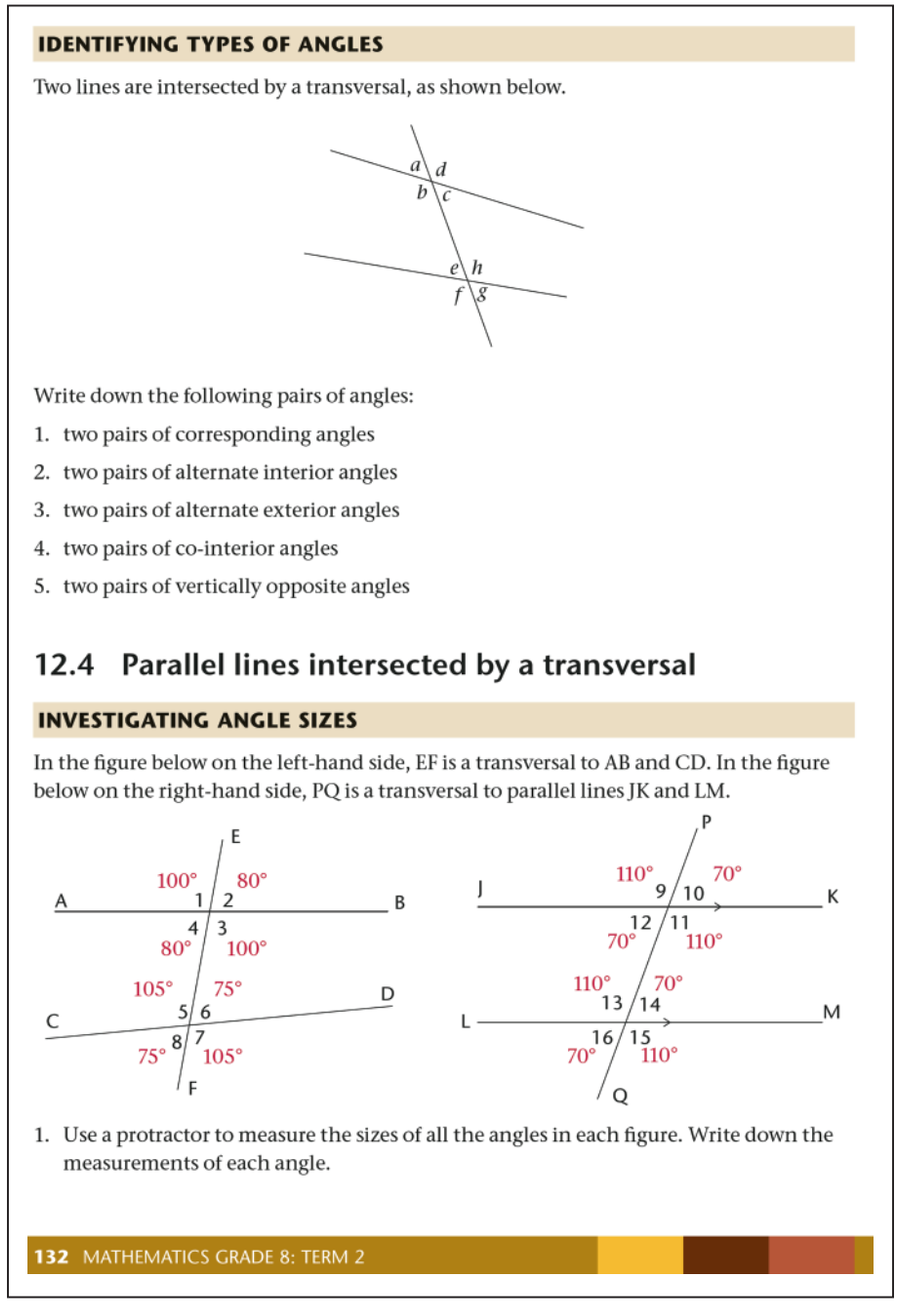
An extension of these questions is to give the angle sizes and ask learners to find out whether the lines are parallel or not.

Misconceptions

Learners often think that because angles are corresponding (or alternate) they are equal and that co-interior angles are supplementary. The measuring activity and the table that follows should help to resolve this misconception.

Answers

1. See the answers on LB page 132 alongside.



IDENTIFYING TYPES OF ANGLES

Two lines are intersected by a transversal, as shown below.

Write down the following pairs of angles:

1. two pairs of corresponding angles
2. two pairs of alternate interior angles
3. two pairs of alternate exterior angles
4. two pairs of co-interior angles
5. two pairs of vertically opposite angles

12.4 Parallel lines intersected by a transversal

INVESTIGATING ANGLE SIZES

In the figure below on the left-hand side, EF is a transversal to AB and CD. In the figure below on the right-hand side, PQ is a transversal to parallel lines JK and LM.

1. Use a protractor to measure the sizes of all the angles in each figure. Write down the measurements of each angle.

Answers

2. See the completed table on LB page 133 alongside.
3. Corresponding angles are equal.
   Alternate interior angles are equal.
   Alternate exterior angles are equal.
   Co-interior angles add up to 180°.

IDENTIFYING ANGLES ON PARALLEL LINES

Teaching guidelines

The fact that lines are parallel and cut by a transversal makes it possible to say that corresponding and alternate angles are equal and that co-interior angles are supplementary.

Remind learners that if lines are parallel they will be marked by small arrows in the same direction.

Answers

1. See the completed diagrams on LB page 133 alongside.

2. Copy the table and use your measurements to complete it.

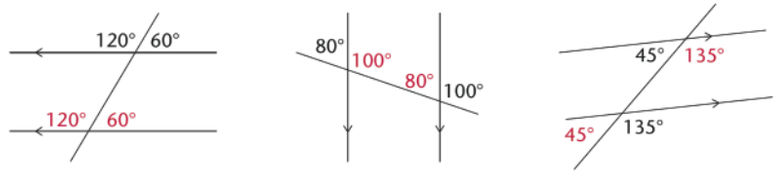
Angles	When two lines are not parallel	When two lines are parallel
Corr. ∠s	$\hat{1} = 100°$; $\hat{5} = 105°$ $\hat{4} = 80°$; $\hat{8} = 75°$ $\hat{2} = 80°$; $\hat{6} = 75°$ $\hat{3} = 100°$; $\hat{7} = 105°$	$\hat{9} = 110°$; $\widehat{13} = 110°$ $\widehat{12} = 70°$; $\widehat{16} = 70°$ $\widehat{10} = 70°$; $\widehat{14} = 70°$ $\widehat{11} = 110°$; $\widehat{15} = 110°$
Alt. int. ∠s	$\hat{4} = 80°$; $\hat{6} = 75°$ $\hat{3} = 100°$; $\hat{5} = 105°$	$\widehat{12} = 70°$; $\widehat{14} = 70°$ $\widehat{11} = 110°$; $\widehat{13} = 110°$
Alt. ext. ∠s	$\hat{1} = 100°$; $\hat{7} = 105°$ $\hat{2} = 80°$; $\hat{8} = 75°$	$\hat{9} = 110°$; $\widehat{15} = 110°$ $\widehat{10} = 70°$; $\widehat{16} = 70°$
Co-int. ∠s	$\hat{4} + \hat{5} = 185°$ $\hat{3} + \hat{6} = 175°$	$\widehat{12} + \widehat{13} = 180°$ $\widehat{11} + \widehat{14} = 180°$

3. Look at your completed table in question 2. What do you notice about the angles formed when a transversal intersects parallel lines?

When lines are parallel:
- corresponding angles are equal
- alternate interior angles are equal
- alternate exterior angles are equal
- co-interior angles add up to 180°.

IDENTIFYING ANGLES ON PARALLEL LINES

1. Copy these drawings and fill in the corresponding angles to those given:



Answers

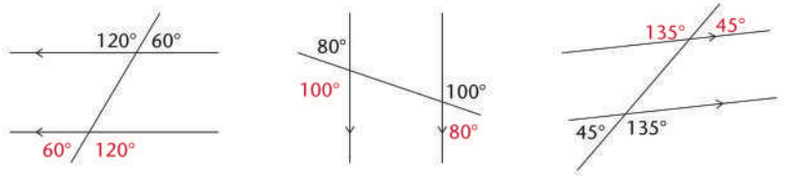
2. to 3. See the completed diagrams on LB page 134 alongside.

4. (a) See the completed diagrams on LB page 134 alongside.

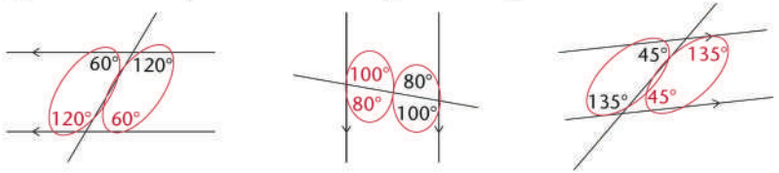
   (b) A: At the top point of the intersection, the vertically opposite angles are y on the left of the transversal and x on the right of the transversal. The angles at the bottom point of intersection are the same sizes as those at the top and are in the same order; corresponding pairs of angles.
   B: The same reasoning as for situation A.

5. See the completed diagrams on LB page 134 alongside.

2. Copy the following drawings and fill in the alternate exterior angles:

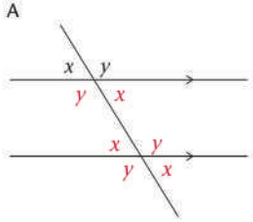


3. (a) Copy the drawings and fill in the alternate interior angles.
   (b) Circle the two pairs of co-interior angles in each figure.

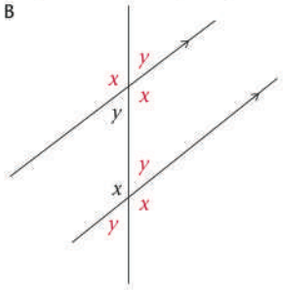


4. (a) Copy the drawings below. Without measuring, fill in all the angles in the following figures that are equal to x and y.
   (b) Explain your reasons for each x and y that you filled in to your partner.

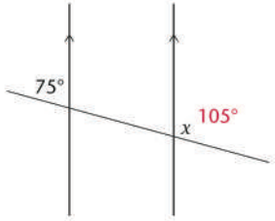
A

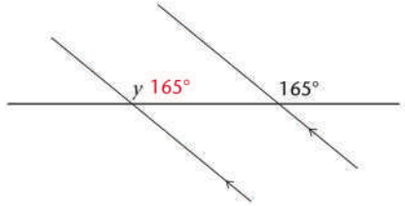


B



5. Give the value of x and y below:





12.5 Finding unknown angles on parallel lines

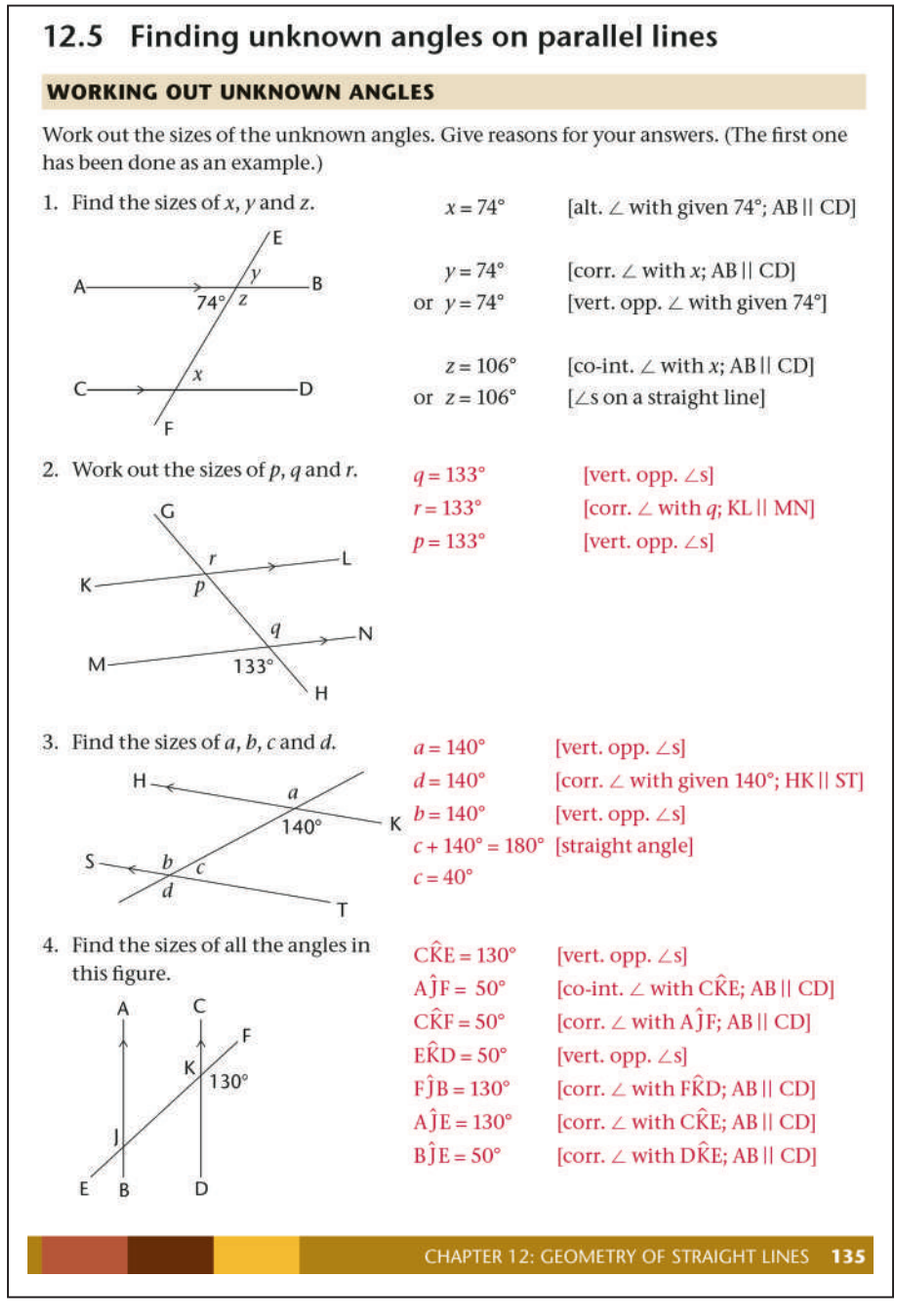
WORKING OUT UNKNOWN ANGLES

Teaching guidelines

Work through question 1 or do an example on the board so that learners can see how to write out the answers with the correct reasons.

Answers

2. to 4. See the answers on LB page 135 alongside.



12.5 Finding unknown angles on parallel lines

WORKING OUT UNKNOWN ANGLES

Work out the sizes of the unknown angles. Give reasons for your answers. (The first one has been done as an example.)

1. Find the sizes of x, y and z.

$x = 74°$ [alt. ∠ with given 74°; AB || CD]

$y = 74°$ [corr. ∠ with x; AB || CD]
or $y = 74°$ [vert. opp. ∠ with given 74°]

$z = 106°$ [co-int. ∠ with x; AB || CD]
or $z = 106°$ [∠s on a straight line]

2. Work out the sizes of p, q and r.

$q = 133°$ [vert. opp. ∠s]
$r = 133°$ [corr. ∠ with q; KL || MN]
$p = 133°$ [vert. opp. ∠s]

3. Find the sizes of a, b, c and d.

$a = 140°$ [vert. opp. ∠s]
$d = 140°$ [corr. ∠ with given 140°; HK || ST]
$b = 140°$ [vert. opp. ∠s]
$c + 140° = 180°$ [straight angle]
$c = 40°$

4. Find the sizes of all the angles in this figure.

$C\hat{K}E = 130°$ [vert. opp. ∠s]
$A\hat{J}F = 50°$ [co-int. ∠ with $C\hat{K}E$; AB || CD]
$C\hat{K}F = 50°$ [corr. ∠ with $A\hat{J}F$; AB || CD]
$E\hat{K}D = 50°$ [vert. opp. ∠s]
$F\hat{J}B = 130°$ [corr. ∠ with $F\hat{K}D$; AB || CD]
$A\hat{J}E = 130°$ [corr. ∠ with $C\hat{K}E$; AB || CD]
$B\hat{J}E = 50°$ [corr. ∠ with $D\hat{K}E$; AB || CD]

Answer

5. See the answers on LB page 136 alongside.

EXTENSION

See the answers on LB page 136 alongside.

SUM OF THE ANGLES IN A QUADRILATERAL

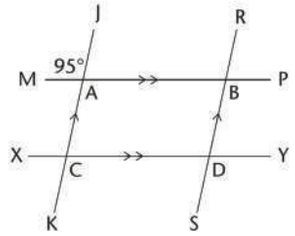
Teaching guidelines

This is a confirmation of the fact about the sum of the interior angles of a quadrilateral.

Answers

1. A parallelogram. Two pairs of opposite sides parallel.
2. See the answers on LB page 136 alongside.

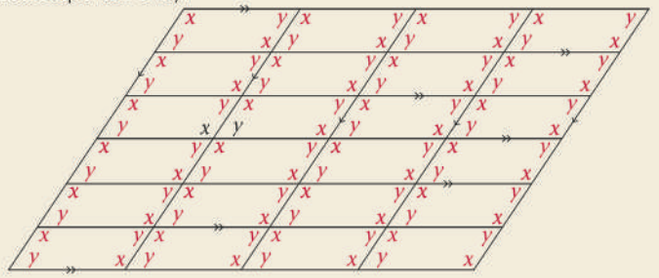
5. Find the sizes of all the angles. (Can you see two transversals and two sets of parallel lines?)



$K\hat{A}M = 85°$	[straight angle]
$J\hat{C}X = 95°$	[co-int. ∠ with $K\hat{A}M$; MP || XY]
$K\hat{C}X = 85°$	[corr. ∠ with $K\hat{A}M$; MP || XY]
$J\hat{A}P = 85°$	[vert. opp. ∠s]
$P\hat{A}K = 95°$	[vert. opp. ∠s]
$J\hat{C}Y = 85°$	[straight angle]
$Y\hat{C}K = 95°$	[straight angle]
$R\hat{B}M = 95°$	[corr. ∠ with $J\hat{A}M$; JK || RS]
$M\hat{B}S = 85°$	[corr. ∠ with $M\hat{A}K$; JK || RS]
$R\hat{D}X = 95°$	[corr. ∠ with $J\hat{C}X$; JK || RS]
$X\hat{D}S = 85°$	[corr. ∠ with $X\hat{C}K$; JK || RS]
$R\hat{B}P = 85°$	[corr. ∠ with $J\hat{A}P$; JK || RS]
$P\hat{B}S = 95°$	[corr. ∠ with $P\hat{A}K$; JK || RS]
$R\hat{D}Y = 85°$	[corr. ∠ with $J\hat{C}Y$; JK || RS]
$Y\hat{D}S = 95°$	[corr. ∠ with $P\hat{B}S$; JK || RS]

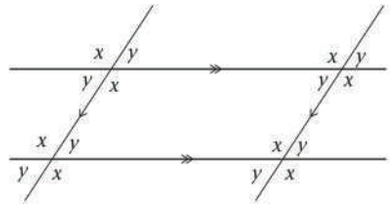
EXTENSION

Two angles in the following diagram are given as x and y. Copy the diagram and fill in all the angles that are equal to x and y.



SUM OF THE ANGLES IN A QUADRILATERAL

The diagram on the right is a section of the previous diagram.



1. What kind of quadrilateral is in the diagram? Give a reason for your answer.
2. Look at the top left intersection. Complete the following equation:

   Angles around a point = 360°

   $\therefore\ x + y + \ldots x \ldots + \ldots y \ldots = 360°$

Answers

3. See the answers on LB page 137 alongside.

12.6 Solving more geometric problems

ANGLE RELATIONSHIPS ON PARALLEL LINES

Teaching guidelines

Work through an example with learners. These questions are more challenging than the ones learners tackled before.

Answers

1. to 3. See the answers on LB page 137 alongside.

3. Look at the interior angles of the quadrilateral on page 136. Copy and complete the following equations:

Sum of angles in the quadrilateral $= x + y +$ ..x.. $+$ ..y..

From question 2: $x + y +$ ..x.. $+$ ..y.. $= 360°$

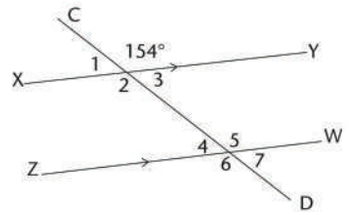
$\therefore$ Sum of angles in a quadrilateral = ..360°..

Can you think of another way to use the diagram on page 136 to work out the sum of the angles in a quadrilateral?

12.6 Solving more geometric problems

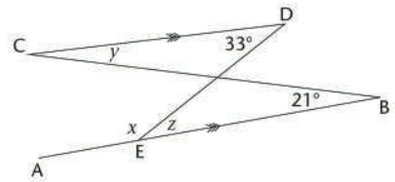
ANGLE RELATIONSHIPS ON PARALLEL LINES

1. Calculate the sizes of $\hat{1}$ to $\hat{7}$.



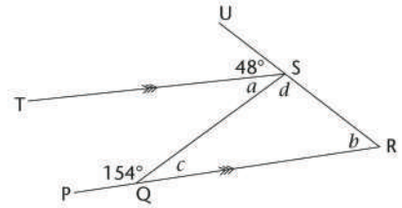
$\hat{1} = 26°$ [straight angle]
$\hat{2} = 154°$ [straight angle]
$\hat{3} = 26°$ [straight angle]
$\hat{4} = 26°$ [corr. $\angle$ with $\hat{1}$; XY || ZW]
$\hat{5} = 154°$ [corr. $\angle$ with given 154°; XY || ZW]
$\hat{6} = 154°$ [corr. $\angle$ with $\hat{2}$; XY || ZW]
$\hat{7} = 26°$ [corr. $\angle$ with $\hat{3}$; XY || ZW]

2. Calculate the sizes of x, y and z.



$y = 21°$ [alternate int. $\angle$ with given 21°, CD || AB]
$z = 33°$ [alternate int. $\angle$ with given 33°, CD || AB]
$x = 147°$ [straight line]

3. Calculate the sizes of a, b, c and d.

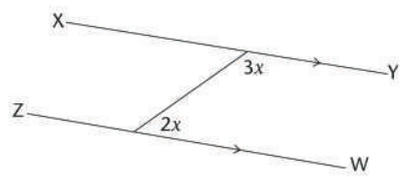


$c = 26°$ [straight line]
$a = 26°$ [alternate int. $\angle$ with c; PR || TS]
$b = 48°$ [corr. $\angle$ with given 48°; PR || TS]
$d = 180° - 26° - 48°$ [$\angle$s of a triangle]
$= 106°$

Answers

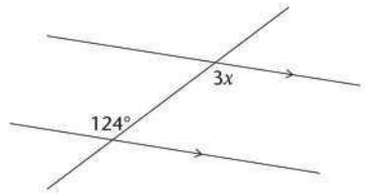
4. to 7. See the answers on LB page 138 alongside.

4. Calculate the size of x.



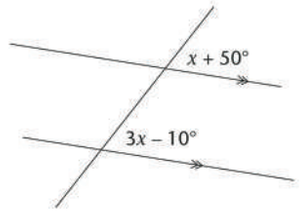
$3x + 2x = 180°$ [co-int. angles; ZW || XY]
$5x = 180°$
$x = 36°$

5. Calculate the size of x.



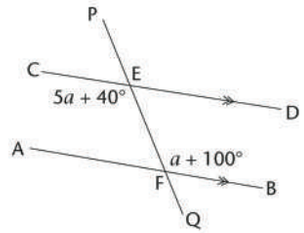
$3x = 124°$ [alt. angles between || lines]
$x = 41{,}33°$

6. Calculate the size of x.



$x + 50° = 3x - 10°$ [corresponding angles above || lines]
$60° = 2x$
$x = 30°$

7. Calculate the sizes of a and $C\hat{E}P$.



$5a + 40° = a + 100°$ [alt. int. ∠s; CD || AB]
$4a = 60°$
$a = 15°$

$C\hat{E}P + 5a + 40° = 180°$ [∠s on straight line]
$C\hat{E}P + 5(15°) = 180° - 40°$
$C\hat{E}P = 140° - 75°$
$C\hat{E}P = 65°$

INCLUDING PROPERTIES OF TRIANGLES AND QUADRILATERALS

Teaching guidelines

In these questions the properties of parallel lines, triangles and quadrilaterals will be used to find solutions.

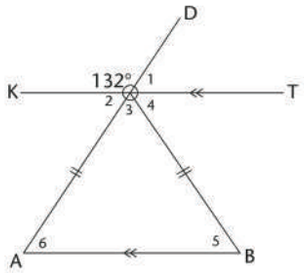
Work through an example with learners to show them how to use the information given and the knowledge they have about the properties of the shapes to find the required answers with reasons.

Answers

1. to 4. See the answers on LB page 139 alongside.

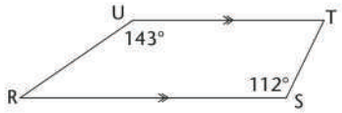
INCLUDING PROPERTIES OF TRIANGLES AND QUADRILATERALS

1. Calculate the sizes of $\hat{1}$ to $\hat{6}$.



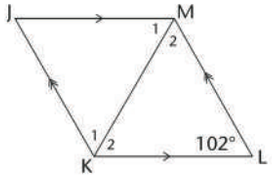
$\hat{1} = 48°$ [straight line]
$\hat{6} = 48°$ [corr. angles with $\hat{1}$; KT || AB]
$\hat{2} = 48°$ [vertically opp. angles]
$\hat{5} = \hat{6} = 48°$ [isosceles triangle]
$\hat{3} + 48° + 48° = 180°$ [angles of triangle]
$\hat{3} = 84°$
$84° + \hat{4} = 132°$ [vert. opp. angles]
$\hat{4} = 48°$

2. RSTU is a trapezium. Calculate the sizes of $\hat{T}$ and $\hat{R}$.



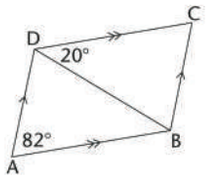
$\hat{R} = 37°$ [co-int. angles with $\hat{U}$; RS || UT]
$\hat{T} = 68°$ [co-int. angles with $\hat{S}$; RS || UT]

3. JKLM is a rhombus. Calculate the sizes of $J\hat{M}L$, $\hat{M}_2$ and $\hat{K}_1$.



$\hat{M}_2 = \hat{K}_2$ [isosceles triangle as LM = LK as JKLM is a rhombus]
$\hat{M}_2 + \hat{M}_2 + 102° = 180°$ [∠s of triangle]
$\hat{M}_2 = 39°$
$\hat{K}_1 = \hat{M}_2 = 39°$ [alt. int. ∠s; JK || LM]
$\hat{M}_1 = \hat{K}_2 = 39°$ [alt. int. ∠s; JM || KL]
$J\hat{M}L = 39° + 39° = 78°$

4. ABCD is a parallelogram. Calculate the sizes of $A\hat{D}B$, $A\hat{B}D$, $\hat{C}$ and $D\hat{B}C$.



$A\hat{B}D = C\hat{D}B = 20°$ [alt. int. ∠s; AB || DC]
$A\hat{D}B = 180° - 82° - 20°$ [∠s of triangle]
$= 78°$
$D\hat{B}C = A\hat{D}B = 78°$ [alt. int. ∠s; AD || BC]
$\hat{C} = 82°$ [opp. ∠s of parallelogram]

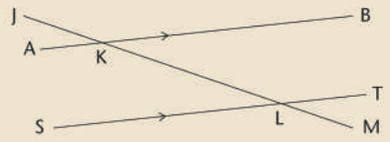
WORKSHEET

Answers

1. (a) $J\hat{K}B$ and $M\hat{K}A$; $K\hat{L}T$ and $S\hat{L}M$; $J\hat{K}A$ and $B\hat{K}L$; $K\hat{L}S$ and $T\hat{L}M$
   (b) $J\hat{K}A$ and $J\hat{L}S$; $A\hat{K}L$ and $S\hat{L}M$; $J\hat{K}B$ and $K\hat{L}T$; or $B\hat{K}L$ and $T\hat{L}M$
   (c) $B\hat{K}L$ and $S\hat{L}K$; or $A\hat{K}L$ and $K\hat{L}T$
   (d) $B\hat{K}L$ and $K\hat{L}T$; or $A\hat{K}L$ and $K\hat{L}S$
2. See the answers on LB page 140 alongside.
3. See the answers on LB page 140 alongside.

WORKSHEET

1. Look at the drawing below. Name the items listed alongside it.



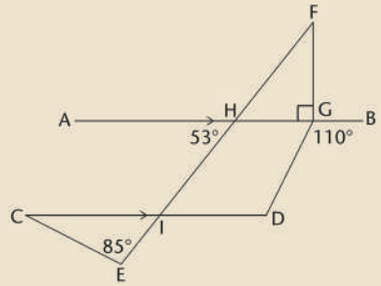
(a) a pair of vertically opposite angles

(b) a pair of corresponding angles

(c) a pair of alternate interior angles

(d) a pair of co-interior angles

2. In the diagram, AB II CD. Calculate the sizes of $F\hat{H}G$, $\hat{F}$, $\hat{C}$ and $\hat{D}$. Give reasons for your answers.



$C\hat{I}E = A\hat{H}I = 53°$ [corr. ∠s; AB II CD]

$\hat{C} = 180° - 85° - 53°$ [∠s of triangle]

$\hat{C} = 42°$

$F\hat{H}G = 53°$ [vertically opp. ∠s]

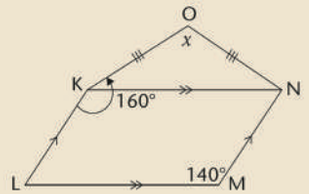
$\hat{F} = 180° - 53° - 90°$ [∠s of triangle]

$\hat{F} = 37°$

$\hat{D} = B\hat{G}D = 110°$ [alt. int. ∠s; AB II CD]

3. In the diagram, OK = ON, KN II LM, KL II MN and $L\hat{K}O = 160°$.

   Calculate the value of x. Give reasons for your answers.



$\hat{L} = 40°$ [co-int. with $\hat{M}$; KL II MN]

$N\hat{K}L = 140°$ [co-int. with $\hat{L}$; KN II LM]

$O\hat{K}N = 160° - 140° = 20°$

$O\hat{N}K = O\hat{K}N = 20°$ [isosceles triangle]

$x = 180° - 20° - 20°$ [∠s of triangle]

$x = 140°$

GEOMETRY OF STRAIGHT LINES

Term 3

Chapter 13: Common fractions 167
13.1 Equivalent fractions 168
13.2 Adding and subtracting fractions 173
13.3 Tenths, hundredths and thousandths 175
13.4 Fraction of a fraction 178
13.5 Division by a fraction 181

Chapter 14: Fractions in decimal notation 185
14.1 Equivalent forms 186
14.2 Ordering and comparing decimal fractions 189
14.3 Rounding off decimal fractions 190
14.4 Calculations with decimal fractions 191
14.5 Solving problems 192

Chapter 15: The theorem of Pythagoras 193
15.1 The lengths of sides of right-angled triangles 194
15.2 Working with the theorem of Pythagoras 197
15.3 Finding the missing sides in right-angled triangles 199
15.4 Are the triangles right-angled? 202

Chapter 16: Perimeter and area of 2D shapes 205
16.1 Perimeter of squares and rectangles 206
16.2 Area of polygons 207
16.3 Perimeter of circles 210
16.4 Area of circles 213
16.5 Converting between square units 217

Chapter 17: Surface area and volume of 3D objects 219
17.1 From 2D to 3D measurements 220
17.2 Surface area of 3D objects 221
17.3 Volume of 3D objects 224
17.4 Relationship between surface area and volume 226
17.5 Converting between cubic units 227
17.6 Capacity of 3D objects 229

Chapter 18: Collect, organise and summarise data 231
18.1 Collecting data 232
18.2 Organising data 236
18.3 Summarising data: measures of central tendency and dispersion 240

Chapter 19: Represent data 247
19.1 Bar graphs and double bar graphs 248
19.2 Histograms 252
19.3 Pie charts 255
19.4 Broken-line graphs 257

Chapter 20: Interpret, analyse and report on data 261
20.1 Critically analysing how data is collected 262
20.2 Critically analysing how data is represented 264
20.3 Critically analysing summary statistics 265

Grade 8 Term 3 Chapter 13 Common fractions

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
13.1 Equivalent fractions	Sharing chocolates in different ways; fraction notation	Pages 141 to 146
13.2 Adding and subtracting fractions	Common denominator	Pages 146 to 148
13.3 Tenths, hundredths and thousandths	A useful family of fraction units	Pages 148 to 151
13.4 Fraction of a fraction	Parts of holes; parts of parts; squares, cubes and roots of fractions	Pages 151 to 154
13.5 Division by a fraction	Serving juice; doing juice calculations more quickly	Pages 154 to 157

CAPS time allocation	7 hours
CAPS content specification	Pages 100 to 102

Mathematical background

- A **fraction** is a measure of how something is to be divided up or shared out. In this chapter you will learn more about fractions and what these numbers are used for.
- Fractions were invented so that **quantities** such as the following can be described accurately:
  - **measures**, for example "a learner is 1,54 m tall"
  - **parts of whole objects**, for example "a quarter of an apple" or "half a loaf of bread"
  - **parts of collections**, for example "three eighths of the learners in a school"
  - **parts of non-physical quantities**, for example "63 hundredths of the available marks", normally written in percentage notation.
- Fractions can be described using various **notations** such as the following:
  - **common fraction notation**, for example "three fifths" of a cake is left over
  - **decimal notation**, for example "0,6" of a cake is left over
  - **percentage notation**, for example "60%" of a cake is left over
- **Equivalent fractions** are fractions that have the same value but are different in form. They enable us to:
  - **convert** a fraction to another notation
  - **reduce** a common fraction by writing it in its simplest form
  - **compare** two or more common fractions by writing them with a common denominator
  - **add and subtract** common fractions by writing them with a common denominator.

13.1 Equivalent fractions

SHARING CHOCOLATE IN DIFFERENT WAYS

Background information

It is useful to distinguish **three phases** in the development of the concept of equivalent fractions in learners' minds.

- Awareness that the **same part** of a whole, collection, quantity or unit of measurement can be described with different fractions, for example 24 forty-eighths of a slab of chocolate can be described as 12 twenty-fourths, eight sixteenths, six twelfths, four eighths, three sixths, two quarters or one half of the slab.
- The ability to **specify equivalent fractions**, for example 24 forty-eighths = 12 twenty-fourths = eight sixteenths = six twelfths = four eighths = three sixths = two quarters = one half.
- Producing equivalent fractions using a strategy, for example by either multiplying or dividing both the numerator and denominator of a fraction by the same number (using the property of 1).

Teaching guidelines

Start by counting the number of small pieces in the chocolate slab alongside.

Misconceptions

Learners could read something like $\frac{5}{8}$ simply as "five over eight" without recognising it as five eighths, which is what it actually means. It is critical that learners use fraction names to ensure a sound understanding of fractions.

Answers

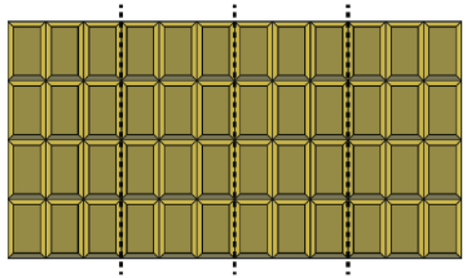
1. (a) 36 small pieces
   (b) 48 small pieces
   (c) They eat the same amount of chocolate. One eighth is six small pieces, so six eighths is 36 small pieces, which is the same as three quarters of the slab.
2. 3
3. one forty-eighth
4. (a) Yes (b) Yes (c) Yes
5. (a) 24 (b) 16 (c) 12 (d) 8

CHAPTER 13

Common fractions

13.1 Equivalent fractions

SHARING CHOCOLATE IN DIFFERENT WAYS



1. (a) John eats three quarters of a chocolate slab like the one above. How many small pieces of chocolate is that?
   (b) How many small pieces are there in the whole slab of chocolate?
   (c) Ratti eats six eighths of a chocolate slab like the one above. Who eats more, Ratti or John, or do they eat the same amount of chocolate? Explain your answer.
2. A slab of chocolate like the above one has to be shared fairly between 16 people. That means each person should get one sixteenth of the slab. How many small pieces of chocolate should each person get?
3. What fraction of the whole slab is one of the small pieces?
4. (a) Is it true that each person in question 2 should get one sixteenth of the slab?
   (b) Is it true that each person in question 2 should get three forty-eighths of the slab?
   (c) Is one sixteenth of the slab of chocolate precisely the same amount of chocolate as three forty-eighths of the slab?
5. How many forty-eighths of a slab will each person get in each of the following cases, if the slab is equally shared among the number of people indicated?
   (a) between two people (b) between three people
   (c) between four people (d) between six people

CHAPTER 13: COMMON FRACTIONS 141

Note on question 6

This question should make learners aware that the **same part** of a whole can be described with different fractions. The drawings as a unit resemble a fraction wall. Point out the relationship between the following fractions:

- (a), (f) and (k): one fifth = two tenths = four twentieths
- (b), (g) and (l): one sixth = two twelfths = four twenty-fourths.

Note on question 8

This question evaluates the learners' ability to **specify equivalent fractions**, for example:

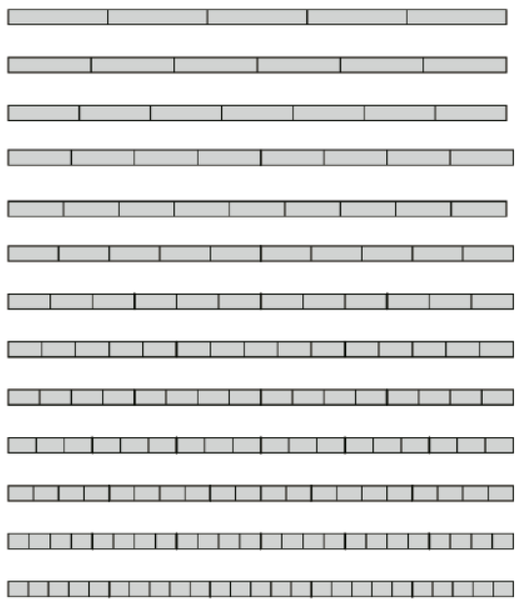
- one twelfth = four forty-eighths
- one eighth = six forty-eighths
- one third = 16 forty-eighths
- one twenty-fourth = two forty-eighths
- one sixth = eight forty-eighths
- one sixteenth = three forty-eighths.

Answers

5. (e) 6 (f) 4 (g) 3 (h) 2
6. (a) fifths (b) sixths (c) sevenths
   (d) eighths (e) ninths (f) tenths
   (g) twelfths (h) fifteenths (i) sixteenths
   (j) eighteenths (k) twentieths (l) twenty-fourths
   (m) twenty-fifths
7. (a) eight people (b) 12 people (c) 16 people
8. (a) four forty-eighths (48 ÷ 12 = 4) (b) six forty-eighths (48 ÷ 8 = 6)
   (c) 16 forty-eighths (48 ÷ 3 = 16) (d) two forty-eighths (48 ÷ 24 = 2)
   (e) eight forty-eighths (48 ÷ 6 = 8) (f) three forty-eighths (48 ÷ 16 = 3)
9. (a) 5 × 4 = 20 forty-eighths (b) 3 × 6 = 18 forty-eighths
   (c) 2 × 16 = 32 forty-eighths (d) 17 × 2 = 34 forty-eighths
   (e) 5 × 8 = 40 forty-eighths (f) 13 × 3 = 39 forty-eighths

(e) between eight people (f) between 12 people
(g) between 16 people (h) between 24 people

6. In each case below, state what the smaller parts of the grey strip may be called.



(a) fifths
(b) sixths
(c) sevenths
(d) eighths
(e) ninths
(f) tenths
(g) twelfths
(h) fifteenths
(i) sixteenths
(j) eighteenths
(k) twentieths
(l) twenty-fourths
(m) twenty-fifths

7. (a) A whole slab of chocolate is divided equally between a number of people and each person gets one eighth of the slab. How many people are there?
   (b) How many people are there if each person gets one twelfth of the slab?
   (c) How many people are there if each person gets one sixteenth of the slab?
8. If each small piece is one forty-eighth of a slab of chocolate, how many pieces are there in each of the following?
   (a) one twelfth of a slab (b) one eighth of a slab
   (c) one third of a slab (d) one twenty-fourth of a slab
   (e) one sixth of a slab (f) one sixteenth of a slab
9. If each small piece is one forty-eighth of a slab of chocolate, how many pieces are there in each of the following?
   (a) five twelfths of a slab (b) three eighths of a slab
   (c) two thirds of a slab (d) 17 twenty-fourths of a slab
   (e) five sixths of a slab (f) 13 sixteenths of a slab

Note on questions 13 to 17

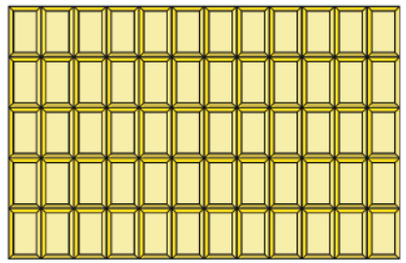
If a slab of chocolate is made up of 60 small pieces, then one piece of the slab is one sixtieth; two pieces are two sixtieths, or one thirtieth; three pieces are three sixtieths, or one twentieth; four pieces are four sixtieths, or one fifteenth; and so on.

Answers

10. (a) Five sixths is more because it is 40 forty-eighths, and 13 sixteenths is only 39 forty-eighths.
    (b) Five twelfths is more because it is 20 forty-eighths, and three eighths is only 18 forty-eighths.
    (c) 17 twenty-fourths is more because it is 34 forty-eighths, and two thirds is only 32 forty-eighths.
11. (a) One third + one eighth = 16 forty-eighths + six forty-eighths = 22 forty-eighths.
    (b) One sixth + three eighths = eight forty-eighths + 18 forty-eighths = 26 forty-eighths.
    (c) Five sixths + seven eighths = 40 forty-eighths + 42 forty-eighths = 82 forty-eighths. 82 forty-eighths is one whole slab and 34 forty-eighths.
12. (a) 18 forty-eighths is three eighths, because one eighth is six forty-eighths and $3 \times 6 = 18$.
    (b) 32 forty-eighths is four sixths, because one sixth is eight forty-eighths and $4 \times 8 = 32$.
13. One sixtieth
14. (a) 12 sixtieths; it is one long side of the slab; it can be calculated as $60 \div 5$.
    (b) Five sixtieths; it is one short side of the slab; it can be calculated as $60 \div 12$.
15. $60 \div 5$ and $60 \div 12$
16. (a) One twentieth is $60 \div 20$ = three sixtieths of the slab.
    (b) One sixth is $60 \div 6 = 10$ sixtieths of the slab.
    (c) One twentieth is three sixtieths, so nine twentieths is $9 \times 3 = 27$ sixtieths of the slab.
17. (a) 14 twentieths is as much as seven tenths, because each tenth is two twentieths.
    (b) 13 twentieths is $13 \times 3 = 39$ sixtieths and nine fifteenths is $9 \times 4 = 36$ sixtieths. So 13 twentieths of a slab is more chocolate than nine fifteenths.
    (c) Three fifths is $3 \times 12 = 36$ sixtieths and seven twelfths is $7 \times 5 = 35$ sixtieths. So three fifths of a slab is more chocolate than seven twelfths.

10. In each of the following say which fraction of the slab gives you more chocolate, or whether the two quantities are the same. How do you know this?
    (a) five sixths of a slab or 13 sixteenths of a slab
    (b) five twelfths of a slab or three eighths of a slab
    (c) two thirds of a slab or 17 twenty-fourths of a slab
11. (a) How many forty-eighths of a slab is one third of a slab and one eighth of a slab together?
    (b) How much of a slab is one sixth of a slab and three eighths of a slab together?
    (c) How much chocolate is five sixths of a slab and seven eighths of a slab together?
12. (a) How many eighths of a slab is 18 forty-eighths of a slab? How did you work this out?
    (b) How many sixths of a slab is 32 forty-eighths of a slab? How did you work this out?

Now, here is a different slab of chocolate:



13. What fraction of the whole slab is each one of the small pieces?
14. How many sixtieths of the yellow 60-piece slab is each of the following?
    (a) one fifth of the slab
    (b) one twelfth of the slab
15. To answer question 14, you may just have counted the small pieces on the diagram. What calculations could you have done to find the answers for question 14?
16. How many sixtieths of the yellow 60-piece slab is each of the following?
    (a) one twentieth of the slab
    (b) one sixth of the slab
    (c) nine twentieths of the slab
17. In each case below, state which is more chocolate, or whether the two fractions of the slab are the same amount of chocolate. How do you know?
    (a) 14 twentieths or seven tenths
    (b) 13 twentieths or nine fifteenths
    (c) three fifths or seven twelfths

Answers

18. (a) 14 twentieths + seven tenths = 14 twentieths + 14 twentieths = 28 twentieths = 14 tenths.
    (b) 13 twentieths + nine fifteenths = 39 sixtieths + 36 sixtieths = 75 sixtieths. 75 sixtieths is one whole slab and 15 sixtieths, which is one and a quarter slabs.
    (c) Three fifths + seven twelfths = 36 sixtieths + 35 sixtieths = 71 sixtieths = one whole slab and 11 sixtieths of a slab.

USING FRACTION NOTATION

Background information

In this section the **three phases** in the development of the concept of equivalent fractions are highlighted by the questions listed below:

- The awareness that the **same part** of something can be described with different fractions: question 5.
- The ability to **specify equivalent fractions**: questions 6 to 8.
- How to produce equivalent fractions: question 9.

Teaching guidelines

Discuss the following concepts:

- **common fraction notation**: five forty-eighths can be written as $\frac{5}{48}$
- **denominator**: shows into how many parts (the fraction unit into which) the whole was divided
- **numerator**: indicates the number of pieces (forty-eighths)
- **mixed number**: made up of a whole number and a fraction.

Answers

1. (a) $\frac{7}{20}$ (b) $3\frac{5}{8}$ (c) $2\frac{7}{9}$ (d) $1\frac{7}{10}$
2. (a) 23 hundredths (b) three and five thirtieths
   (c) two and five eighteenths (d) 17 twenty-fifths
3. (a) one fifth (b) 30 (c) one thirtieth
4. (a) one tenth (b) 40 (c) one fortieth

18. In each case below, work out how much of a slab is made up of the two parts together:
    (a) 14 twentieths and seven tenths. At the end, give your answer as a number of tenths.
    (b) 13 twentieths and nine fifteenths. Give your final answer as wholes and quarters.
    (c) three fifths and seven twelfths

USING FRACTION NOTATION

Instead of writing five forty-eighths, we may write $\frac{5}{48}$.
This is called the **common fraction notation**.

The number 48 below the line is called the **denominator** and it shows that the whole was divided into 48 equal pieces, so each piece is one forty-eighth of the whole. The denominator shows the **unit** in which the number is expressed.

The number 5 above the line is called the **numerator** and it indicates the **number** of pieces.

A number that is made up of a whole number and a fraction, like two and three fifths, can be written as a **mixed number**: $2\frac{3}{5}$.

1. Write each of the following numbers in fraction notation:
   (a) seven twentieths (b) three and five eighths
   (c) two and seven ninths (d) one and seven tenths
2. Write each of the following numbers in words:
   (a) $\frac{23}{100}$ (b) $3\frac{5}{30}$ (c) $2\frac{5}{18}$ (d) $\frac{17}{25}$
3. (a) The strip below is divided into five equal parts.
   What part of the whole strip is each of the five parts?



   (b) If you divide each fifth into six smaller equal parts, how many smaller parts will there be altogether?
   (c) What fraction of the whole strip is each of these smaller parts?
4. (a) The strip below is divided into ten equal parts.
   What part of the whole strip is each of the ten parts?



   (b) If you divide each tenth into four smaller equal parts, how many smaller parts will there be altogether?
   (c) What fraction of the whole strip is each of these smaller parts?

Teaching guidelines (continued)

Discuss the following concept:

- **Equivalent fractions** are fractions that look different but have the same value.

Note on question 6

This question starts to evaluate the learners' ability to **specify equivalent fractions**.

Answers

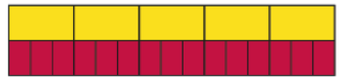
4. (d) 50
   (e) one fiftieth
   (f) 100
   (g) one hundredth
5. (a) two tenths = one fifth
   (b) three fifteenths = one fifth
   (c) three fifths = nine fifteenths
   (d) four twentieths = one fifth
   (e) five twentieths = one quarter
   (f) 15 twentieths = three quarters
   (g) one tenth = two twentieths
6. (a) $\frac{2}{10}=\frac{1}{5}$ (b) $\frac{3}{15}=\frac{1}{5}$ (c) $\frac{3}{5}=\frac{9}{15}$ (d) $\frac{4}{20}=\frac{1}{5}$
   (e) $\frac{5}{20}=\frac{1}{4}$ (f) $\frac{15}{20}=\frac{3}{4}$ (g) $\frac{1}{10}=\frac{2}{20}$
7. (a) The statement is false. 15 twentieths = three quarters. Five twentieths make up one quarter precisely, so 15 twentieths is the same as three quarters.
   (b) The statement is false. Nine fifteenths = three fifths. Three fifteenths make up exactly one fifth, so nine fifteenths is the same as three fifths.
   (c) The statement is false. One third is exactly four twelfths, so two thirds is eight twelfths, which is more than seven twelfths.

(d) If you divide each tenth into five smaller equal parts, how many smaller parts will there be altogether?
(e) What fraction of the whole strip is each of these smaller parts?
(f) If you divide each tenth into ten smaller equal parts, how many smaller parts will there be altogether?
(g) What fraction of the whole strip is each of these smaller parts?

5. (a) How many tenths make up one fifth? You may use the diagram on the right to figure this out.



(b) How many fifteenths are there in one fifth?
(c) How many fifteenths are there in three fifths?
(d) How many twentieths are there in one fifth? If you need help with this, draw a diagram like those in question 5(a) and (b) to help you. Your diagram need not be accurate.



(e) How many twentieths are there in one quarter?
(f) How many twentieths are there in three quarters?
(g) How many twentieths do you think will make up one tenth? If you need help, make marks on the diagram in question 5(a) to help you.

Your answers for question 5 can also be written in fraction notation. For example, your answer for question 5(c) can be written as $\frac{3}{5}=\frac{9}{15}$.

6. Write each of your other answers for question 5 in fraction notation.

7. In this question write the fractions in words. Decide whether each statement is true or false and give reasons for your answers.
   (a) $\frac{15}{20}$ of the red strip below is longer than $\frac{3}{4}$ of the strip



   (b) $\frac{9}{15}$ is a bigger number than $\frac{3}{5}$
   (c) $\frac{2}{3}$ is a smaller number than $\frac{7}{12}$

The same number can be expressed in different units. For example, the number $\frac{3}{4}$ can be expressed in eighths as $\frac{6}{8}$, in twentieths as $\frac{15}{20}$, in sixtieths as $\frac{45}{60}$ and in many other units. $\frac{3}{4}$, $\frac{6}{8}$, $\frac{15}{20}$ and $\frac{45}{60}$ are all different ways of expressing the same number. Hence they are called **equivalent fractions**.

Equivalent fractions let us write the same number in different ways, for example: $\frac{3}{4}=\frac{6}{8}=\frac{15}{20}=\frac{45}{60}$

Note on question 9

This question introduces learners to a way of creating equivalent fractions. Multiplying the numerator and the denominator by the same number means multiplying the fraction by 1.

Answers

8. (a) Three eighths is six sixteenths or 15 fortieths.
   $\frac{3}{8} = \frac{6}{16} = \frac{15}{40}$
   (b) Three fifths is six tenths or 12 twentieths or 24 fortieths or 60 hundredths.
   $\frac{3}{5} = \frac{6}{10} = \frac{12}{20} = \frac{24}{40} = \frac{60}{100}$
   (c) Seven tenths is 28 fortieths or 35 fiftieths or 70 hundredths.
   $\frac{7}{10} = \frac{28}{40} = \frac{35}{50} = \frac{70}{100}$

9. (a) $\frac{6}{8}$; yes (b) $\frac{3}{4} = \frac{9}{12}$; yes
   (c) $\frac{12}{16}$; yes (d) $\frac{18}{24}$; yes

10. (a) $\frac{5}{8}$ is bigger. Five eighths is 25 fortieths and three fifths is 24 fortieths.
   (b) $\frac{7}{10}$ is bigger. Seven tenths is 56 eightieths and five eighths is 50 eightieths.
   (c) $\frac{7}{8}$ is bigger. Seven eighths is 35 fortieths and four fifths is 32 fortieths.

13.2 Adding and subtracting fractions

Background information

Common fractions can be added and subtracted if they are expressed with a **common denominator**, which can be either the:

- product of the denominators
- or the LCM (lowest common multiple) of the denominators.

Teaching guidelines

Discuss how to find a common denominator for fractions.

8. Write your answers in words and in fraction notation, and explain your answers.
   (a) Express $\frac{3}{8}$ in sixteenths and in fortieths.
   (b) Express $\frac{3}{5}$ in tenths, twentieths, fortieths and hundredths.
   (c) Express $\frac{7}{10}$ in fortieths, fiftieths and hundredths.
9. Consider the fraction three quarters. It can be written as $\frac{3}{4}$.
   (a) Multiply both the numerator and the denominator by 2 to form a "new" fraction. Is the "new" fraction equivalent to $\frac{3}{4}$? You can check your answer on this diagram.



   (b) Multiply both the numerator and the denominator of $\frac{3}{4}$ by 3 to form a "new" fraction. Is the "new" fraction equivalent to $\frac{3}{4}$?
   (c) Multiply both the numerator and the denominator of $\frac{3}{4}$ by 4 to form a "new" fraction. Is the new fraction equivalent to $\frac{3}{4}$?
   (d) Multiply both the numerator and the denominator of $\frac{3}{4}$ by 6 to form a "new" fraction. Is the new fraction equivalent to $\frac{3}{4}$?

$\frac{15}{20}$ is equivalent to $\frac{3}{4}$ because there are five twentieths in one quarter, and so there are 15 twentieths in three quarters. $\frac{9}{16}$ is not equivalent to $\frac{3}{4}$ because there are four sixteenths in one quarter. Therefore, three quarters is 12 sixteenths; not nine sixteenths.

10. Decide whether the two given numbers are equal or not. Explain your answer. If they are not equal, state which one is bigger and explain why you say so. You may first write the fractions in words if that helps you.
   (a) $\frac{5}{8}$ and $\frac{3}{5}$ (Hint: express both numbers in fortieths)
   (b) $\frac{7}{10}$ and $\frac{5}{8}$ (c) $\frac{4}{5}$ and $\frac{7}{8}$

13.2 Adding and subtracting fractions

To add or subtract fractions, all the fractions must be expressed in the same unit.

1. Calculate each of the fractions on page 147. The work that you did in question 10 above may help you.

Misconceptions

Avoid mistakes like $\frac{2}{4}+\frac{1}{4}=\frac{3}{8}$ by:

- using the correct fraction names: two quarters + one quarter = three quarters
- using a diagram to illustrate the correct way to add fractions.

Answers

1. See the answers on LB page 147 alongside.
2. (a) $8 \times 3 = 24$; $\frac{5}{8} = \frac{15}{24}$; $\frac{2}{3} = \frac{16}{24}$; $\frac{2}{3}$ is bigger
   (b) $8 \times 6 = 48$; $\frac{5}{6} = \frac{40}{48}$; $\frac{7}{8} = \frac{42}{48}$; $\frac{7}{8}$ is bigger
   (c) $4 \times 5 = 20$; $\frac{3}{4} = \frac{15}{20}$; $\frac{4}{5} = \frac{16}{20}$; $\frac{4}{5}$ is bigger
   (d) $4 \times 3 = 12$; $\frac{5}{12} = \frac{5}{12}$; $\frac{2}{3} = \frac{8}{12}$; $\frac{2}{3}$ is bigger
   (e) $8 \times 12 = 96$; $\frac{7}{12} = \frac{56}{96} = \frac{28}{48}$; $\frac{3}{8} = \frac{36}{96} = \frac{18}{48}$; $\frac{7}{12}$ is bigger
   (f) $20 \times 3 = 60$; $15 \times 4 = 60$; $\frac{9}{20} = \frac{27}{60}$; $\frac{4}{15} = \frac{16}{60}$; $\frac{9}{20}$ is bigger
   (g) $10 \times 4 = 40$; $\frac{3}{10} = \frac{12}{40}$; $\frac{1}{4} = \frac{10}{40}$; $\frac{3}{10}$ is bigger
   (h) $10 \times 8 = 80$; $\frac{7}{10} = \frac{56}{80}$; $\frac{5}{8} = \frac{50}{80}$; $\frac{7}{10}$ is bigger
   (i) $13 \times 17 = 221$; $\frac{9}{13} = \frac{153}{221}$; $\frac{11}{17} = \frac{143}{221}$; $\frac{9}{13}$ is bigger
3. (a) $\frac{15}{24}+\frac{16}{24}=\frac{31}{24}=1\frac{7}{24}$
   (b) $\frac{40}{48}+\frac{42}{48}=\frac{82}{48}=1\frac{34}{48}=1\frac{17}{24}$
   (c) $\frac{15}{20}+\frac{16}{20}=\frac{31}{20}=1\frac{11}{20}$
   (d) $\frac{5}{12}+\frac{8}{12}=\frac{13}{12}=1\frac{1}{12}$
   (e) $\frac{56}{96}+\frac{36}{96}=\frac{92}{96}$ or $\frac{28}{48}+\frac{18}{48}=\frac{46}{48}$ or $\frac{14}{24}+\frac{9}{24}=\frac{23}{24}$
   (f) $\frac{27}{60}+\frac{16}{60}=\frac{43}{60}$
   (g) $\frac{12}{40}+\frac{10}{40}=\frac{22}{40}=\frac{11}{20}$
   (h) $\frac{56}{80}+\frac{50}{80}=\frac{106}{80}=1\frac{26}{80}=1\frac{13}{40}$
   (i) $\frac{153}{221}+\frac{143}{221}=\frac{296}{221}=1\frac{75}{221}$

(a) $\frac{5}{8}+\frac{3}{5}=\frac{25}{40}+\frac{24}{40}=\frac{49}{40}=1\frac{9}{40}$

(b) $\frac{7}{10}+\frac{5}{8}=\frac{56}{80}+\frac{50}{80}=\frac{106}{80}=\frac{53}{40}=1\frac{13}{40}$

(c) $\frac{7}{10}+\frac{3}{8}=\frac{56}{80}+\frac{30}{80}=\frac{86}{80}=\frac{43}{40}=1\frac{3}{40}$

(d) $\frac{5}{8}-\frac{3}{5}=\frac{25}{40}-\frac{24}{40}=\frac{1}{40}$

(e) $\frac{7}{10}-\frac{3}{8}=\frac{56}{80}-\frac{30}{80}=\frac{26}{80}=\frac{13}{40}$

(f) $6 \times \frac{5}{8}$ (which is $\frac{5}{8}+\frac{5}{8}+\frac{5}{8}+\frac{5}{8}+\frac{5}{8}+\frac{5}{8}$) $= 6 \times 5 = 30$. Therefore it is $\frac{30}{8}=\frac{15}{4}=3\frac{3}{4}$

(g) $8 \times \frac{7}{10} = 8 \times 7 = 56$. Therefore it is 56 tenths $= \frac{56}{10} = 5\frac{6}{10}$ $(5\frac{3}{5})$

To compare, add or subtract fractions, for example $\frac{5}{8}$ and $\frac{3}{5}$, find a fraction unit in which both fractions can be expressed so that you can compare them. We call this a **common denominator**. The "product" of the two denominators is helpful to find such a unit. In this case, $5 \times 8 = 40$. Since one eighth is five fortieths, $\frac{5}{8}$ is 25 fortieths or $\frac{25}{40}$. Since one fifth is eight fortieths, $\frac{3}{5}$ is 24 fortieths or $\frac{24}{40}$. So, $\frac{5}{8}$ is bigger than $\frac{3}{5}$.

2. In each question, explain why the two given numbers are equal or why they are not equal. If they are not equal, state which one is bigger and explain why you say so. You may first write the fractions in words if that will help you.

   (a) $\frac{5}{8}$ and $\frac{2}{3}$ (b) $\frac{5}{6}$ and $\frac{7}{8}$ (c) $\frac{3}{4}$ and $\frac{4}{5}$

   (d) $\frac{5}{12}$ and $\frac{2}{3}$ (e) $\frac{7}{12}$ and $\frac{3}{8}$ (f) $\frac{9}{20}$ and $\frac{4}{15}$

   (g) $\frac{3}{10}$ and $\frac{1}{4}$ (h) $\frac{7}{10}$ and $\frac{5}{8}$ (i) $\frac{9}{13}$ and $\frac{11}{17}$

3. Add the two fractions given in each part of question 2. Show how you work it out.

   (a) $\frac{5}{8}+\frac{2}{3}$ (b) $\frac{5}{6}+\frac{7}{8}$ (c) $\frac{3}{4}+\frac{4}{5}$

   (d) $\frac{5}{12}+\frac{2}{3}$ (e) $\frac{7}{12}+\frac{3}{8}$ (f) $\frac{9}{20}+\frac{4}{15}$

   (g) $\frac{3}{10}+\frac{1}{4}$ (h) $\frac{7}{10}+\frac{5}{8}$ (i) $\frac{9}{13}+\frac{11}{17}$

Answers

4. (a) $\frac{16}{24}-\frac{15}{24}=\frac{1}{24}$ (b) $\frac{42}{48}-\frac{40}{48}=\frac{2}{48}=\frac{1}{24}$

   (c) $\frac{16}{20}-\frac{15}{20}=\frac{1}{20}$ (d) $\frac{8}{12}-\frac{5}{12}=\frac{3}{12}=\frac{1}{4}$

   (e) $\frac{56}{96}-\frac{36}{96}=\frac{20}{96}=\frac{5}{24}$ or $\frac{28}{48}-\frac{18}{48}=\frac{10}{48}=\frac{5}{24}$ or $\frac{14}{24}-\frac{9}{24}=\frac{5}{24}$

   (f) $\frac{27}{60}-\frac{16}{60}=\frac{11}{60}$ (g) $\frac{12}{40}-\frac{10}{40}=\frac{2}{40}=\frac{1}{20}$

   (h) $\frac{56}{80}-\frac{50}{80}=\frac{6}{80}=\frac{3}{40}$ (i) $\frac{153}{221}-\frac{143}{221}=\frac{10}{221}$

5. See the answers on LB page 148 alongside.

13.3 Tenths, hundredths and thousandths

A USEFUL FAMILY OF FRACTION UNITS

Background information

Any fraction can be expressed as a sum of other fractions, for example, $\frac{1}{3}=\frac{7}{30}+\frac{1}{10}$. This property of fractions provides the basis for the decimal motivation. In the decimal notation a fraction is represented as a sum of tenths, hundredths, thousandths, and so on. For example, $\frac{5}{8}=\frac{6}{10}+\frac{2}{100}+\frac{5}{1\,000}$.

This section is intended to make learners aware of the possibility of expressing fractions in the above way, hence empowering them to make sense of the decimal notation in the next chapter.

Teaching guidelines

Discuss how to convert a common fraction to its simplest form (refer to LB page 149 question 3).

Answers

1. (a) See the answer on LB page 148 alongside.
   (b) 10
   (c) 100
   (d) hundredths

4. Now subtract the smaller number from the bigger number in each part of question 2.

   (a) $\frac{2}{3}-\frac{5}{8}$ (b) $\frac{7}{8}-\frac{5}{6}$ (c) $\frac{4}{5}-\frac{3}{4}$

   (d) $\frac{2}{3}-\frac{5}{12}$ (e) $\frac{7}{12}-\frac{3}{8}$ (f) $\frac{9}{20}-\frac{4}{15}$

   (g) $\frac{3}{10}-\frac{1}{4}$ (h) $\frac{7}{10}-\frac{5}{8}$ (i) $\frac{9}{13}-\frac{11}{17}$

5. Calculate each of the following:

   (a) $3\frac{2}{3}-1\frac{5}{6}=\frac{11}{3}-\frac{11}{6}=\frac{22}{6}-\frac{11}{6}=\frac{11}{6}=1\frac{5}{6}$

   (b) $5\frac{6}{7}+\frac{3}{8}=5+\frac{6}{7}+\frac{3}{8}=5+\frac{48}{56}+\frac{21}{56}=5+\frac{69}{56}=5+1\frac{13}{56}=6\frac{13}{56}$

   (c) $12\frac{5}{8}+7\frac{4}{9}=12+7+\left(\frac{5}{8}+\frac{4}{9}\right)=19+\left(\frac{45}{72}+\frac{32}{72}\right)=19+\frac{77}{72}=20\frac{5}{72}$

   (d) $4\frac{5}{12}-2\frac{3}{10}=2+\left(\frac{50}{120}-\frac{36}{120}\right)=2+\frac{14}{120}=2\frac{7}{60}$

   (e) $1\frac{3}{10}-\frac{2}{3}=\frac{13}{10}-\frac{2}{3}=\frac{39}{30}-\frac{20}{30}=\frac{19}{30}$

   (f) $2\frac{7}{15}-1\frac{3}{8}=1+\left(\frac{56}{120}-\frac{45}{120}\right)=1+\frac{11}{120}=1\frac{11}{120}$

   (g) $\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}=\frac{35}{8}=4\frac{3}{8}$

   (h) $\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}=\frac{56}{8}=7$

   (i) $\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}+\frac{7}{8}=\frac{91}{8}=11\frac{3}{8}$

   (j) $2\frac{4}{12}+2\frac{4}{12}+2\frac{4}{12}+2\frac{4}{12}+2\frac{4}{12}+2\frac{4}{12}+2\frac{4}{12}+2\frac{4}{12}=16+\frac{32}{12}=18\frac{8}{12}=18\frac{2}{3}$

13.3 Tenths, hundredths and thousandths

A USEFUL FAMILY OF FRACTION UNITS

1. (a) Shade three tenths of the following strip:



   (b) Into how many smaller parts is each tenth of the above strip divided?
   (c) How many of these smaller parts are there in the whole strip?
   (d) What is each of these smaller parts called?

Note on question 4

To add and/or subtract fractions, follow these steps:

Step 1: Convert all fractions to the same fraction unit (common denominator).

Step 2: Add and/or subtract the numerators and divide the answer by the common denominator.

Step 3: Write the answer in its simplest form.

Note on question 5(b)

$\frac{7}{100}$ of R250 = $\frac{7}{100}$ × R250 = R250 ÷ 100 × 7 = R2,50 × 7 = R17,50

Answers

1. (e) 40
   (f) 25
   (g) Learners shade 37 hundredths of the strip. See LB page 149 alongside.
2. (a) $\frac{80}{100}$ (b) $\frac{5}{100}$ (c) $\frac{35}{100}$
   (d) $\frac{4}{100}$ (e) $\frac{68}{100}$ (f) $\frac{14}{100}$
3. (a) $\frac{3}{4}$ (b) $\frac{3}{5}$ (c) $\frac{13}{20}$ (d) $\frac{9}{10}$
4. (a) $\frac{12}{100} + \frac{20}{100} = \frac{32}{100} = \frac{8}{25}$
   (b) $\frac{24}{100} + \frac{30}{100} = \frac{54}{100} = \frac{27}{50}$
   (c) $\frac{7}{100} = \frac{14}{200}$ so $\frac{7}{100} + \frac{9}{200} = \frac{14}{200} + \frac{9}{200} = \frac{23}{200}$
5. (a) R4
   (b) 7 × R2,50 = R17,50
   (c) R150
   (d) R150
   (e) R280
   (f) R280

(e) How many hundredths make up two fifths of the strip?
(f) How many hundredths make up one quarter of the strip?
(g) Shade 37 hundredths of the following strip:



2. Express each of the following numbers as a number of hundredths, and write your answers in fraction notation:
   (a) four fifths (b) one twentieth
   (c) seven twentieths (d) one twenty-fifth
   (e) 17 twenty-fifths (f) seven fiftieths

Because one twentieth is five hundredths, seven twentieths is 35 hundredths.
This can also be expressed in fraction notation: $\frac{35}{100} = \frac{7}{20}$.

$\frac{7}{20}$ is called the **simplest form** of $\frac{35}{100}$ because $\frac{35}{100}$ cannot be expressed with a smaller numerator than 7.

3. Express each of the following fractions in its simplest form:
   (a) $\frac{75}{100}$ (b) $\frac{60}{100}$
   (c) $\frac{65}{100}$ (d) $\frac{90}{100}$
4. Calculate each of the following, and express your answer in its simplest form:
   (a) $\frac{3}{25} + \frac{4}{20}$ (b) $\frac{6}{25} + \frac{6}{20}$ (c) $\frac{7}{100} + \frac{9}{200}$
5. (a) How much is $\frac{1}{100}$ of R400?
   (b) How much is $\frac{7}{100}$ of R250?
   (c) How much is $\frac{25}{100}$ of R600?
   (d) How much is $\frac{1}{4}$ of R600?
   (e) How much is $\frac{40}{100}$ of R700?
   (f) How much is $\frac{2}{5}$ of R700?

Instead of writing $\frac{40}{100}$ of R700, we may write $\frac{40}{100}$ × R700.

Teaching guidelines (continued)

Remind learners that "per cent" is another name for "hundredths".

Answers

6. $\frac{40}{100} = \frac{4}{10} = \frac{2}{5}$ or 40 hundredths is the same as four tenths or two fifths.

7. (a) $\frac{80}{100} \times R900 = R720$ (b) $\frac{80}{100} \times R650 = R520$

   (c) $\frac{80}{100} \times R250 = R200$ (d) $\frac{80}{100} \times R3\,400 = R2\,720$

8. (a) R72 (b) R52 (c) R20 (d) R272

9. (a) R135 (b) R97,50 (c) R37,50 (d) R510

10. (a) 1 000 (b) thousandths

11. (a) R600 (b) R60 (c) R6 (d) R600

    (e) R600 (f) R420 (g) R420 (h) R0,6 = 60 cents

12. (a) $\frac{300}{1\,000} + \frac{625}{1\,000} = \frac{925}{1\,000} = \frac{37}{40}$

    (b) $5\frac{11}{10} = 6\frac{1}{10}$

    (c) $\frac{30}{100} + \frac{7}{100} = \frac{37}{100}$

    (d) $\frac{30}{100} + \frac{70}{100} = \frac{100}{100} = 1$

    (e) $\frac{300}{1\,000} + \frac{7}{1\,000} = \frac{307}{1\,000}$

    (f) $\frac{300}{1\,000} + \frac{70}{1\,000} = \frac{370}{1\,000} = \frac{37}{100}$

13. (a) $\frac{374}{1\,000} = \frac{187}{500}$

    (b) $\frac{300}{1\,000} + \frac{700}{1\,000} + \frac{400}{1\,000} = \frac{1\,400}{1\,000} = \frac{14}{10} = 1\frac{2}{5}$

    (c) $\frac{600}{1\,000} + \frac{200}{1\,000} + \frac{700}{1\,000} = \frac{1\,500}{1\,000} = 1\frac{1}{2}$

    (d) $\frac{200}{1\,000} + \frac{50}{1\,000} + \frac{4}{1\,000} = \frac{254}{1\,000} = \frac{127}{500}$

6. Explain why your answers for question 5(e) and (f) are the same.

Another word for *hundredth* is *per cent*.
Instead of saying:
*Miriam received **32 hundredths** of the prize money*,
we can say:
*Miriam received **32 per cent** of the prize money.*
The symbol for per cent is % (percentage).

7. How much is 80% of each of the following?
   (a) R900 (b) R650
   (c) R250 (d) R3 400

8. How much is 8% of each of the amounts in 7(a), (b), (c) and (d)?

9. How much is 15% of each of the amounts in 7(a), (b), (c) and (d)?



The above strip is divided into hundredths. Imagine that each of the hundredths is divided into ten equal parts (they will be almost impossible to see).

10. (a) How many of these very small parts will there be in the whole strip?
    (b) What could each of these very small parts be called?

11. How much is each of the following?
    (a) one tenth of R6 000 (b) one hundredth of R6 000
    (c) one thousandth of R6 000 (d) ten hundredths of R6 000
    (e) 100 thousandths of R6 000 (f) seven hundredths of R6 000
    (g) 70 thousandths of R6 000 (h) one ten thousandth of R6 000

12. Calculate each of the following:
    (a) $\frac{3}{10} + \frac{5}{8}$ (b) $3\frac{3}{10} + 2\frac{4}{5}$
    (c) $\frac{3}{10} + \frac{7}{100}$ (d) $\frac{3}{10} + \frac{70}{100}$
    (e) $\frac{3}{10} + \frac{7}{1\,000}$ (f) $\frac{3}{10} + \frac{70}{1\,000}$

13. Calculate each of the following:
    (a) $\frac{3}{10} + \frac{7}{100} + \frac{4}{1\,000}$ (b) $\frac{3}{10} + \frac{70}{100} + \frac{400}{1\,000}$
    (c) $\frac{6}{10} + \frac{20}{100} + \frac{700}{1\,000}$ (d) $\frac{2}{10} + \frac{5}{100} + \frac{4}{1\,000}$

Answers

14. (a) True: $\frac{100}{1\,000} + \frac{230}{1\,000} + \frac{346}{1\,000} = \frac{676}{1\,000} = \frac{600}{1\,000} + \frac{30}{1\,000} + \frac{46}{1\,000} = \frac{6}{10} + \frac{3}{100} + \frac{46}{1\,000}$

(b) Not true: LHS = $\frac{676}{1\,000}$ from (a) but $\frac{700}{1\,000} + \frac{20}{1\,000} + \frac{46}{1\,000} = \frac{766}{1\,000}$

(c) True: LHS = $\frac{676}{1\,000}$ from (a) and RHS = $\frac{600}{1\,000} + \frac{70}{1\,000} + \frac{6}{1\,000} = \frac{676}{1\,000}$

(d) True: $\frac{676}{1\,000} = \frac{600}{1\,000} + \frac{70}{1\,000} + \frac{6}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$

13.4 Fraction of a fraction

CALCULATE PARTS OF WHOLES AND PARTS OF PARTS

Background information

To calculate a **fraction of a quantity**:

- divide the quantity by the denominator and then
- multiply by the numerator.

Example: $\frac{7}{20}$ of R500 = R500 ÷ 20 × 7 = R25 × 7 = R175

Teaching guidelines

Discuss how to calculate a fraction of a quantity (refer to the example above):

- divide the quantity by the denominator to get one twentieth
- multiply the one twentieth by the numerator to get the answer.

Note on question 1

Learners have to find a fraction of a quantity.

Answers

1. (a) to (c) See the answers on LB page 151 alongside.
2. $\frac{2}{5}$ of 1 000 = R400 and 400 ÷ 8 = R50
3. (a) 400 ÷ 8 × 7 = R350
   (b) 350 ÷ 5 × 2 = R140
   (c) 400 ÷ 20 × 7 = R140

14. In each case investigate whether the statement is true or not, and give reasons for your decision.

(a) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{3}{100} + \frac{46}{1\,000}$ True

(b) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{7}{10} + \frac{2}{100} + \frac{46}{1\,000}$ Not true

(c) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$ True

(d) $\frac{676}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$ True

13.4 Fraction of a fraction

CALCULATE PARTS OF WHOLES AND PARTS OF PARTS

To calculate $\frac{7}{20}$ (seven twentieths) of R500 you can first calculate one twentieth, and then multiply by 7:

one twentieth of R500 is R500 ÷ 20 = R25, so $\frac{7}{20}$ of R500 is 7 × R25 = R175.

This means that to calculate $\frac{7}{20}$ of R500, you work out (500 ÷ 20) × 7. You divide by the denominator and then multiply by the numerator.

$\frac{7}{20}$ of 500 is the same as $\frac{7}{20}$ × 500.

1. Calculate each of the following:

(a) $\frac{9}{25}$ of R500 500 ÷ 25 × 9 = 20 × 9 = R180

(b) $\frac{9}{20}$ of R500 500 ÷ 20 × 9 = 25 × 9 = R225

(c) $\frac{9}{125}$ of R500 500 ÷ 125 × 9 = 4 × 9 = R36

2. A small choir of eight members won the second prize in a competition and they received two fifths of the total prize money. They shared the money equally between themselves. The total prize money was R1 000. How much prize money did each member of the choir get?

3. (a) How much is $\frac{7}{8}$ of 400?

   (b) How much is $\frac{2}{5}$ of your answer for (a)?

   (c) How much is $\frac{7}{20}$ of 400?

Background information (continued)

To calculate a **fraction of a fraction of a quantity**:

- convert the second fraction to a multiple of the first denominator
- multiply the fractions
- divide the quantity by the new denominator and
- multiply by the new numerator.

  Example: $\frac{1}{8}$ of $\frac{2}{5}$ of R1 000 = $\frac{1}{8}$ of $\frac{8}{20}$ of R1 000 = $\frac{1}{20}$ of R1 000 = R50

Teaching guidelines (continued)

Using the example above ($\frac{1}{8}$ of $\frac{2}{5}$ of R1 000), discuss how to calculate a fraction of a fraction of a quantity:

- convert $\frac{2}{5}$ to a multiple of 8: $\frac{8}{20}$
- find $\frac{1}{8}$ of $\frac{8}{20}$: $\frac{1}{20}$
- divide the quantity by 20: R50
- multiply by 1: R50.

Note on question 6

Learners have to find a fraction of a fraction of a quantity.

Answers

4. (a) Learners check and try again if necessary.
   (b) 1 000 ÷ 20 = R50
   (c) One eighth of two fifths is difficult to calculate. However, it is easy to find one eighth of a number which is a multiple of 8. Write two fifths as 16 fortieths and find one eighth of 16 fortieths, which is two fortieths or one twentieth.
5. Refer to the completed table on LB page 152 alongside.
6. (a) R24 (b) R54 (c) R78 (d) R108 (e) R720

4. Here is Nathi's answer to question 2 on page 151:
   One fifth of R1 000 is R200, so two fifths is R400. So the choir team received R400 in total. Each member received one eighth of the R400, which is R400 ÷ 8 = R50.
   (a) Compare your own answer to Nathi's answer. If they are different, work them out again and find out who is right.
   (b) Do you agree that $\frac{1}{20}$ of R1 000 is R50?
   (c) Explain why the answer for question 2 is the same as $\frac{1}{20}$ of R1 000.
5. Copy the table below and write your answers in it. For each of the following numbers, 80, 180, 260, 360 and 2 400, work out:
   (a) How much is $\frac{2}{5}$ of each of the numbers?
   (b) How much is $\frac{3}{4}$ of each of your answers for (a)?
   (c) How much is $\frac{6}{20}$ of each of the numbers?

Number	80	180	260	360	2 400
$\frac{2}{5}$ of the number	80 ÷ 5 × 2 = 16 × 2 = 32	180 ÷ 5 × 2 = 36 × 2 = 72	260 ÷ 5 × 2 = 52 × 2 = 104	360 ÷ 5 × 2 = 72 × 2 = 144	2 400 ÷ 5 × 2 = 480 × 2 = 960
$\frac{3}{4}$ of the answer	32 ÷ 4 × 3 = 8 × 3 = 24	72 ÷ 4 × 3 = 18 × 3 = 54	104 ÷ 4 × 3 = 26 × 3 = 78	144 ÷ 4 × 3 = 36 × 3 = 108	960 ÷ 4 × 3 = 240 × 3 = 720
$\frac{6}{20}$ of the number	80 ÷ 20 × 6 = 4 × 6 = 24	180 ÷ 20 × 6 = 9 × 6 = 54	260 ÷ 20 × 6 = 13 × 6 = 78	360 ÷ 20 × 6 = 18 × 6 = 108	2 400 ÷ 20 × 6 = 120 × 6 = 720

6. Use your answers for question 5 to answer the following questions:
   (a) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R80?
   (b) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R180?
   (c) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R260?
   (d) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R360?
   (e) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R2 400?
7. To calculate $\frac{3}{4}$ of $\frac{2}{5}$ of a number you did this: the number ÷ 4 × 3 ÷ 5 × 2.
   (a) Investigate whether the number × 3 × 2 ÷ 5 ÷ 4 will give the same results as the number ÷ 4 × 3 ÷ 5 × 2, for the numbers in question 5 or any other numbers you may choose.

Note on question 7

This question offers an **alternative method** to calculate a fraction of a fraction of a quantity: $\frac{1}{8}$ of $\frac{2}{5}$ means $\frac{1}{8} \times \frac{2}{5}$ or $\frac{1 \times 2}{8 \times 5}$, which is $\frac{2}{40}$ or $\frac{1}{20}$.

Example: $\frac{1}{8}$ of $\frac{2}{5}$ of R1 000 = $\frac{1 \times 2}{8 \times 5}$ of R1 000 = $\frac{2}{40}$ of R1 000 = $\frac{1}{20}$ of R1 000 = R50

Answers

7. (a) Yes it will. (b) Yes it will. (c) Yes it will.

Note on question 8

This question explains how to multiply by a **mixed number**: Express the whole number part in the same fraction units as the fraction part.

Example: $2\frac{7}{8}$ = two wholes + seven eighths = 16 eighths + seven eighths = $\frac{23}{8}$

Answers

8. (a) $\frac{36}{250} = \frac{18}{125}$ (b) $\frac{20}{630} = \frac{2}{63}$
   (c) $\frac{5}{6} \times \frac{6}{7} = \frac{30}{42} = \frac{5}{7}$ (d) $\frac{6}{24} = \frac{1}{4}$
   (e) $\frac{13}{5} \times \frac{5}{6} = \frac{65}{30} = \frac{13}{6} = 2\frac{1}{6}$ (f) $\frac{11}{4} \times \frac{17}{5} = \frac{187}{20} = 9\frac{7}{20}$
   (g) $\frac{8}{3} \times \frac{8}{3} = \frac{64}{9} = 7\frac{1}{9}$ (h) $\frac{42}{5} \times \frac{10}{3} = \frac{420}{15} = 28$
   (i) $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$; $\frac{6}{7} \times \frac{5}{6} = \frac{6 \times 5}{7 \times 6} = \frac{5}{7}$ (j) $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$
   (k) $\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$; $\frac{6}{7} \times \frac{1}{6} = \frac{1}{7}$ (l) $\frac{3}{7} - \frac{2}{7} = \frac{1}{7}$
   (m) $\frac{9}{6} \times \frac{1}{5} = \frac{9}{30} = \frac{3}{10}$ (n) $\frac{5}{30} + \frac{2}{15} = \frac{5}{30} + \frac{4}{30} = \frac{9}{30} = \frac{3}{10}$
   (o) $\frac{3}{4} - \frac{10}{30} = \frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$ (p) $\frac{7}{8} \times (\frac{20}{35} + \frac{14}{35}) = \frac{7}{8} \times \frac{34}{35} = \frac{34}{40} = \frac{17}{20}$

SQUARES AND CUBES AND ROOTS OF FRACTIONS

Background information

- The **square of a fraction** is equal to the square of the numerator divided by the square of the denominator.

(b) Investigate whether or not the number × 6 ÷ 20 will give the same results as the number × 3 × 2 ÷ 5 ÷ 4.
(c) Investigate whether or not the number × 3 ÷ 10 will give the same results as the number × 6 ÷ 20.

Instead of $\frac{3}{4}$ of $\frac{2}{5}$ we may write $\frac{3}{4} \times \frac{2}{5}$.
$\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5}$

To multiply by a mixed number like $2\frac{7}{8}$, it is good practice to express the whole number part in the same fraction units as the fraction part, for example:
two wholes is 16 eighths, so $2\frac{7}{8}$ is $\frac{16}{8} + \frac{7}{8} = \frac{23}{8}$.

8. Calculate each of the following:
   (a) $\frac{3}{10} \times \frac{12}{25}$ (b) $\frac{5}{18} \times \frac{4}{35}$
   (c) $\left(\frac{1}{3} + \frac{1}{2}\right) \times \frac{6}{7}$ (d) $\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4}$
   (e) $2\frac{3}{5} \times \frac{5}{6}$ (f) $2\frac{3}{4} \times 3\frac{2}{5}$
   (g) $2\frac{2}{3} \times 2\frac{2}{3}$ (h) $8\frac{2}{5} \times 3\frac{1}{3}$
   (i) $\frac{6}{7} \times \left(\frac{1}{3} + \frac{1}{2}\right)$ (j) $\frac{6}{7} \times \frac{1}{3} + \frac{6}{7} \times \frac{1}{2}$
   (k) $\frac{6}{7} \times \left(\frac{1}{2} - \frac{1}{3}\right)$ (l) $\frac{6}{7} \times \frac{1}{2} - \frac{6}{7} \times \frac{1}{3}$
   (m) $\left(\frac{5}{6} + \frac{2}{3}\right) \times \frac{1}{5}$ (n) $\frac{5}{6} \times \frac{1}{5} + \frac{2}{3} \times \frac{1}{5}$
   (o) $\frac{3}{4} - \frac{2}{5} \times \frac{5}{6}$ (p) $\frac{7}{8} \times \left(\frac{4}{7} + \frac{2}{5}\right)$

SQUARES AND CUBES AND ROOTS OF FRACTIONS

1. Calculate each of the following:
   (a) $\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$ (b) $\frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{27}{1\,000}$
   (c) $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$ (d) $\left(\frac{5}{9}\right)^2 = \frac{25}{81}$
   (e) $\left(\frac{3}{5}\right)^3 = \frac{27}{125}$ (f) $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

Example: $\left(\frac{4}{5}\right)^2 = \frac{4^2}{5^2} = \frac{16}{25}$

- The **cube of a common fraction** is the cube of the numerator divided by the cube of the denominator.

  Example: $\left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3} = \frac{64}{125}$

- The **square root of a common fraction** is the square root of the numerator divided by the square root of the denominator.

  Example: $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$

- The **cube root of a common fraction** is the cube root of the numerator divided by the cube root of the denominator.

  Example: $\sqrt[3]{\frac{64}{125}} = \frac{\sqrt[3]{64}}{\sqrt[3]{125}} = \frac{4}{5}$

Teaching guidelines

Revise the concepts listed above.

Answers

1. See the answers on LB page 153 above and LB page 154 alongside.
2. $\frac{3}{4}$
3. See answers on LB page 154 alongside.

13.5 Division by a fraction

SERVING JUICE

Background information

To **divide a quantity by a fraction**, follow these steps:

- Multiply the quantity by the denominator to find the number of fraction units available.
- Divide the number of fraction units by the numerator to find the number of groups of the numerator that can be made up.

  Example: $36 \div \frac{3}{4} = (36 \times 4 \text{ quarters}) \div \text{groups of } 3 = (36 \times 4) \div 3 = 48$

(g) $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$ (h) $\left(\frac{4}{7}\right)^2 = \frac{16}{49}$

(i) $\left(\frac{5}{8}\right)^3 = \frac{125}{512}$ (j) $\left(\frac{5}{8}\right)^2 = \frac{25}{64}$

(k) $\left(\frac{5}{12}\right)^3 = \frac{125}{1\,728}$ (l) $\left(\frac{5}{12}\right)^2 = \frac{25}{144}$

2. What number multiplied by itself will give $\frac{9}{16}$?

This number is called the square root of $\frac{9}{16}$. It can be written as $\sqrt{\frac{9}{16}}$.

3. Find each of the following. In some cases, your answers to question 1 will help you.

(a) $\sqrt{\frac{4}{9}} = \frac{2}{3}$ (b) $\sqrt[3]{\frac{27}{64}} = \frac{3}{4}$

(c) $\sqrt{\frac{25}{81}} = \frac{5}{9}$ (d) $\sqrt[3]{\frac{125}{343}} = \frac{5}{7}$

(e) $\sqrt{\frac{25}{36}} = \frac{5}{6}$ (f) $\sqrt[3]{\frac{125}{216}} = \frac{5}{6}$

(g) $\sqrt{\frac{9}{100}} = \frac{3}{10}$ (h) $\sqrt[3]{\frac{27}{1\,000}} = \frac{3}{10}$

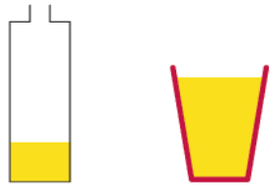
13.5 Division by a fraction

SERVING JUICE

Jamie pours juice from bottles into glasses.



He uses three quarters of a bottle of juice to fill one glass.



Teaching guidelines

Discuss how to divide a quantity by a fraction (refer to the example above):

- Find the number of fraction units in the quantity:
  36 × 4 quarters = 144 quarters
- Find the number of groups of three in this total:
  144 quarters ÷ 3 = 48 groups of three

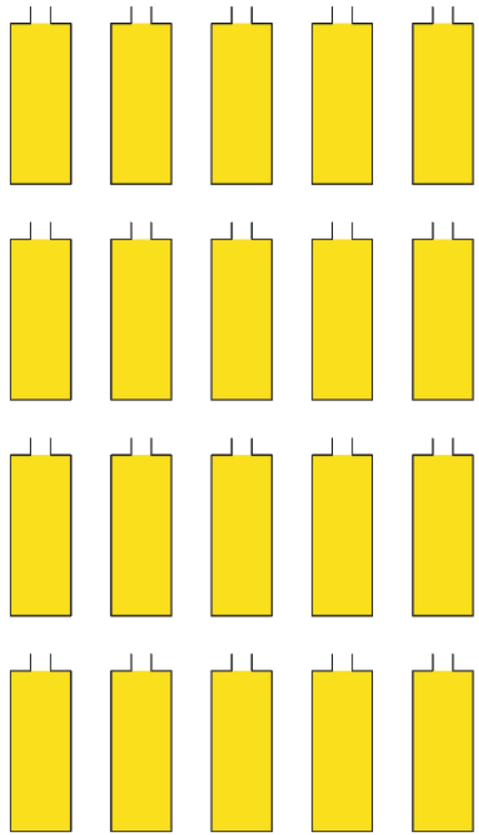
Note on question 6

80 quarters ÷ 3 = 26 groups of three quarters each + two quarters left over
= 26 glasses + two thirds of a glass

Answers

1. $\frac{3}{4} \times 10 = \frac{30}{4} = 7\frac{1}{2}$
2. $\frac{3}{4} \times 30 = \frac{90}{4} = 22\frac{1}{2}$
3. $\frac{3}{4} \times 100 = 300 \div 4 = 75$
4. $\frac{3}{4} \times 180 = 540 \div 4 = 135$
5. $\frac{3}{4} \times 37 = \frac{111}{4} = 27\frac{3}{4}$
6. 20 bottles give 20 × 4 = 80 quarters of a bottle
   80 quarters ÷ 3 = 26 glasses filled and two quarters left over (that is two thirds of a glass)
7. 36 × 4 = 144 quarters
   144 ÷ 3 = 48 three quarters of a bottle = 48 glasses

1. How many bottles will Jamie need to fill ten glasses?
2. How many bottles will Jamie need to fill 30 glasses?
3. How many bottles will Jamie need to fill 100 glasses?
4. How many bottles will Jamie need to fill 180 glasses?
5. How many bottles will Jamie need to fill 37 glasses?
6. How many glasses can Jamie fill from 20 full bottles of juice?



7. How many glasses can Jamie fill from 36 full bottles of juice?

Answers

8. $50 \times \frac{5}{8} = 250$ eighths; $250 \div 8 = 31\frac{2}{8}$. So he will need 32 bottles.
9. $25 \times 8 = 200$ eighths; $200 \div 5 = 40$ five eighths = 40 glasses can be filled
10. $44 \times \frac{7}{10} = \frac{308}{10} = 30\frac{4}{5}$. So he will need 31 bottles.
11. 25 bottles: $25 \times 10 = 250$ tenths; $250 \div 7 = 35$ glasses with seven tenths in each and five tenths left over. He can fill 35 glasses and five tenths of a bottle is left over. He can fill five sevenths of a glass with what is left, so he could fill 35 and five sevenths of a glass.
12. $36 \times 4 = 144$ quarters; $144 \div 3 = 48$ three quarters of a bottle = 48 glasses

DOING THE JUICE CALCULATIONS MORE QUICKLY

Background information

To divide a quantity by a fraction:

- first multiply it by the denominator of the fraction
- then divide the product by the numerator.

  Example: $36 \div \frac{3}{4} = (36 \times 4) \div 3 = 48$

Teaching guidelines

Discuss how to divide with fractions more quickly.

Answers

1. Division. She has to calculate $R850 \div 67$.
2. (a) $16 \times 4 = 64$ quarters
   (b) $64 \div 3 = 21$ glasses and there is one quarter of a bottle left (one third of a glass)
3. 20 bottles give $20 \times 8 = 160$ eighths.
   $160 \div 5 = 32$ glasses can be filled
4. 25 bottles have $25 \times 5 = 125$ fifths.
   He can fill $125 \div 3 = 41$ glasses and two fifths of a bottle is left over.
   With two fifths of a bottle, he can fill two thirds of a glass.
   He can fill 41 glasses and one more glass will be only two thirds full.

On another day Jamie uses different size glasses. He needs five eighths of a bottle of juice to fill one of these glasses.

8. How many bottles of juice does Jamie need to fill 50 of these glasses?
9. How many of these glasses can Jamie fill from 25 full bottles of juice?

Jamie changes glasses again. For the new glasses, he needs $\frac{7}{10}$ of a full bottle of juice to fill one glass.

10. How many bottles of juice does Jamie need to fill 44 of these glasses?
11. How many of these glasses can Jamie fill from 25 full bottles of juice?
12. How many glasses can Jamie fill from 36 full bottles of juice if he needs three quarters of a bottle to fill one glass?

DOING THE JUICE CALCULATIONS MORE QUICKLY

1. Ria has R850 and chickens cost R67 each. What operation does she need to do to work out how many chickens she can buy?
2. Jamie has 16 bottles of juice and needs three quarters of a bottle to fill one glass.
   (a) How many quarters of a bottle of juice are there in 16 full bottles?
   (b) How many glasses can he fill with these quarters?

In question 2 you have worked out how many glasses, each taking $\frac{3}{4}$ of a bottle, can be filled from 16 bottles. You did this by first working out the total number of quarters in 16 bottles, and then dividing by 3 to find out how many glasses can be filled.
Do questions 3 and 4 in the same way.

3. Jamie has 20 bottles of juice and needs five eighths of a bottle to fill one glass. To work out how many glasses he can fill, he needs to work out 20 divided by $\frac{5}{8}$. Work in the same way you did for question 2 to find out.
4. Jamie has 25 bottles of juice and needs $\frac{3}{5}$ of a bottle to fill one glass. How many glasses can he fill?

In questions 2, 3 and 4 you have actually done the following calculations:
In question 2 you have calculated $16 \div \frac{3}{4}$, by doing $16 \times 4 \div 3$.
In question 3 you have calculated $20 \div \frac{5}{8}$, by doing $20 \times 8 \div 5$.
In question 4 you have calculated $25 \div \frac{3}{5}$, by doing $25 \times 5 \div 3$.

Answers

5. (a) $9 \times \frac{3}{2} = \frac{27}{2} = 13\frac{1}{2}$ (b) $12 \times \frac{8}{3} = \frac{96}{3} = 32$
   (c) $15 \times \frac{10}{7} = \frac{150}{7} = 21\frac{3}{7}$ (d) $2 \times \frac{20}{3} = \frac{40}{3} = 13\frac{1}{3}$
   (e) $20 \times \frac{12}{7} = \frac{240}{7} = 34\frac{2}{7}$ (f) $120 \times \frac{5}{18} = \frac{600}{18} = 33\frac{1}{3}$
6. (a) $\frac{27}{2} = 13\frac{1}{2}$ (b) $\frac{96}{3} = 32$
   (c) $\frac{150}{7} = 21\frac{3}{7}$ (d) $\frac{40}{3} = 13\frac{1}{3}$
   (e) $\frac{240}{7} = 34\frac{2}{7}$ (f) $\frac{600}{18} = 33\frac{1}{3}$
7. The answers are exactly the same.

Background information

To divide by a fraction:

- multiply by the **reciprocal** (**multiplicative inverse**) of the fraction.
  Example: $36 \div \frac{3}{4} = 36 \times \frac{4}{3} = \frac{144}{3} = 48$

Teaching guidelines

Discuss the following:

- The concept of reciprocity (the multiplicative inverse) of a fraction.
- How to divide by a fraction using its reciprocal.

Answers

8. (a) $\frac{7}{10} \times \frac{20}{3} = \frac{140}{30} = 4\frac{2}{3}$ (b) $\frac{9}{10} \times \frac{18}{3} = \frac{162}{30} = 5\frac{2}{5}$
   (c) $\frac{17}{20} \times \frac{7}{2} = \frac{119}{40} = 2\frac{39}{40}$ (d) $\frac{27}{10} \times \frac{5}{3} = \frac{135}{30} = 4\frac{1}{2}$
   (e) $\frac{39}{8} \times \frac{3}{2} = \frac{117}{16} = 7\frac{5}{16}$ (f) $\frac{47}{8} \times \frac{5}{13} = \frac{235}{104} = 2\frac{27}{104}$

To divide by a fraction, you multiply by the denominator and divide by the numerator.

5. Calculate each of the following:
   (a) $9 \div \frac{2}{3}$ (b) $12 \div \frac{3}{8}$
   (c) $15 \div \frac{7}{10}$ (d) $2 \div \frac{3}{20}$
   (e) $20 \div \frac{7}{12}$ (f) $120 \div 3\frac{3}{5}$
6. Calculate each of the following:
   (a) $9 \times \frac{3}{2}$ (b) $12 \times \frac{8}{3}$
   (c) $15 \times \frac{10}{7}$ (d) $2 \times \frac{20}{3}$
   (e) $20 \times \frac{12}{7}$ (f) $120 \times \frac{5}{18}$
7. What do you notice about your answers for questions 5 and 6?

To divide by a fraction, we may turn the fraction around and multiply!
For example, $15 \div \frac{7}{10} = 15 \times \frac{10}{7}$.
$\frac{10}{7}$ is the **reciprocal** (also called the multiplicative inverse) of $\frac{7}{10}$.

Division is the inverse of multiplication.

The method of dividing by multiplying by the reciprocal also works when a fraction is divided by a fraction. For example $\frac{5}{18} \div \frac{7}{10}$ can be calculated by doing $\frac{5}{18} \times \frac{10}{7}$.

8. Calculate each of the following:
   (a) $\frac{7}{10} \div \frac{3}{20}$ (b) $\frac{9}{10} \div \frac{3}{18}$
   (c) $\frac{17}{20} \div \frac{2}{7}$ (d) $2\frac{7}{10} \div \frac{3}{5}$
   (e) $4\frac{7}{8} \div \frac{2}{3}$ (f) $5\frac{7}{8} \div 2\frac{3}{5}$

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
14.1 Equivalent forms	Conversions between fractions in common fraction, decimal and percentage notations	Pages 158 to 160
14.2 Ordering and comparing decimal fractions	Decimal numbers on a number line; decimal numbers in ascending and descending order	Pages 160 to 162
14.3 Rounding off decimal fractions	Rounding to the nearest whole number, tenth or hundredth	Pages 162 to 163
14.4 Calculations with decimal fractions	Addition and subtraction of decimal fractions; multiplication and division of decimal fractions	Pages 163 to 164
14.5 Solving problems	Real-life problems	Page 164

CAPS time allocation	6 hours
CAPS content specification	Pages 103 to 105

Mathematical background

It is important to always keep in mind that the **common fraction notation**, the **decimal notation** and the **percentage notation** are just different ways to represent the same numbers.

- **Decimals** are used to describe a limited range of fractional units, namely tenths, hundredths, thousandths, and so on. In this chapter you will do more work with fractions written in decimal notation. When fractions are written in the decimal form, calculations can be done in the same way as for whole numbers.
- **Percentages** make up an even more limited system of describing quantities, in the sense that they allow hundredths only. In practice, however, percentages are extended to include fractional percentages, for example 25,5%. On LB page 160, in question 2, learners express numbers in three notations. One of these is the common fraction notation, using hundredths, if possible.

The **calculator** is an efficient teaching aid for learners to explore and investigate decimals. Learners should know how to program their calculators to add or subtract a decimal number repeatedly. This could assist them in evaluating their ability to count forwards or backwards in decimals. It can also be used to explore the rules for multiplication and division by powers of 10, for example: 10; 100; 1 000; 0,1; 0,01 and 0,001 as well as to discover general rules such as the following:

- If you multiply a number by a number less than 1, the answer will be less than the original number.
- When you divide a number by a number between 0 and 1 ($0 < n < 1$), the answer is more than the original number.

14.1 Equivalent forms

FRACTIONS IN DECIMAL NOTATION

Background information

In this section learners convert common fractions to decimal fractions.

Teaching guidelines

Revise the following with the learners.

- $\frac{1}{10}$ (common fraction notation) = 0,1 (decimal notation) therefore $\frac{2}{10}$ = 0,2; $\frac{3}{10}$ = 0,3; and so on.
- $\frac{1}{100}$ (common fraction notation) = 0,01 (decimal notation) therefore $\frac{2}{100}$ = 0,02; $\frac{3}{100}$ = 0,03; and so on.
- one tenth is equivalent to ten hundredths therefore 0,1 = 0,10.

Discuss why 0,35 (35 hundredths) can be expressed as any of the following:

- 1 tenth + 25 hundredths (10 hundredths + 25 hundredths)
- 2 tenths + 15 hundredths (20 hundredths + 15 hundredths)
- 3 tenths + 5 hundredths (30 hundredths + 5 hundredths)

  The normal way of expressing these quantities is the third option because, by international agreement, the number of hundredths should be kept below 10.

Answers

1. See the completed table on LB page 158 alongside.

CHAPTER 14

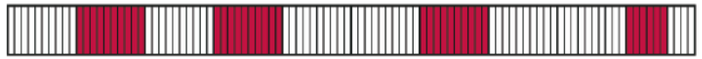
Fractions in decimal notation

14.1 Equivalent forms

FRACTIONS IN DECIMAL NOTATION

1. What fraction of each rectangle is coloured in? Copy the table and write your answers in it.

(a)



(b)



(c)



(d)



Coloured in	Fraction notation	Decimal notation
(a) Red	$\frac{36}{100}$	0,36
(b) Blue	$\frac{17}{100}$	0,17
Yellow	$\frac{28}{100}$	0,28
(c) Blue	$\frac{17}{100}$	0,17
Yellow	$\frac{28}{100}$	0,28
(d) Yellow	$\frac{34}{100}$	0,34
Blue	$\frac{38}{100}$	0,38

158 MATHEMATICS GRADE 8: TERM 3

Answers

2. See the completed table on LB page 159 alongside.

Background information (continued)

- To **convert a common fraction to a decimal fraction**: Express it with a power of ten (10, 100, 1 000, etc.) as the denominator.

  Example: $\frac{9}{20} = \frac{9}{20} \times \frac{5}{5} = \frac{45}{100} = 0{,}45$

- To **convert a decimal fraction to a common fraction**: Express it as a common fraction with a power of ten (10, 100, 1 000, etc.) as the denominator and then simplify if necessary.

  Example: $0{,}65 = \frac{65}{100} = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$

Teaching guidelines (continued)

Revise how to convert between fractions in common fraction form and decimal fraction form.

Answers

3. $\frac{1}{2} = 0{,}5$ $\quad \frac{3}{4} = 0{,}75$ $\quad \frac{4}{5} = 0{,}8$

   $\frac{7}{5} = 1{,}4$ $\quad \frac{7}{2} = 3{,}5$ $\quad \frac{65}{100} = 0{,}65$

4. (a) $20 + 1 + 0{,}3 = 21{,}3$
   (b) $3 + 0{,}06 = 3{,}06$
   (c) $0{,}03$
   (d) $0{,}007$

5. $0{,}2 = \frac{2}{10} = \frac{1}{5}$

   $0{,}85 = \frac{85}{100} = \frac{17}{20}$

   $0{,}07 = \frac{7}{100}$

   $12{,}04 = 12\frac{4}{100} = 12\frac{1}{25}$

   $40{,}006 = 40\frac{6}{1\,000} = 40\frac{3}{500}$

2. Now find out what fraction in each rectangle in question 1 is not coloured in. Copy the table and fill in yours answers.

Not coloured in	Fraction notation	Decimal notation
(a)	$\frac{64}{100}$	0,64
(b)	$\frac{55}{100}$	0,55
(c)	$\frac{55}{100}$	0,55
(d)	$\frac{28}{100}$	0,28

Decimal fractions and common fractions are simply different ways of expressing the same number. We call them different **notations**.

To write a **common fraction as a decimal fraction**, we must first express the common fraction with a power of ten (10, 100, 1 000, etc.) as the denominator.

For example: $\frac{9}{20} = \frac{9}{20} \times \frac{5}{5} = \frac{45}{100} = 0{,}45$

If you have a calculator, you can also divide the numerator by the denominator to get the decimal form of a fraction, for example: $\frac{9}{20} = 9 \div 20 = 0{,}45$.

To write a **decimal fraction as a common fraction**, we must first express it as a common fraction with a power of ten as the denominator and then simplify if necessary.

For example: $0{,}65 = \frac{65}{100} = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$

3. Give the decimal form of each of the following numbers:

   $\frac{1}{2}$ $\quad \frac{3}{4}$ $\quad \frac{4}{5}$ $\quad \frac{7}{5}$ $\quad \frac{7}{2}$ $\quad \frac{65}{100}$

4. Write the following as decimal fractions:

   (a) $2 \times 10 + 1 \times 1 + \frac{3}{10}$ $\quad$ (b) $3 \times 1 + 6 \times \frac{1}{100}$

   (c) three hundredths $\quad$ (d) $7 \times \frac{1}{1\,000}$

5. Write each of the following numbers as fractions in their simplest form:

   0,2 $\quad$ 0,85 $\quad$ 0,07 $\quad$ 12,04 $\quad$ 40,006

Answers

6. (a) $5 + 1{,}2 = 6{,}2$ (b) $2 + 0{,}3 + 0{,}17 = 2{,}47$
   (c) $0{,}13 + 0{,}015 = 0{,}145$ (d) $0{,}07 + 1{,}54 = 1{,}61$

HUNDREDTHS, PERCENTAGES AND DECIMALS

Background information

- **Per cent** is another word for hundredths.
  We do not say: "How many **per cent** of a drawing is blue?"
  We say: "What **percentage** of the drawing is blue?"
- Fractions written in words, for example three tenths, can be expressed in decimal notation (0,3), percentage notation (30%) and common fraction notation ($\frac{3}{10}$).

Teaching guidelines

Revise the following with the learners.

- Per cent is another word for hundredths.
- The symbol for per cent is %.
- Six hundredths can be described as six per cent and can be written as 6%.

Discuss why 0,37 and 37% and $\frac{37}{100}$ are different symbols for 37 hundredths.

Note on question 2

One eighth has to be written as hundredths, which is not possible. However, it can be written as thousandths and therefore as a percentage (12,5%). This is an opportunity for learners to become aware of fractional percentages.

Answers

1. (a) 0,8; 80%; $\frac{80}{100}$ (b) 0,05; 5%; $\frac{5}{100}$
   (c) 0,6; 60%; $\frac{60}{100}$ (d) 0,35; 35%; $\frac{35}{100}$
2. See the completed table on LB page 160 alongside.

6. Write each of the following in the decimal notation:
   (a) five + 12 tenths (b) two + three tenths + 17 hundredths
   (c) 13 hundredths + 15 thousandths (d) seven hundredths + 154 hundredths

HUNDREDTHS, PERCENTAGES AND DECIMALS

It is often difficult to compare fractions with different denominators. Fractions with the same denominator are easier to compare. For this and other reasons, fractions are often expressed as hundredths. A fraction expressed as hundredths is called a **percentage**.

Instead of six hundredths we can say 6 per cent, $\frac{6}{100}$ or 0,06.
6 per cent, $\frac{6}{100}$ and 0,06 are just three different ways of writing the same number.

The symbol % is used for per cent.
Instead of writing "17 per cent", we may write 17%.

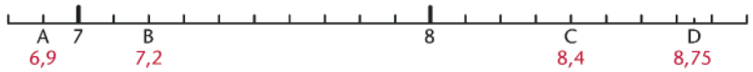
1. Write each of the following in three ways: in decimal notation, in percentage notation and in common fraction notation. Leave your answers in hundredths.
   (a) 80 hundredths (b) five hundredths
   (c) 60 hundredths (d) 35 hundredths
2. Copy and complete the following table:

Common fraction	Decimal fraction	Percentage
$\frac{3}{10}$	0,3	30%
$\frac{1}{4}$	0,25	25%
$\frac{15}{100}$	0,15	15%
$\frac{1}{8}$	0,125	12,5%
$\frac{55}{100}$	0,55	55%
$\frac{1}{100}$	0,01	1%

14.2 Ordering and comparing decimal fractions

BIGGER, SMALLER OR THE SAME?

1. Copy the number lines. Write the values of the marked points (A to D) in as accurately as possible in **decimal notation**. Write the values **beneath** the letters A to D.
   (a)



14.2 Ordering and comparing decimal fractions

BIGGER, SMALLER OR THE SAME?

Background information

Decimal fractions are compared by looking at their number of tenths first, then at their hundredths, then at their thousandths, etc.

The numbers $\frac{1}{10}$, $\frac{10}{100}$ and $\frac{100}{1\,000}$ are equivalent. Therefore, written in decimal notation, 0,1, 0,10 and 0,100 are also equivalent. Note that the zeros at the end of 0,10 and 0,100 do not add to the value of the number.

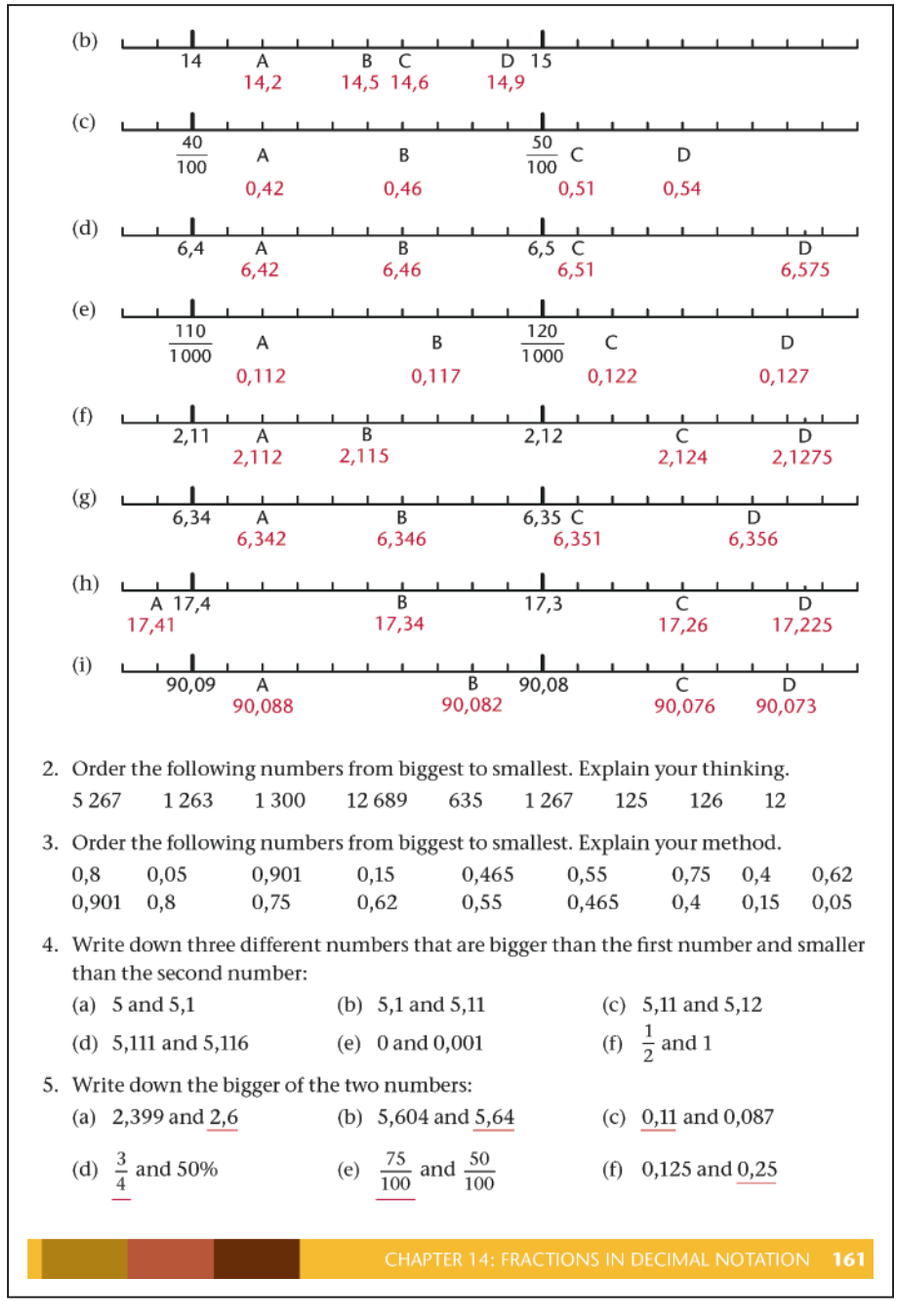
Teaching guidelines

Explain why 0,1, 0,10 and 0,100 are equivalent decimal fractions. Discuss how to order decimal fractions, for example:

- to order up to **two-digit decimal fractions**, add zeros to the end of the fractions so that each fraction represents hundredths.
- to order up to **three-digit decimal fractions**, add zeros to the end of the fractions so that each fraction represents thousandths.

Answers

1. (a) See the completed number line on LB page 160 on the previous page.
   (b) to (i) See the completed number lines on LB page 161 alongside.
2. 12 689 5 267 1 300 1 267 1 263 635 126 125 12
   Learners' own answers, for example: I looked at the number of digits and then at the place value of the digits.
3. Learners' methods may differ. Sample explanation: I added zeros to the end of the numbers to the right of the comma so that each number represents thousandths. That made it easier for me to compare the numbers.
4. All example answers.
   (a) 5,05 (b) 5,103 (c) 5,115
   (d) 5,115 (e) 0,0002 (f) $\frac{3}{4}$
5. See the underlined numbers on LB page 161 alongside.



2. Order the following numbers from biggest to smallest. Explain your thinking.
   5 267 1 263 1 300 12 689 635 1 267 125 126 12

3. Order the following numbers from biggest to smallest. Explain your method.
   0,8 0,05 0,901 0,15 0,465 0,55 0,75 0,4 0,62
   0,901 0,8 0,75 0,62 0,55 0,465 0,4 0,15 0,05

4. Write down three different numbers that are bigger than the first number and smaller than the second number:
   (a) 5 and 5,1 (b) 5,1 and 5,11 (c) 5,11 and 5,12
   (d) 5,111 and 5,116 (e) 0 and 0,001 (f) $\frac{1}{2}$ and 1

5. Write down the bigger of the two numbers:
   (a) 2,399 and 2,6 (b) 5,604 and 5,64 (c) 0,11 and 0,087
   (d) $\frac{3}{4}$ and 50% (e) $\frac{75}{100}$ and $\frac{50}{100}$ (f) 0,125 and 0,25

Answers

6. Klitschko has the advantage of height (he is taller), weight (he is heavier) and his reach is also further (longer).
7. See answers on LB page 162 alongside.
8. Infinitely many

14.3 Rounding off decimal fractions

LET'S ROUND OFF

Background information

Decimal fractions can be rounded off to the nearest whole number or to one, two, three, etc. digits after the decimal comma, for example:

rounding to the nearest whole number: If the tenths digit is:

- 5 or more, round up to the next whole number
- less than 5, round down to the previous whole number.

rounding to one decimal place: If the hundredths digit is:

- 5 or more, round up to the next tenth
- less than 5, round down to the previous tenth.

rounding to two decimal places: If the thousandths digit is:

- 5 or more, round up to the next hundredth
- less than 5, round down to the previous hundredth.

Teaching guidelines

Revise the procedure for rounding off a number to the nearest multiple, for example: 0,253 and 0,257 to the nearest hundredth are 0,250 and 0,260 respectively.

Answers

1. 29 4 14 3 39 565 2
2. 19,5 421,3 490,0 24,4 6,8
3. 8,35 6,63 5,56 34,24 21,90
4. He can pay R34,42 for four months (R137,68) and R34,41 for two months (R68,82).
5. (a) Two women will get 3,3 kg and one woman will get 3,4 kg.
   (b) R8,26 × 2 + R8,25 × 1

6. The table gives information about two world-champion heavyweight boxers. If they fight against one another, who would you expect to have the advantage, and why?

	Wladimir Klitschko	Alexander Povetkin
Height (m)	1,98	1,88
Weight (kg)	112	103,3
Reach (m)	2,03	1,91

7. Copy and fill in <, > or =:
   (a) 3,09 [<] 3,9 (b) 3,9 [=] 3,90 (c) 2,31 [<] 3,30
   (d) 3,197 [<] 3,2 (e) 4,876 [<] 5,987 (f) 123,321 [>] 123,3
8. How many numbers are there between 3,1 and 3,2?

14.3 Rounding off decimal fractions

Decimal fractions can be rounded in the same way as whole numbers. They can be rounded to the nearest whole number or to one, two, three, etc. figures after the comma.

If the last digit of the number is 5 or bigger, it is rounded **up** to the next number. For example: 13,5 rounded to the nearest whole number is 14; 13,526 rounded to two figures after the comma is 13,53. If the last digit is 4 or less, it is rounded **down** to the previous number. For example: 13,4 rounded to the nearest whole number is 13.

LET'S ROUND OFF

1. Round each of the following numbers off to the nearest whole number:
   29,34 3,65 14,452 3,299 39,1 564,85 1,768
2. Round each of the following numbers off to one decimal place:
   19,47 421,34 489,99 24,37 6,77
3. Round each of the following numbers off to two decimal places:
   8,345 6,632 5,555 34,239 21,899
4. Mr Peters buys a radio for R206,50. The shop allows him to pay it off over six months. How must he pay back the money?
5. (a) Mrs Smith buys a box of 10 kg grapes at the market for R24,77. She must divide it between herself and two friends. How much does each person get?
   (b) How much must each person pay Mrs Smith for the grapes?

Answers

6. (a) About $(1,5)^2 = 2,3$ (b) About $4 \times 4 = 16$

14.4 Calculations with decimal fractions

LET'S DO CALCULATIONS!

Background information

To add and subtract decimal fractions:

- tenths may be added to tenths, hundredths to hundredths, and so on.
- tenths may be subtracted from tenths, hundredths from hundredths, and so on.

Teaching guidelines

Revise different methods to add and subtract decimal fractions, for example:

- **Using common fractions**: First convert the decimal fractions to common fractions, then convert all common fractions to tenths, hundredths or thousandths, then add and subtract and convert the answer to a decimal fraction.
- **Using columns**: Refer to the answers to questions 5 and 6 below.

Answers

1. 85,7 km
2. (a) 18,87 (b) 25,90 (c) 26,30 (d) 32,98 (e) 0,145 (f) 10,98
3. (a) 22,42 (b) 21,87 (c) 89,15 (d) 0,011 (e) 0,855 (f) 28,1
4. (a) 583,2 cm ≈ 583 cm (b) 585 cm
   (c) 4(a) is the sum of the actual measurements; the rounding off was done at the end, so the total error is less than 0,5.
5. $\begin{array}{r} 0,70 \\ -\ 0,07 \\ \hline 0,63 \end{array}$
6. $\begin{array}{r} 18,4 \\ -\ 0,75 \\ \hline 17,65 \end{array}$

6. Estimate the answers for each of the following by rounding off the numbers:
   (a) 1,43 × 1,62 (b) 3,89 × 4,21

14.4 Calculations with decimal fractions

To **add** and **subtract** decimal fractions:

- tenths may be added to tenths
- tenths may be subtracted from tenths
- hundredths may be added to hundredths
- hundredths may be subtracted from hundredths, etc.

LET'S DO CALCULATIONS!

1. Four consecutive stages in a cycling race are 21,4 km, 14,7 km, 31 km and 18,6 km long. How long is the whole race?
2. Calculate each of the following:
   (a) 16,52 + 2,35 (b) 16,52 + 9,38 (c) 16,52 + 9,78
   (d) 30,08 + 2,9 (e) 0,042 + 0,103 (f) 9,99 + 0,99
3. Calculate each of the following:
   (a) 45,67 – 23,25 (b) 45,67 – 23,80 (c) 187,6 – 98,45
   (d) 1,009 – 0,998 (e) 0,9 – 0,045 (f) 65,7 – 37,6
4. The following set of measurements (in cm) was recorded during an experiment:
   56,8; 55,4; 78,9; 57,8; 34,2; 67,6; 45,5; 34,5; 64,5; 88
   (a) Find the sum of the measurements and round it off to the nearest whole number.
   (b) First round off each measurement to the nearest whole number and then find the sum.
   (c) Which of your answers in 4(a) and (b) is closest to the actual sum? Explain why.
5. By how much is 0,7 greater than 0,07?
6. The difference between two numbers is 0,75. The bigger number is 18,4. What is the other number?

To **multiply** fractions written as decimals, convert the fractions to whole numbers by multiplying by powers of 10 (for example, 0,3 × 10 = 3), do your calculations with the whole numbers, and then convert back to decimals again.

Background information (continued)

To **multiply fractions written as decimals**, convert the fractions to whole numbers by multiplying by powers of 10, do your calculations with the whole numbers and then convert back to decimals again.

Example: 13,1 × 1,01 = 131 × 101 ÷ 10 ÷ 100 = 13 231 ÷ 1 000 = 13,231

To **divide fractions written as decimals**, first multiply both numbers by the same number to make the working easier.

Example: 21,7 ÷ 0,7 = (21,7 × 10) ÷ (0,7 × 10) = 217 ÷ 7 = 31

Teaching guidelines (continued)

Revise multiplication and division strategies for decimal fractions.

Answers

7. (a) 0,036 (b) 0,0036 (c) 0,36 (d) 15,05
   (e) 0,001505 (f) 0,0208 (g) 2,08 (h) 0,208
8. (a) 3,965 (b) 396,5 (c) 0,03965 (d) 3 965
   (e) 1,3 (f) 130 (g) 305 (h) 0,0305
9. (a) 150,5 (b) 15,05 (c) 0,1505
   (d) 0,1505 (e) 43 (f) 350
10. (a) 25 (b) 2,5 (c) 25 (d) 0,25

14.5 Solving problems

Teaching guidelines

Recall real-life problems where decimal fractions are used, for example:

- times and distances recorded during the Olympic Games
- fuel consumption of vehicles or price of fuel per litre.

Answers

1. (a) R6,35 (b) seven weeks
2. (a) 2,42 (b) 1,5
3. (a) $x = 1{,}69$ (b) $x = 4{,}34$ (c) $x = 1{,}88$
   (d) $x = 24{,}50$ (e) $x = 2{,}61$ (f) $x = 347{,}83$
4. (a) 49,75 kg; 0,4975 kg (b) R114,53; R28,66

For example: 13,1 × 1,01

13,1 × **10** × 1,01 × **100** = 131 × 101 = 13 231; 13 231 ÷ **10** ÷ **100** = 13,231

When you do **division** you can first multiply the number and the divisor by the same number to make the working easier.

For example: 21,7 ÷ 0,7 = (21,7 × **10**) ÷ (0,7 × **10**) = 217 ÷ 7 = 31

7. Calculate each of the following. You may use fraction notation if you wish.
   (a) 0,12 × 0,3 (b) 0,12 × 0,03 (c) 1,2 × 0,3
   (d) 350 × 0,043 (e) 0,035 × 0,043 (f) 0,13 × 0,16
   (g) 1,3 × 1,6 (h) 0,13 × 1,6
8. 30,5 × 1,3 = 39,65. Use this answer to work out each of the following:
   (a) 3,05 × 1,3 (b) 305 × 1,3 (c) 0,305 × 0,13
   (d) 305 × 13 (e) 39,65 ÷ 30,5 (f) 39,65 ÷ 0,305
   (g) 39,65 ÷ 0,13 (h) 3,965 ÷ 130
9. 3,5 × 4,3 = 15,05. Use this answer to work out each of the following:
   (a) 3,5 × 43 (b) 0,35 × 43 (c) 3,5 × 0,043
   (d) 0,35 × 0,43 (e) 15,05 ÷ 0,35 (f) 15,05 ÷ 0,043
10. Calculate each of the following. You may convert to whole numbers to make it easier.
    (a) 62,5 ÷ 2,5 (b) 6,25 ÷ 2,5
    (c) 6,25 ÷ 0,25 (d) 0,625 ÷ 2,5

14.5 Solving problems

1. (a) Divide R44,45 between seven people so that each one receives the same amount.
   (b) John saves R15,25 every week. He now has R106,75 saved up. For how many weeks has he been saving?
2. (a) Calculate 14,5 ÷ 6, correct to two decimal places.
   (b) Calculate 7,41 ÷ 5, correct to one decimal place.
3. Determine the value of x. (Give answers rounded to two decimal places.)
   (a) $7{,}1 \div x = 4{,}2$ (b) $x \div 0{,}7 = 6{,}2$ (c) $12 \div x = 6{,}4$
   (d) $x \div 3{,}5 = 7$ (e) $2{,}3 \times x = 6$ (f) $0{,}023 \times x = 8$
4. (a) 1 ℓ of water weighs almost 0,995 kg. What will 50 ℓ of water weigh? What will 0,5 ℓ of water weigh?
   (b) Mince meat costs R36,65 per kilogram (kg). What will 3,125 kg of mince meat cost? What will 0,782 kg of mince meat cost?

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
15.1 The lengths of sides of right-angled triangles	Types of triangles; hypotenuse of a right-angled triangle; relationship between the lengths of the sides of right-angled triangles	Pages 165 to 167
15.2 Working with the theorem of Pythagoras	Theorem of Pythagoras	Pages 168 to 170
15.3 Finding the missing sides in right-angled triangles	The length of the hypotenuse; the length of any side in a right-angled triangle	Pages 170 to 173
15.4 Are the triangles right-angled?	Converse of the theorem of Pythagoras	Pages 173 to 174

CAPS time allocation	5 hours
CAPS content specification	Page 105

Mathematical background

- A right-angled triangle has **one interior angle** equal to 90°.
- The side opposite the right angle of a right-angled triangle is called the **hypotenuse**.
- Right-angled triangles have a **special feature** that does not apply to other types of triangles. This feature has come to be known as the theorem of Pythagoras.
- A **theorem** is a statement which has been proved to be true.
- A **converse** is a statement that swaps around what is given in a theorem and what must be determined.
- The **theorem of Pythagoras** states the following: *If any triangle is a right-angled triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.* The theorem of Pythagoras enables us to find:
  - the **length of the hypotenuse** of a right-angled triangle if the lengths of the other two sides are given
  - the **length of any side** of a right-angled triangle if the lengths of the hypotenuse and one other side are given.
- The **converse of the theorem of Pythagoras** states the following: *If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the longest side, then the triangle is a right-angled triangle.* The converse of the theorem of Pythagoras enables us to determine whether a triangle is:
  - an **acute-angled** triangle
  - a **right-angled** triangle, or
  - an **obtuse-angled** triangle.

15.1 The lengths of sides of right-angled triangles

WHAT DO YOU REMEMBER ABOUT TRIANGLES?

Background information

- The size of an **acute angle** is between 0° and 90°.
- The size of a **right angle** is 90°.
- The size of an **obtuse angle** is between 90° and 180°.
- **Acute-angled triangles** have three interior acute angles.
- **Right-angled triangles** have one interior right angle. The word **hypotenuse** indicates the side opposite to the right angle in a right-angled triangle.
- **Obtuse-angled triangles** have one interior obtuse angle.

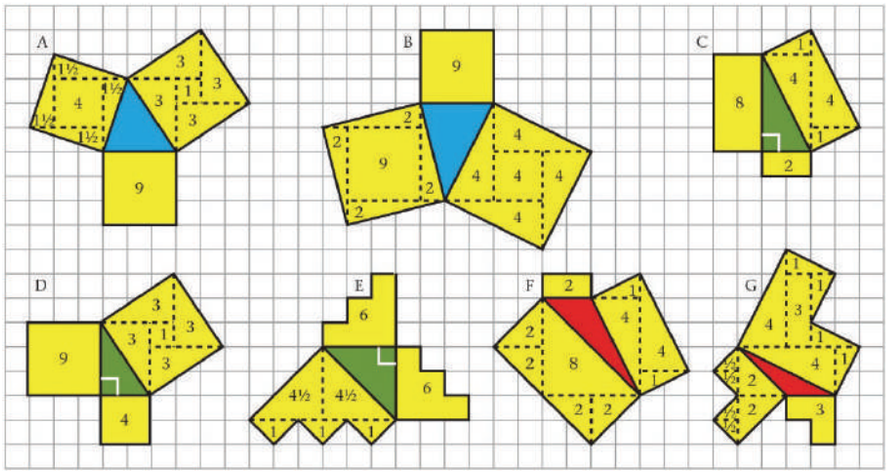
Teaching guidelines

Revise the features of angles and triangles listed above.

INVESTIGATING THE RELATIONSHIP BETWEEN THE LENGTHS OF SIDES

Background information

- **Similar figures** have the same shape but may differ in size.
- Similar figures can be drawn with **matching sides** on the sides of triangles.



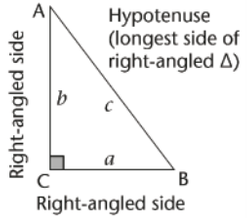
CHAPTER 15

The theorem of Pythagoras

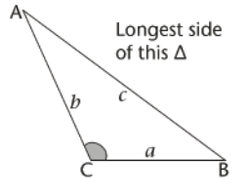
15.1 The lengths of sides of right-angled triangles

WHAT DO YOU REMEMBER ABOUT TRIANGLES?

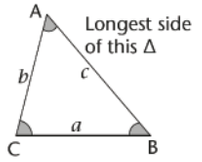
Right-angled triangle (Δ): one angle is 90°.



Obtuse-angled triangle (Δ): one angle is obtuse (between 90° and 180°).



Acute-angled triangle (Δ): all angles are acute (less than 90°).



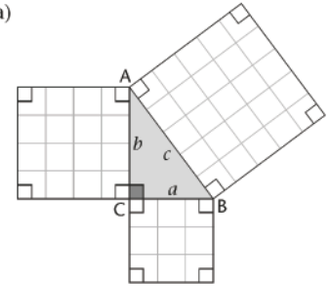
If the vertices of a triangle are labelled A, B and C, the sides opposite these vertices are often labelled as *a*, *b* and *c*, as shown in the above diagrams.

We use the word **hypotenuse** to indicate the side opposite the 90° angle of a right-angled triangle. The hypotenuse is always the longest side of a right-angled triangle. A triangle with no right angle, therefore, does not have a hypotenuse.

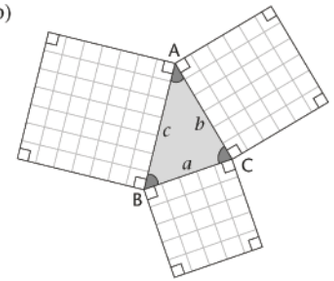
INVESTIGATING THE RELATIONSHIP BETWEEN THE LENGTHS OF SIDES

1. Study the figures below. Each triangle in the following four figures has a square drawn on each of its sides. So, in figure (a), *a* = three units, *b* = four units and *c* = five units long.

(a)



(b)



From the drawings on the previous page, the following statements can be formulated:

- Figures A and B: The area of a figure drawn on the longest side of an acute-angled triangle is **smaller than** the sum of the areas of similar figures drawn on the other two sides.
- Figures C to E: The area of a figure drawn on the longest side of a right-angled triangle is **equal to** the sum of the areas of similar figures drawn on the other two sides.
- Figures F and G: The area of a figure drawn on the longest side of an obtuse-angled triangle is **larger than** the sum of the areas of similar figures drawn on the other two sides.
- The easiest figure to construct on the sides of a triangle is a **square** and its area is the easiest to calculate.

Teaching guidelines

- Use square grid paper and draw similar figures on the sides of different types of triangles.
- Investigate the relationship between the areas of the similar figures on the sides of each triangle.

Note on question 1

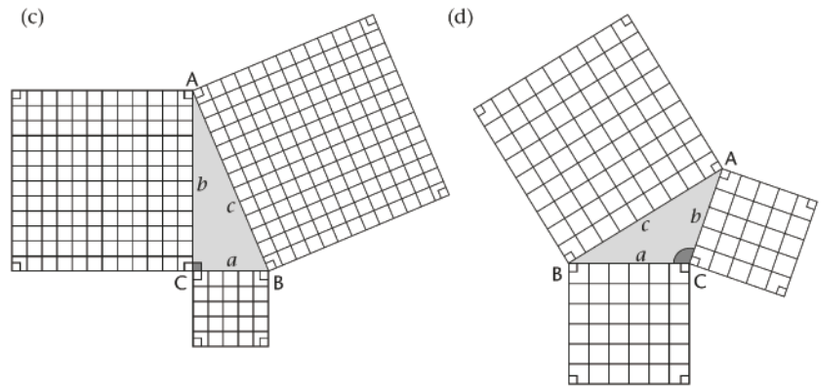
Different sized grids are used in the four drawings. However, the same grid size is used to draw the squares in each drawing.

Note on question 2

The lengths *a*, *b* and *c* are expressed in terms of the grid size used in each drawing.

Answers

1. See the two figures on LB page 165 on the previous page and the other two figures on LB page 166 alongside.
2. See the completed table on LB page 166 alongside.
3. See the answers on LB page 166 alongside.
4. Statements A and C are correct.



2. Copy the following table. Refer to the four figures from question 1(a) to (d) to complete the table.

Figure	Type of triangle	Length of side *a*	Length of side *b*	Length of side *c*	a^2	b^2	c^2
(a)	Right-angled	3	4	5	9	16	25
(b)	Acute-angled	5	6	7	25	36	49
(c)	Right-angled	5	12	13	25	144	169
(d)	Obtuse-angled	6	5	9	36	25	81

3. Look at your completed table and then copy the statements below and insert =, > or <.

   $a^2 + b^2$ [>] c^2 when ΔABC is an acute-angled triangle.

   $a^2 + b^2$ [<] c^2 when ΔABC is an obtuse-angled triangle.

   $a^2 + b^2$ [=] c^2 when ΔABC is a right-angled triangle.

4. Which of the following statements are correct?

   A. In any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

   B. If a triangle is acute-angled, then the square of the length of the longest side is equal to the sum of the squares of the lengths of the other two sides.

   C. If a triangle is right-angled, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

   D. In any obtuse-angled triangle, the area of the square on the longest side is equal to the sum of the area of the squares on the other two sides.

Note on question 5

This question focuses on the converse (or the reverse) of the theorem of Pythagoras.

Answers

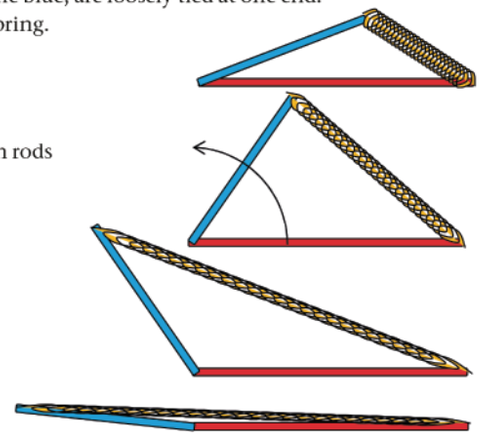
5. See the completed table on LB page 167 alongside.
6. The bigger the angle, the longer the spring.

5. The following table gives the side lengths a, b and c of ten triangles. Copy and complete the table to decide what type of triangle each triangle is (acute-angled, obtuse-angled or right-angled).

a	b	c	$a^2 + b^2$	c^2	Fill in =, < or >	Type of triangle
7	8	10	$7^2 + 8^2 = 113$	$10^2 = 100$	$a^2 + b^2 > c^2$	Acute-angled
4	5	8	$4^2 + 5^2 = 41$	$8^2 = 64$	$a^2 + b^2 < c^2$	Obtuse-angled
6	8	10	$6^2 + 8^2 = 100$	$10^2 = 100$	$a^2 + b^2 = c^2$	Right-angled
8	13	17	$8^2 + 13^2 = 233$	$17^2 = 289$	$a^2 + b^2 < c^2$	Obtuse-angled
3	4	5	25	25	$a^2 + b^2 = c^2$	Right-angled
5	6	7	61	49	$a^2 + b^2 > c^2$	Acute-angled
5	12	13	169	169	$a^2 + b^2 = c^2$	Right-angled
15	8	17	289	289	$a^2 + b^2 = c^2$	Right-angled
11	60	61	3 721	3 721	$a^2 + b^2 = c^2$	Right-angled
12	35	37	1 369	1 369	$a^2 + b^2 = c^2$	Right-angled

6. Two pieces of wood, one red and one blue, are loosely tied at one end. The two free ends are linked by a spring.

The angle between the two wooden rods can be changed.



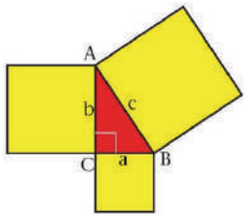
Describe how this angle affects the length of the spring.

15.2 Working with the theorem of Pythagoras

WORKING WITH THE FORMULA

Background information

- The ancient Egyptians, Babylonians and Chinese were aware of a special relationship between the lengths of the sides of triangles. They often made use of this, but never proved it.
- The first person to prove the special relationship between the lengths of the sides of a right-angled triangle was **Pythagoras**, a Greek mathematician and philosopher (582 – 500 BC). He established a school of philosophy in Italy where he and his followers expressed views that were considered strange at the time, for example, the view that the Earth rotates around its own axis and that the planets revolve around the sun! As a result of these "strange" theories, Pythagoras was forced to flee to Egypt, where he was later murdered. His theorem on right-angled triangles is called the theorem of Pythagoras.
- The **theorem of Pythagoras** can be expressed in:
  - **word form**: If a triangle is a right-angled triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.
  - **diagrammatic form**:



  - **symbolic form:**

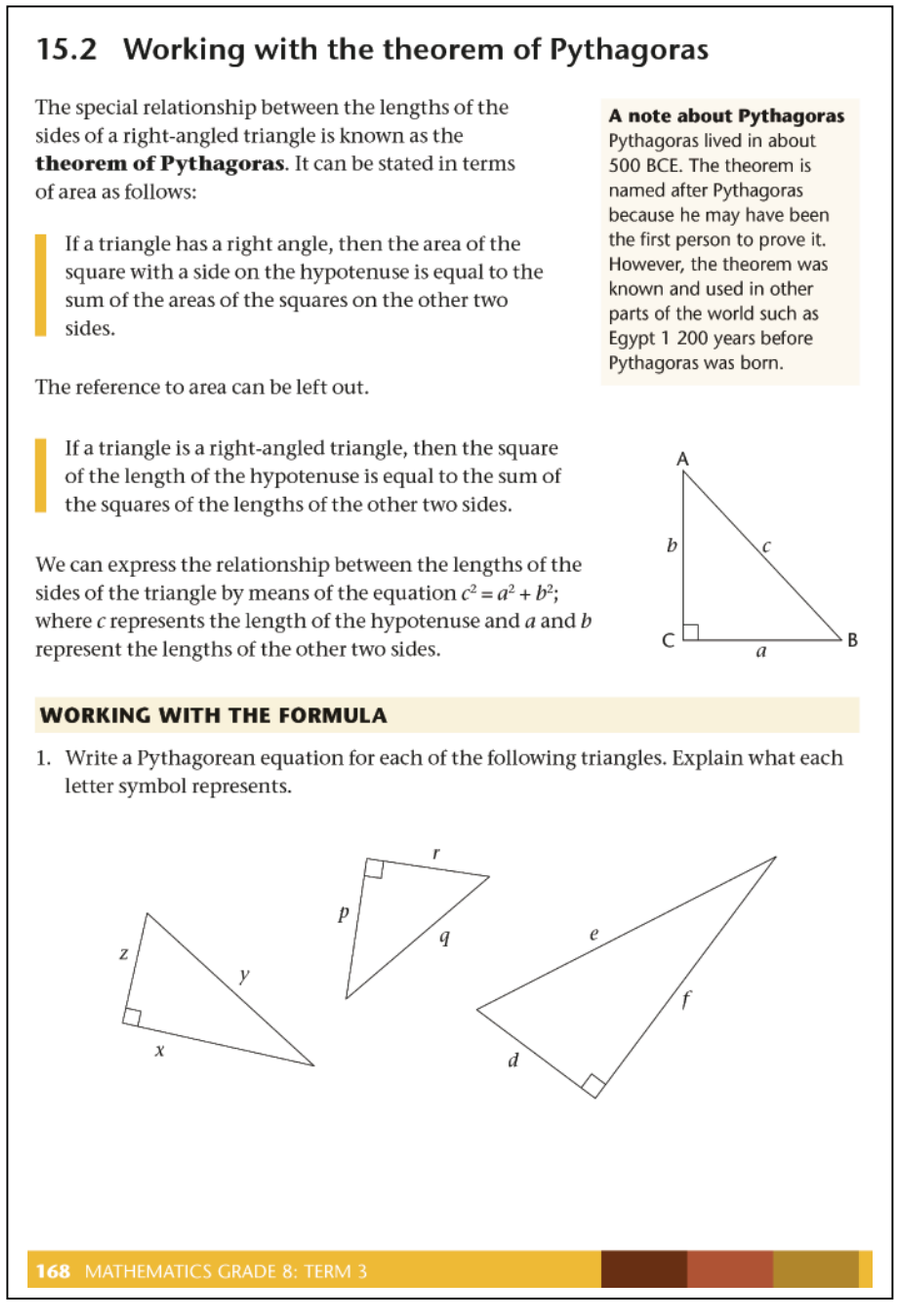
    If ΔABC has a right angle, $\hat{C} = 90°$, then
    $AB^2 = BC^2 + CA^2$ or $c^2 = a^2 + b^2$.

Teaching guidelines

Express the theorem of Pythagoras in word, diagrammatic and symbolic form.

Answers

1. $y^2 = x^2 + z^2$ $\quad q^2 = r^2 + p^2$ $\quad e^2 = d^2 + f^2$
   y, q and e represent the lengths of the hypotenuses and x, z, r, p, d and f represent the lengths of the other sides.



15.2 Working with the theorem of Pythagoras

The special relationship between the lengths of the sides of a right-angled triangle is known as the **theorem of Pythagoras**. It can be stated in terms of area as follows:

> If a triangle has a right angle, then the area of the square with a side on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

The reference to area can be left out.

> If a triangle is a right-angled triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

We can express the relationship between the lengths of the sides of the triangle by means of the equation $c^2 = a^2 + b^2$; where c represents the length of the hypotenuse and a and b represent the lengths of the other two sides.

A note about Pythagoras
Pythagoras lived in about 500 BCE. The theorem is named after Pythagoras because he may have been the first person to prove it. However, the theorem was known and used in other parts of the world such as Egypt 1 200 years before Pythagoras was born.

WORKING WITH THE FORMULA

1. Write a Pythagorean equation for each of the following triangles. Explain what each letter symbol represents.

Teaching guidelines (continued)

Discuss the layout of the solution in the example on LB page 169 alongside.

Note on Pythagoras

Learners could be challenged to do some research on Pythagoras and to write a short report on his life and contributions to the field of Mathematics.

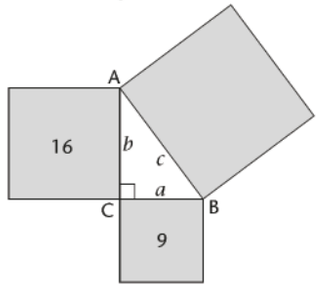
Answers

2. See the example on LB page 169 alongside.
3. (a) See the answer on LB page 169 alongside.
   (b) See the answer on LB page 169 alongside.

2. Study the following worked example:

Example

Consider the triangle below. Side a is three units long and side b is four units long. What is the length of side c?

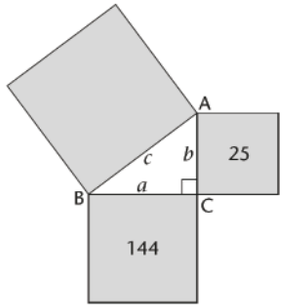


If side a is three units long, and side b is four units long, then, according to Pythagoras' theorem:

$c^2 = a^2 + b^2$
$c^2 = 3^2 + 4^2$
$c^2 = 9 + 16$
$c^2 = 25$
$\sqrt{c^2} = \sqrt{25}$
c = five units

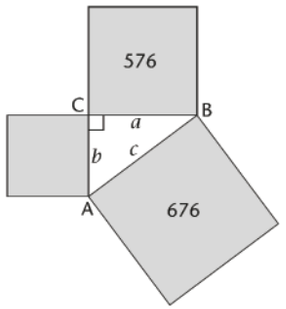
3. The areas of some of the following squares are given. Calculate the areas of each of the squares that are not given and the lengths of all the sides.

(a)



According to Pythagoras' theorem:
$c^2 = a^2 + b^2$
$c^2 = 144 + 25$
$c^2 = 169$
$\sqrt{c^2} = \sqrt{169}$
c = 13 units
a = 12 units; b = 5 units

(b)



According to Pythagoras' theorem:
$c^2 = a^2 + b^2$
$676 = 576 + b^2$
$676 - 576 = b^2$
$100 = b^2$
$\sqrt{100} = \sqrt{b^2}$
b = 10 units
a = 24 units; c = 26 units

Answers

4. See the completed table on LB page 170 alongside.

15.3 Finding the missing sides in right-angled triangles

Background information

The theorem of Pythagoras enables us to find:

- the **length of the hypotenuse** of a right-angled triangle if the lengths of the other two sides are given, for example:

  If ΔABC has a right angle, $\hat{C} = 90°$, and BC and CA are given, then
  $c^2 = a^2 + b^2$.

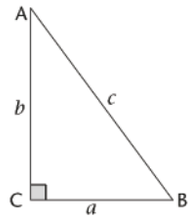
- the **length of any side** of a right-angled triangle if the lengths of the hypotenuse and one other side are given, for example:

  If ΔABC has a right angle, $\hat{C} = 90°$, and AB and BC are given, then
  $b^2 = c^2 - a^2$.

Teaching guidelines

Discuss the layout of the solutions in the two examples on LB page 170 alongside.

4. The following table gives information about the sides of five right-angled triangles. The letter symbol c represents the length of the hypotenuse in all cases. Use Pythagoras' theorem to complete the table, leaving answers in surd form if necessary.



a	b	c	a^2	b^2	$a^2 + b^2$	c^2
7	24	25	49	576	625	625
16	30	34	256	900	1 156	1 156
10	24	26	100	576	676	676
4	7	$\sqrt{65}$	16	49	65	65
1	1	$\sqrt{2}$	1	1	2	2

15.3 Finding the missing sides in right-angled triangles

We can use the theorem of Pythagoras to calculate the length of the third side of a right-angled triangle if we know the lengths of the other two sides.

Example 1

A right-angled triangle has side a = six units and side b = eight units. Calculate the length of side c.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 6^2 + 8^2 \\ &= 36 + 64 \\ &= 100 \\ \sqrt{c^2} &= \sqrt{100} \\ c &= 10 \\ \therefore c &= 10 \text{ units} \end{aligned}$$

Example 2

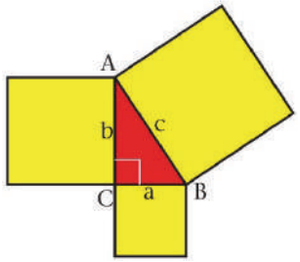
A right-angled triangle has side a = five units and side b = three units. Calculate the length of side c.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 3^2 \\ &= 25 + 9 \\ &= 34 \\ \sqrt{c^2} &= \sqrt{34} \\ c &= \sqrt{34} \text{ (leave in surd form)} \\ \therefore c &= \sqrt{34} \text{ units} \end{aligned}$$

CALCULATING THE LENGTH OF THE HYPOTENUSE

Background information

The theorem of Pythagoras enables us to find the **length of the hypotenuse** of a right-angled triangle if the lengths of the other two sides are given, for example:



If ΔABC has a right angle, $\hat{C} = 90°$, and BC and CA are given, then:

- $AB^2 = BC^2 + CA^2$ or
- $c^2 = a^2 + b^2$.

Teaching guidelines

- Revise how to find the length of the hypotenuse of a right-angled triangle if the lengths of the other two sides are given.
- Start each solution by writing the theorem of Pythagoras in symbolic notation.

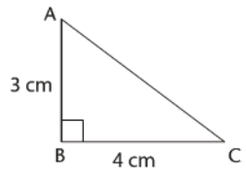
Answers

1. to 5. See the answers on LB page 171 alongside.

CALCULATING THE LENGTH OF THE HYPOTENUSE

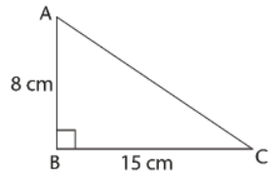
Use the formula for the theorem of Pythagoras to calculate the length of the hypotenuse. Leave answers in surd form if necessary.

1.



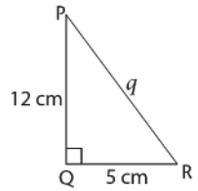
$AC^2 = AB^2 + BC^2$
$= (3\text{ cm})^2 + (4\text{ cm})^2$
$= 9\text{ cm}^2 + 16\text{ cm}^2$
$= 25\text{ cm}^2$
$\sqrt{AC^2} = \sqrt{25\text{ cm}^2}$
$AC = 5\text{ cm}$

2.



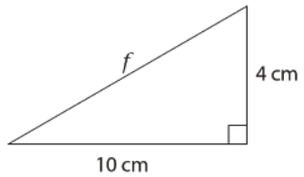
$AC^2 = AB^2 + BC^2$
$= (8\text{ cm})^2 + (15\text{ cm})^2$
$= 64\text{ cm}^2 + 225\text{ cm}^2$
$= 289\text{ cm}^2$
$\sqrt{AC^2} = \sqrt{289\text{ cm}^2}$
$AC = 17\text{ cm}$

3.



$q^2 = p^2 + r^2$
$= (5\text{ cm})^2 + (12\text{ cm})^2$
$= 25\text{ cm}^2 + 144\text{ cm}^2$
$= 169\text{ cm}^2$
$\sqrt{q^2} = \sqrt{169\text{ cm}^2}$
$q = 13\text{ cm}$

4.



$f^2 = (4\text{ cm})^2 + (10\text{ cm})^2$
$= 16\text{ cm}^2 + 100\text{ cm}^2$
$= 116\text{ cm}^2$
$\sqrt{f^2} = \sqrt{116\text{ cm}^2}$
$f = \sqrt{116}\text{ cm}$

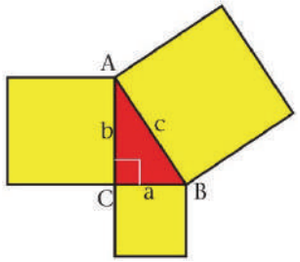
5. A right-angled triangle with hypotenuse c and sides of the following lengths:
$a = 9$ cm and $b = 40$ cm.

$c^2 = a^2 + b^2$
$= (9\text{ cm})^2 + (40\text{ cm})^2$
$= 81\text{ cm}^2 + 1\,600\text{ cm}^2$
$= 1\,681\text{ cm}^2$
$c = 41\text{ cm}$

CALCULATING THE LENGTH OF ANY SIDE IN A RIGHT-ANGLED TRIANGLE

Background information

The theorem of Pythagoras enables us to find the **length of any side** of a right-angled triangle if the lengths of the hypotenuse and another side are given, for example:



If ΔABC has a right angle, $\hat{C} = 90°$, and AB and BC are given, then:

- $AC^2 = AB^2 - BC^2$ or
- $b^2 = c^2 - a^2$.

If ΔABC has a right angle, $\hat{C} = 90°$, and AB and CA are given, then:

- $BC^2 = AB^2 - CA^2$ or
- $a^2 = c^2 - b^2$.

Teaching guidelines

- Revise how to find the length of the third side of a right-angled triangle if the lengths of the hypotenuse and another side are given.
- Start each solution by writing the theorem of Pythagoras in symbolic notation.

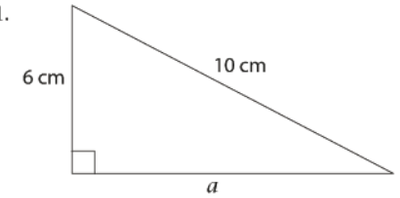
Answers

1. to 4. See the answers on LB page 172 alongside.

CALCULATING THE LENGTH OF ANY SIDE IN A RIGHT-ANGLED TRIANGLE

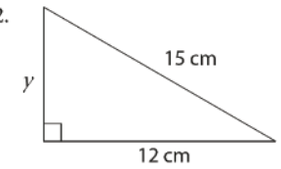
Calculate the missing sides in the following triangles. Do not use a calculator and leave the answers in the simplest surd form where necessary.

1.



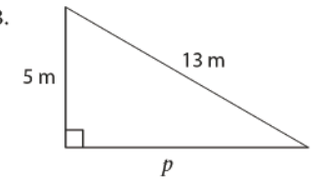
$(10\text{ cm})^2 = a^2 + (6\text{ cm})^2$
$100\text{ cm}^2 = a^2 + 36\text{ cm}^2$
$100\text{ cm}^2 - 36\text{ cm}^2 = a^2$
$64\text{ cm}^2 = a^2$
$\sqrt{64\text{ cm}^2} = \sqrt{a^2}$
$8\text{ cm} = a$
$a = 8\text{ cm}$

2.



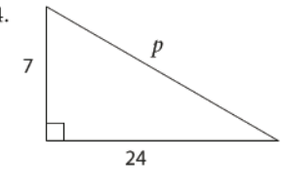
$(15\text{ cm})^2 = y^2 + (12\text{ cm})^2$
$225\text{ cm}^2 = y^2 + 144\text{ cm}^2$
$225\text{ cm}^2 - 144\text{ cm}^2 = y^2$
$81\text{ cm}^2 = y^2$
$\sqrt{81\text{ cm}^2} = \sqrt{y^2}$
$9\text{ cm} = y$
$y = 9\text{ cm}$

3.



$(13\text{ m})^2 = p^2 + (5\text{ m})^2$
$169\text{ m}^2 = p^2 + 25\text{ m}^2$
$169\text{ m}^2 - 25\text{ m}^2 = p^2$
$144\text{ m}^2 = p^2$
$\sqrt{144\text{ m}^2} = \sqrt{p^2}$
$12\text{ m} = p$
$p = 12\text{ m}$

4.



$p^2 = (7)^2 + (24)^2$
$p^2 = 49 + 576$
$p^2 = 625$
$\sqrt{p^2} = \sqrt{625}$
$p = 25\text{ units}$

Answers

5. See the answer on LB page 173 alongside.

15.4 Are the triangles right-angled?

Background information

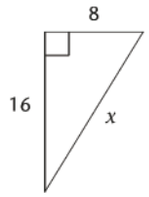
- A **converse** is a statement that swaps around what is given in a theorem and what must be determined.
- The **converse of the theorem of Pythagoras** states the following: "If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the longest side, then the triangle is a right-angled triangle."
- The converse of the theorem of Pythagoras enables us to determine whether a triangle is **acute-angled**, **right-angled** or **obtuse-angled**.
  - If the square on the longest side is **smaller** than the sum of the squares on the other two sides, the triangle is **acute-angled**.
  - If the square on the longest side is **equal to** the sum of the squares on the other two sides, the triangle is **right-angled**.
  - If the square on the longest side is **larger than** the sum of the squares on the other two sides, the triangle is **obtuse-angled**.
- **Pythagorean triplets** are sets of three numbers that match the Theorem of Pythagoras. Here are some examples:

3; 4; 5	6; 8; 10	9; 12; 15	12; 16; 20	15; 20; 25
5; 12; 13	10; 24; 26	15; 36; 39	20; 48; 52	25; 60; 65
8; 15; 17	7; 24; 25	12; 35; 37	20; 21; 29	24; 45; 51

Teaching guidelines

- Learners discuss the following:
  - What a converse of a theorem does.
  - How to use the converse of the theorem of Pythagoras to determine the type of a triangle.
- Learners find some examples of Pythagorean triplets.
- Learners study the layout of the example on LB page 173 alongside.

5.



$x^2 = (8)^2 + (16)^2$
$x^2 = 64 + 256$
$x^2 = 320$
$\sqrt{x^2} = \sqrt{320}$
$x = \sqrt{64 \times 5}$
$x = 8\sqrt{5}$ units

15.4 Are the triangles right-angled?

You learnt in Sections 15.1 and 15.2 that in a right-angled triangle the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

How can we tell whether a triangle is right-angled if we are given the lengths of the sides? One way is to use the "converse" of the Pythagoras theorem.

The **converse** states that if the sum of the squares of the lengths of two sides equals the square of the length of the longest side, then the triangle is a right-angled triangle.

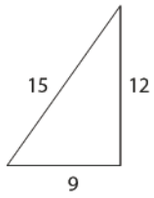
A converse is a statement that swaps around **what is given** in a theorem and **what must be determined**.

We can also state the converse as follows:

If a triangle has side lengths a, b and c such that $c^2 = a^2 + b^2$, then the triangle is a right-angled triangle.

In the questions that follow, you have to determine whether triangles are right-angled or not. Study the following example first:

Example: Determine whether the triangle is right-angled or not.



(Length of longest side)^2 = $(15)^2$ = 225
Sum of the squares of the lengths of the other two sides
= $9^2 + 12^2$
= 81 + 144
= 225
(Longest side length)^2 = Sum of squares of other two sides lengths
And this can be written as $15^2 = 9^2 + 12^2$
∴ The triangle is right-angled.

RIGHT-ANGLED OR NOT?

Background information (continued)

Multiples of Pythagorean triplets are also Pythagorean triplets.

For example: 3; 4; 5 are Pythagorean triplets, as are their multiples: 6; 8; 10 or 9; 12; 15 or 12; 16; 20.

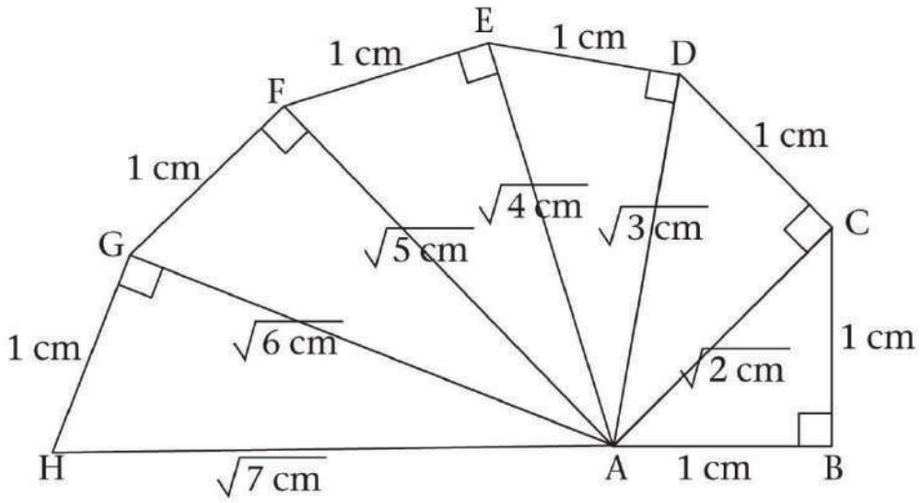
Answers

1. to 3. See the answers on LB page 174 alongside.

4. (b), (c), (e) and (f) are right-angled triangles.

   They are multiples of right-angled triangles whose sides are three, four and five units long.

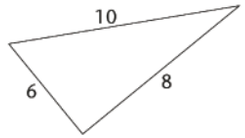
An interesting application of the theorem of Pythagoras



RIGHT-ANGLED OR NOT?

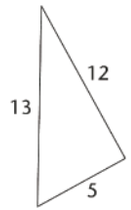
Determine whether the triangles are right-angled or not.

1.



(Length of longest side)^2 = 10^2 = 100
Sum of the squares of the lengths of the other two sides
= $8^2 + 6^2$
= 64 + 36
= 100
This can be written as $10^2 = 8^2 + 6^2$
∴ The triangle is a right-angled triangle.

2.



(Length of longest side)^2 = 13^2 = 169
Sum of the squares of the lengths of the other two sides
= $12^2 + 5^2$
= 144 + 25
= 169
This can be written as $13^2 = 12^2 + 5^2$
∴ The triangle is a right-angled triangle.

3. A triangle has sides measuring six, nine and 15 units.

(Length of longest side)^2 = 15^2 = 225
Sum of the squares of the lengths of the other two sides
= $9^2 + 6^2$
= 81 + 36
= 117
This can be written as $15^2 \neq 9^2 + 6^2$
∴ The triangle is not a right-angled triangle.

4. Which of the following lengths of sides of a triangle will form a right-angled triangle? Answer without doing any calculations and explain your answer.

(a) 4, 2, 2	(b) 6, 8, 10	(c) 9, 12, 15
(d) 3, 4, 6	(e) $3x$, $4x$, $5x$	(f) 30, 40, 50

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
16.1 Perimeter of squares and rectangles	Explaining the formulae for perimeter; calculating perimeters using formulae	Pages 175 to 176
16.2 Area of polygons	Areas of squares and rectangles; more perimeter and area problems; area of triangles and problems involving the area of triangles; area of composite shapes	Pages 176 to 179
16.3 Perimeter of circles	Parts of a circle; the relationship between a circle's circumference and diameter; pi (π) and the formula for the circumference of a circle; using the formula for the circumference of a circle	Pages 179 to 182
16.4 Area of circles	Investigating the formula for the area of a circle; using the formula for the area of a circle	Pages 182 to 186
16.5 Converting between square units	Converting from one square unit to another	Pages 186 to 187

CAPS time allocation	5 hours
CAPS content specification	Pages 106 to 107

Mathematical background

Perimeter means the distance along the outer edge of a shape or figure. It is measured in units of length – i.e. mm, cm, m and km. The formula for the perimeter of a:

- square is $P = 4s$, where s is the side length of the square
- rectangle is $P = 2(l + b)$, where l is the length and b is the breadth (or height) of the rectangle
- circle is called its circumference and is given by $C = 2\pi r$, where r is the radius of the circle.

Area is a measure of the size of the flat surface of a shape. It is measured in square units of area – i.e. mm^2, cm^2, m^2 and km^2. The formula for the area of a:

- square is $A = s^2$, where s is the side length of the square
- rectangle is $A = lb$, where l is the length and b is the breadth (or height) of the rectangle
- triangle is $A = \frac{1}{2}bh$, where b is the base of the triangle and h is the perpendicular height on that base
- circle is $A = \pi r^2$, where r is the radius of the circle.

Parts of a circle:

- the midpoint of a circle is called its **centre**
- the line drawn between two points on the edge of the circle and that passes through the centre is called the **diameter**
- the line drawn from any point on the circumference of the circle to the centre is called the **radius** of the circle.

16.1 Perimeter of squares and rectangles

EXPLAINING THE FORMULAE FOR PERIMETER

Teaching guidelines

Learners can physically count the number of units in the perimeter or may know the formula. If they do not know the formula, they can discuss their results in the table with a partner and develop the formula.

Remind learners that perimeter is a length and is therefore measured in units of length – i.e. millimetres, centimetres, metres and kilometres.

Answers

1. See the completed table on LB page 175 alongside.

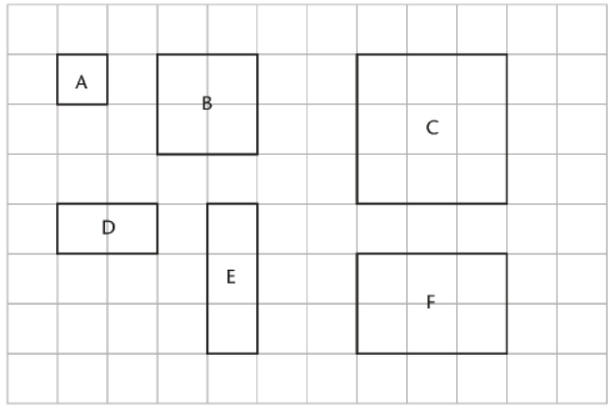
CHAPTER 16
Perimeter and area of 2D shapes

16.1 Perimeter of squares and rectangles

The **perimeter** (P) of a flat shape is the distance around a shape. We measure it in units such as millimetres (mm), centimetres (cm), metres (m) and kilometres (km).

EXPLAINING THE FORMULAE FOR PERIMETER

1. Each block in the grid below measures 1 cm × 1 cm. Calculate the perimeter of each shape by adding up the lengths and breadths.



Copy and complete the table based on shapes A to F above.

Shape	A	B	C	D	E	F
Length	1 cm	2 cm	3 cm	2 cm	1 cm	3 cm
Breadth	1 cm	2 cm	3 cm	1 cm	3 cm	2 cm
Perimeter	4 cm	8 cm	12 cm	6 cm	8 cm	10 cm

Answers

2. Learners' own work.
3. A = $4s = 4 \times 1$ cm = 4 cm B = $4s = 4 \times 2$ cm = 8 cm
   C = $4s = 4 \times 3$ cm = 12 cm D = $2(l + b) = 2 \times 3$ cm = 6 cm
   E = $2(l + b) = 2 \times 4$ cm = 8 cm F = $2(l + b) = 2 \times 5$ cm = 10 cm

CALCULATING PERIMETERS USING FORMULAE

Teaching guidelines

Learners use the following formulae to calculate the perimeter of the squares or rectangles.

- Square: $P = 4s$
- Rectangle: $P = 2(l + b)$

Answers

1. $P = 4s$ 4×4 cm = 16 cm
2. $P = 2(l + b)$ $2(5 \text{ cm} + 3 \text{ cm}) = 2(8 \text{ cm}) = 16$ cm
3. $P = 2(l + b)$ $2(4{,}5 \text{ cm} + 6 \text{ cm}) = 2(10{,}5 \text{ cm}) = 21$ cm
4. $P = 4s$ 4×7 cm = 28 cm
5. $P = 4s$ $4 \times 1{,}5$ cm = 6 cm
6. $P = 2(l + b)$ $2(12 \text{ cm} + 8 \text{ cm}) = 2(20 \text{ cm}) = 40$ cm

16.2 Area of polygons

AREA OF SQUARES AND RECTANGLES

Teaching guidelines

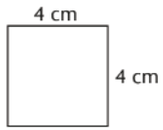
Learners can count the number of 1 cm^2 squares that cover the shape to find the area, or they can use the formula if they know it.

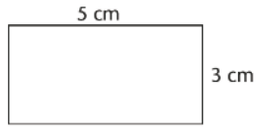
Show learners that the area of a rectangle, for example, can be deduced by counting the number of square blocks in a row and multiplying that number by the number of rows. This leads to the formula and an understanding of what the formula means.

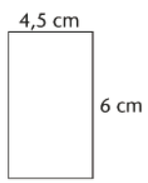
2. Explain to a partner why the following formulae for perimeter are correct:
   Perimeter of a square = $4s$ or (4 × length of a side)
   Perimeter of a rectangle = $2(l + b)$ or $2l + 2b$ (l is the length and b is the breadth)
3. Use the formulae in question 2 to calculate the perimeters of shapes A to F above.

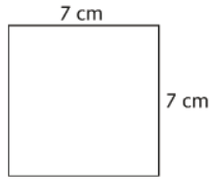
CALCULATING PERIMETERS USING FORMULAE

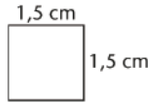
Use formulae to calculate the perimeters of the following shapes.

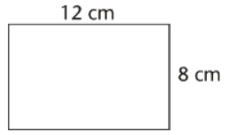
1. 

2. 

3. 

4. 

5. 

6. 

16.2 Area of polygons

We use square units such as mm^2, cm^2, m^2 and km^2 to measure the **area** (A), or the size of a flat surface of a shape.

AREA OF SQUARES AND RECTANGLES

1. How many square units make up the area of the shapes on page 177?

Answers

1. See the answers on LB page 177 alongside.
2. A = 1 cm^2 B = 4 cm^2 C = 9 cm^2 D = 2 cm^2 E = 3 cm^2 F = 6 cm^2
3. Area of shape C: $s^2 = (3\text{ cm})^2 = 3\text{ cm} \times 3\text{ cm} = 9\text{ cm}^2$
   Area of shape E: $l \times b = 1\text{ cm} \times 3\text{ cm} = 3\text{ cm}^2$
   Area of shape F: $l \times b = 3\text{ cm} \times 2\text{ cm} = 6\text{ cm}^2$

SOLVING MORE PERIMETER AND AREA PROBLEMS

Teaching guidelines

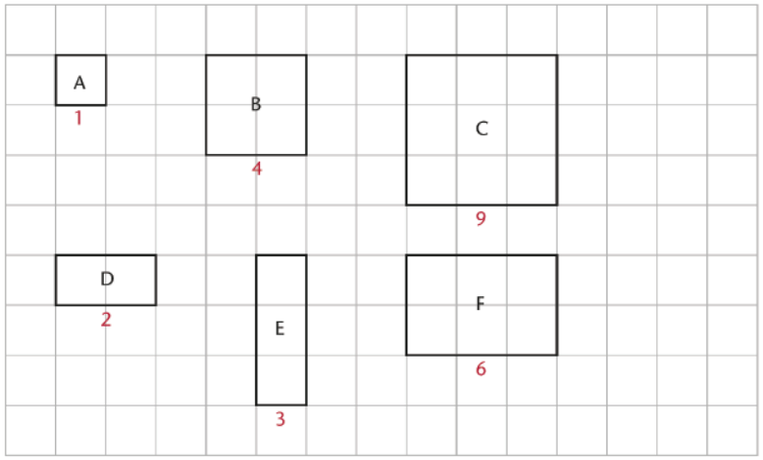
Learners may find it easier if they draw a sketch of the situations.

Learners need to work out what the missing information is that they need and how to get that information (see, for example, questions 5 and 6).

Learners must remember that the units for area also need to be squared.

Answers

1. $P = 4s$ $\quad 8\text{ cm} = 4s$
   $2\text{ cm} = s$ $\quad \therefore$ Length of each side = 2 cm
2. $A = l \times b$ $\quad 40\text{ cm}^2 = 8\text{ cm} \times b$
   $5\text{ cm} = b$ $\quad \therefore$ Breadth of rectangle = 5 cm
3. $P = 4s$ $\quad 32\text{ cm} = 4s$
   $8\text{ cm} = s$
   $A = s^2$ $\quad (8\text{ cm})^2 = 64\text{ cm}^2$ $\quad \therefore$ Length = 8 cm; area = 64 cm^2
4. $A = l \times b$ $\quad 60\text{ cm}^2 = 12\text{ cm} \times b$
   $5\text{ cm} = b$
   $P = 2(l + b)$ $\quad 2(17\text{ cm}) = 34\text{ cm}$ $\quad \therefore$ Breadth = 5 cm; perimeter = 34 cm
5. $l = 600\text{ m}^2 \div 20\text{ m} = 30\text{ m}$
   $P = 2(l + b)$ $\quad 2(30\text{ m} + 20\text{ m})$
   $= 2(50\text{ m}) = 100\text{ m}$ $\quad \therefore$ Length = 30 m; perimeter = 100 m
6. $A = s^2 = 10\,000\text{ m}^2$
   $\sqrt{s^2} = \sqrt{10\,000\text{ m}^2}$
   $s = 100\text{ m}$
   $P = 4s$ $\quad 4 \times 100\text{ m} = 400\text{ m}$ $\quad \therefore$ Perimeter = 400 m



2. Each square on the grid above measures 1 cm × 1 cm (or 1 cm^2). Write down the area of each shape above in square centimetres (cm^2).

Below are formulae for calculating area:

Area of a square = s^2
Area of a rectangle = $l \times b$

3. Calculate the areas of shapes C, E and F in question 1 using the formulae.

SOLVING MORE PERIMETER AND AREA PROBLEMS

1. The perimeter of a square is 8 cm. What is the length of each side?
2. The area of a rectangle is 40 cm^2 and its length is 8 cm. What is its breadth?
3. The perimeter of a square is 32 cm. What is its length and area?
4. The area of a rectangle is 60 cm^2 and its length is 12 cm. What is its breadth and perimeter?
5. A rectangular yard has an area of 600 m^2. If the breadth is 20 m, find the length and the perimeter.
6. A square has an area of 10 000 m^2. What is the perimeter?

AREA OF TRIANGLES

Teaching guidelines

Learners often have a problem if the triangle is oriented in a way that the base they have to work with is not horizontal. Show learners a few triangles in different orientations and talk about the base and the height so that learners realise that the height they need to work with is perpendicular to the chosen base.

PROBLEMS INVOLVING THE AREA OF TRIANGLES

Teaching guidelines

Learners complete the exercises that should help them to choose a base and the associated height that goes with it. In question 2 (b), for example, the height is given as 11 cm and it is on the base (3 + 17) cm.

Answers

1. See the answers on LB page 178 alongside.
2. (a) $A = \frac{1}{2}(b \times h)$ $= \frac{1}{2}(8\text{ cm} \times 6\text{ cm})$ $= 24\text{ cm}^2$

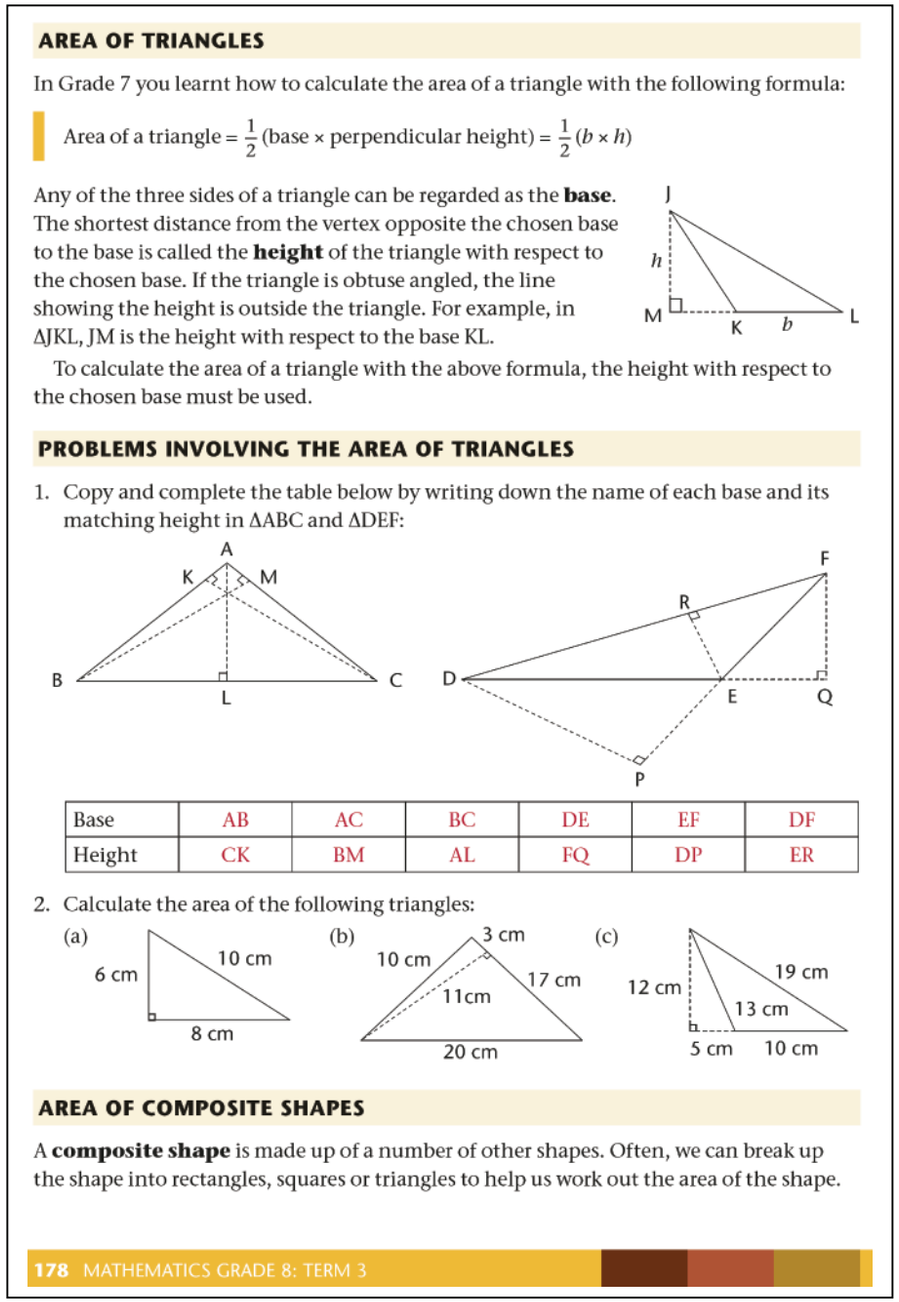
   (b) $A = \frac{1}{2}(b \times h)$ $= \frac{1}{2}(20\text{ cm} \times 11\text{ cm})$ $= 110\text{ cm}^2$

   (c) $A = \frac{1}{2}(b \times h)$ $= \frac{1}{2}(10\text{ cm} \times 12\text{ cm})$ $= 60\text{ cm}^2$

AREA OF COMPOSITE SHAPES

Teaching guidelines

Learners know the formulae for rectangles, squares and triangles. Hence shapes that are made up of these shapes can be subdivided and their areas can be calculated by adding the areas of the rectangles, squares or triangles that can be identified.



AREA OF TRIANGLES

In Grade 7 you learnt how to calculate the area of a triangle with the following formula:

Area of a triangle = $\frac{1}{2}$(base × perpendicular height) = $\frac{1}{2}(b \times h)$

Any of the three sides of a triangle can be regarded as the **base**. The shortest distance from the vertex opposite the chosen base to the base is called the **height** of the triangle with respect to the chosen base. If the triangle is obtuse angled, the line showing the height is outside the triangle. For example, in ΔJKL, JM is the height with respect to the base KL.

To calculate the area of a triangle with the above formula, the height with respect to the chosen base must be used.

PROBLEMS INVOLVING THE AREA OF TRIANGLES

1. Copy and complete the table below by writing down the name of each base and its matching height in ΔABC and ΔDEF:

Base	AB	AC	BC	DE	EF	DF
Height	CK	BM	AL	FQ	DP	ER

2. Calculate the area of the following triangles:

(a)

(b)

(c)

AREA OF COMPOSITE SHAPES

A **composite shape** is made up of a number of other shapes. Often, we can break up the shape into rectangles, squares or triangles to help us work out the area of the shape.

Answers

1. and 2. Below and see completed diagrams on LB page 179 alongside.

(a) Area of shape $= A$ of rectangle $+ A$ of triangle

$= l \times b + \frac{1}{2}(b \times h)$

$= 6\text{ cm} \times 4\text{ cm} + \frac{1}{2}(3\text{ cm} \times 4\text{ cm})$

$= 24\text{ cm}^2 + 6\text{ cm}^2 = 30\text{ cm}^2$

(b) Area of one square $= 2{,}5\text{ cm} \times 2{,}5\text{ cm}$

$= 6{,}25\text{ cm}^2$

Area of shape $= 5 \times 6{,}25\text{ cm}^2$

$= 31{,}25\text{ cm}^2$

(c) $a = b = 20$ cm [Congruent triangles, RHS]

$h^2 = (20\text{ cm})^2 - (5\text{ cm})^2 = 375\text{ cm}^2$

$\therefore h \approx 19{,}36$ cm

Area of rectangle $= 15\text{ cm} \times 19{,}36\text{ cm} = 290{,}4\text{ cm}^2$

Area of triangle $= \frac{1}{2}(5\text{ cm} \times 19{,}36\text{ cm}) = 48{,}4\text{ cm}^2$

Area of shape $= 2 \times 48{,}4\text{ cm}^2 + 290{,}4\text{ cm}^2 = 387{,}2\text{ cm}^2$

(d) $h^2 = (10\text{ cm})^2 - (6\text{ cm})^2 = 64\text{ cm}^2$

$\therefore h = 8$ cm

$a^2 = 64\text{ cm}^2 - 25\text{ cm}^2 = 39\text{ cm}^2$

$\therefore a \approx 6{,}24$ cm

Area of left triangle $= \frac{1}{2}(5 \times 6{,}24)\text{ cm}^2 \approx 15{,}6\text{ cm}^2$

Area of rectangle $= 20\text{ cm} \times 8\text{ cm} = 160\text{ cm}^2$

Area of right triangle $= \frac{1}{2}(6\text{ cm} \times 8\text{ cm}) = 24\text{ cm}^2$

Area of shape $= 15{,}6\text{ cm}^2 + 160\text{ cm}^2 + 24\text{ cm}^2 = 199{,}6\text{ cm}^2$

16.3 Perimeter of circles

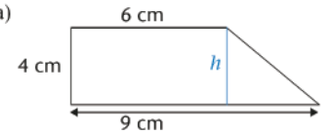
PARTS OF A CIRCLE

Teaching guidelines

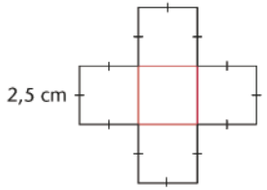
Make sure that learners know the parts of a circle as described on LB page 179 alongside. You could let them make a poster to illustrate the information.

1. Trace the following shapes. Use a ruler and pencil to divide each of the shapes into rectangles, squares and/or triangles. The first one has been done for you.
2. Work out the length of the sides you need and then calculate the area of the shapes. Round off your answers to two decimal places where necessary.

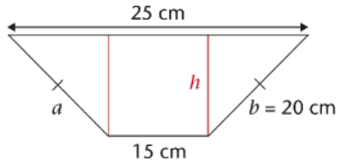
(a)



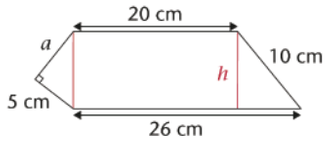
(b)



(c)



(d)



16.3 Perimeter of circles

PARTS OF A CIRCLE

In Grade 7, you learnt about the different parts of a circle, including the following:

The **centre** of a circle is the point in the middle (centre) of the circle.

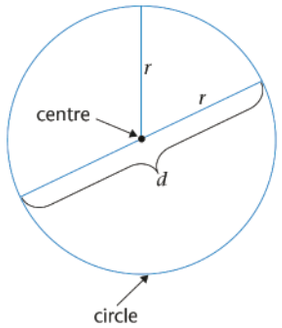
The **circumference** (C) is the distance around the circle. It is the length of the curved line that forms the circle.

The **radius** (r) is the line segment drawn from the centre of the circle to any point on the circle.

The **diameter** (d) is the line segment passing through the centre of the circle and joining any two points on the circle.

The length of the radius is always half the length of the diameter: $r = \frac{1}{2}d$

The length of the diameter is always twice the length of the radius: $d = 2r$



Answers

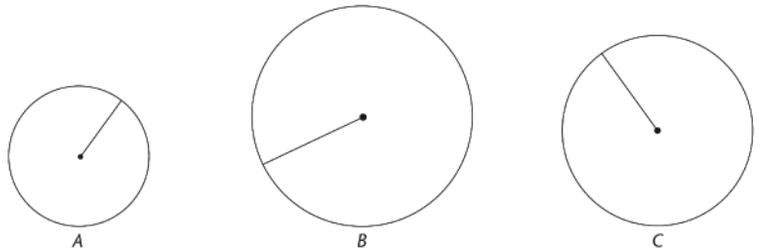
1. See the answers on LB page 180 alongside.
2. (a) $d = 16$ cm (b) $d = 2$ m (c) $d = 9$ cm (d) $d = 12{,}4$ m

RELATIONSHIP BETWEEN A CIRCLE'S CIRCUMFERENCE AND DIAMETER

Teaching guidelines

When learners have completed the activity (which entails measuring the circumference and diameters) discuss the value of the ratio circumference : diameter with learners. Some learners may have noticed that the decimal part of the number seems to continue indefinitely (an irrational number). Discuss the need to round the irrational number to two or three decimal places for calculation purposes.

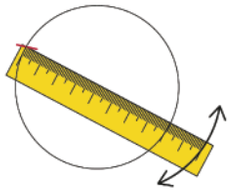
1. Use a ruler to measure the radii (plural of radius) given below. Then copy the table and write down the lengths of both the radii and diameters of the circles in the table.



Circle	A	B	C
Radius (mm)	14	22	19
Diameter (mm)	28	44	38

2. Write down the diameters of circles with the following radii:
   (a) $r = 8$ cm (b) $r = 1$ m (c) $r = 4{,}5$ cm (d) $r = 6{,}2$ m

RELATIONSHIP BETWEEN A CIRCLE'S CIRCUMFERENCE AND DIAMETER

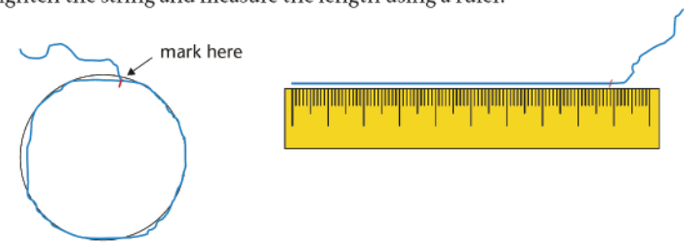


If you do not know where the **centre** of a circle is, you can determine it by measuring the diameter as follows:

- Mark a point on the circle from which to measure.
- Keeping the '0' of the ruler in place, move the other end of the ruler until you find the longest distance. This is the diameter.

You can get a rough measurement of the **circumference** of a circle as follows:

- Use a string and lay it around the edge of the circle as accurately as possible.
- Mark the string when you reach the point where you first started measuring.
- Straighten the string and measure the length using a ruler.



Answers

1. and 2. See the answers on LB page 181 alongside.

3. The circumference of any circle divided by its diameter is equal to 3,14, accurate to two decimal places.

PI (π) AND THE FORMULA FOR THE CIRCUMFERENCE OF A CIRCLE

Teaching guidelines

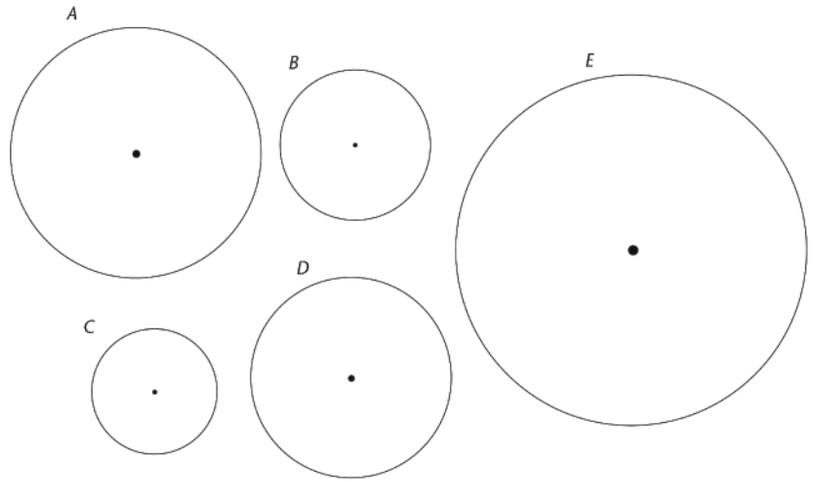
Refer to the previous activity for this discussion. Explain the concept of an irrational number.

Discuss the reason why $\frac{22}{7}$ can be used for the value of pi. The reason is that both values rounded off to the second decimal, give 3,14.

Work through the change of the formula from circumference ÷ diameter = π, to the form $C = d\pi = 2\pi r$.

Circles of different sizes are given below. The circumferences are shown in the table in question 2, rounded off to two decimal places. Copy the table.

1. Measure the diameter of each circle and write it in the table.



2. Use a calculator to work out the answers in the last column. (Round off to two decimal places.)

Circle	Diameter (cm)	Circumference (cm)	Circumference ÷ diameter
A	5	15,71	3,14
B	3	9,42	3,14
C	2,5	7,85	3,14
D	4	12,57	3,14
E	7	21,99	3,14

3. What do you notice?

PI (π) AND THE FORMULA FOR THE CIRCUMFERENCE OF A CIRCLE

In the previous activity, you should have found that the circumference of a circle divided by its diameter is always equal to the same number. This number is a constant value and is called **pi**. *Pi* is a Greek letter and its symbol is π.

USING THE FORMULA FOR THE CIRCUMFERENCE OF A CIRCLE

Teaching guidelines

Do an example of each type of calculation with learners. Some learners may find it difficult to find the radius if the circumference is given. You could do a specific example of such a calculation.

Answers

1. (a) $C = 2\pi r$
$= 2(3,14)(2\text{ cm}) = 12,56\text{ cm}$
(b) $C = 2\pi r$
$= 2(3,14)(10\text{ mm}) = 62,8\text{ mm}$
(c) $C = \pi d$
$= (3,14)(8\text{ cm}) = 25,12\text{ cm}$
(d) $C = \pi d$
$= (3,14)(25\text{ mm}) = 78,5\text{ mm}$
(e) $C = 2\pi r$
$= 2(3,14)(40\text{ m}) = 251,2\text{ m}$
(f) $C = \pi d$
$= (3,14)(100\text{ m}) = 314\text{ m}$
2. (a) $r = d \div 2$
$= 125\text{ mm} \div 2 = 62,5\text{ mm}$
$C = \pi d$
$= (3,14)(125\text{ mm}) = 392,5\text{ mm}$
(b) $r = d \div 2$
$= 70\text{ cm} \div 2 = 35\text{ cm}$
$C = \pi d$
$= (3,14)(70\text{ cm}) = 219,8\text{ cm}$
3. (a) $C = 2\pi r$
$110\text{ cm} = 2(3,14)r$
$110\text{ cm} = 6,28r$
$r = 17,52\text{ cm}$
(b) $C = 2\pi r$
$200\text{ m} = 2(3,14)r$
$200\text{ m} = 6,28r$
$r = 31,85\text{ m}$

16.4 Area of circles

INVESTIGATING THE FORMULA FOR THE AREA OF A CIRCLE

Teaching guidelines

The discussion about the investigation is general although it is about the area of a circle. The method of covering the area with a square grid and counting the squares can be used for any area – it was used for the area of squares and rectangles.

Talk about the fact that a curved area makes it difficult to determine the area accurately. The smaller the squares in the grid, the more accurate the estimate of the area.

You also worked with values rounded off to two decimal places (hundredths). However, π is an **irrational number**. This means that the numbers after the decimal comma go on and on without ending and without repeating. On a calculator, you will find that the value for π is given as 3,141592654 (correct to nine decimal places).

When we use π in our calculations, we usually round it off as $\pi \approx \frac{22}{7}$ or 3,14.

In the previous activity, you found that, for any circle, $\frac{C}{d} = \pi$ (the circumference divided by its diameter is equal to the constant, π). Therefore, if we multiply the diameter of a circle by π, we should get the circumference of the circle:

Circumference of a circle $(C) = \pi d$
$= \pi(2r)$
$= 2\pi r$

USING THE FORMULA FOR THE CIRCUMFERENCE OF A CIRCLE

In the following calculations, use $\pi = 3,14$ and round off your answers to two decimal places where necessary.

1. Calculate the circumference of a circle with:
   (a) a radius of 2 cm
   (b) a radius of 10 mm
   (c) a diameter of 8 cm
   (d) a diameter of 25 mm
   (e) a radius of 40 m
   (f) a diameter of 100 m
2. Calculate the radius and circumference of a circle with a diameter of:
   (a) 125 mm
   (b) 70 cm
3. Calculate the radius of a circle with a circumference of:
   (a) 110 cm
   (b) 200 m

16.4 Area of circles

INVESTIGATING THE FORMULA FOR THE AREA OF A CIRCLE

1. Each square in the grid on the following page measures 1 cm by 1 cm (1 cm^2).
   (a) Count the number of squares inside the circle. Estimate what the parts of squares add up to. What is the area inside the circle?
   (b) What is the radius (r) of the circle?
   (c) How accurate is the above method for finding the area of a circle?
   (d) How can we improve on this method of using squares to approximate the area of a circle?

Answers

1. (a) About 50 cm^2
   (b) 4 cm (read off the grid)
   (c) The purpose of the question is to raise the question of accuracy in learners' minds.
   (d) Use smaller squares.
   (e) The one where we use squares that are 0,5 cm by 0,5 cm. We can fit in more squares inside the circle.
   (f) The one where we use 0,25 cm by 0,25 cm squares.

Teaching guidelines

A more accurate way to determine the area of a circle is shown in the discussion on LB page 183.

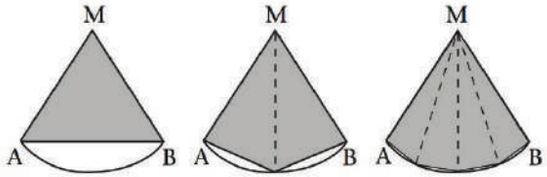
The circle is changed into a parallelogram by rearranging the sectors of the circle. This method is more accurate if the sectors are small. The smaller the sectors, the closer the height of the sector to the radius of the circle and height of the parallelogram.

Show learners that we have formed a rectangle by taking all the steps described on LB page 183. We know how to calculate the area of a rectangle. The length of this rectangle is half of the circumference of the circle and the height of the rectangle is the radius of the circle.

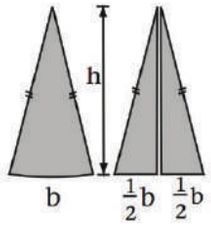
The area $= \frac{1}{2}$ circumference $\times r = \frac{1}{2} \times 2\pi r \times r = \pi r^2$.

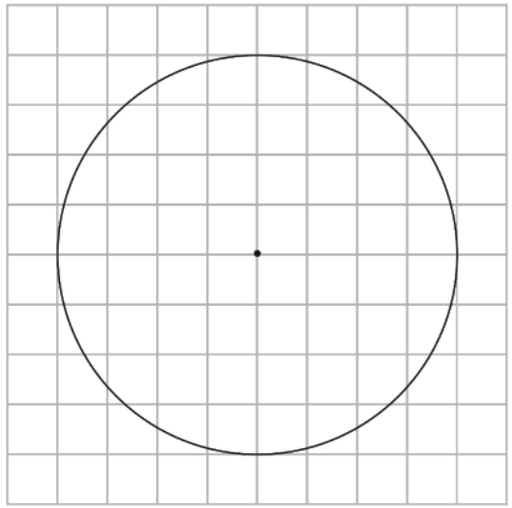
An alternative way to develop the formula is shown below.

The diagrams below show equal sectors of a circle. Each successive sector is divided into smaller sectors. The parts that are shaded darker show the part of the circle that is not covered by a triangle.



Imagine a very small sector and the triangle that fits into it. The base of the triangle will be almost the same length as the arc of the sector and the height will be almost the same as the radius of the circle.





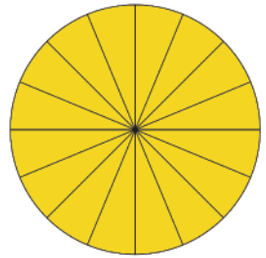
(e) Suppose instead of using 1 cm by 1 cm squares we use 0,5 cm by 0,5 cm squares to measure the area of the circle above. Which of the two measurements of area will be more accurate? Explain.
(f) Now suppose we use squares that are 0,25 cm by 0,25 cm. Which measurement will be the best estimate of the three?

We can estimate area by placing a square grid over the surface of which we want to estimate. We can then count approximately how many squares are needed to cover the surface we wish to measure.

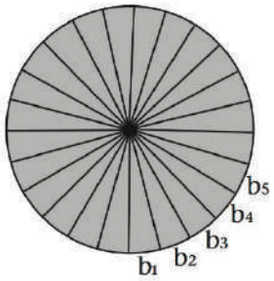
In the case of a curved surface like a circle, the area cannot be accurately determined in this way; it can only be estimated. The accuracy of the estimate depends on the size of the squares used.

In this activity we are going to develop a formula for calculating the area of a circle.

Consider the circle alongside. It has been divided into 16 identical sectors. We will use a technique that mathematicians sometimes use to transform a shape into one that they know something about in order to solve a problem.



Imagine dividing the circle into p very small triangles. As the number of triangles increase, the heights of the triangles are the same and more equal to the radius of the circle. Therefore:



Area of circle $= \frac{1}{2}h(b_1 + b_2 + b_3 + ...)$

The larger the number of small triangles (p) is, the closer the height (h) is to the radius (r) of the circle, and the closer $b_1 + b_2 + b_3 + ...$ is to the circumference of the circle.

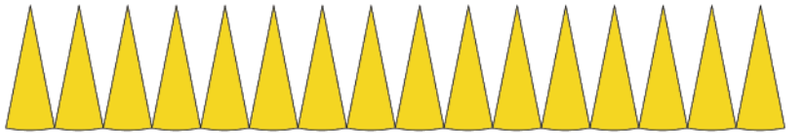
Area of a circle $= \frac{1}{2}$ × length of the radius × circumference of the circle

$$= \frac{1}{2} \times r \times 2\pi r$$
$$= \pi r^2$$

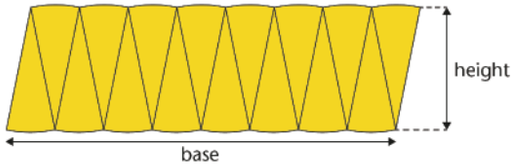
Answers

2. A parallelogram
3. (a) The radius
   (b) Half of the circumference
4. Consider learners' answers. Suggested answer: We can divide the circle into more sectors so that we can form a rectangular shape.
5. (a) Two congruent shapes that approximate right-angled triangles
   (b) A shape that approximates a rectangle
6. (a) The radius
   (b) Half of the circumference

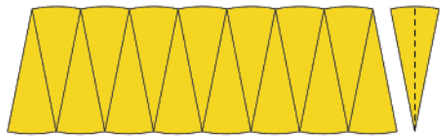
The challenge here is that we want to find a way to calculate the area of a circle. We know how to find the area of a rectangle. Is there a way that we can redraw a circle so that it looks something like a rectangle? One way to go about this is to divide the circle into 16 identical sectors. We then cut the circle into 16 different pieces as shown below:



We then re-arrange the sectors like this:



2. We have transformed the circle by cutting it into identical sectors and re-arranging them. What does this shape look like?
3. What does the:
   (a) height of the shape above match in the original circle?
   (b) base of the shape match in the original circle?
4. Is there a way in which we can make the challenge easier for ourselves?
5. The last sector in the arrangement below is further divided in half.
   (a) What shapes are formed from dividing the sector?



   (b) What new shape will be formed by placing each half of the sector on either side of the shape above?
6. To what does the:
   (a) height of the new shape correspond in the original circle?
   (b) base of the new shape correspond in the original circle?

Teaching guidelines

Remind learners that the value of π should be taken as 3,14.

Answers

7. (a) $A = 3{,}14 \times (4\text{ cm})^2 = 50{,}24\text{ cm}^2$
   (b) Consider the learners' answers.
8. (a) 1,5 cm
   (b) 2,25 cm^2
   (c) 2,25 cm^2
   (d) See the answer on LB page 185 alongside.

USING THE FORMULA FOR THE AREA OF A CIRCLE

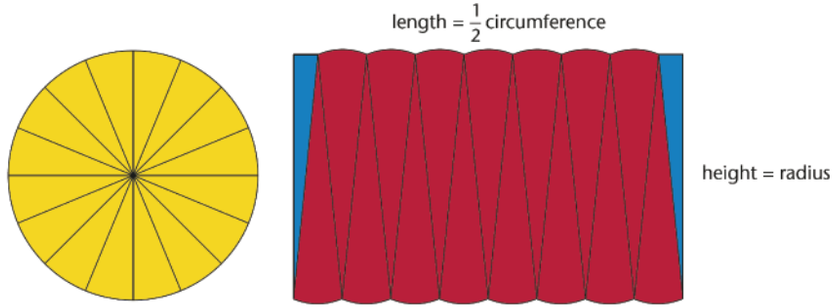
Teaching guidelines

Learners now begin to use the formula. They use the formula as it is to calculate the area of a circle.

They also need to learn how to change the formula in order to calculate the radius if the area is given. Do an example with learners so that they can see how it should be done.

Questions 3(a) and (b) require a degree of problem solving by the learners. Discuss the strategies they can follow to find the required answers. Let them talk in pairs about how to tackle the problem.

You have probably noticed that when we divide a circle into many sectors and then re-arrange the sectors, they form a rectangular shape. Try to make sense of the argument presented below.



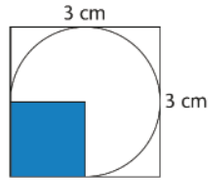
$C = 2\pi r$

Area = $l \times b$
Area = $\frac{1}{2} \times 2 \times \pi \times r \times r$
Area = πr^2

7. (a) Use the formula $A = \pi r^2$ to calculate the area of a circle with a radius of 4 cm. Use $\pi = 3{,}14$.
   (b) How close is this answer to the number of squares you calculated inside the circle in question 1 on page 182?

From now onwards we will use the **formula** $A = \pi r^2$ to calculate the **area of a circle**, where r is the length of the radius. You will be given the value of π to use in the calculations. The value of π is usually given correct to two decimal places as 3,14.

8. How can we interpret r^2 in the formula $A = \pi r^2$? Use the figure on the right to answer the following questions:



   (a) What is the radius of the circle?
   (b) The length of the blue square is 1,5 cm. What is its area?
   (c) What is the value of r^2?
   (d) If r is the radius of a circle, then r^2 is ... a square whose sides are r units long.

USING THE FORMULA FOR THE AREA OF A CIRCLE

In the following calculations, use 3,14 as an approximation for π and round your answers off to two decimal places. Use a calculator where necessary.

Answers

1. (a) $A = \pi r^2$
$A = 3{,}14 \times (8\text{ cm})^2$
$= 3{,}14 \times 64\text{ cm}^2$
$= 200{,}96\text{ cm}^2$

(b) $A = \pi r^2$
$A = 3{,}14 \times (4{,}5\text{ cm})^2$
$= 3{,}14 \times 20{,}25\text{ cm}^2$
$= 63{,}59\text{ cm}^2$

2. (a) $100\text{ m}^2 = \pi r^2$
$100\text{ m}^2 = 3{,}14 \times r^2$
$31{,}85\text{ m}^2 = r^2$
$r = 5{,}64\text{ m}$

(b) $76\text{ m}^2 = \pi r^2$
$76\text{ m}^2 = 3{,}14 \times r^2$
$24{,}2\text{ m}^2 = r^2$
$r = 4{,}92\text{ m}$

3. (a) Area of large circle $= 3{,}14 \times (12\text{ cm})^2 = 3{,}14 \times 144\text{ cm}^2 = 452{,}16\text{ cm}^2$
Area of small circle $= 3{,}14 \times 49\text{ cm}^2 = 153{,}86\text{ cm}^2$
Area of shaded part = Area of large circle – area of small circle
$= 452{,}16\text{ cm}^2 - 153{,}86\text{ cm}^2 = 298{,}3\text{ cm}^2$

(b) Area of large circle $= 3{,}14 \times (10\text{ cm})^2 = 3{,}14 \times 100\text{ cm}^2 = 314\text{ cm}^2$
Area of small circle $= 3{,}14 \times 36\text{ cm}^2 = 113{,}04\text{ cm}^2$
Area of shaded part = Area of large circle – area of small circle
$= 314\text{ cm}^2 - 113{,}04\text{ cm}^2 = 200{,}96\text{ cm}^2$

16.5 Converting between square units

Teaching guidelines

To convert from the larger unit to the smaller unit, we multiply by the conversion factor, and from the smaller unit to the larger unit, we divide by the conversion factor. For example, to change centimetres (the larger unit) to millimetres (the smaller unit), multiply the amount by 10.

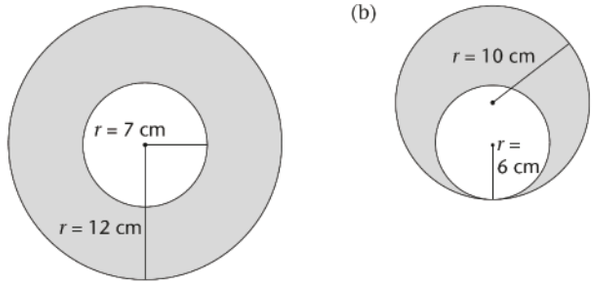
Misconceptions

Learners become confused about the relative size of units and would divide centimetres by 10 to get millimetres. Let them ask the question: If I change this unit, for example centimetres to metres, will the number be more or less than it is. In this case, the answer is less, so we will divide cm by 100 to get metres.

Answers

1. to 4. See the answers on LB page 186 alongside.

1. Calculate the area of a circle with a radius of:
   (a) $r = 8$ cm (b) $r = 4{,}5$ cm
2. Calculate the radius of a circle with the following area:
   (a) 100 m² (b) 76 m²
3. Work out the area of the shaded parts of the following shapes:
   (a) (b)



16.5 Converting between square units

You already know how to convert between units we use to measure lengths or distances, for example mm, cm, m and km:

To convert	Do this	To convert	Do this
cm to mm	× 10	mm to cm	÷ 10
m to cm	× 100	cm to m	÷ 100
km to m	× 1 000	m to km	÷ 1 000

Use this knowledge to work out how to convert between square units (mm², cm², m² and km²). Copy and complete the following conversion:

1. Convert cm² to mm²
   1 cm² = 1 cm × 1 cm
   = 10 mm × 10 mm
   = 100 mm²

2. Convert m² to cm²
   1 m² = 1 m × 1 m
   = 100 cm × 100 cm
   = 10 000 cm²

3. Convert km² to m²
   1 km² = 1 km × 1 km
   = 1 000 m × 1 000 m
   = 1 000 000 m²

4. Convert mm² to cm²
   1 mm² = 1 mm × 1 mm
   = 0,1 cm × 0,1 cm
   = 0,01 cm²

Answers

5. to 7. See the answers on LB page 187 alongside.

5. Convert cm^2 to m^2

$1\ cm^2 = \underline{1}\ cm \times \underline{1}\ cm$

$= 0{,}01\ m \times 0{,}01\ m$

$= 0{,}0001\ m^2$

6. Convert m^2 to km^2

$1\ m^2 = \underline{1}\ m \times \underline{1}\ m$

$= 0{,}001\ km \times 0{,}001\ km$

$= 0{,}000001\ km^2$

7. Copy and complete the following table:

To convert	Do this	To convert	Do this
cm^2 to mm^2	× 100	mm^2 to cm^2	÷ 100
m^2 to cm^2	× 10 000	cm^2 to m^2	÷ 10 000
km^2 to m^2	× 1 000 000	m^2 to km^2	÷ 1 000 000

Grade 8 Term 3 Chapter 17 Surface area and volume of 3D objects

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
17.1 From 2D to 3D measurements	Defining the surface area and volume of an object; investigating the surface area and volume of an object	Pages 188 to 189
17.2 Surface area of 3D objects	Using nets to calculate the surface area of objects; deducing formulae for surface areas; calculating surface areas	Pages 189 to 191
17.3 Volume of 3D objects	Deriving formulae to calculate the volume of objects; calculating volumes	Pages 192 to 193
17.4 Relationship between surface area and volume	Investigating the relationship between surface area and volume	Pages 194 to 195
17.5 Converting between cubic units	Determining that the number of cubic units change when the dimensions are increased; converting between units	Pages 195 to 196
17.6 Capacity of 3D objects	Defining the difference between volume and capacity; calculations about displacement and volume	Pages 196 to 197

CAPS time allocation	5 hours
CAPS content specification	Page 108

Mathematical background

- The **surface area** of an object is the sum of the areas of all the faces; it is measured in square units. The **net** of an object can be used to calculate its surface area.
- Another way to interpret the formula of the net of a prism is to say: Surface area = 2 × area of the base + the perimeter × the height of the prism.
- The **volume** of an object is the amount of space it takes up. Volume = area of base × height. Three dimensions are involved: length, breadth and height. So, volume is measured in cubic units, for example cubic centimetres (cm^3) and cubic metres (m^3).
- A **cubic centimetre** is a cube with sides that are 1 cm each.
- A **cubic metre** is a cube with sides that are 1 m each.
- An object can have both volume and capacity. The difference between volume and capacity is:
  - volume is the amount of space an object takes up
  - **capacity** is how much a container can hold.
- If volume is measured in cubic centimetres (cm^3) the measure of capacity is millilitres. A millilitre is the content of a cube with sides measuring 1 cm.

17.1 From 2D to 3D measurements

Background information

The surface area of an object is the sum of the areas of all its faces. Area is measured in square units.

The volume of an object is the amount of space it takes up. Three dimensions are involved, namely length, breadth and height. Volume is measured in cubic units.

A prism is an object that has two identical faces, the bases that are parallel to each other, and these are joined by parallelograms (the side faces or lateral faces).

A right prism is an object of which the side faces are perpendicular to the bases. For the purpose of this work, a prism means a right prism.

A rectangular or cubic prism could have any of its faces as the base; the side faces will be perpendicular to the chosen base.

In this definition, the base of a triangular prism would be its triangular faces on which the rectangular faces are perpendicular.

INVESTIGATING THE SURFACE AREA AND VOLUME OF A BOOK

Teaching guidelines

This activity should make learners aware of the difference between surface area and volume.

Answers

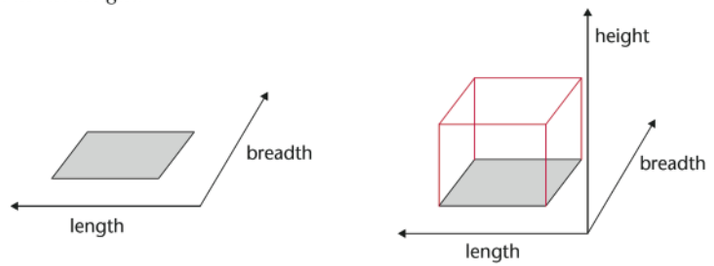
1. Six surfaces (or faces)
2. Consider learners' answers.

CHAPTER 17

Surface area and volume of 3D objects

17.1 From 2D to 3D measurements

Remember that 2D shapes have only length and breadth, while 3D objects have length, breadth and height.



2D shape *3D object*

A 2D shape has only one surface. We call the size of this flat surface the **area** of the shape.

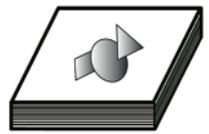
A 3D object has more than one surface. For example, a cube has six surfaces, or faces. The sizes of these surfaces on the outside of the 3D object are called its **surface area**.

A 2D shape is flat, so it takes up space in only two directions. But a 3D object has height as well, so it takes up space in a third direction also. The space that a 3D object takes up is called its **volume**.

INVESTIGATING THE SURFACE AREA AND VOLUME OF A BOOK

Work with a partner. Choose a book each. The books must be different sizes.

1. Run your hand over all the outside surfaces of your book. How many surfaces (or faces) does your book have?
2. Estimate whether the surface area of your book is bigger or smaller than that of your partner's book.



Answers

3. If wrapping paper covers each surface of the book exactly, then the least amount of paper needed to cover the book is the sum of the areas of each of the faces of the book.
4. Consider learners' answers.

17.2 Surface area of 3D objects

USING NETS TO EXPLORE SURFACE AREA

Teaching guidelines

Let learners bring discarded rectangular or cubic boxes, such as cereal boxes or milk cartons, to class to cut open. Let them arrange the faces so that if they were to fold them up again, they would get the original box. Refer to the illustrations in question 2 on LB page 189.

Ask learners measure the sides of the containers they brought to class and get them to write the measurements along the sides of the net.

Let them work out the area of each face of the container. All the boxes will be rectangular or cubic and will therefore have six faces each.

To find the surface area of the container, they must add up all the areas they calculated.

Answers

1. and 2.

A: Area of each face = 3 cm × 3 cm = 9 cm^2

Surface area = 6 × 9 cm^2 = 54 cm^2

B: Areas of faces: two faces each 2 cm × 4 cm = 8 cm^2

two faces each 4 cm × 7 cm = 28 cm^2

two faces each 2 cm × 7 cm = 14 cm^2

Surface area = 2 × (8 cm^2 + 28 cm^2 + 14 cm^2) = 2 × 50 cm^2 = 100 cm^2

Teaching guidelines (for C on LB page 190 on following page)

In order to work out the area of the two rectangular side faces, the side length of the triangular face has to be known. To work out the unknown side of the triangle, learners must use the theorem of Pythagoras. The height of the triangular side divides the triangle into two congruent right-angled triangles with right-angled sides of 5 cm and 2 cm respectively. The unknown side is the hypotenuse of each of these two right-angled triangles.

3. If you were to cover the book with wrapping paper, explain how you would calculate the minimum size of paper you would need.
4. Estimate whose book takes up the most space. How could you calculate which book really takes up the most space?

17.2 Surface area of 3D objects

USING NETS TO EXPLORE SURFACE AREA

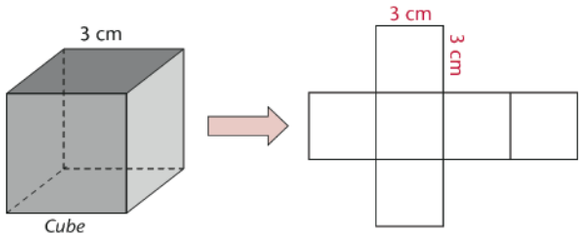
The **surface area** of an object is equal to the sum of the areas of all its faces. So we can use the net of an object to investigate its surface area.

A net is a flat shape that can be folded to make a 3D object.

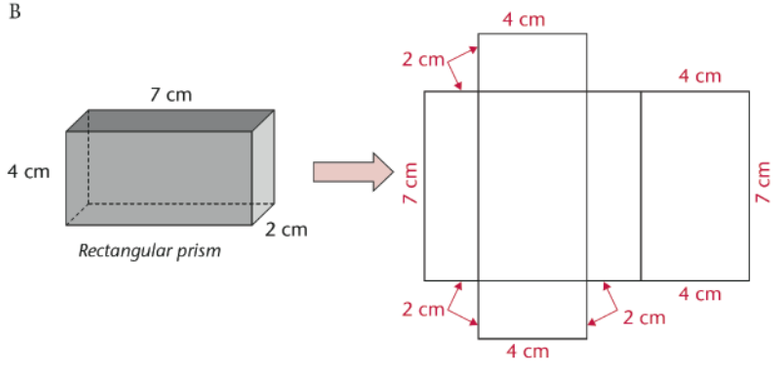
The following diagrams show 3D objects with their matching nets:

1. Use the measurements given to calculate the area of each face shown by the net. (Use your calculator if necessary and round off to two decimal places.)
2. Add up the areas to calculate the surface area of each object.

A



B



Answers

C: Unknown side of the triangle: $\sqrt{5^2 + 2^2} = \sqrt{29}\ \text{cm}^2 \approx 5{,}39\ \text{cm}$

Area of each triangular face $= \frac{1}{2}(4\ \text{cm} \times 5\ \text{cm}) = 10\ \text{cm}^2$

Area of face on which triangular prisms rests $= 4\ \text{cm} \times 7\ \text{cm} = 28\ \text{cm}^2$

Area of each of the other two faces $= 7\ \text{cm} \times 5{,}39\ \text{cm} = 37{,}73\ \text{cm}^2$

Surface area $= (2 \times 10\ \text{cm}^2) + (2 \times 37{,}73\ \text{cm}^2) + 28\ \text{cm}^2$

$= 20\ \text{cm}^2 + 75{,}46\ \text{cm}^2 + 28\ \text{cm}^2 = 123{,}46\ \text{cm}^2$

DEDUCING FORMULAE FOR SURFACE AREAS

Teaching guidelines

Learners work with a formula that is derived by adding the areas of all the faces of the prism. There is another way to use the formula. See the note on LB page 191.

Answers

1. (a) The third choice: $6s^2$ (b) Consider learners' answers.
2. (a) Surface area $= 2(bh + lb + lh)$
   (b) Consider learners' answers. A sample answer is given below.
   When a rectangular prism is opened flat, it is composed of three sets of two identical rectangles. Two of them are the front and back faces of the prism, and the other four rectangles connect the front and the back faces.
3. (a) Surface area = Area of two triangular faces (bases) + Area of three rectangles (lateral faces)
   (b) Consider learners' answers. A sample answer is given below.
   When a triangular prism is opened flat, it is composed of two triangles, the front and back faces, and three rectangles which connect the triangular faces.
4. Surface area of cube: $6 \times (8\ \text{mm})^2 = 6 \times 64\ \text{mm}^2 = 384\ \text{mm}^2$

   Surface area of rectangular prism:

   $2 \times (10\ \text{mm} \times 12\ \text{mm}) + 2 \times (30\ \text{mm} \times 10\ \text{mm}) + 2 \times (30\ \text{mm} \times 12\ \text{mm})$

   $= 2 \times (120\ \text{mm}^2 + 300\ \text{mm}^2 + 360\ \text{mm}^2) = 2 \times 780\ \text{mm}^2 = 1\,560\ \text{mm}^2$

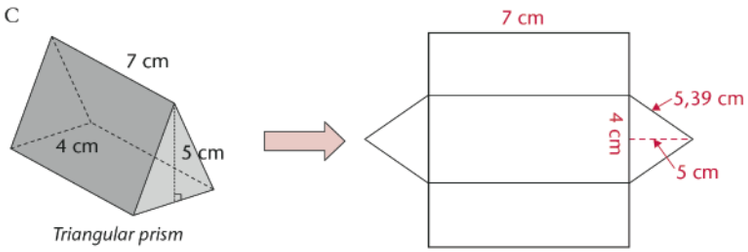
   Surface area of triangular prism:

   $2(\frac{1}{2} \times 12\ \text{mm} \times 9\ \text{mm}) + (12\ \text{mm} \times 14\ \text{mm}) + (15\ \text{mm} \times 14\ \text{mm}) + (9\ \text{mm} \times 14\ \text{mm})$

   $= 2(54\ \text{mm}^2) + 168\ \text{mm}^2 + 210\ \text{mm}^2 + 126\ \text{mm}^2$

   $= 108\ \text{mm}^2 + 168\ \text{mm}^2 + 210\ \text{mm}^2 + 126\ \text{mm}^2 = 612\ \text{mm}^2$

C

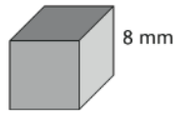


DEDUCING FORMULAE FOR SURFACE AREAS

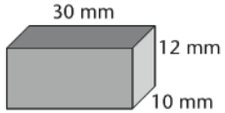
The surface area of a prism = the sum of the areas of all its faces

1. (a) Use the general formula above and the work you did on the cube on page 189 to determine which of the following formulae are correct. Write down the correct one(s).
   - Surface area of a cube $= 4 \times s$
   - Surface area of a cube $= s \times s \times s \times s$
   - Surface area of a cube $= 6 \times s^2$
   - Surface area of a cube $= s^6$

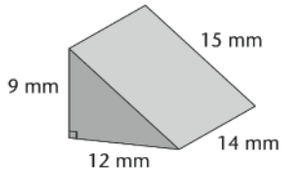
   (b) Explain your choice above.



2. (a) Write a formula for the surface area of any rectangular prism.
   (b) Explain your formula.



3. (a) Write a formula for the surface area of any triangular prism.
   (b) Explain your formula.



4. Use the formulae in questions 1 to 3 to calculate the surface areas of the cube, rectangular prism and triangular prism shown in questions 1 to 3.

   Surface area of cube: ...

   Surface area of rectangular prism: ...

   Surface area of triangular prism: ...

Teaching guidelines (Note extended from page 189)

If you look at the net of a prism, the side faces form a rectangle of which, the length is the perimeter of the prism, and the height is the length of the prism. See the drawings below.

1. Rectangular prism

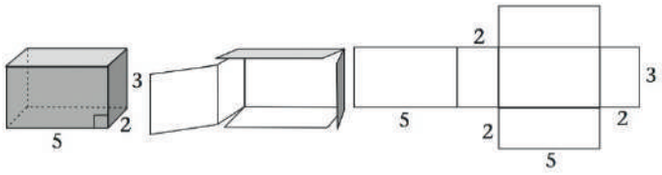
   Area of base:

   $= 5 \times 2$

   Area of side faces:

   $= 2(5 + 2) \times 3$

   = perimeter × height



2. Triangular prism

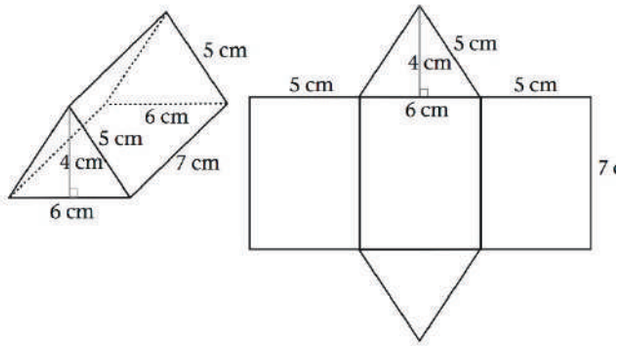
   Area of base:

   $= \frac{1}{2} \times 6 \times 4\ \text{cm}^2$

   Area of side faces:

   $= (2 \times 5 + 6) \times 7\ \text{cm}^2$

   = perimeter × height



Another way to interpret the formula from the net of a prism is to say:

Surface area = 2 × area of the base + the perimeter × the height of the prism

SURFACE AREA CALCULATIONS

Teaching guidelines

Learners should use the formulae. Do an example so that they can see how to present their answers.

Answers

1. Surface area:

   $= 2 \times (10\ \text{mm} \times 2\ \text{mm}) + 2 \times (10\ \text{mm} \times 40\ \text{mm}) + 2 \times (2\ \text{mm} \times 40\ \text{mm})$

   $= 2 \times 20\ \text{mm}^2 + 2 \times 400\ \text{mm}^2 + 2 \times 80\ \text{mm}^2$

   $= 40\ \text{mm}^2 + 800\ \text{mm}^2 + 160\ \text{mm}^2 = 1\ 000\ \text{mm}^2 = 10\ \text{cm}^2$

2. to 4. See the answers on LB page 191 alongside.

SURFACE AREA CALCULATIONS

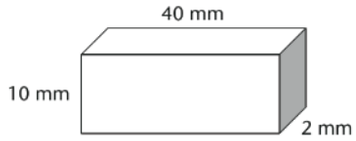
Work out the surface areas of the following four objects.
Give all answers in cm².

Remember:

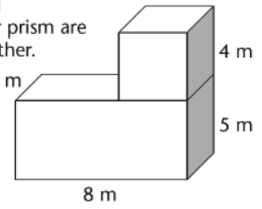
1 cm² = 100 mm²	1 mm² = 0,01 cm²
1 m² = 10 000 cm²	1 cm² = 0,0001 m²
1 km² = 1 000 000 m²	1 m² = 0,000001 km²

It may be a good idea to sketch the net for each object before doing the calculations.

1.



2. A cube and rectangular prism are glued together.



Surface area:

$2 \times (4\ \text{m} \times 5\ \text{m}) + 2 \times (8\ \text{m} \times 5\ \text{m}) + (8\ \text{m} \times 4\ \text{m}) + (4\ \text{m} \times 4\ \text{m}) + 5 \times (4\ \text{m})^2$

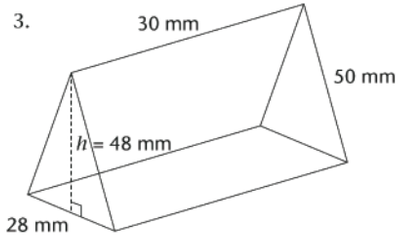
$= 2 \times 20\ \text{m}^2 + 2 \times 40\ \text{m}^2 + 32\ \text{m}^2 + 16\ \text{m}^2 + 80\ \text{m}^2$

$= 40\ \text{m}^2 + 80\ \text{m}^2 + 32\ \text{m}^2 + 16\ \text{m}^2 + 80\ \text{m}^2$

$= 248\ \text{m}^2$

$= 2\ 480\ 000\ \text{cm}^2$

3.



Surface area:

$2 \times (\frac{1}{2} \times 28\ \text{mm} \times 48\ \text{mm}) + (28\ \text{mm} \times 30\ \text{mm}) + 2 \times (30\ \text{mm} \times 50\ \text{mm})$

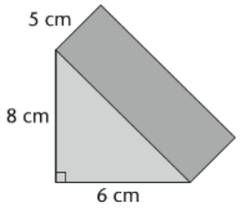
$= 2 \times 672\ \text{mm}^2 + 840\ \text{mm}^2 + 2 \times 1\ 500\ \text{mm}^2$

$= 1\ 344\ \text{mm}^2 + 840\ \text{mm}^2 + 3\ 000\ \text{mm}^2$

$= 5\ 184\ \text{mm}^2$

$= 51{,}84\ \text{cm}^2$

4.



Surface area:

$2 \times (\frac{1}{2} \times 6\ \text{cm} \times 8\ \text{cm}) + (6\ \text{cm} \times 5\ \text{cm}) + (8\ \text{cm} \times 5\ \text{cm}) + (5\ \text{cm} \times 10\ \text{cm}^*)$

$= 48\ \text{cm}^2 + 30\ \text{cm}^2 + 40\ \text{cm}^2 + 50\ \text{cm}^2$

$= 168\ \text{cm}^2$

[*Use Pythagoras]

17.3 Volume of 3D objects

DERIVING FORMULAE TO CALCULATE VOLUME

Teaching guidelines

Another way to think about the volume of a prism is to imagine the area of the base as a number of cubes along the length, and this row repeated along the width to cover the base. This is the area of the "floor" of the object, the bottom layer. As more layers are stacked on the bottom one, the height grows. The number of cubic centimetres (or units) grows in multiples of the number in the bottom layer, as you add more layers.

As learners think of building up the prisms by adding layers, they may see the relationship between the measurements of the prism and its volume.

Volume = area of base × height

This formula can be confusing when learners work with a triangular prism as there are two heights to consider – the height of the triangular base and the height (or length) of the prism. Stress this point to help learners avoid making a mistake.

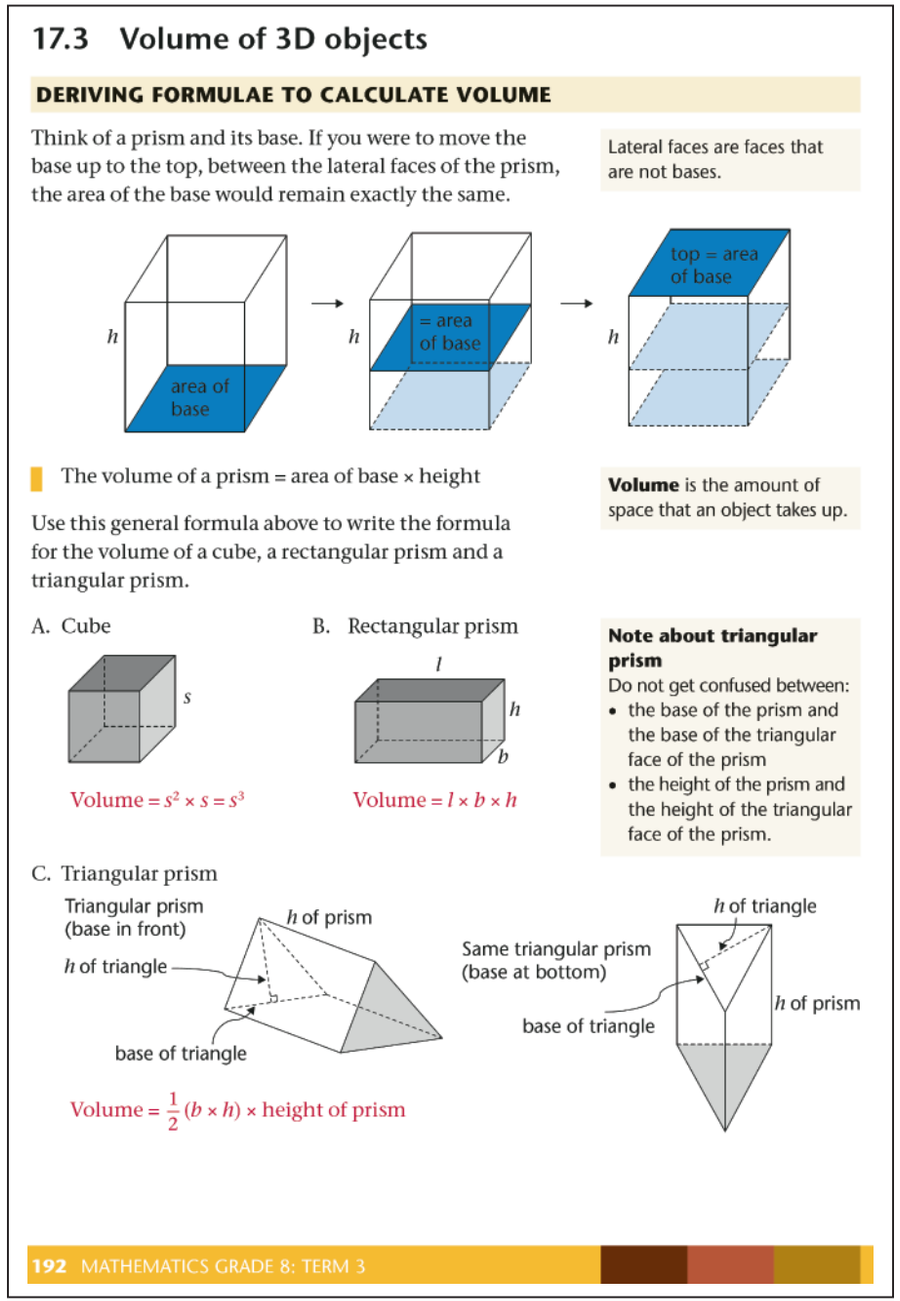
The formula: volume = area of base × height is the general form of the formula and will work for any prism. What will change when we work with a particular object, is the area of the base. This depends on the shape of the base of the object. For example, the volume of a cylinder will still be given by the area of the base multiplied by the height, but the area of the base will be the area of a circle.

Misconceptions

Learners do not know which height to use when they work with a triangular prism.

Answers

See the formulae on LB page 192 alongside.



17.3 Volume of 3D objects

DERIVING FORMULAE TO CALCULATE VOLUME

Think of a prism and its base. If you were to move the base up to the top, between the lateral faces of the prism, the area of the base would remain exactly the same.

Lateral faces are faces that are not bases.

The volume of a prism = area of base × height

Volume is the amount of space that an object takes up.

Use this general formula above to write the formula for the volume of a cube, a rectangular prism and a triangular prism.

A. Cube

Volume = $s^2 \times s = s^3$

B. Rectangular prism

Volume = $l \times b \times h$

Note about triangular prism

Do not get confused between:

- the base of the prism and the base of the triangular face of the prism
- the height of the prism and the height of the triangular face of the prism.

C. Triangular prism

Volume = $\frac{1}{2}(b \times h) \times$ height of prism

192 MATHEMATICS GRADE 8: TERM 3

VOLUME CALCULATIONS

Teaching guidelines

Once learners understand the formula they can apply it to any object. If the object consists of two joined prisms, as in question 1, they work out the volume of each object and add the two answers to get the volume of the combined object.

Answers

1. to 3. See the answers on LB page 193 alongside.

You should have found the following volume formulae:

Volume of a cube = s^3 or $s \times s \times s$

Volume of a rectangular prism = $l \times b \times h$

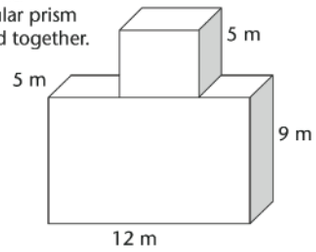
Volume of a triangular prism = $\frac{1}{2}$ (base × h) × height of prism

Because we multiply three dimensions, the units used are cubic units, such as mm³, cm³ or m³.

VOLUME CALCULATIONS

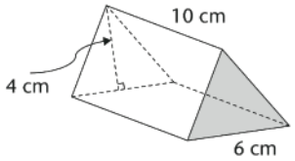
Calculate the volume of the following objects using the formulae given above:

1. A cube and rectangular prism are glued together.



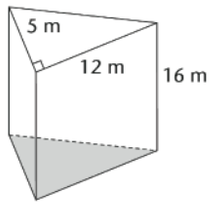
Volume = 12 m × 5 m × 9 m + (5 m)³
= 540 m³ + 125 m³
= 665 m³

2.



Volume = $\frac{1}{2}$ × (6 cm × 4 cm) × 10 cm
= 120 cm³

3.



Volume = $\frac{1}{2}$ × (12 m × 5 m) × 16 m
= 480 m³

17.4 Relationship between surface area and volume

Teaching guidelines

Suppose we want to make a box with a fixed volume, for example 24 m^3, but we want to use as little material as possible (in other words, the net must be as small as possible). This means that we want the surface area to be as small as possible. The question that could be asked is: Will the surface area change if the shape of the object changes but the volume remains the same?

The investigation should help learners answer the question.

Notes on the questions

You could ask learners to investigate the relationship between the surface area and the volume of an object that has as a volume of, for example, 64 cm^3. They should find that the possible arrangements of cubic centimetre blocks will be in the prime factors of 64. The arrangement that gives the smallest surface area will be a cube with sides 4 cm. For example, if the measurements are given (l, b, h) some of the possible combinations are:

(64, 1, 1; 32, 2, 1; 16, 4, 1; 16, 2, 2; 8, 8, 1; 8, 4, 2; 8, 2, 4; 4, 4, 4) etc.

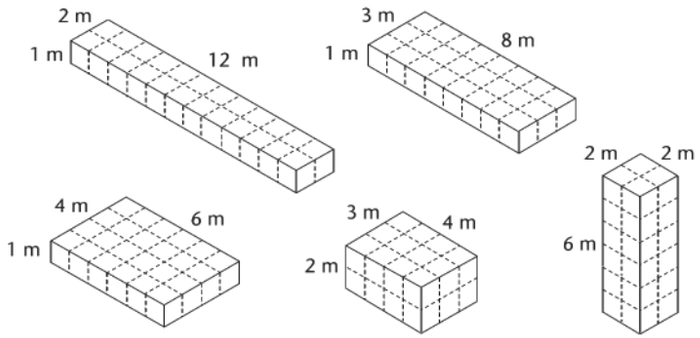
Answers

1. (a) See the answers on LB page 194 alongside.
   (b) Consider learners' answers. One possibility is given in the last row of the table shown on LB page 194 alongside.
2. When the volume is constant, the surface area depends on the shape of the object being considered.
3. Consider learners' answers. Two possible answers are given below.
   A. length = 2 m; breadth = 1 m; height = 4 m
   B. length = 8 m; breadth = 1 m; height = 1 m

17.4 Relationship between surface area and volume

Do objects with the same volume always have the same surface area? Do the investigation below in order to find out.

1. (a) Calculate the surface area and volume of the following five rectangular prisms by copying and completing the table below:



Length (m)	Breadth (m)	Height (m)	Surface area (m^2)	Volume (m^3)
12	2	1	76	24
8	3	1	70	24
6	4	1	68	24
4	3	2	52	24
2	2	6	56	24
1	1	24	98	24

(b) In the last row of the table, write another set of dimensions (l, b and h) that will give the same volume but a different surface area as the ones already recorded.

2. Look at the completed table. What can you conclude about the surface area and volume of objects?
3. A rectangular prism has a volume of 8 m^3. Write down two possible sets of dimensions. Draw the prisms below with their dimensions written on the drawings.
4. The table on the following page shows surface area and volume calculations for cubes with different side lengths.

Answers

4. (a) The surface area increases as the side length of the cube increases.
   (b) The volume increases as the side length of the cube increases.
   (c) The volume increases more rapidly when the side length of the cube increases.
   (d) See the answer on LB page 195 alongside.

17.5 Converting between cubic units

HOW MANY CUBES?

Teaching guidelines

Volume is measured in cubic units, for example cubic centimetres (cm^3) and cubic metres (m^3).

- A cubic centimetre is a cube with sides that are 1 cm each.
- A cubic metre is a cube with sides that are 1 m each.

Make sure that learners know how to read the measurements of a cube or a rectangular prism.

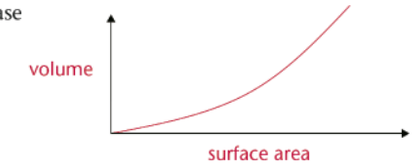
Misconceptions

Some learners find it very difficult to realise that if you increase the sides of a cube the volume will increase threefold. For example, if a cube has side lengths of 2 cm, its volume is $(2\text{ cm})^3 = 8\text{ cm}^3$. Now double the side lengths of the cube. What is the volume of the new cube? Learners find it difficult to believe that the new volume is not 16 cm^3, double the previous volume.

Show learners that the new volume = (new side length)^3 = $(4\text{ cm})^3 = 64\text{ cm}^3$.

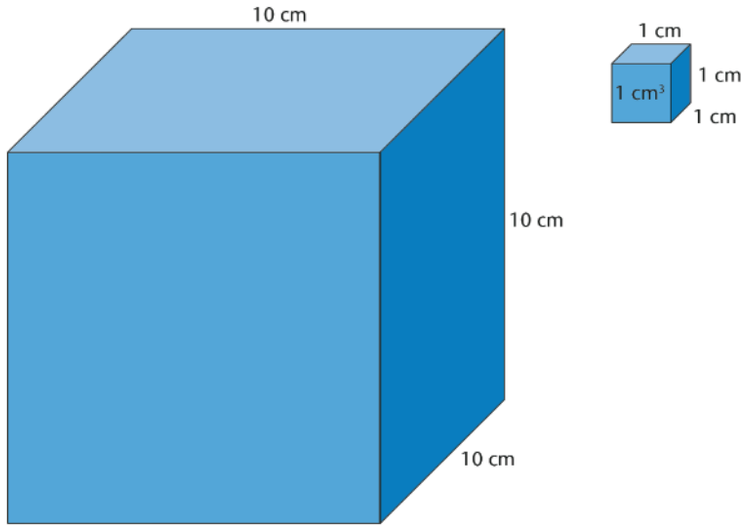
Side length of cube (m)	Surface area (m^2)	Volume (m^3)
1	6	1
2	24	8
3	54	27
5	150	125
8	384	512
10	600	1 000

(a) Look at the surface area column. Does the surface area increase or decrease as the side length of the cube increases?
(b) Look at the volume column. Does the volume increase or decrease as the side length of the cube increases?
(c) Does **volume** or **surface area** increase more rapidly when the side length of the cube increases?
(d) Sketch a global graph of the volume of a cube versus its surface area.



17.5 Converting between cubic units

HOW MANY CUBES?



Answers

1. 1 000 cubes
2. 1 000 cubes
3. (a) There will be 10 × 10 × 10 = 1 000 cubes of 1 000 cm^3
   (b) 1 000 × 1 000 cm^3 = 1 000 000 cm^3
   (c) 1 000 000 cubes
4. Neither: they have the same volume.
   100 cm makes a metre, so 100 cm × 100 cm × 100 cm = 1 000 000 cm^3 which is equivalent to 1 m × 1 m × 1 m = 1 m^3.
5. (a) 1 000 cubes (b) 1 000 mm^3

PRACTISE CONVERTING BETWEEN UNITS

Teaching guidelines

Learners should be aware that if the answer is to be given in cubic centimetres, for example, all the units have to be in centimetres. Sometimes units have to be converted to one unit.

Help learners to decide which unit to convert everything to. The chosen unit should make the answer manageable. For example, changing kilometres to centimetres and getting an answer like 14 560 000 cm^3 is not quite manageable. All units should rather be changed to metres or kilometres, unless the problem demands answers in cubic centimetres.

Misconceptions

Learners do not realise that to change 300 mm^3 to cubic centimetres they have to divide by 1 000. Their reasoning is: there are 10 mm in a centimetre, so to convert they should divide by 10, where they should rather be reasoning 300 mm^3 = 300 ÷ (10 × 10 × 10) cm^3 = 0,3 cm^3.

Answers

1. (a) 0,003 cm^3 (b) 0,045 cm^3
   (c) 600 000 cm^3 (d) 1 220 000 cm^3
2. (a) 20 000 mm^3 (b) 151 000 mm^3
   (c) 4 700 mm^3 (d) 89 500 mm^3
3. (a) 0,000009 m^3 (b) 0,00005 m^3
   (c) 0,000643 m^3 (d) 0,001967 m^3
4. (a) 4 000 000 cm^3 + 68 cm^3 = 4 000 068 cm^3
   (b) 12 000 000 cm^3 + 143 cm^3 = 12 000 143 cm^3

1. The small cube has the dimensions of 1 cm × 1 cm × 1 cm and a volume of 1 cm^3. How many 1 cm^3 cubes will you need to form a large cube with dimensions 10 cm × 10 cm × 10 cm like the one shown on the previous page?
2. How many 10 cm × 10 cm × 10 cm cubes will form a 100 cm × 100 cm × 100 cm cube?
3. (a) To form a 1 000 cm^3 cube you need 1 000 cubes with a volume of 1 cm^3. If cubes of 1 000 cm^3 (10 cm × 10 cm × 10 cm) are then used to form a cube of 100 cm × 100 cm × 100 cm, how many 1 000 cm^3 cubes will there be?
   (b) What is the volume of this new cube?
   (c) How many cubes of 1 cm^3 will form a cube with a volume of 1 000 000 cm^3?
4. Which of the cubes given below has a bigger volume? Explain.
   A. A cube with a volume of 1 m^3
   B. A cube with a volume of 1 000 000 cm^3
5. (a) How many 1 mm × 1 mm × 1 mm cubes (1 mm^3) are needed to form a 1 cm × 1 cm × 1 cm cube?
   (b) What is the total volume of the 1 mm^3 cubes forming the 1 cm^3 cube?

PRACTISE CONVERTING BETWEEN UNITS

When working with volume, you often have to convert between different cubic units. Here are two examples of how you can work out equivalent units.

Converting cm^3 to mm^3:

1 cm^3 = 1 cm × 1 cm × 1 cm
= 10 mm × 10 mm × 10 mm
= 1 000 mm^3

∴ multiply by 1 000

Converting cm^3 to m^3:

1 cm^3 = 1 cm × 1 cm × 1 cm
= 0,01 m × 0,01 m × 0,01 m
= 0,000001 m^3

∴ multiply by 0,000001 or divide by 1 000 000

1. Write the following volumes in cm^3:
   (a) 3 mm^3 (b) 45 mm^3 (c) 0,6 m^3 (d) 1,22 m^3
2. Write the following volumes in mm^3:
   (a) 20 cm^3 (b) 151 cm^3 (c) 4,7 cm^3 (d) 89,5 cm^3
3. Write the following volumes in m^3:
   (a) 9 cm^3 (b) 50 cm^3 (c) 643 cm^3 (d) 1 967 cm^3
4. Write the following answers in cm^3:
   (a) 4 m^3 + 68 cm^3 (b) 12 m^3 + 143 cm^3

17.6 Capacity of 3D objects

DIFFERENCE BETWEEN CAPACITY AND VOLUME

Capacity is the amount of space available *inside* an object.

Volume is the amount of space that the object itself takes up.

17.6 Capacity of 3D objects

DIFFERENCE BETWEEN CAPACITY AND VOLUME

Teaching guidelines

There is often confusion when people work with the concepts of volume and capacity. The volume is the amount of space an object takes up, while capacity is the amount of the content a container can hold when filled to capacity.

An object can have both volume and capacity.

Volume is measured in cubic centimetres (cm^3) and capacity is measured in millilitres. A millilitre is the content of a cube with sides of 1 cm. For example: a measuring cup has a capacity of 250 ml, but there is a volume of 125 ml of water in the cup.

Answers

1. (a) Volume = 6 000 cm^3
   (b) Sides = 2,5 cm; bottom = 2 cm
   (c) Capacity = 25 cm × 15 cm × 8 cm = 3 000 cm^3
   (d) 3 000 cm^3
2. Capacity = 28 cm × 18 cm × 9 cm = 4 536 cm^3 = 4 536 ml = 4,536 ℓ

DISPLACEMENT AND MORE CAPACITY CALCULATIONS

Teaching guidelines

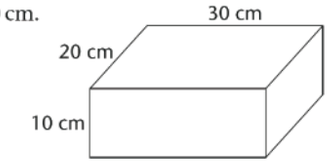
This method can be used to determine the volume of irregular objects (for example, a stone) or of objects like marbles of which the volume is difficult to find otherwise.

Answers

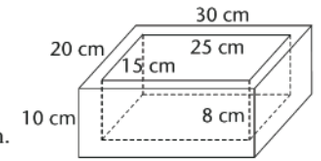
1. (a) Capacity = 8 cm × 2 cm × 5 cm
   = 80 cm^3
   = 80 ml
   (b) Capacity = 10 m × 17 m × 8 m
   = 1 360 m^3
   = 1 360 kl
2. Consider learners' answers.

1. A solid block of wood measures 30 cm × 20 cm × 10 cm.
   (a) What is its volume?

   The same solid block of wood is carved out to make a hollow container. The measurements inside the container are 25 cm × 15 cm × 8 cm.
   (b) How thick are the walls of the container?
   (c) What is the capacity of the container?
   (d) If you filled the container with water, what volume of water would the container hold?



2. More of the wood is carved out of the container to make walls 1 cm thick at the sides and the bottom. Calculate the capacity of the container in litres.

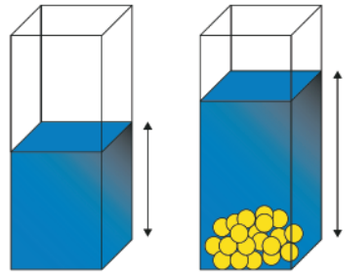


DISPLACEMENT AND MORE CAPACITY CALCULATIONS

Consider a glass vase half full of water. As soon as you place marbles into the water, the level of the water rises. This is not because the amount of water has changed, but rather because the marbles have taken the place of the water and have pushed the water higher up in the vase.

If one of the marbles has a volume of 1 cm^3, it would **displace** 1 ml of water.

∴ We know that:

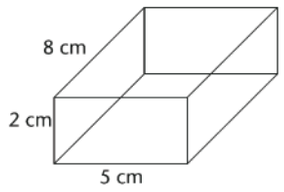


1 cm^3 = 1 ml
1 m^3 = 1 kl

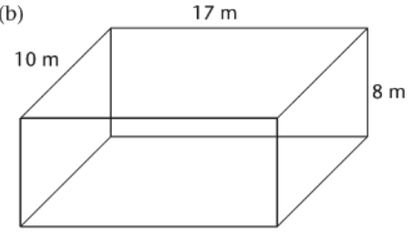
To displace means to move something out of its place.

1. Calculate the capacities of the following containers. The *inside* measurements are given. Write your answers in ml or kl.

(a)



(b)



2. Work out a possible set of inside measurements for a container with a capacity of 12 kl. Draw a sketch and write the measurements on it.

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
18.1 Collecting data	Data handling; sources of data collection; populations and samples; random samples; questionnaires; respondents	Pages 198 to 202
18.2 Organising data	Tally marks, tables and stem-and-leaf displays; frequency; grouped data; class intervals	Pages 202 to 206
18.3 Summarising data: measures of central tendency and dispersion	Mode; median; mean; range; outliers (extremes of the data)	Pages 206 to 211

CAPS time allocation	4 hours
CAPS content specification	Page 109

Mathematical background

Data handling is the part of Mathematics that deals with numbers and facts that we collect about the world around us. Data can be about a wide variety of situations, for example: people's opinions on politics or the success rates of treating people with a certain kind of medicine. We use data to help us make decisions and solve problems about the world around us.

The **data handling cycle** consists of the following phases:

- **Pose a question**: Identify a real-life problem and pose (formulate) a question that requires the collection of data.
- **Collect data**: Identify the data source (the population), which is the whole group you are asking the question about. If the population is too large to handle, select a smaller group (the sample) to represent the population. Find the most suitable method to collect the data, for example, through observation (by watching something closely) or by using a questionnaire (a list of questions) or research material. Decide whether to use a data collection sheet (during observation) or a questionnaire (during interviews) to collect the data.
- **Classify and organise the data**: Identify whether the data is categorical (words) or numerical (numbers) and whether the numerical data is discrete (fixed numbers) or continuous (measurements). Sort the data into categories or into ungrouped or grouped intervals. Organise the data using tallies in frequency tables as well as stem-and-leaf displays.
- **Summarise the data**: Find the mode, median and mean, which are measures of central tendency (balance); the range, which is a measure of spread (width); and outliers (extremes) of the data set.
- **Represent the data**: Draw a graph of the data, for example a bar graph, double bar graph, histogram, pie chart or a broken-line graph.
- **Interpret and analyse the data**: Ask questions about the data and identify and describe trends or patterns in the data in order to draw conclusions about the data.
- **Report on the data**: Explain what the data tells about the problem or question and predict how the data can be used to solve problems about the world around us.

18.1 Collecting data

SOURCES OF DATA COLLECTION

Background information

The **data handling cycle** is started by identifying a real-life problem and posing (formulating) a question that requires the collection of data. Before data is collected questions should be asked about the following:

- The **question**: What should be asked?
- The **population**: Who or what is the whole group the question is about?
- The **sample**: Must a smaller group be chosen to represent the population?
- The **data collection method**: Which is the best method to obtain the data:
  - **observation** (by watching something closely), or
  - **interview** (by talking to someone face-to-face or over the telephone)?
- The **data collection instrument**: How will the data be recorded:
  - on a **data collection sheet** (during observation), or
  - on a **questionnaire** (during an interview or survey)?
- The **source of information** (**data**) is determined by the question asked.
  - If the question involves only learners in your class, the source of information is the learners in your class.
    Example: What is the average shoe size of boys in my class?
  - If the question involves only learners in your grade, the source of information is the learners in your grade.
    Example: What is the average shoe size of girls in my grade?
  - If the question involves only people in your community, the source of information is the people in your community.
    Example: What is the average income per household in my community?
  - If the question involves people in South Africa, the source of information is the people in South Africa.
    Example: How many people in South Africa have access to electricity?

CHAPTER 18

Collect, organise and summarise data

The term **data handling** is used to describe certain ways of trying to make sense of large collections of observations (data) about things in the real world. Data can be about many different things, for example people's opinions on politics or the success rates of treating people with a certain kind of medicine. We use data to help us make decisions and solve problems about the world around us.

18.1 Collecting data

To find out more about any situation, we need to start by asking questions and collecting data. When you collect data, you need to consider:

- what you want to find out or the questions that you want to answer
- where you will find the data to answer the questions (for example, from people such as learners in your school, your family and community; or from published sources such as newspapers, books or magazines)
- who you will collect the data from (all of the people or a sample)
- how you will collect the data (such as using questionnaires or interviews).

SOURCES OF DATA COLLECTION

In some cases you can use data that has already been collected by another person or organisation.

Example 1

Your question is:

What is the most common form of transport that learners in South Africa use to travel to school?

For this question, you will find that this data already exists in a publication called *Census @ School 2009*, published by Statistics South Africa. You can then present and interpret the existing data.

Example 2

Your question is:

What is the most common form of transport that learners at my school use to travel to school?

198 MATHEMATICS GRADE 8: TERM 3

Teaching guidelines

Discuss the following:

- The points which are to be considered when collecting data (as discussed on LB page 198 on the previous page).
- The data sources, as explained in Examples 1 and 2 on LB page 198 on the previous page.

Answers

1. to 9. Refer to the table on LB page 199 alongside.

POPULATIONS AND SAMPLES

Background information

- The **population** of a statistical investigation is the whole group of people or things that you want to find out about.
  - The **size of a population** depends on what you need to find out.
  - The **larger the population**, the more difficult it becomes to involve every member of that population in the data collection process.
  - When the population is large, a **sample** (smaller group) can be chosen from the population to represent the whole population.
  - The sample should represent all the **features of the whole population** and should be chosen with care.

Teaching guidelines

Discuss the following:

- the concept of **population** of a statistical investigation.
- the concept of **sample** of a statistical investigation.
- examples 1 and 2 given at the top of LB page 200 on the following page.

Check with the principal whether such data has already been collected by the school. If the data does not exist, or is very old, you need to decide where to get the data from. You could then decide to collect the data yourself from your peers.

Copy the following table. For each of the following investigation questions, write down what or who would most likely be an appropriate source of information.

Question	Appropriate source of data collection
1. What is the favourite type of music amongst teenagers in my community?	Peers (or teenagers in my community)
2. How much money do workers at Dress Factory earn per week?	The workers at Dress Factory
3. To what height do baobab trees usually grow?	Existing data (probably obtained from botany books or organisations)
4. What are the ages of learners in Grades R to 7 in South Africa?	Existing data (probably obtained from the South African government)
5. How many people in South Africa have access to electricity?	Existing information (you can find this kind of information on the Statistics South Africa website)
6. How many people in different African countries had malaria during the last five years?	Existing information (you can find this kind of information on the World Health Organization's website)
7. Has my school recycled more or fewer glass bottles this year than last year?	Perhaps from existing data held by the school, or perhaps new data to be collected from staff in charge of recycling, or from learners who recycle glass bottles
8. What kind of household chores do seven- to ten-year-olds in my neighbourhood usually do?	7- to 10-year-olds in my neighbourhood
9. How many children in South Africa under the age of ten have been vaccinated to protect them from childhood diseases?	The South African Department of Health might have this data. Or health researchers might have carried out such a survey.

POPULATIONS AND SAMPLES

The whole group of people (or things) that you want to find out about is usually referred to as the **population**.

A population is often quite large. The size of the population depends on what you need to find out. The larger your population, the more difficult it becomes to ask each member of that population the questions you want to ask.

RANDOM SAMPLES

Background information

- A **random sample** reflects all the features of a whole population.
- During random sampling every member of the population has an **equal chance** of being chosen.
- The following are **random sampling methods**:
  - **Simple random sampling**: Choose the sample by drawing names from a bag that contains the names of all the learners in the school.
  - **Systematic random sampling**: Choose a number from 1 to 20 at random, say 14. Then choose every fourteenth name on an alphabetical list of all the learners in the school.
  - **Stratified random sampling**: Determine the ratio of boys to girls in the school. Now choose a random sample from the whole school in that ratio.
  - **Cluster random sampling**: Divide the learners in the school into grades and each grade into classes. Choose one class at random from each grade.
- The following are **biased sampling methods** and usually lead to the collection of unreliable data:
  - **Convenience sampling**: Choose only your friends to be part of the sample.
  - **Self-selection sampling**: Ask who in your school would like to be part of the sample.
  - **Quota sampling**: Choose only boys in Grade 12 to be part of the sample.

Teaching guidelines

Discuss the following:

- the concept of **random sample** (refer to the drawings on LB page 200 alongside)
- **methods of random sampling** (refer to the examples at the bottom of LB page 200 alongside).

Answers

1. See the completed table on LB page 200 alongside.

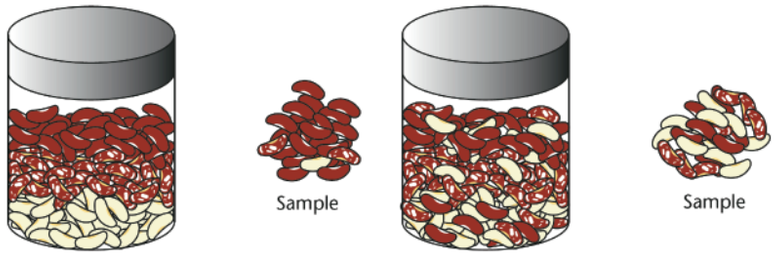
You could choose a smaller group of individuals from the population. Such a group, that is used to represent the whole population, is called a **sample**.

Examples

1. You may want to find out how much time the learners in a school spend on doing homework. If there are many learners in the school, you may be unable to ask them all about it. What you can do in this case is to talk to some learners in each class in the school. You may, for instance, speak to five learners from each class.
2. Health researchers may collect information about children by doing a survey of households selected randomly in each community.

RANDOM SAMPLES

A sample has to be chosen carefully to make sure that it represents the population. To understand what this means, think about what happens if you choose some beans from a jar in which the different kinds of beans are in separate layers. If you take a sample of the beans at the top, the sample is not representative. If the beans are all mixed up, then each bean has an equal chance of being chosen. The sample will be representative.



Example

You could select a random sample of learners from your school in the following two ways:

1. You may write the names of all the learners on separate paper strips. You then put all the strips in a plastic bag, mix them up, and draw 30 strips without looking at the names before you have finished.
2. You could select every fifth name from each of the class lists.

Look at the investigation questions. Which of the samples given do you think will reflect the whole population more appropriately? Copy the following table. Tick your choice and give a reason.

Sample 1	Sample 2	Reason
1. What is the type of music liked by most teenagers in my community?		
50 teenagers at a local school	25 teenagers each from two different local schools ✓	If you only choose one school, then you are not representing the whole population.

Answers

2. to 5. See the completed table on LB page 201 alongside.

QUESTIONNAIRES

Background information

- A **questionnaire** is a sheet with questions used to collect data.
- The person who answers these questions is called the **respondent**.
- When a questionnaire is constructed, different **types of questions** can be asked, for example:
  - questions with "yes" or "no" responses (answers)
  - multiple choice questions with a selection of answers
  - questions that ask for a rating like "never", "sometimes" or "always"
  - open questions where the respondents may enter their own opinion or information.
- If you use the wrong method or instrument to collect data, the data may be **flawed**. This could lead to unreliable conclusions and predictions.
- The type of questions that you choose depends on the data you want to collect. The following hints will **help you construct a questionnaire**:
  - Ask clear and short questions.
  - Start with easy questions.
  - Avoid leading questions which tell the respondent what the answer should be.
  - Avoid biased questions which favour someone or something.
  - Avoid offensive questions which are too personal or upsetting.
  - Give clear instructions.
  - Make the questionnaire as short as possible.
- **Multiple choice questions** should have the following features:
  - The options should **not overlap**.
  - There should be **no gaps** between the options.
  - The options should cover **all possible answers**.

Teaching guidelines

Discuss the background information listed above.

2. How much money do the 200 workers at Dress Factory earn per week?		
The workers at every fourth workstation in the factory ✓	The 50 workers that gather outside during lunch break	If you choose workers who behave in a particular way, the sample is not random.
3. To what height do baobab trees usually grow?		
All baobab trees in a marked-off area ✓	Every second baobab tree in a marked-off area	If you can access all of the trees in an area, then they can all be measured.
4. What are the ages of learners in Grades R to 7 in South Africa?		
All the Grade R to 7 learners in my school	Ten learners in each grade from Grade R to 7 at three different schools ✓	To represent all South African schools, you need to select a sample from different areas, and choose enough learners for the sample.
5. Has my school recycled more or fewer glass bottles this year than last year?		
All the glass bottles recycled in one month this year and in the same month last year ✓	The glass bottles recycled in one month this year and in any other month last year	Compare the same month each time, so that you don't introduce other things that could influence the results.

QUESTIONNAIRES

We can use different methods to collect data, for example questionnaires, face-to-face interviews or telephonic interviews. In this section, you will work with questionnaire questions that have multiple-choice responses.

Here are two questions with multiple-choice responses from which a **respondent** will choose.

A respondent is a person who responds to the questions.

How satisfied are you with our level of service?	What is the colour of your eyes?
☐ Not satisfied at all	☐ Brown
☐ Fairly satisfied	☐ Green
☐ Very satisfied	☐ Blue
	☐ Other

1. Write a suitable question with multiple-choice responses to find out the following information:
   (a) What do teenagers spend their money on?
   (b) How much time do Grade 8s spend on homework every day?

Answers

1. Sample answers:

(a)	(b)
On which of the following items do you spend most of your money? ☐ food ☐ clothing ☐ air time ☐ make-up or toiletries ☐ entertainment	How much time do you spend doing homework every day? ☐ less than 1 hour ☐ between 1 and 2 hours ☐ between 2 and 3 hours ☐ more than 3 hours

2. Sample answers:
   (a) 50 teenagers from my school, or 50 teenagers from the local community.
   (b) All the Grade 8s in my school, or 20 Grade 8 learners from two schools in the area.

18.2 Organising data

Background information

The way data is organised depends on the type of data (categorical or numerical) and what we want to find out from the data.

Teaching guidelines

Distinguish between categorical data (words), discrete data (fixed numbers) and continuous data (measurements).

Answers

1. Sample answer: We want to know on which day the highest number of learners want to practise. We need to write down how many times each day appears.
2. Sample answer: We need to find out if these body weights are healthy compared to other children's body weights. We could arrange the body weights in order and then summarise the whole set by using measures of central tendency and range. Grouping the data might be useful to show the number of body weights between 10 kg and 15 kg, 15 kg and 20 kg, and so on.
3. Sample answer: We want to know how long different learners take to answer a question. We need to arrange the times in order and then look at a representation of the whole set. We could group the numbers to see how many are between one second and ten seconds, ten seconds and 20 seconds, 20 seconds and 30 seconds, etc. to answer the question.

2. Choose one of the questions from question 1 on page 201. Write down what you think would be the best sample to use if you had to conduct this investigation.
3. Use the multiple-choice question that you chose in question 2 to collect the data. Keep the results for the next section.

18.2 Organising data

The way we organise and summarise data depends on the kind of data that we have. It also depends on what we want to find out from the data. Work in groups to explore this. Do not worry about getting the answers right at this stage. You will learn about the different ways in which to organise and summarise data in this chapter.

Look at the following sets of data. For each one, discuss with your group and write down what you want to find out and what you think you would need to do to the data.

1. Data collected to find out which day would be best to have a soccer club practice:

   Twenty-five learners' preferred day for soccer practice

   Tuesday Tuesday Tuesday Wednesday Monday Thursday Tuesday Friday Friday Friday
   Tuesday Thursday Wednesday Wednesday Tuesday Tuesday Wednesday Monday
   Thursday Tuesday Tuesday Wednesday Monday Thursday Tuesday

2. Data collected to find out whether five-year-old children in a certain village have healthy body weights:

   Body weights of twenty-five children in kilograms, rounded off to the nearest 0,5 kg

   17 kg 16,5 kg 13,5 kg 14 kg 18 kg 18 kg 14 kg 21 kg 13,5 kg 15 kg 15 kg 14,5 kg
   15,5 kg 19,5 kg 17 kg 17,5 kg 14 kg 14 kg 20 kg 14,5 kg 16 kg 18 kg 12 kg 16 kg 19 kg

3. Data collected to find out how many learners answer a certain type of question in less than 20 seconds:

   Time taken (in seconds) by a group of learners to answer a question

   20 25 24 33 13 26 10 19 39 31 11 16 21 17 11 34 14 15 21 18 17 38 16 21 25

4. Data collected to analyse the monthly salaries of employees of a small business:

   The monthly salaries of ten employees

   R8 000 R2 500 R75 000 R6 000 R7 500 R5 200 R4 800 R10 300 R15 000 R9 500

Answers

4. Sample answer: This is a small data set of only ten values. We could arrange the data in order and then summarise it using measures of central tendency and range.

TALLY MARKS, TABLES AND STEM-AND-LEAF DISPLAYS

Background information

- Before data is organised it has to be:
  - **classified** as categorical, discrete or continuous data
  - **sorted** into categories or class intervals.
- Data can be **organised** in ordinary tables, frequency tables and stem-and-leaf displays.
- **Ordinary tables** show **raw data**, which is data as it is collected, before any sorting is done.
- **Frequency tables** show data sorted into categories or intervals.
  - Column 1 shows the different **categories or intervals** of data.
  - Column 2 shows the **counts** of each category or interval by means of tally marks (|) recorded in clusters of five (𝍸) to make it easier to find the total number of counts for a category. This column is optional.
  - Column 3 shows the **total number of counts** in each category and is called the **frequency** of that category.
- A **stem-and-leaf display** shows numerical data listed in two columns separated by a vertical line.
  - The **stem column** (on the left of the vertical line) shows the tens and hundreds of all the data values in numerical order, including those which are missing from the sequence.
  - The **leaf column** (on the right of the vertical line) shows the units of all the data values in numerical order and in line with their relevant stems, which means that something like 23|0, 5 represents the data values 230 and 235.

TALLY MARKS, TABLES AND STEM-AND-LEAF DISPLAYS

In Grade 7, you learnt about using tally tables and stem-and-leaf displays. We revise these two ways of organising data here.

We can use **tally tables** to record data in different categories. We draw a tally mark (|) for each item we count. We group tally marks in groups of five to count them quickly.

A **stem-and-leaf display** is a way of listing numerical data. If the numbers in a set of data consist of digits for tens and units (such as 23, 25, 34), the column on the right (the leaf column) shows the units digits of the numbers, and the column on the left (the stem column) shows the tens digits of the numbers.

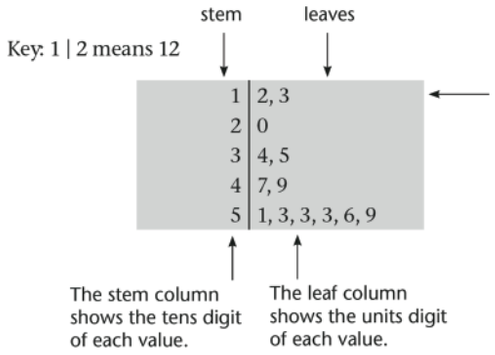
Examples of tally marks:
A count of three = |||
A count of five = 𝍸
A count of seven = 𝍸 ||

If the numbers in a data set consist of three digits (such as 324, 428, 526), both the hundreds and tens digits are written in the stem column and the units digits are written in the leaf column, for example: 32 | 4 would show 324.

Let's see how to display these numbers using the stem-and-leaf method:

12, 13, 20, 34, 35, 47, 49, 51, 53, 53, 53, 56, 59

The numbers range from 12 to 59, so the first digits represent the numbers 10 to 50.



Values with the same stem are written in the same row. Different leaves with the same stem are separated by a space or a comma.
In the first row, 1 | 2, 3 shows 12 and 13.

Notice that the stem-and-leaf display also shows us what the data set looks like. We can quickly see that most numbers are in the 50s and that there is only one number in the 20s.

1. Look back at the three sets of data on page 202. Copy and fill in the table on page 204 to show which form of data organisation you can use for each of these. Write a short explanation.

Teaching guidelines

Revise how to use a:

- frequency table to organise categorical data into categories
- frequency table to organise ungrouped numerical data
- stem-and-leaf displays to organise ungrouped numerical data.

Answers

1. See the table on LB page 204 alongside.
2. (a) See the frequency table on LB page 204 alongside.
   (b) Tuesday. The highest number of learners chose Tuesday (11 out of 25 learners).
   (c) Friday would be the worst day, because only two of the 25 learners chose it.
3. (a) Key: 4 | 8 means 48

4	8, 8, 9
5	1, 4, 5, 5, 8, 9, 9, 9
6	0, 1, 2, 2, 3, 4, 4, 7
7	0

	Tally table	Stem-and-leaf display
A. Preferred day for soccer practice	Yes, we can use tallies to count how many children prefer each of the days.	No, the data is not numerical.
B. Body weight of children	No, the numbers do not repeat often. If you grouped them, you could use tallies to count the number in each group.	Yes. It organises the numbers and also shows how they are distributed.
C. Time taken to answer a question	No, only if the data was grouped.	Yes. It organises the numbers and also shows how they are distributed.

2. Use the data set about preferred days for soccer practice:

Twenty-five learners' preferred day for soccer practice

Tuesday Tuesday Tuesday Wednesday Monday Thursday Tuesday Friday Tuesday Friday Tuesday Thursday Wednesday Wednesday Tuesday Tuesday Wednesday Monday Thursday Tuesday Tuesday Wednesday Monday Thursday Tuesday

(a) Copy the following tally table. Organise the data using the tally table.

Preferred day	Tally	Frequency
Monday	|||	3
Tuesday	𝍸 𝍸 |	11
Wednesday	𝍸	5
Thursday	||||	4
Friday	||	2

(b) Which day should they choose to have soccer practice? Why?
(c) Which day would be the worst day for soccer practice? Why?

3. Zandile collected data about the number of garments that each of her workers produced per day. The answers were as follows:

61, 58, 48, 59, 49, 51, 54, 67, 55, 70, 59, 60, 62, 59, 62, 63, 64, 48, 64, 55

(a) Record the data in the form of a stem-and-leaf display.

Key:

Answers

3. (b) 50s and 60s.
   (c) Fastest worker: 70; Slowest worker(s): 48
4. (a) Learners should choose a tally table to organise the data as the stem-and-leaf display works with numerical data, not data in categories. Learners record the data in a tally table.
   (b) Learners should be able to say what the most common and least common categories are.

GROUPING DATA IN INTERVALS

Background information

- Sets of **numerical data** are usually grouped when:
  - the data set contains a large number of data values
  - the data values are very different in magnitude (size).
- A **class** of data is one of the **groups** in a collection of grouped data.
- The **class limits** are the **two values** which define the two ends of a class.
- The **class interval** is the **width** of a class, or the difference between the two class limits. The upper class limit is **excluded** from the class interval.
- Data is grouped into **intervals** to make it easier to handle. The following are **features** of class intervals:
  - Intervals should **not overlap**.
  - There should be **no gaps** between intervals.
- Once data is grouped, the **original data values** cannot be found again.

Teaching guidelines

Discuss the concept of **class intervals**. Point out that the highest number in each class interval is excluded from that class interval to avoid any overlaps.

Answers

1. (a) 55–60 kg, 60–65 kg, 65–70 kg, 70–75 kg, 75–80 kg
   (b) See the frequency table on LB page 205 alongside.
   (c) The highest numbers of athletes are in the 60–65 kg and the 65–70 kg intervals.

(b) Complete: Most values occur in the ...
(c) How many garments do the fastest and slowest workers make?

4. Use the data you collected in the investigation in question 3 at the top of page 202.
   (a) Decide whether a tally table or a stem-and-leaf display will organise the data best, and record the data.
   (b) What does your tally table or stem-and-leaf display show about your data?

GROUPING DATA IN INTERVALS

When there are many values in a data set, it is often useful to group the data items into **class intervals**.

Example

Height in centimetres	Frequency
130–140	6
140–150	13
150–160	31
160–170	30
170–180	10

The class interval does not include the highest number in each case. So, the height of 150 cm falls into the interval 150–160 cm; not into the interval 140–150 cm.

This is a grouped frequency table. It represents 90 data values, but the values themselves are not shown. Instead, we show the frequency or the number of values falling into that interval.

1. The table shows the body weights (in kilograms) of athletes competing in a tournament.

55,2	56,1	58,4	59,3	60,6	61,2	61,7	63,4
63,2	64,2	65,9	66,5	66,7	67,3	67,8	68,0
70,5	72,9	73,4	74,1	74,8	75,9	76,7	78,7

   (a) Group the weights into 5 kg intervals. List the intervals.
   (b) Use a table to show the frequency of each class interval. It is useful to fill in the tallies first and then count up the frequencies, so that you do not leave any data items out.

Body weights of athletes	Tally	Frequency
55–60 kg	𝍸	4
60–65 kg	𝍸 |	6
65–70 kg	𝍸 |	6
70–75 kg	𝍸	5
75–80 kg	|||	3

   (c) In which intervals are the highest numbers of athletes?

Answers

2. (a) Sample intervals: 30–35 min, 35–40 min, 40–45 min, 45–50 min, 50–55 min
   (b) See the frequency table on LB page 206 alongside.
   (c) Between 40 and 45 minutes
3. (a) See the frequency table on LB page 206 alongside.
   (b) No. All learners answered the question in less than 40 seconds.
   (c) Yes. Twelve learners answered in less than 20 seconds, and 8 + 5 = 13 learners took 20 seconds or more.

18.3 Summarising data: measures of central tendency and dispersion

ONE NUMBER SPEAKS FOR MANY: THE MODE AND THE MEDIAN

Background information

- We **summarise** data by finding a few numbers that, together, show us more about the whole data set. Some of these numbers involve all data values. Others involve only a few.
- **Measures of central tendency** tell us more about the balance in a data set.
- The **first** measure of central tendency is the **mode**. It shows the **most common data value** in an *ordered data set*. It is equal to the data value with the highest frequency.
  - A data set can have **more than one mode**.
  - If the frequencies of all data values are equal, the data set has **no mode**.

Teaching guidelines

- Revise the concept of the **mode** of a data set after learners have completed question 1.
- Point out that some data sets have more than one mode, and many data sets have none.

2. The following data shows the time taken (in minutes and seconds) by runners to complete a race:

34:30	34:59	35:36	36:58	40:08	40:55	41:33	43:18
44:26	45:40	48:13	48:49	49:15	50:08	52:09	53:36

(a) Group the times into suitable intervals. List the intervals.
(b) Record the grouped data in the form of a table.

Race times	Frequency
30–35 min	2
35–40 min	2
40–45 min	5
45–50 min	4
50–55 min	3

(c) How long did the highest number of runners take to finish the race?

3. Take another look at the data about the time (in seconds) that learners took to answer a certain question:

20 25 24 33 13 26 10 19 39 31 11 16 21 17 11 34 14 15 21 18 17 38 16 21 25

(a) Group the data into three intervals of ten seconds. Copy and fill in the following table to show the grouped data:

Answer times (seconds)	Tally	Frequency
10–20	𝍸 𝍸 ||	12
20–30	𝍸 |||	8
30–40	𝍸	5

(b) Do you think that learners will need at least 40 seconds to answer this type of question? Explain.
(c) Were there more learners who took at least 20 seconds or more to answer the question than learners who took less than 20 seconds? Explain.

18.3 Summarising data: measures of central tendency and dispersion

ONE NUMBER SPEAKS FOR MANY: THE MODE AND THE MEDIAN

1. A farmer wants to know whether he used good quality seed when he planted pumpkins. So he counts the number of pumpkins on each of a sample of 20 pumpkin plants. The numbers of pumpkins are given below:

6 7 3 7 4 7 7 8 7 5 7 7 6 7 8 5 4 7 6 7

Background information (continued)

- The **second** measure of central tendency is the **median**. It **cuts** an *ordered data set* **in half**.
  - If the data set contains an **odd** number of data values, the median is the **data value in the middle** of the *ordered data set*.
  - If the data set contains an **even** number of data values, the median is the **value between the two data values in the middle** of the *ordered data set*.

Teaching guidelines (continued)

- Revise the concept of the **median** of a data set after learners have completed question 2.
- Distinguish between the median of an even number of data values and that of an odd number of data values.
- Point out that the median might not be equal to any of the data values.

Answers

1. (a) 3 4 4 5 5 6 6 6 7 7 7 7 7 7 7 7 7 8 8
   (b) No definite answer is required. The purpose of the question is to make the learners aware of the idea of the mode.
   (c) Yes. Consider learners' answers.
2. Bongile is right. The scores are arranged from smallest to largest below.
   3 4 5 6 7 7 7 7 7 7 8 8 9 9 10 11 13 15 19 23 25
   Half of the marks are 8 or more, and half of the marks are less than 8.
3. Sample answer: 4 6 5 8 12 24 25 27 29 30 31

(a) Arrange the data values from smallest to biggest, to get a clearer picture.
(b) The farmer says to his wife: *Most of the plants have seven pumpkins, so it is not too bad.* Do you think this is a good summary of the data, or should the farmer give some more information?

In some data sets some values or items are repeated often. The value or item that occurs most often is called the **mode**. Some data sets have more than one mode, and many data sets have none.

Instead of "most often" we can also say "most frequently".

(c) Do you think that if the farmer had said the following, his wife would be somewhat better informed about the pumpkin plants?

The number of pumpkins varies from three to eight, but there are seven pumpkins on most of the plants.

2. Look at the Mathematics test results, out of 30, of a small class of 21 learners:

15 7 11 7 13 4 8 9 3 7 25
7 6 10 8 9 23 19 7 5 7

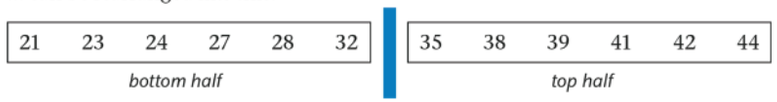
Bongile scored 9 out of 30 in the test, which is poor. Can he claim that his mark is in the top half of the class? Explain your answer well.

A set of data can be separated into a top half and a bottom half by arranging the data items from smallest to largest and finding the number between the two halves.

For example, the following data set:

23 35 44 21 28 32 38 41 39 42 24 27

... can be rearranged like this:



The number that sits halfway between the upper item in the bottom half and the lower item in the top half in this case is 33,5 (calculation: [32 + 35] ÷ 2 = 33,5).

The number that separates a set of data into an upper half and a lower half is called the **median**.

Half of the items are above the median and half of the items are below the median. To find the median, the data items need to be arranged from smallest to largest.

If a numerical data set has an odd number of items, the median is equal to the number in the middle of the set, when the items are arranged from smallest to largest:

3 4 5 6 7 7 7 7 7 7 **8** 8 9 9 10 11 13 15 19 23 25

3. Write down any 11 different numbers so that the median is 24.

Answers

4. Bottom: 12 13,5 13,5 14 14 14 14 14,5 14,5 15 15 15,5 16
   Top: 16 16,5 17 17 17,5 18 18 18 19 19,5 20 21
   The median is 16 kg.
5. (a) Yes, the mode is 7.
   (b) Tuesday

IF THEY WERE ALL EQUAL ... BUT THEY ARE NOT

Background information

- The **third** measure of central tendency is the **mean**. It is the "average" of a data set.
- **Calculate** the mean by adding all the data values and dividing the total by the number of data values.

Teaching guidelines

- Revise the concept of the **mean** of a data set after learners have completed question 3.
- Point out that the mean might not be equal to any of the data values.

Answers

1. This is a very open question. It is intended to induce thought from the learners to empower them to make good sense of the concept of the mean, when it is formally introduced later on in this activity.

2. (a) Yes, because:
   R16 + R16 + R18 + R15 + R14 + R14 + R16 + R14 + R13 + R14 = R150
   (b) R150 ÷ 10 = R15 for each watermelon

3. (a) 7 + 15 + 10 + 16 + 9 + 15 = 72 = 12 + 12 + 12 + 12 + 12 + 12 (or, 12 × 6)
   Susan's answer is correct.

4. The body weights of 25 children are given below, all rounded off to the nearest 0,5 kg. This data was collected to find out whether five-year-old children in a certain village have healthy body weights or not.

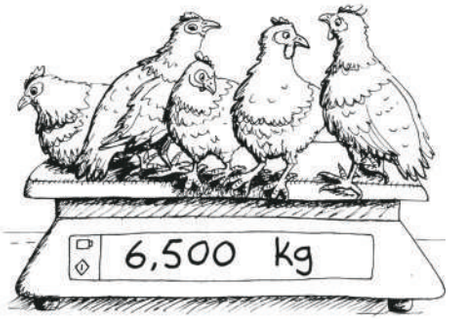
17	16,5	13,5	14	18	18	14	21	13,5
15	15	14,5	15,5	19,5	17	17,5	14	
14	20	14,5	16	18	12	16	19	

   Rearrange these data items into a bottom half and a top half, and state what the median body weight is.

5. (a) Does the data set shown above question 2 on page 207 have a mode, and if it has what is it?
   (b) What is the mode of the data set in question 2 on page 204?

IF THEY WERE ALL EQUAL ... BUT THEY ARE NOT

1. Five chickens are placed on a scale and the scale shows 6,500 kg, which is the same as 6 500 g. What can you say if someone asks you:
   What does each of the chickens weigh?



2. A roadside stall owner has ten watermelons to sell. They are not all of the same size and he did not pay the farmer the same price for each watermelon. So the stall owner weighs the watermelons and decides to sell them at the following prices:

R16	R16	R18	R15	R14	R14	R16	R14	R13	R14

   (a) Check whether you agree that he will get R150 for the ten watermelons together.
   (b) The stall owner now decides to make the prices of all the watermelons the same, to simplify advertising and selling. What should he make the price of each watermelon, if he still wants to receive R150 for all of them together?

3. Susan bought six pumpkins at the market. Her husband, Abraham, asks her what she paid for each pumpkin. Susan says:
   They actually came at different prices, and I have forgotten the prices now. But I know I paid R72 in total so it would have been the same if I paid R12 each. So you can say that on average I paid R12 each.
   (a) Check if Susan's answer to her husband is correct. The actual prices she paid for the different pumpkins are given below:

R7	R15	R10	R16	R9	R15

Answers

3. (b) A good answer would be to say she added up and divided by 6. However, Susan could also have "shifted rand from one pumpkin to another" until she had the same price for each pumpkin.
   (c) Yes. 11 + 12 + 13 + 11 + 12 + 13 = 72 = 12 + 12 + 12 + 12 + 12 + 12
4. Learners may now use the idea of the mean, and state that the mean mass of the chickens is 1 300 g. While this is a good answer, it is important that they realise that none of the chickens may actually weigh exactly 1 300 g.
5. (a) Learners may cite a variety of reasons for stating that bread is cheaper in city A: Only three of the ten shops in City A charge R9 or more, while seven of the ten shops in City B charge R9 or more. Or: The two lowest prices out of 20 are in City A, and the two highest prices are in City B.
   (b) Mean white bread price in sample of City A shops
   = 8 909 ÷ 10 = 890,9 cents
   Mean white bread price in sample of City B shops
   = 9 099 ÷ 10 = 909,9 cents
   (c) City A: 839 861 880 885 888 889 899 904 927 937
   City B: 872 872 890 900 908 910 924 933 942 948
   Median white bread price in sample of City A shops:
   (888 + 889) ÷ 2 = 888,5 cents
   Median white bread price in sample of City B shops:
   (908 + 910) ÷ 2 = 909 cents
6. (a) 21 × R830 = R17 430
   (b) R17 430 – R4 800 = R12 630
   The mean price of the 20 goats was R12 630 ÷ 20 = R631,50.
7. (a) The mean is 150 ÷ 15 = 10 and the median is 1.
   (b) Many different answers are possible.
   (c) Many different answers are possible.
   (d) Many different answers are possible.

(b) How do you think Susan came to the R12 she used when she answered her husband's question?
(c) Check if Susan's answer would have been correct if the actual prices of the pumpkins were as follows:
R11 R12 R13 R11 R12 R13

When she gave an answer to her husband's question, Susan used the number 12 as a "summary" to represent the six different numbers 7; 15; 10; 16; 9 and 15. The number 12 is a good representation of 7; 15; 10; 16; 9 and 15 together because:

7 + 15 + 10 + 16 + 9 + 15
= 12 + 12 + 12 + 12 + 12 + 12

If each value in a data set is replaced by the same number and the total remains the same, the "replacement number" is called the **mean** or **average**.

It can be calculated by dividing the total (sum) of the values by the number of values in the data set:
Mean = sum of values ÷ number of values. (In the example above, the mean is 72 ÷ 6 = 12.)

Like the median, the mean (average) may not be equal to any of the actual values in the data set.

4. Look again at question 1 on page 208 about the five chickens. If you want to now give a different answer than before, write it down.
5. A journalist investigated the price of white bread at different stores in two large cities. The prices are given in cents at ten different shops in each city:

City A:	927	885	937	889	861	904	899	888	839	880
City B:	890	872	908	910	942	924	900	872	933	948

   (a) If you just look at the above data, do you think one can say that white bread is cheaper in the one city than in the other? Look carefully, and give reasons for your answer.
   (b) Calculate the mean price of white bread for the sample in each of the two cities.
   (c) Find the median bread price in the sample for each of the two cities.
6. Geoffrey is a stock famer. He buys 21 goats at a mean price of R830 each.
   (a) How much do the 21 goats cost, in total?
   (b) One of the goats was a stud goat for which Geoffrey paid R4 800. What was the mean price of the other 20 goats?
7. (a) Find the mean and the median of the following data set:
   1 1 1 1 1 1 1 1 2 2 2 2 2 2 130
   (b) Write ten numbers so that the mean is much smaller than the median.
   (c) Write ten numbers so that the mean is much bigger than the median.
   (d) Write ten different numbers so that the mean is equal to the median.

Answers

8. Grade 8B works faster, but it is very difficult to notice this in the unorganised data sets. Learners may arrange the numbers from smallest to largest and compare, as shown below, or they may calculate the mean time for both classes. Both methods are sound for making this comparison. The mean times are 25,8 seconds and 22,1 seconds for Grades 8A and 8B respectively, and the medians are 27 seconds and 20 seconds respectively.

   The times taken by Grade 8A learners:

   11 13 14 14 15 16 18 20 20 22 24 24 27 29 29 29 29 30 35 36 37 38 38 38 39

   The times taken by Grade 8B learners:

   10 11 11 13 14 14 14 15 16 16 17 20 20 20 22 22 26 26 28 31 35 36 37 39 39

HOW WIDE IS THE DATA SPREAD?

Background information

- **Measures of spread** tell us how far apart the data values in an *ordered data set* lie.
- The **range** is the **difference** between the largest and smallest data values in an *ordered data set*.

Teaching guidelines

- Revise the concept of **range** after learners have completed question 1.

Answers

1. (a) Learners may say that there is no real difference, which is a good answer.
   (b) Egg masses from farm A: 45 45 46 49 51 53 54 54 54 55
   Mean mass of farm A eggs = 506 ÷ 10 = 50,6 g. Median mass is 52,0 g.
   Egg masses from farm B: 41 43 44 47 52 52 53 55 57 59
   Mean mass of farm B eggs = 503 ÷ 10 = 50,3 g. Median mass is 52,0 g.
   (c) The egg masses from farm A vary from 45 to 55, a difference of 10.
   The egg masses from farm B vary from 41 to 59, a difference of 18.
2. (a) See the answers on LB page 210 alongside.
   (b) See the answers on LB page 210 alongside.

8. Look at the times taken by the different learners in Grade 8A in a certain school, in seconds, to do question 7(b) on page 209:

   20 30 36 14 20 14 29 39 15 37 35 24
   29 29 18 16 38 13 24 27 22 38 29 11 38

   Look at the times taken by the different learners in Grade 8B in the same school, in seconds, to do question 7(b) on page 209:

   20 22 39 22 16 37 36 15 14 13 16 10 14
   26 11 14 31 17 11 28 39 20 35 26 20

   Which class works the fastest, Grade 8A or Grade 8B? Explain your answer very well.

HOW WIDE IS THE DATA SPREAD?

1. Two samples were taken from the eggs produced on two different egg farms, to investigate the masses of the eggs coming from the two farms.
   The mean mass of the eggs from farm A is 50,6 g and the median mass is 52,0 g.
   The mean mass of the eggs from farm B is 50,3 g and the median mass is 52,0 g.
   (a) Do these figures indicate that the eggs from the two farms are similar, or that they differ?
   (b) The actual masses of the eggs in the two samples are given below. Check whether the mean and median masses given above are correct.
   Masses of the sample of eggs from farm A, in grams:

   51 54 45 53 49 54 55 46 54 45

   Masses of the sample of eggs from farm B, in grams:

   53 52 55 44 57 41 59 43 47 52

   (c) In what way do the masses of the eggs from the two farms differ?

The **range** of a set of data is the **difference** between the **maximum** (highest or top value) and the **minimum** (lowest or bottom value).

The values in the data set below vary from 36 to 60, hence the range is 60 – 36 = 24:

36 36 39 39 43 45 46 47 52 52 53 55 57 60

2. The following data shows the exam marks of two groups of learners:
   Group 1: 30 31 35 50 55 58 60 70 78 80 88 88 90 90
   Group 2: 55 55 56 57 59 59 59 67 69 75 80 80 80 81
   Compare the two groups by copying and completing the following statements:
   (a) In group 1 the marks vary from 30 to 90, a range of 60.
   (b) In group 2 the marks vary from 55 to 81, a range of 26.

Answers

3. (a) Learners may observe that prices are generally lower in Town B, except for two houses that cost more than a million rand each.

   (b) Hopefully learners will decide to arrange the prices in each list in either ascending or descending order as shown below. A statement such as the following will illuminate the key features of the two data sets:

   In Town A the prices range from R181 000 to R405 000.

   In Town B most prices are much lower. They range from R88 000 to R175 000, but there are two houses that cost more than a million rand each.

Town A:	R181 000	R199 000	R248 000	R283 000
	R299 000	R303 000	R315 000	R321 000
	R322 000	R380 000	R405 000	
Town B:	R88 000	R100 000	R105 000	R107 000
	R122 000	R146 000	R151 000	R166 000
	R175 000	R1 114 000	R1 199 000	

4. (a) One good response would be that the statement may create the impression that houses in Town B are slightly more expensive. This is not true, since nine of the 11 houses in Town B are cheaper than the cheapest house in Town A.

   (b) The prices of the two houses that are so much higher than the other nine prices.

Background information (continued)

- **Outliers** are data values that are much lower or higher than the trend.

Teaching guidelines (continued)

- Discuss the concept of an **outlier** after learners have completed question 4.
- Point out that the **mean** is not a good way to summarise a data set which contains outliers.

Answers

5. (a) Yes, the salary of R75 000 is clearly an outlier.

   (b) Yes. The median is close to the majority of the prices for both towns.

3. These two sets of data show the prices of houses that have been sold in Towns A and B in one month:

Town A:	R321 000	R199 000	R181 000	R303 000
	R299 000	R248 000	R283 000	R315 000
	R405 000	R380 000	R322 000	
Town B:	R88 000	R122 000	R175 000	R166 000
	R107 000	R105 000	R1 114 000	R100 000
	R151 000	R1 199 000	R146 000	

   (a) Read through the prices in each list and write down anything that comes to your mind when you look at the two sets of figures.

   (b) You have been asked by the local newspaper to write a short paragraph on the prices of houses in the two towns. You want to make it quick and easy for the readers to get some sense of the prices in the two towns. Write your newspaper paragraph neatly.

The mean price for houses in the list for Town A is R296 000. This is very close to the median of R303 000. All the prices in Town A are within R115 000 of the mean.

The mean price for houses in the list for Town B is R315 727, which is more than double the median price of R146 000. Nine of the 11 houses in Town B cost far less than the mean, while the prices in Town A are more evenly spread on both sides of the mean.

4. Someone asks you about the house prices in Towns A and B, and you say: *The mean house price in Town A is R296 000, and the mean house price in Town B is R315 727.*

   (a) Does this statement provide good information about the difference in pricing of houses in the two towns? In what way may it actually be misleading?

   (b) What causes the mean to be a misleading way of describing the data for the house prices in Town B?

Data items like the house prices of R1 114 000 and R1 199 000 in the list for Town B in question 3 are called **outliers** (or extreme values). Outliers are data values that are much lower or much higher than any other values in the data set. The mean is not a good way to summarise a set of data with outliers.

5. (a) Is there an outlier in this set of monthly salaries of the employees at a small business?

R8 000	R2 500	R75 000	R6 000	R7 500
R5 200	R4 800	R10 300	R15 000	R9 500

   (b) Would the median prices of the two data sets be a good way to indicate the main difference between house prices in Towns A and B in question 3? Explain your answer.

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
19.1 Bar graphs and double bar graphs	Revision; representations of bar graphs and double bar graphs	Pages 212 to 214
19.2 Histograms	Features of a histogram; representations of histograms	Pages 214 to 216
19.3 Pie charts	Sizes of slices in a pie chart; representations of pie charts	Pages 217 to 218
19.4 Broken-line graphs	Introduction of broken-line graphs; representations of broken-line graphs; comparison of representations of data	Pages 218 to 221

CAPS time allocation	3 hours
CAPS content specification	Page 110

Mathematical background

- In Chapter 18 we covered the following **phases in the data handling cycle**:
  - **Pose a question** about a real-life problem that requires the collection of data.
  - **Collect and record data** on data recording sheets during observations and on questionnaires during interviews.
  - **Classify, sort and organise data** in categories or intervals on frequency tables and stem-and-leaf displays.
  - **Summarise data** by finding the mode, median, mean and range of the data set and taking note of outliers.
- In this chapter the focus is on **representing data**, which is the next phase in the data handling cycle.
- Data is represented by **drawing a picture** of the tabulated data. This is done for the following reasons:
  - A picture makes information **easier to understand**.
  - It is easier to **identify patterns** in a picture than in a table. People find it easier to see patterns in pictures than in raw numbers.
  - It is easier to **identify trends** in a picture than in a table. These trends can be upward or downward, and they can even be cyclical. Trends are easier to see in pictures than in tables.
  - People are more attracted to visual information that is presented in an **aesthetically pleasing manner** in the media.
- A large **variety of statistical displays** are used by statisticians and in the media to convey information, for example:
  - dot plots, pictographs, bar graphs, double bar graphs, pie charts, line and broken-line graphs and scatter plots
  - histograms, frequency polygons, ogives (cumulative frequency polygons), regression functions, normal distributions, and so on.
- In this chapter we focus on bar graphs, double bar graphs, histograms, pie charts and broken-line graphs.

19.1 Bar graphs and double bar graphs

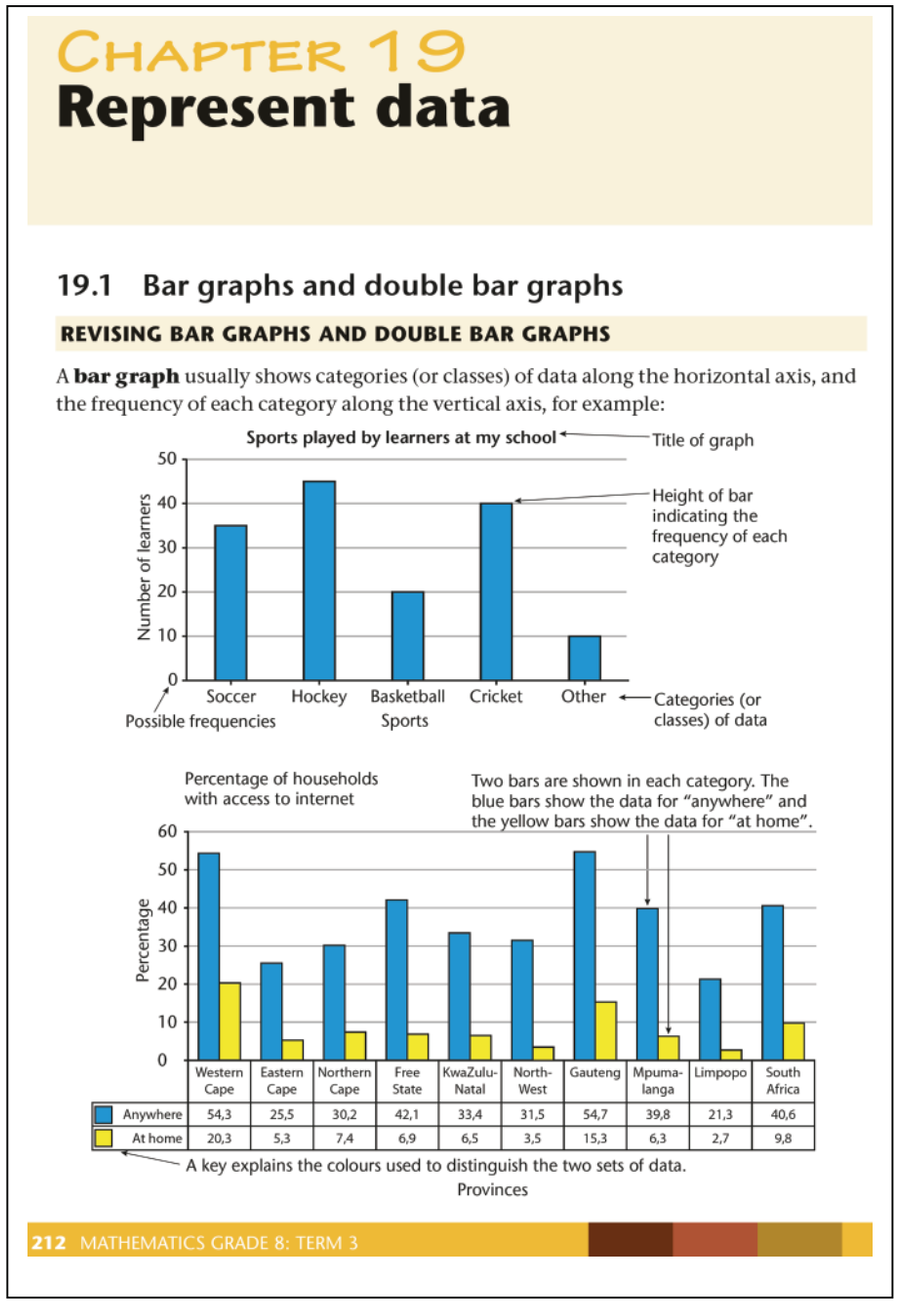
REVISING BAR GRAPHS AND DOUBLE BAR GRAPHS

Background information

- A **bar graph** usually shows categories of data along the horizontal axis, and the frequency of each category along the vertical axis. Here are some **features of a bar graph**:
  - It represents only one set of data.
  - It represents either categorical or ungrouped numerical data.
  - It never represents grouped numerical data.
  - It uses bars to show the frequencies of the different categories.
  - The heights or lengths of the bars represent the frequencies of the different categories.
  - The title (heading) tells what the bar graph is about.
- A **double bar graph** shows two sets of data in the same categories on the same set of axes. Here are some **features of a double bar graph**:
  - It represents two sets of data with matching categories.
  - It represents either categorical or ungrouped numerical data.
  - It never represents grouped numerical data.
  - It uses separate pairs of bars to show the frequencies of matching categories.
  - The heights or lengths of the bars represent the frequencies of the matching categories.
  - The title (heading) tells what the double bar graph is about.
  - The key explains the colours used to distinguish the two sets of data.
- **Vertical bar graphs** are usually used to show change over time at discrete times, for example, absentees per day of the week.
- **Horizontal bar graphs** are usually used to compare or rank items at one point in time, such as absentees per grade on a specific day.

Teaching guidelines

- Use the bar graph on LB page 212 alongside to revise the features of a bar graph.
- Use the double bar graph on LB page 212 alongside to revise the features of a double bar graph.



CHAPTER 19
Represent data

19.1 Bar graphs and double bar graphs

REVISING BAR GRAPHS AND DOUBLE BAR GRAPHS

A **bar graph** usually shows categories (or classes) of data along the horizontal axis, and the frequency of each category along the vertical axis, for example:

	Western Cape	Eastern Cape	Northern Cape	Free State	KwaZulu-Natal	North-West	Gauteng	Mpuma-langa	Limpopo	South Africa
Anywhere	54,3	25,5	30,2	42,1	33,4	31,5	54,7	39,8	21,3	40,6
At home	20,3	5,3	7,4	6,9	6,5	3,5	15,3	6,3	2,7	9,8

212 MATHEMATICS GRADE 8: TERM 3

REPRESENTING DATA IN BAR GRAPHS AND DOUBLE BAR GRAPHS

Background information

- **Bar graphs are suitable** for:
  - comparing frequencies in a set of data
  - identifying patterns in the data
  - describing trends shown by the data.
- To **draw a bar graph**, remember the following:
  - All bars should be equally wide.
  - Gaps between bars should be equally wide, but narrower than the bars, unless a bar is missing from the sequence.
  - The first bar should not touch the frequency axis.
  - A title should explain what the bar graph is about.

Teaching guidelines

Revise how to draw a bar graph.

Answers

1. (a) See LB page 213 alongside.
   (b) See bar graph given on TG page 250.
   (c) There was an increase in the road traffic deaths for all four years in a row.
   (d) No. The heights of bars are not accurate representations and they can only show differences of about one hundred clearly.
2. (a) It is possible that the type of vehicle was not recorded in these cases.
   (b) We don't know how many of these accidents were fatal. We also don't know how many people were involved in each accident.
   (c) Cars are involved in the most accidents. This does not necessarily mean that they are the least safe. You would also need to take into account how many of each kind of vehicle there are on the road. More data is needed to draw a conclusion.
   (d) See the table on LB page 213 alongside, and see bar graph given on TG page 250.

A **double bar graph** shows two sets of data in the same categories on the same set of axes. The graph at the bottom of the previous page shows the percentage of households *with* access to the internet at home, or for which at least one member has access elsewhere, by province in 2012.

REPRESENTING DATA IN BAR GRAPHS AND DOUBLE BAR GRAPHS

1. Road accidents are a big problem in South Africa, especially during the holiday season. Statistics about road accidents are published to make people aware of this problem.
   (a) Copy the table. Round off the numbers in the second column to the nearest hundred and write the result in the third column.

Year	Number of road accident deaths	Rounded-off number
2002	3 661	3 700
2003	4 445	4 400
2004	5 234	5 200
2005	5 443	5 400
2006	5 639	5 600

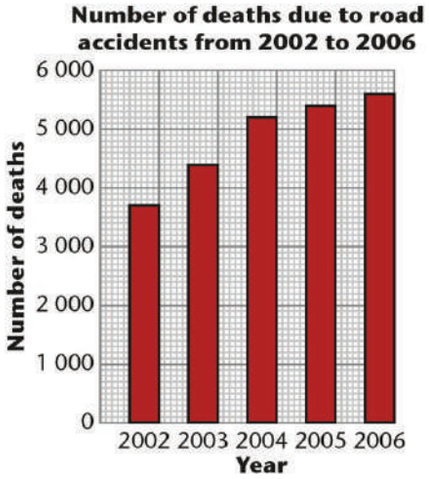
   (b) Draw a bar graph of the rounded-off numbers.
   (c) What trend do you notice in this data?
   (d) For this form of representation, do you think it makes a difference that you have rounded the data off? Explain.

2. Road accident data can be analysed in different ways. The table below shows the kinds of vehicles and the number of accidents that they were involved in, for 2011. The data is from the "Arrive Alive" campaign. Copy the table:

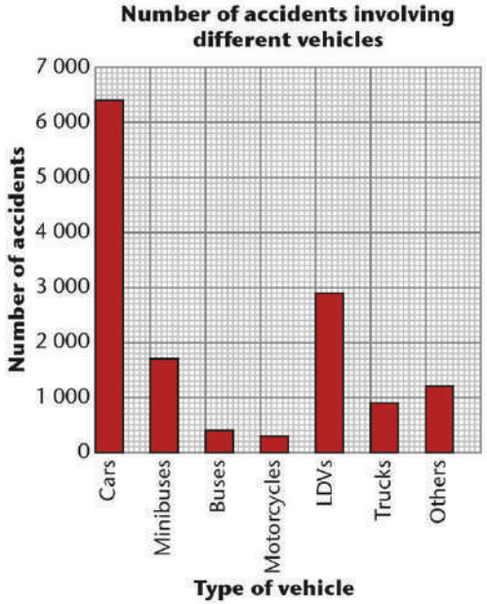
Vehicle type	Number of accidents	Rounded-off number
Cars	6 381	6 400
Minibuses	1 737	1 700
Buses	406	400
Motorcycles	289	300
LDVs and Bakkies	2 934	2 900
Trucks	861	900
Other and unknown	1 161	1 200
Total	**13 769**	**13 800**

Answers (bar graphs of questions from Section 19.1)

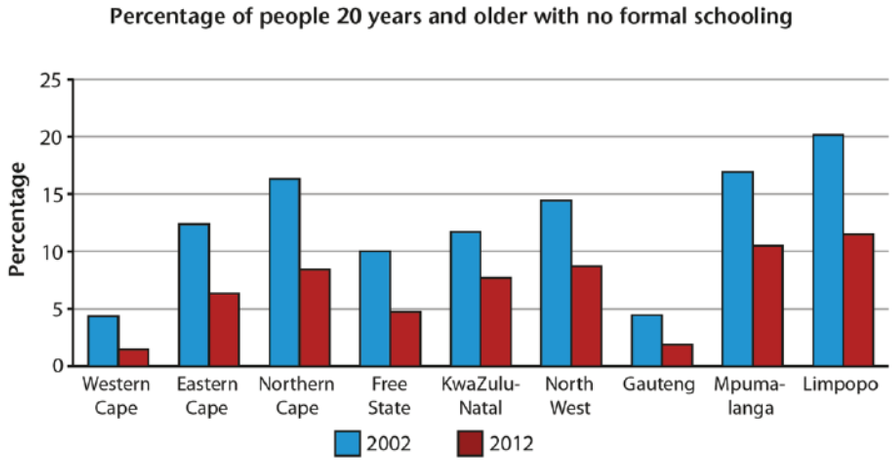
1. (b)



2. (d)



3. (b)



Background information (continued)

- **Double bar graphs are suitable** for:
  - comparing frequencies of matching categories in two data sets
  - identifying similar and different patterns in the two data sets
  - describing similar and different trends shown by the two data sets.
- To **draw a double bar graph**, remember the following:
  - All bars should be equally wide.
  - Gaps between pairs of bars should be equally wide, but narrower than the bars, unless a bar is missing from one or both sequences.
  - The first bar should not touch the frequency axis.
  - A title should explain what the double bar graph is about.
  - A key (legend) should explain the colours used to distinguish the two sets of data.

Teaching guidelines (continued)

- Revise how to draw a double bar graph.
- Point out that, in some data sets, some of the bars could be missing if the frequency for a specific category is equal to 0.

Answers

3. (a) The researchers wanted to find out whether there was an improvement in the situation over ten years.
   (b) See the double bar graph given on TG page 250.
   (c) See the answers on LB page 214 alongside.
   (d) The Western Cape and Gauteng, because there is the smallest difference between the heights of these bars. The percentages of people with no formal schooling were already low in 2002 compared to the other provinces, and so the improvement is less dramatic.

(a) A large proportion of the data involves "other and unknown" vehicle types. What could the reason be for this?
(b) What information is missing from the table? What would we need to know to get a better picture of these accidents?
(c) Which vehicle was involved in the highest number of accidents? Does this mean that this vehicle is the least safe? Explain.
(d) Round off the data in the table to the nearest 100. Then draw a bar graph of the rounded-off data.

3. Statistics South Africa released the data below in their 2012 General Household Survey (GHS):

Percentage of people 20 years and older with no formal schooling

	WC	EC	NC	FS	KZN	NW	GAU	MPU	LIM
2002	4,4	12,5	16,5	10,0	11,8	14,6	4,5	17,1	20,1
2012	1,5	6,4	8,5	4,8	7,8	8,8	1,9	10,6	11,6

(a) Why do you think the data from 2012 is compared to 2002?
(b) Plot a double bar graph of this data.
(c) Explain the data for Limpopo, by copying the sentences below and filling in the missing percentages:
The percentage of people over 20 who had no formal schooling in Limpopo in 2002 was 20,1% . In 2012, the survey showed that the percentage of people with no formal schooling was 11,6% . The difference in the percentages is 8,5% .
(d) From the graph, which provinces showed the least change in the percentage of people with no formal schooling? Explain how you know this and give a suggestion about why this could be so.

19.2 Histograms

WHAT HISTOGRAMS REPRESENT

A histogram is a graph of the frequencies of data in different **class intervals**, as shown in the example on the following page. Each class interval is used for a range of values. The different class intervals are consecutive and cannot have values that overlap. The data may result from counting or from measurement.

A histogram looks somewhat like a bar graph, but histograms are normally drawn without gaps between the bars.

19.2 Histograms

WHAT HISTOGRAMS REPRESENT

Background information

- A **histogram** shows grouped numerical data on the horizontal axis and the frequencies of data in different class intervals on the vertical axis. Here are some **features of a histogram**:
  - It represents only one set of data.
  - It represents numerical data, which is grouped in class intervals.
  - Each class interval is used for a range of data values.
  - The different class intervals are consecutive and cannot have values that overlap. This means that either the upper boundary or the lower boundary of each class interval is excluded from that interval.
  - It uses bars to show the frequencies of the different class intervals. There are no gaps between the bars because the class intervals are consecutive.
  - The heights of the bars represent the frequencies of the different class intervals.

Teaching guidelines

- Use the example on LB page 215 to revise the features of a histogram.
- Point out that the top value (also called the upper boundary) of each class interval is not included in the interval. For example, the value of 400 is included in the interval 400–600 and not in the interval 200–400.

Misconceptions

"The upper class limit is always excluded from an interval." This is NOT true. In some cases, the lower class limit is excluded. The strategy used is determined by the question we want to answer, for example:

- If we want to know how many learners scored **50% or more** in a test, 50 should be the **lower class limit** of the class interval **50–60**.
- If we want to know how many learners scored **50% or less**, 50 should be the **upper class limit** of the class interval **40–50**.

Example

Look at the following numbers of oranges harvested from 60 trees in an orchard:

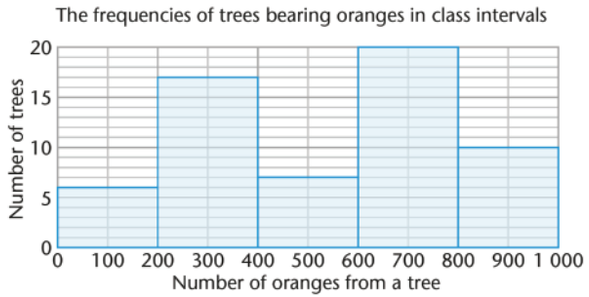
830	102	57	726	400	710	333	361	295	674	927	945
276	792	787	765	540	785	305	104	88	203	224	974
852	716	790	145	755	661	728	637	319	221	766	764
397	734	856	775	330	659	211	918	345	360	518	822
818	727	346	279	804	478	626	324	478	471	69	462

The frequencies of trees with numbers of oranges in specific class intervals are shown in the following table:

Number of oranges	Number of trees
0–200	6
200–400	17
400–600	7
600–800	20
800–1 000	10

We follow the convention that the top value (also called the **upper boundary**) of each class interval is not included in the interval. The value of 400 is therefore included in the interval 400–600 and not in the interval 200–400.

Lets look at how this data is then represented in the form of a histogram:



REPRESENTING DATA IN HISTOGRAMS

Background information

- **Histograms are suitable** for:
  - frequencies of categories of grouped numerical data
  - identifying patterns in grouped numerical data
  - describing trends shown by grouped numerical data.
- To **draw a histogram**, remember the following:
  - All bars should be equally wide if all class intervals are equally wide.
  - There are no gaps between bars unless the frequency of a specific class interval is equal to 0.
  - The title should explain what the histogram is about.

Teaching guidelines

- Revise how to draw a histogram.
- Point out that there may not be any gaps between class intervals, and that class intervals may not overlap. By convention, the upper class boundary is excluded from a class interval.

Answers

1. (a) The interval indicated as 30–40 minutes, not the interval 20–30 minutes. By convention, the top value of each class interval is not included in the interval.
   (b) See the histogram given on TG page 254.
   (c) Most learners take between 10 and 20 minutes to get to school, and many learners take up to 10 minutes, 20 to 30 minutes or 30 to 40 minutes. A few learners take between 40 and 50 minutes and even fewer take more than 50 minutes.
   (d) In a farming area it is likely that the bars will be higher in the higher intervals, as many learners will need to travel much further to get to school.
2. (a) See the histogram given on TG page 254.
   (b) See the histogram given on TG page 254.
   (c) The shapes of the histograms are similar. It may be useful to collect more data about the bulbs made by Company B. Company B tested only 60 light bulbs so it isn't a fair comparison. It looks as if Company A produces light bulbs that last longer than those of Company B. None of Company B's light bulbs lasted longer than 500 hours.

REPRESENTING DATA IN HISTOGRAMS

1. In the 2009 Census@School, the learners from Grades 3 to 7 at a certain school were asked how long (in minutes) it takes them to travel to school. The following table shows the results from a sample of 120 learners:

Time in minutes	Frequency
0–10	15
10–20	48
20–30	34
30–40	14
40–50	6
50–60	1
60–70	2

   (a) In which interval is a travel time of 30 minutes counted?
   (b) Draw a histogram to illustrate this data.
   (c) Describe in your own words what the histogram shows.
   (d) What would you expect this data to look like for a school in a farming area?

2. Company A manufactures light bulbs. They want to see how many hours (h) their light bulbs last, as they would like to use that data to promote their light bulbs. They investigate a sample of 200 light bulbs straight from the factory. Look at the following data they collect:

Lifetime (h)	300–350	350–400	400–450	450–500	500–550
Frequency	15	25	70	50	40

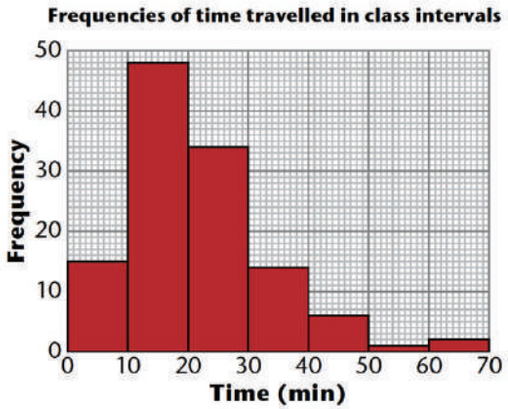
   (a) Draw a histogram to represent this data.
   (b) Company B, which makes similar light bulbs, carries out a similar experiment and gets the following results. Draw a histogram to represent this data:

Lifetime (h)	300–350	350–400	400–450	450–500	500–550
Frequency	7	11	24	18	0

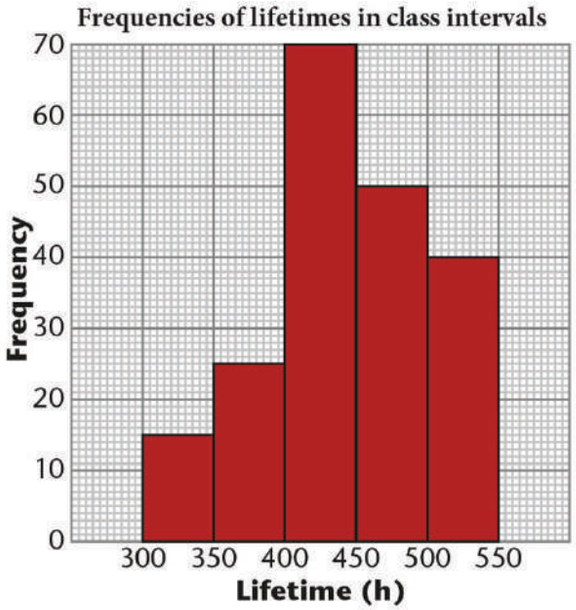
   (c) Comment on the differences between the two histograms.

Answers (histograms of questions from Section 19.2)

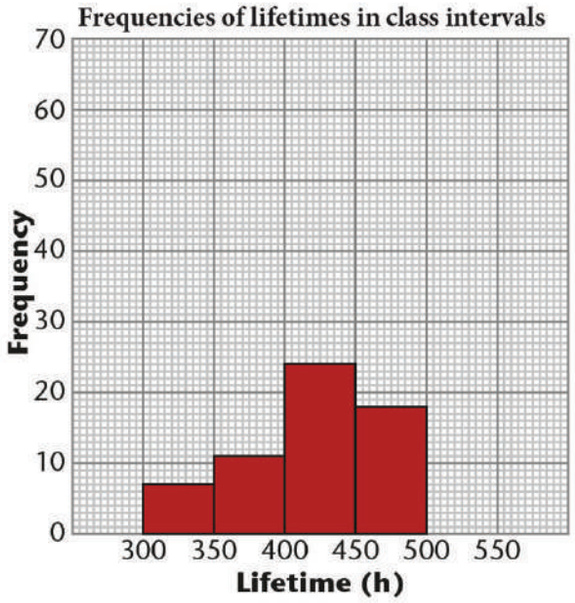
1. (b)



2. (a)



(b)



19.3 Pie charts

ESTIMATING THE SIZE OF SLICES IN A PIE CHART

Background information

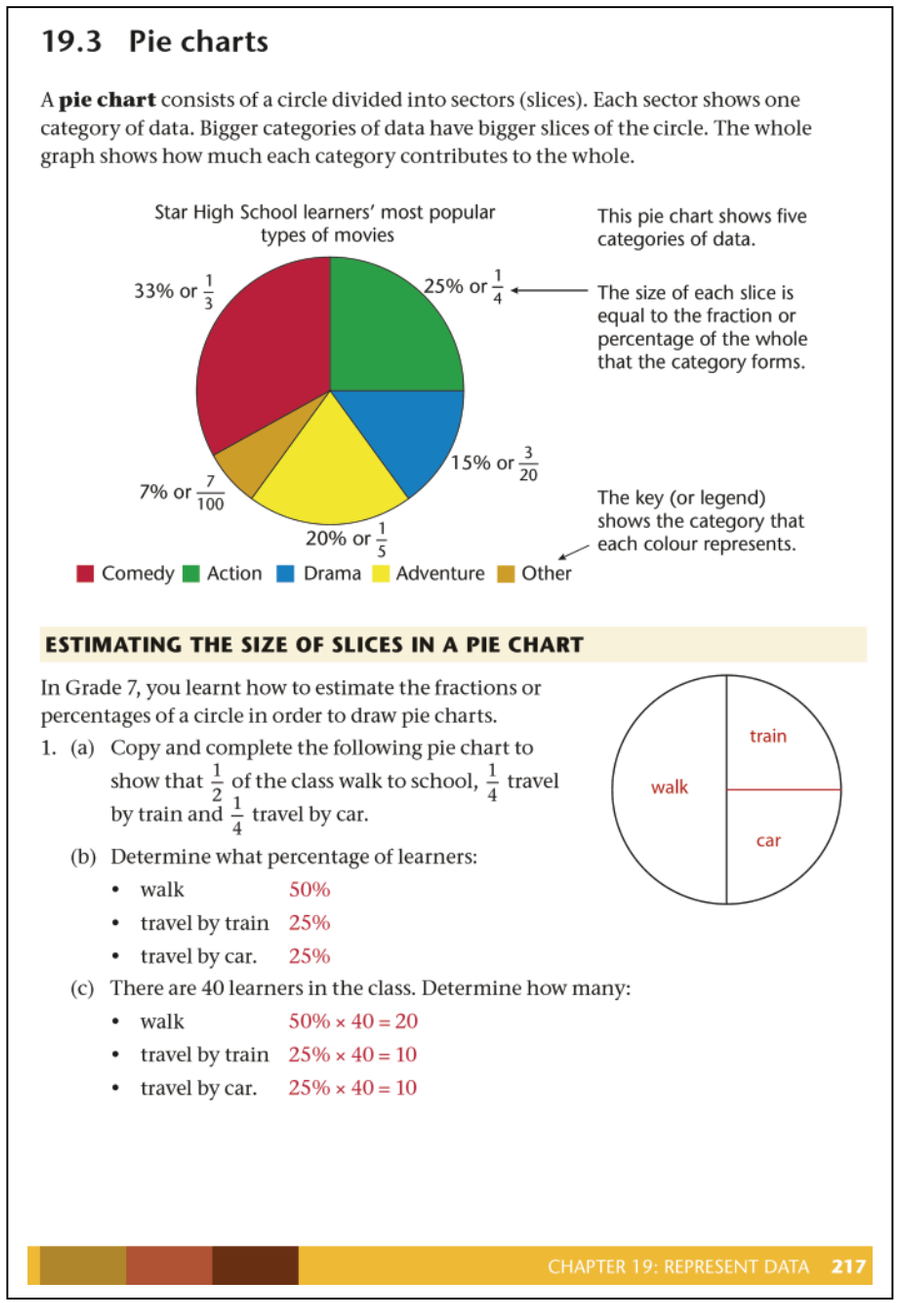
- A **pie chart** consists of a circle divided into sectors (slices). Here are some **features of a pie chart**:
  - It represents only one set of data.
  - It can be used to display any type of data.
  - It uses sectors of the same circle to compare the frequencies of the different categories within a data set. Bigger categories of data have bigger slices of the circle.
  - It shows how the data set is divided up into different categories and what fraction of the data set each category represents. The whole graph shows how much each category contributes to the whole.
  - The sizes of the sectors are proportional to the frequencies of the different categories (in other words, the fraction or percentage of the whole that the category forms).
  - It works best if the frequencies of the different categories in the data set are expressed as percentages.
  - The title (heading) shows what the data set is about.
  - The key (legend) shows the category that each sector represents.

Teaching guidelines

Use the first pie chart on LB page 217 alongside to discuss the features of a pie chart.

Answers

1. (a) See the second pie chart on LB page 217 alongside.
   (b) See the answers on LB page 217 alongside.
   (c) See the answers on LB page 217 alongside.



19.3 Pie charts

A **pie chart** consists of a circle divided into sectors (slices). Each sector shows one category of data. Bigger categories of data have bigger slices of the circle. The whole graph shows how much each category contributes to the whole.

ESTIMATING THE SIZE OF SLICES IN A PIE CHART

In Grade 7, you learnt how to estimate the fractions or percentages of a circle in order to draw pie charts.

1. (a) Copy and complete the following pie chart to show that $\frac{1}{2}$ of the class walk to school, $\frac{1}{4}$ travel by train and $\frac{1}{4}$ travel by car.

(b) Determine what percentage of learners:
- walk 50%
- travel by train 25%
- travel by car. 25%

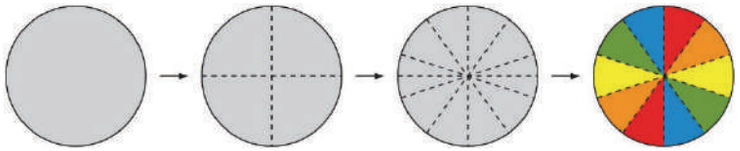
(c) There are 40 learners in the class. Determine how many:
- walk 50% × 40 = 20
- travel by train 25% × 40 = 10
- travel by car. 25% × 40 = 10

CHAPTER 19: REPRESENT DATA 217

Notes on how to draw a pie chart

Refer to question 2(d).

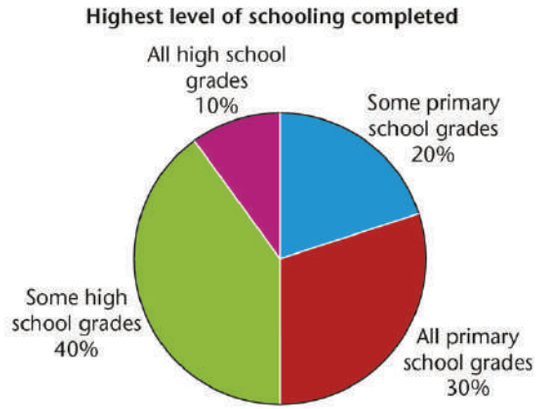
- The first category on the pie chart usually starts at a vertical line at 12 o'clock and turns in a clockwise direction.
- To divide a circle into ten equal slices using estimation, first divide it into quarters and then divide each quarter into $2\frac{1}{2}$ slices.



- To draw an accurate pie chart, each fraction can be converted to degrees by multiplying it by 360°. A protractor can then be used to construct the different sectors accurately, starting at 12 o'clock and moving in a clockwise direction.
- If a key is not provided, each sector should be accompanied by a description of the category as well as the percentage it represents.

Answers

2. (a) 180
   (b) See the third column of the table on LB page 218 alongside.
   (c) See the fourth column of the table on LB page 218 alongside.
   (d) Pie chart:



2. The following data shows the highest level of schooling completed by a group of people:

Highest level of schooling completed	Number of people	Fraction of whole	Percentage of whole
Some primary school grades	36	$\frac{36}{180} = \frac{1}{5}$ or $\frac{2}{10}$	$\frac{36}{180} \times \frac{100}{1} = 20\%$
All primary school grades	54	$\frac{54}{180} = \frac{3}{10}$	$\frac{54}{180} \times \frac{100}{1} = 30\%$
Some high school grades	72	$\frac{72}{180} = \frac{2}{5} = \frac{4}{10}$	$\frac{72}{180} \times \frac{100}{1} = 40\%$
All high school grades	18	$\frac{18}{180} = \frac{1}{10}$	$\frac{18}{180} \times \frac{100}{1} = 10\%$
Total	**180**		**100%**

(a) How many people make up the whole group?
(b) Copy the table. Complete the third column by working out the fraction of the whole group that each category makes up.
(c) Complete the fourth column by working out the percentage of the whole group that each category makes up.
(d) Draw a pie chart showing the data in the completed table. (Estimate the size of the slices.)

19.4 Broken-line graphs

PLOTTING DATA POINTS

The following table shows the average temperature in Bethal recorded every day for one week:

Day	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Temperature (°C)	4	10	12	9	13	13	11

19.4 Broken-line graphs

PLOTTING DATA POINTS

Background information

- A **broken-line graph** uses successive points, joined by line segments, to show the frequencies of different categories in a set of data collected over a period of time. Here are some **features of a broken-line graph**:
  - It represents only one set of data.
  - It represents numerical data collected at specific moments over a period of time, for example, per day over a period of one week.
  - The horizontal axis shows categories (days) in consecutive order.
  - The vertical axis shows the frequencies of the different categories by using points. The heights of these points above the horizontal axis match the frequencies of the different categories.
  - The order of the categories is indicated on the graph by joining successive points using line segments.
  - The title shows what the broken-line graph is about.

Answers

1. See the completed broken-line graph on LB page 219 alongside.

Teaching guidelines

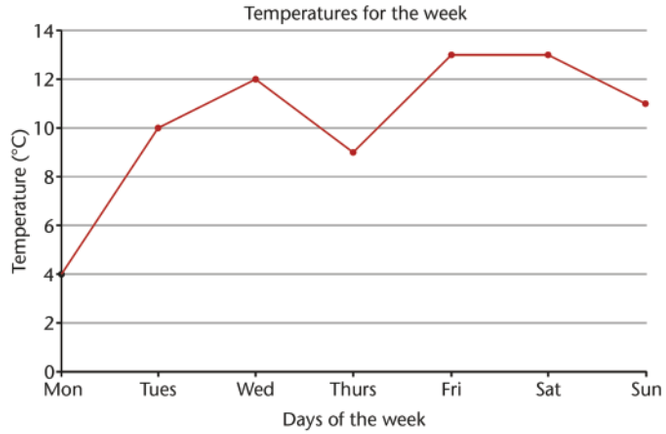
Use the broken-line graph on LB page 219 to discuss its features.

DRAWING BROKEN-LINE GRAPHS

Background information

- **Broken-line graphs are suitable** for:
  - identifying patterns in data collected over a period of time
  - describing trends shown by data collected over a period of time.
- To **draw a broken-line graph**, remember the following:
  - Consecutive categories should be positioned at equal distances from each other on the horizontal axis.
  - The heights of the points above the horizontal axis should match the frequencies of the different categories.
  - Successive points should be joined by line segments.
  - The title should explain what the broken-line graph is about.

1. Copy the axes below. Plot the data on the set of axes given. Make a dot for every point that you plot.



2. Use a ruler to join the dots in order.

You now have drawn a broken-line graph!

A **broken-line graph** is a line that joins consecutive data points plotted on a set of axes. Broken-line graphs are useful to show how something has changed or stayed the same over time.

DRAWING BROKEN-LINE GRAPHS

The following table shows the income of Pam's small business and Luthando's small business over six months:

Month	January	February	March	April	May	June
Pam's income (R)	12 000	12 000	9 000	6 000	7 000	9 000
Luthando's income (R)	6 000	7 000	8 000	8 000	9 000	9 000

Teaching guidelines

- Revise how to draw a broken-line graph.
- Point out that, if the frequency of a specific category is 0, the point that represents this category should be positioned on the horizontal axis.

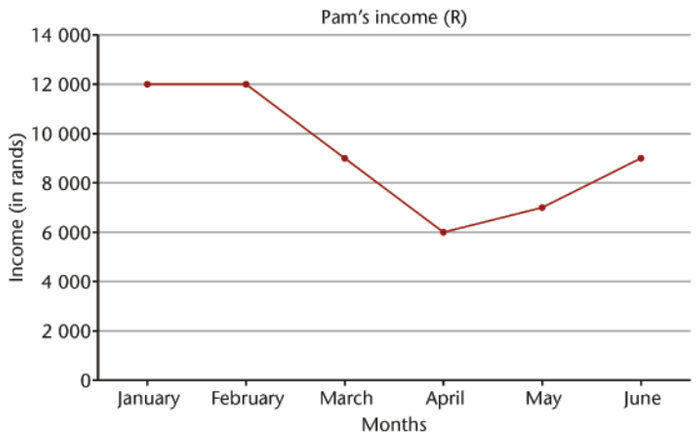
Answers

1. See the first broken-line graph on LB page 220 alongside.
2. See the second broken-line graph on LB page 220 alongside.
3. Luthando's income seems to be increasing steadily per month.

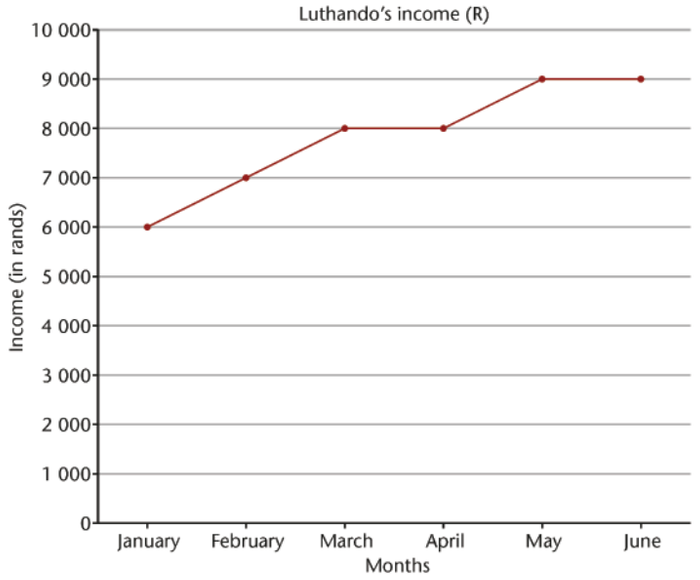
What statistical display to choose

- Different statistical displays are used for different purposes, for example:
  - to compare frequencies of one set of categorical or ungrouped numerical data, use a **bar graph**
  - to compare frequencies of two sets of categorical or ungrouped numerical data, use a **double bar graph**
  - to compare frequencies of one set of grouped numerical data, use a **histogram**
  - to compare categories within a data set, use a **pie chart**
  - to compare the frequencies of numerical data collected over a period of time, use a **broken-line graph**.

1. Copy the axes below and draw a broken-line graph showing Pam's income:



2. Copy the axes below and draw a broken-line graph showing Luthando's income:



3. Whose income seems to be increasing steadily per month?

COMPARING DIFFERENT WAYS OF REPRESENTING DATA

Background information

Refer to the information provided on the previous page under "What statistical display to choose".

Teaching guidelines

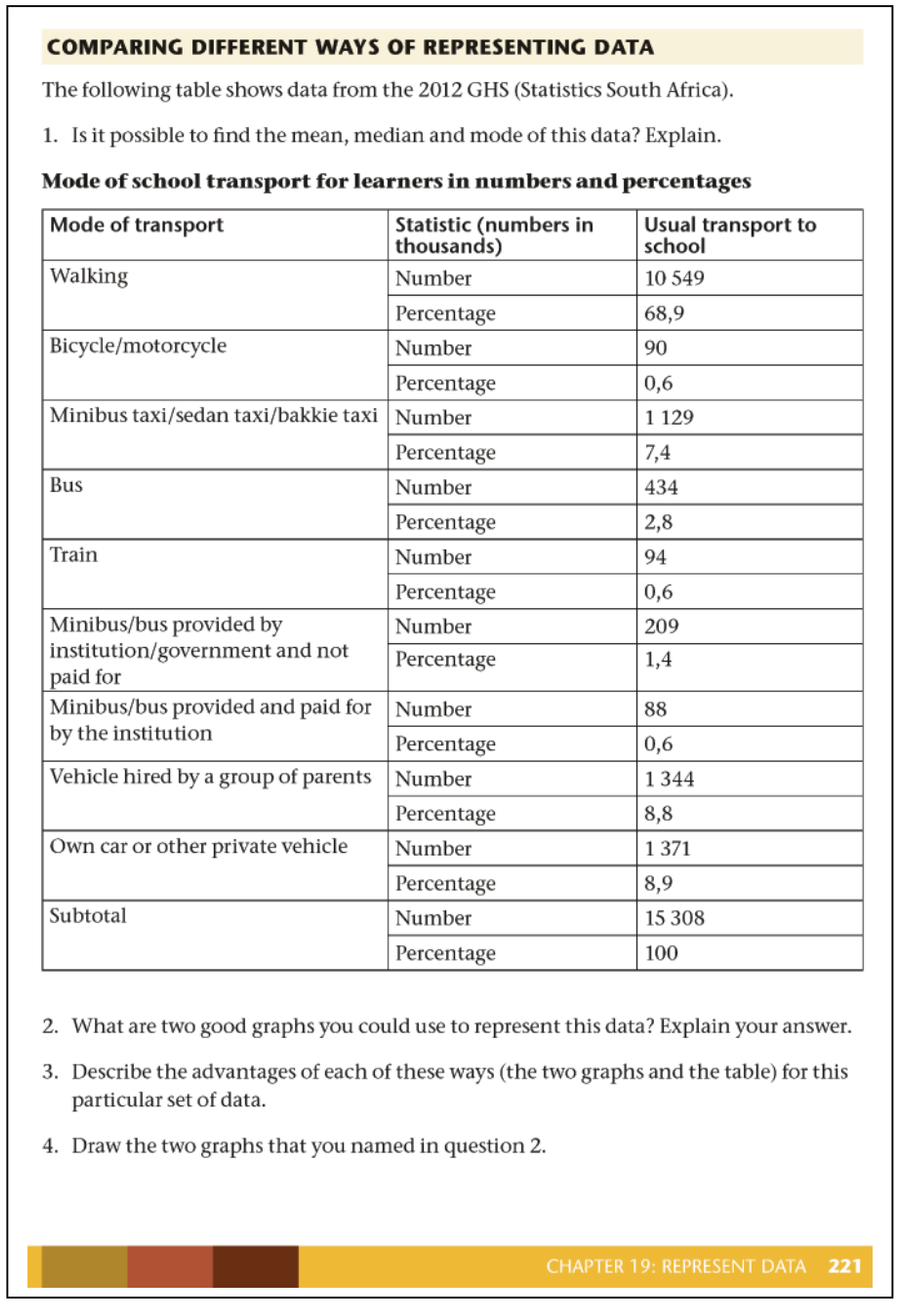
Discuss the information provided on the previous page under "What statistical display to choose".

Answers

1. No, this is not numerical data, as we are looking at the frequency of different categories. A list of data values for ten learners could look like this: walking, walking, bicycle, walking, minibus (paid for), minibus (free), walking, walking, train, bus.
2. A bar chart. The data is in categories so each bar could show the frequency of a different category and the bars could be compared. A pie chart, as we are interested in the percentage that each category contributes to the whole.
3. A bar graph would be useful for comparing the numbers in different categories, but it would be difficult to draw the scale of the graph as the highest number is over 10 000 and some of the numbers are less than 100. It could be more useful to show the percentages on the vertical axis. The smaller numbers will not be shown accurately.

   As the total adds up to 100%, the relative proportions could be shown in a pie chart. A pie chart makes it easier to see the relative contribution of each mode of transport. However, a few of the types of transport will need to be grouped together, as some of the percentages are very small.

   Notice that the table is already a good way to show the data, because there is one very large number and some of the other numbers are very small. This makes it difficult to compare them visually, unless you are only interested in the visual impact of the fact that the "walking" category is very high.
4. Learners draw the two graphs of their choice.



COMPARING DIFFERENT WAYS OF REPRESENTING DATA

The following table shows data from the 2012 GHS (Statistics South Africa).

1. Is it possible to find the mean, median and mode of this data? Explain.

Mode of school transport for learners in numbers and percentages

Mode of transport	Statistic (numbers in thousands)	Usual transport to school
Walking	Number	10 549
	Percentage	68,9
Bicycle/motorcycle	Number	90
	Percentage	0,6
Minibus taxi/sedan taxi/bakkie taxi	Number	1 129
	Percentage	7,4
Bus	Number	434
	Percentage	2,8
Train	Number	94
	Percentage	0,6
Minibus/bus provided by institution/government and not paid for	Number	209
	Percentage	1,4
Minibus/bus provided and paid for by the institution	Number	88
	Percentage	0,6
Vehicle hired by a group of parents	Number	1 344
	Percentage	8,8
Own car or other private vehicle	Number	1 371
	Percentage	8,9
Subtotal	Number	15 308
	Percentage	100

2. What are two good graphs you could use to represent this data? Explain your answer.
3. Describe the advantages of each of these ways (the two graphs and the table) for this particular set of data.
4. Draw the two graphs that you named in question 2.

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
20.1 Critically analysing how data is collected	Manipulation of data sources; manipulation of data collection methods	Pages 222 to 224
20.2 Critically analysing how data is represented	Manipulation in data representations	Page 224
20.3 Critically analysing summary statistics	Misleading summary statistics; manipulation in summary reports	Pages 225 to 226

CAPS time allocation	3,5 hours
CAPS content specification	Page 111

Mathematical background

In this chapter the focus is on interpretation and analysis of data, and reporting on findings.

- To **interpret data** means to extract information directly from the data. This process is often used to determine whether learners understand the given data.
- To **analyse data** means to investigate the given data in order to find patterns and trends shown by the data.
  - **Patterns** can be cyclic, for example, data may show that learners tend to be absent from school on Mondays.
  - **Trends** are increasing or decreasing patterns, for example, the percentage of girls per grade increases in higher grades.
- To **report on data** means to summarise the most important findings from the data in a short paragraph.
- **Bias** is a term which refers to how far the information gained from a sample lies from the information hidden by the population. Bias can already surface during the sampling process (refer to TG page 234) in the data handling cycle.
  - A **biased sampling method** is a method that tends to give non-representative samples. Such samples under-represent or over-represent some characteristics of the population. For biased sampling methods.
  - An **unbiased sampling method** is a method that tends to give representative samples. Such samples give a true reflection of the characteristics of the population.
- **Misleading data** is data which is manipulated in order to favour a specific opinion. This usually occurs when statistical displays in newspapers and financial reports are presented in such a way that the data reflects a positive picture of one party and/or a negative picture of the other party. It can be done in a variety of ways, for example:
  - Starting the scale on the frequency axis at a point other than 0.
  - Using three-dimensional displays to give a better impression of a specific category.
  - Using different scales on the frequency axes of two displays.

20.1 Critically analysing how data is collected

DATA SOURCES AND COLLECTION METHODS

Background information

- We **interpret data** by reading information directly from the data. This proves that we understand the given data.
- We **analyse data** by identifying cyclic patterns and describing increasing and decreasing trends shown by the data.
- We **report on data** by writing a short paragraph to summarise our findings.
- When the source of data is not carefully checked, or the method of data collection is not planned carefully, it can result in bias and misleading data.
    - **Bias** refers to how far the information gained from a sample lies from the information hidden by the population.
    - **Misleading data** refers to the misuse of data to create a picture which favours a specific opinion.
- The **sample** that represents the population must be large enough and must be randomly selected to reflect all features of the population.
    - The **researcher** has to be aware of all places where bias could occur, and should design the data handling process so that it does not happen.
- When **statistical reports** are read, be aware that you need information on the following before you can interpret, analyse and report on the data in a responsible manner:
    - how the sample was chosen (sampling method)
    - where the data was collected (data source)
    - how the data was collected (data collection method)
    - when the data was collected (data changes over time).

Teaching guidelines

Discuss the concepts listed above.

Answers

1. (a) Yes, the percentages are a concern.
   (b) See the answers on LB page 222 alongside.
   (c) No

CHAPTER 20

Interpret, analyse and report on data

20.1 Critically analysing how data is collected

Data collection methods can sometimes result in bias and misleading data. This is not always intended by the researcher – it often happens when the source of the data was not carefully checked, or the method of collecting data has not been planned carefully.

In Chapter 18 you learnt that a sample must be large enough to be representative and must be randomly selected from the population. If data is collected from only one part of a population, it could be biased towards that part. The researcher has to be aware of all the places where bias could occur, and should design the data-handling process so that it does not happen.

When you read reported statistics, always be aware that you need information about *how* the data was collected, *when* it was collected and *how* the sample was chosen.

Data can change over time, so you should also be aware of when it was collected. This information should be given in any report on data.

DATA SOURCES AND COLLECTION METHODS

1. Read the following paragraph and answer the questions that follow:

   A recent study revealed that 50% of high school learners smoke cigarettes, 45% drink alcohol and 60% abuse drugs. This is an indication of the general poor health and social problems of the teenage population in our country.

   (a) Do you agree that the figures are high enough to conclude that the habits of these teenagers are unhealthy?

   (b) Can we tell the following from the data above?

   - What was the sample of this study? No
   - Where was the data collected? No
   - When was the data collected? No

   (c) If the sample consisted of ten teenagers all located in an area known for drug and alcohol abuse, would the data be a good reflection of all teenagers in the country?

222 MATHEMATICS GRADE 8: TERM 3

Answers

1. (d) Possible answer: A sample that consisted of 1 000 teenagers chosen from various areas all over South Africa.

   (e) Situations and facts tend to change over time. Therefore, data also changes over time. For example, even if the data above were true ten years ago, the data is likely to have changed since then. Data should be collected and reported on at the time when it is still applicable. It is useful to compare new data to older data.

2. (a) 90% of households have flush toilets.

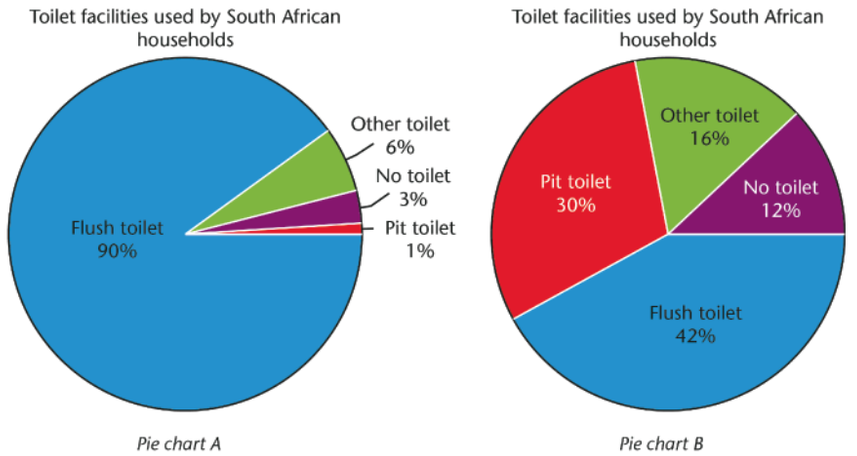
   (b) According to Pie chart B, 42% of households have flush toilets.

   (c) There is a big difference between the two sets of data. Pie chart A shows that 90% of households use flush toilets, 1% use pit toilets, 6% use other toilet facilities and 3% have no toilets. Pie chart B shows that less than half (42%) use flush toilets, 30% use pit toilets, 16% use other toilet facilities and 12% have no toilets.

(d) Describe what you think would be a better sample.

(e) Why is it important to know the date of when this data set was collected?

2. The following pie charts show the toilet facilities in households in South Africa.

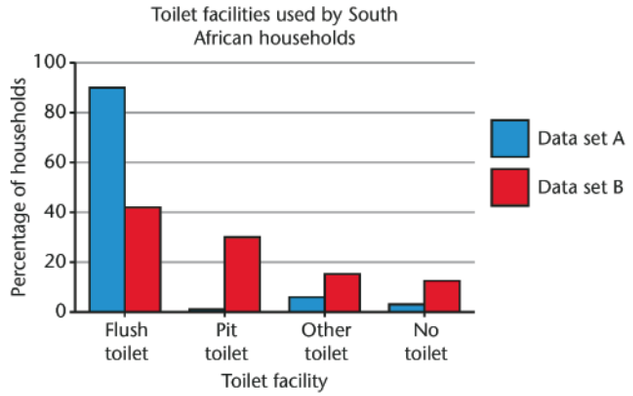


Pie chart A *Pie chart B*

(a) According to Pie chart A, what type of toilet facility do most people have and what percentage of households is this?

(b) How will your answer for (a) be different if you use Pie chart B to answer the question?

(c) Write a short report in one paragraph about the data in the pie charts.



Answers

2. (d) Almost all households have flush toilets.
   (e) Less than half (42%) of all households have flush toilets. Almost a third (30%) use pit toilets.
   (f) Possible answer: The data in Pie chart A was probably collected many years after the data in Pie chart B.
   (g) The double bar graph allows the two sets of data to be more easily compared.
   (h) The pie charts show the percentage of types of toilet facilities better.

20.2 Critically analysing how data is represented

MANIPULATION IN DATA PRESENTATION

Background information

- Statistical displays can be distorted to reflect a specific opinion, for example:
  - The frequency axis does not start at 0.
  - The scale on the frequency axis is either expanded or compressed.
  - The graph is not labelled properly.
  - Some data is left out.
  - The graph is drawn in three dimensions.

Teaching guidelines

Discuss the points listed under background information above.

Answers

1. Yes, they do.
2. Graph A uses a vertical scale that increases in units of 5°. The vertical scale of Graph B does not start at zero and it increases in units of 0,5°. Therefore, the vertical distances between two consecutive data points are much smaller in Graph A (or much bigger in Graph B).
3. They would use Graph B because the steep up and down slopes of the broken-line graph give the impression that there are big variations (changes) between the temperatures.
4. Change the scale to units bigger than 5°, for example units of 10°.
5. The graph shows that the temperatures stay more or less constant, between 16° and 20° over the seven years. Therefore, the temperatures for years 8 and 9 would most likely also stay in this temperature range.

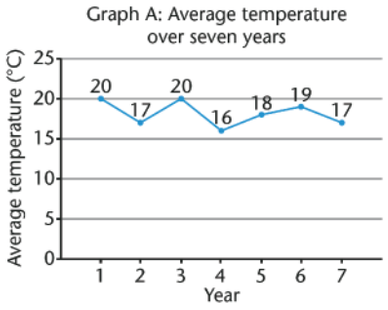
(d) What can you conclude from the data in Pie chart A?
(e) What can you conclude from the data in Pie chart B?
(f) How could the year of data collection account for the differences in the data?
(g) The graph at the bottom of the previous page shows the same data as the two pie charts. Does the diagram of the pie charts or the double bar graph allow us to compare the two sets of data more easily?
(h) Do the pie charts or the double bar graph show the percentage of types of toilet facilities used in South Africa better?

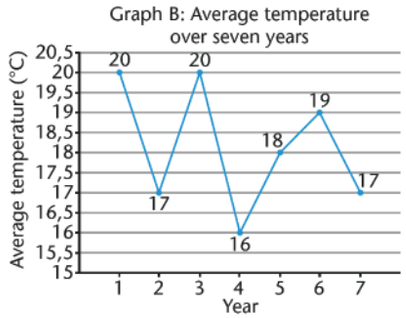
20.2 Critically analysing how data is represented

Graphs do not always show what they seem to show at first glance! If you look a bit closer, you might see that they are misleading you into drawing an incorrect conclusion. Work through the following activity to see how this can happen.

MANIPULATION IN DATA PRESENTATION

These graphs show the average temperatures recorded in the same place, at the same time.





1. Do both graphs show exactly the same data?
2. Why do the graphs look so different?
3. Which of the graphs would people use to emphasise that there are big differences in the temperatures over the years? Explain your answer.
4. Suggest a way to change the vertical scale of Graph A to emphasise even more that there are no big differences between the temperatures over the years.
5. Write a short report on Graph A. Also include a prediction of the temperatures in Years 8 and 9.

20.3 Critically analysing summary statistics

HOW SUMMARY STATISTICS CAN BE MISLEADING

Background information

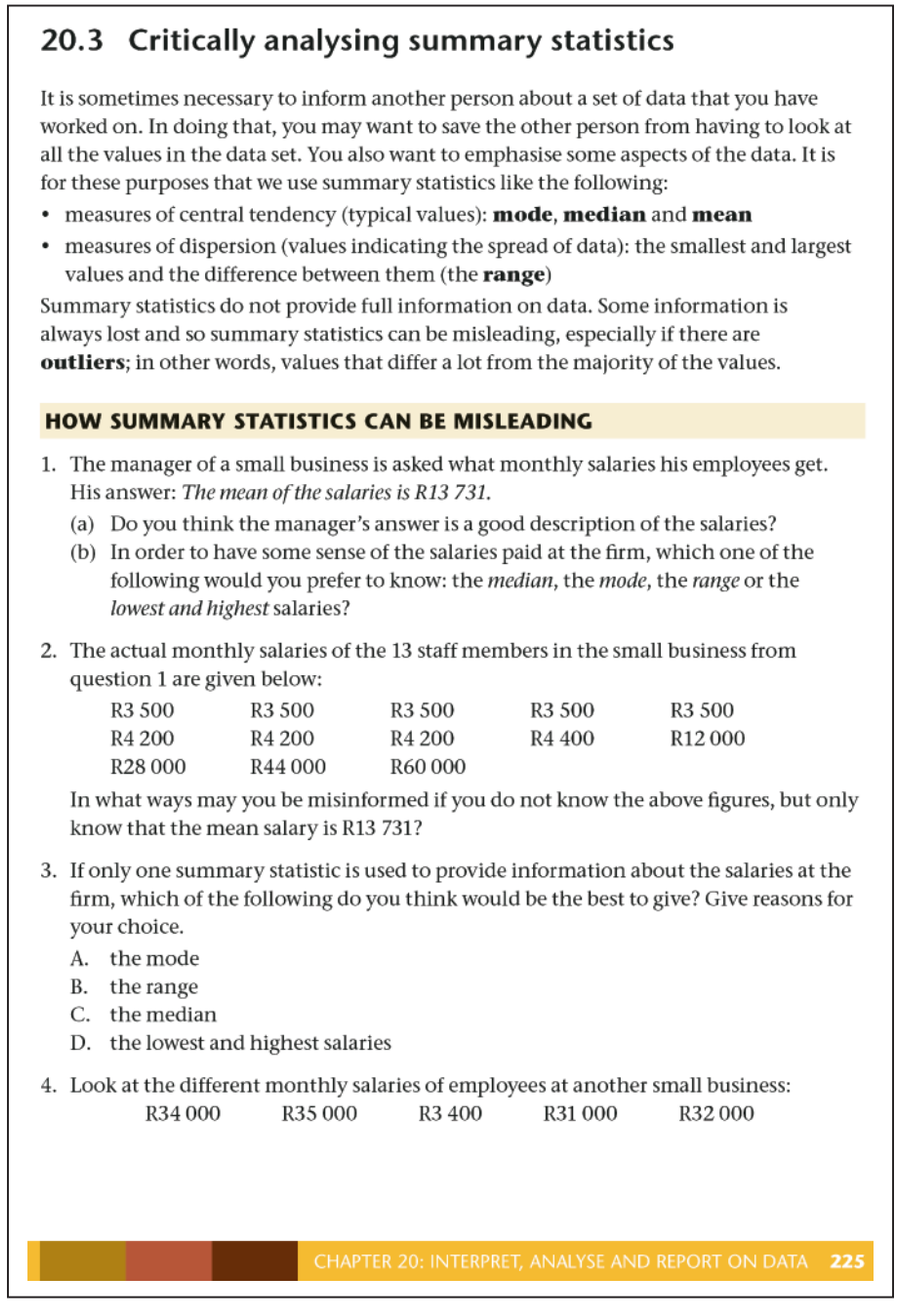
- **Summary statistics** do not provide full information on data.
  - **Measures of central tendency** (mode, median and mean) provide information about the balance in a data set.
  - **Measures of dispersion** (range) provide information on the spread of values in a data set, i.e. the difference between the smallest and largest data values.
- Some summary statistics are influenced by **outliers** (extreme values) within a data set, such as:
  - The **mean** involves all values in a data set and therefore any outlier will have an influence on its value.
  - The **range** involves the smallest and largest data values and therefore any outlier will have an influence on its value.

Teaching guidelines

Discuss how outliers can have an influence on some summary statistics.

Answers

1. (a) It is actually impossible to answer this question. It does provide learners with an opportunity to think about circumstances under which the mean may be a good summary description, and when it may be a poor summary description.
   (b) This is an open question to induce thinking.
2. This is an open question. The following are possible good answers.
   - "One may think that many of the salaries are close to R13 731. Actually only one of 13 salaries (R12 000) is close to the mean."
   - "Only three of the salaries are above the mean, but nine of the salaries are far below the mean."
3. This is an open question, requiring the learners to engage with the concepts again. It would be good to let learners think individually for a while, and then discuss this in small groups. In practice, stating the lowest and highest salaries may be most informative, for example, by saying "The salaries range from R3 500 to R60 000".



20.3 Critically analysing summary statistics

It is sometimes necessary to inform another person about a set of data that you have worked on. In doing that, you may want to save the other person from having to look at all the values in the data set. You also want to emphasise some aspects of the data. It is for these purposes that we use summary statistics like the following:

- measures of central tendency (typical values): **mode**, **median** and **mean**
- measures of dispersion (values indicating the spread of data): the smallest and largest values and the difference between them (the **range**)

Summary statistics do not provide full information on data. Some information is always lost and so summary statistics can be misleading, especially if there are **outliers**; in other words, values that differ a lot from the majority of the values.

HOW SUMMARY STATISTICS CAN BE MISLEADING

1. The manager of a small business is asked what monthly salaries his employees get. His answer: *The mean of the salaries is R13 731.*
   (a) Do you think the manager's answer is a good description of the salaries?
   (b) In order to have some sense of the salaries paid at the firm, which one of the following would you prefer to know: the *median*, the *mode*, the *range* or the *lowest and highest* salaries?

2. The actual monthly salaries of the 13 staff members in the small business from question 1 are given below:

R3 500	R3 500	R3 500	R3 500	R3 500
R4 200	R4 200	R4 200	R4 400	R12 000
R28 000	R44 000	R60 000		

   In what ways may you be misinformed if you do not know the above figures, but only know that the mean salary is R13 731?

3. If only one summary statistic is used to provide information about the salaries at the firm, which of the following do you think would be the best to give? Give reasons for your choice.
   A. the mode
   B. the range
   C. the median
   D. the lowest and highest salaries

4. Look at the different monthly salaries of employees at another small business:

R34 000	R35 000	R3 400	R31 000	R32 000

Answers

4. (a) The outlier R3 400 will cause the mean to be lower than four of the five salaries, and the mean will "hide" the fact that one salary is very low.
   (b) Mean salary = R135 400 ÷ 5 = R27 080
5. (a) The median. The mean will be too high because of the outlier 179.
   (b) The monthly sales were between 38 and 48 with the exception of one month when they were 179.
   (c) Yes. The value of selling 179 boxes of chocolates was an unusual occurrence. So leaving it out of the calculation of summary statistics seems to give a more accurate account of the typical number and range of sales in a month.

MANIPULATION IN SUMMARY REPORTS ON DATA

Background information

Some people choose the summary statistics that do not show typical values, but rather the value that works best for them.

Teaching guidelines

Discuss how people can manipulate summary statistics.

Answers

1. (a) Mean = 6,5 (or 7 when rounded off), therefore Thivha is truthful in his report.
   (b) Mode = 4; Median = 4
   (c) The mode and median are better summary statistics to describe Thivha's sales data because the majority of the values cluster around these figures, except for the value 22, which is not a typical value in the set. Its inclusion increases the mean value to 6,5. Without this extreme value, the mean for the other seven weeks would have been 4,3.
2. (a) Mode = R10; Median = R10; Mean = R13,50; Range = 50
   (b) She would probably use the mean amount, since it is higher than both the mode and the median.
   (c) The mode and median seem best (R10). The upper extreme value of R50 is unusual. So if we were to leave out the upper extreme value, the mode and median will remain R10, and the mean would be R9,44, which is much closer to R10 than the mean that includes the upper extreme.

(a) Why would the mean not be a good way to summarise this data?
(b) Calculate the mean salary.

5. The following data shows the number of boxes of chocolates sold by a store in ten consecutive months:

   42 38 179 40 43 40 48 39 41 42

   (a) Which would be the better summary description of the data, the mean or the median? Explain your answer.
   (b) Write a good summary description of the data without using the median.
   (c) Would it make sense to leave out the outlier, 179, when calculating the mean of the monthly sales? Explain your answer.

MANIPULATION IN SUMMARY REPORTS ON DATA

The mode, median and mean each highlight different bits of information about the same set of data. They can be very different from one another, depending on the kind of data set you have.

Sometimes people choose the statistic that does not show the typical values, but rather the value that works best for them.

1. Thivha sells restored furniture. He reports that he usually sells seven items per week, and that he has the data to prove it. The receipts show that he made 52 sales over a period of eight weeks.
   (a) Can you tell from the data above whether Thivha is truthful or not about the sales?
   (b) You examined the receipts for the eight weeks closely, and find the following number of sales per week:
   3, 4, 4, 4, 4, 5, 6, 22
   Determine the mode and median of the data set.
   (c) Do you think the mode, median or mean is a better reflection of Thivha's typical sales figures per week? Explain your answer.

2. The following data shows the amount of pocket money that a group of learners receives per week:

   R0 R0 R5 R10 R10 R10 R10 R20 R20 R50

   (a) Determine the mode, median, mean and range of the data set.
   (b) The teenager who receives R5 a week wants to convince her parents to give her more money. Which of the summary statistics would she use? Explain your answer.
   (c) Which summary statistic do you think best represents the weekly pocket money in this group of learners? Explain your answer.

Term 4

Chapter 21: Functions and relationships 269
21.1 Calculating output values 270
21.2 Different forms of representation 271
21.3 Completing more tables 273
21.4 Solving some problems 275

Chapter 22: Algebraic equations 279
22.1 Revision 280
22.2 Solving equations 282
22.3 Generating tables of ordered pairs 284

Chapter 23: Graphs 287
23.1 What we can tell with graphs 288
23.2 More features of graphs 293
23.3 Drawing graphs 294

Chapter 24: Transformation geometry 301
24.1 Transformations and coordinate systems 302
24.2 Translation on the coordinate system 305
24.3 Reflection on the coordinate system 307
24.4 Rotation on the coordinate system 309
24.5 Enlargements and reductions 312

Chapter 25: Geometry of 3D objects 317
25.1 Revision: 3D objects 318
25.2 Nets and models of prisms and pyramids 326
25.3 Platonic solids 342

Chapter 26: Probability 349
26.1 How often different things can happen 350
26.2 Probability 356

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
21.1 Calculating output values	Using formulae to calculate input or output values	Pages 227 to 228
21.2 Different forms of representation	Flow diagrams, tables of values, verbal descriptions and formulae are used to represent relationships; moving from one representation to another; using x to represent input values and y to represent output values	Pages 228 to 230
21.3 Completing more tables	Completing tables for different formulae but using the same input values and evaluating the output for different formulae	Pages 230 to 232
21.4 Solving some problems	Working with contextual situations described by formulae and solving particular situations	Pages 232 to 235

CAPS time allocation	6 hours
CAPS content specification	Page 113

Mathematical background

A relationship or a function between two variables can be described:

- verbally: for example, add 7 to the input number and then multiply by 2 to get the corresponding output number
- in a flow diagram: input value $\xrightarrow{\quad + 7 \quad} \xrightarrow{\quad \times 2 \quad}$ output value
- using a formula such as $y = 2(x + 7)$
- a table of values of the two variables.

We can use a formula to calculate output values for given input values.
If the formula is $y = 2x - 3$ and the input values are 5 and 10, we can calculate the corresponding output by substitution:

$y = 2(5) - 3 = 7$ and
$y = 2(10) - 3 = 17$

If the output value 13 is given, we can substitute it in the formula to produce the equation:

$13 = 2x - 3$ which, solved gives the input value as
$x = 8$

In a description of a function, the formula as well as the domain of input values should be given, for example, the input values could be the whole numbers, or the integers, etc. If the function describes a real-life situation, care should be taken that the domain is correct, for example, if the input is number of days and the output is cost to hire a car, it is clear that the input numbers are whole numbers.

21.1 Calculating output values

FORMULAE AND TABLES

Teaching guidelines

In a function a formula like $y = 2x + 6$ shows how an input value is changed in the calculation to give an output value.

If we are asked to choose a formula for a set of values from a list of formulae, we work out what the correct formula is by substituting the input values. The correct formula(e) is the one that gives the same output value as in the table for the input value. Learners should test each of the given formulae in question 1 to find the correct one.

In question 2 they have been given eight formulae and six sets of tables. Substituting the input values and comparing the calculated output value to the output value in the table associated with the input value will show learners which formula is the correct one.

Answers

1. B. $y = -5x + 20$
2. (a) E. $y = 5x + 2$
   (b) A. $y = x^2$
   (c) D. $y = x^2 + 2$

CHAPTER 21
Functions and relationships

21.1 Calculating output values

FORMULAE AND TABLES

The statement $y = 2x + 6$ can be true for any value of x, provided one chooses the appropriate value of y. The statement is true for certain combinations of values of x and y. Such a statement is called a **formula**.

A formula is a description of how the values of a dependent variable can be calculated for any given values of the other variable(s) on which it depends.

1. Which of the following are formulae for the function illustrated in the table?

   A. $y = 15x$ B. $y = -5x + 20$ C. $y = 5(20 - x)$ D. $y = 5x + 10$

x	1	2	3	4	5	6
y	15	10	5	0	–5	–10

2. For each of the tables that follow, determine which of the following formulae could have been used to complete the table. The letter symbol x is used to represent the input numbers and the symbol y represents the output numbers.

   A. $y = x^2$ B. $y = 10x$ C. $y = 10x - 1$
   D. $y = x^2 + 2$ E. $y = 5x + 2$ F. $y = -5x + 2$
   G. $y = 3^x$ H. $y = 3^{x+1}$

(a)

Input value	1	4	11	30	40	60
Output value	7	22	57	152	202	302

(b)

Input value	1	6	9	12	18	20
Output value	1	36	81	144	324	400

(c)

Input value	1	6	9	12	18	20
Output value	3	38	83	146	326	402

Answers

2. (d) B. $y = 10x$ (e) C. $y = 10x - 1$ (f) G. $y = 3^x$

21.2 Different forms of representation

FLOW DIAGRAMS, TABLES, WORDS AND FORMULAE

Teaching guidelines

The different representations of a function each give information about the function. For example, a flow diagram gives information about the calculation instructions; what should be done with the input values to get the output values.

The flow diagram can be rewritten as a formula, for example the flow diagram:

input number → × 4 → + 3 → output number

can be written as:

$y = 4x + 3$

Learners should be able to give the formula in words. The formula above, in words, would be: "multiply the input number by 4 and add 3 to the answer".

With the flow diagram or formula, and with input numbers available, a table of values can be drawn up. Such a table only shows input values and the corresponding output values to each input value. A table shows that there are many pairs of input and output values that can make the equation true.

If only the table is known, the formula could be worked out.

If an output number is known, the formula becomes an equation, for example: if $y = 19$, we can write $19 = 4x + 3$, which is usually written as $4x + 3 = y$. Now there is only one value of x that can make the equation true.

Misconceptions

Learners think of a function formula like $y = 2x + 3$ as an equation. They should be encouraged to think of an equation as having one solution or a limited number of solutions in the case of some non-linear equations.

Answers

1. (a) See the answers on LB page 228 alongside.
   (b) $y = 3x + 2$
   (c) Multiply the input number by 3 and then add 2 to the answer.
   (d) $x = 23$
   (e) $x = 86$
2. (a) See the completed table on LB page 228 alongside.

(d)

Input value	3	11	19	27	45	70
Output value	30	110	190	270	450	700

(e)

Input value	3	11	19	27	45	70
Output value	29	109	189	269	449	699

(f)

Input value	1	2	3	4	5	6
Output value	3	9	27	81	243	729

21.2 Different forms of representation

FLOW DIAGRAMS, TABLES, WORDS AND FORMULAE

1. This question is about the relationship between two variables. Some information about the relationship is given in the following flow diagram:

   input number → × 3 → + 2 → output number

   (a) Copy the table. Use the instructions in the flow diagram to complete the table:

Input value	1	2	3	4	5	10	23	50	86
Output value	5	8	11	14	17	32	71	152	260

   (b) Describe by means of a formula how to calculate the output number for any input number. (Let x represent the input numbers and y the output numbers.)

   (c) Describe verbally how to calculate the output number for any input number.

   (d) What input number will make the statement $3x + 2 = 71$ true?

   (e) What input number will make the statement $3x + 2 = 260$ true?

When there is only one output number for any input number, the relationship between the two variables is called a **function**.

2. Some information about the relation between output and input values in a certain function is given in the flow diagram:

   input number → + 2 → × 3 → output number

   (a) Copy the table. Use the flow diagram to complete the following table:

Input value	1	2	3	4	5	10	23	50	86
Output value	9	12	15	18	21	36	75	156	264

Teaching guidelines

In question 5 the four types of representation learners have worked with so far are summarised. Learners are required to:

- rewrite a formula as a flow diagram
- write the function rule in words
- complete tables using the formula
- use the table to find the other representations.

Answers

2. (b) $y = 3(x + 2)$
   (c) Add 2 to the input number and then multiply the answer by 3.
   (d) Yes, Themba is correct. The formulae $y = 3(x + 2)$ and $y = (x + 2)3$ are one and the same. The formula $y = (x + 2)3$ should, by convention, be written as $y = 3(x + 2)$.
3. See the answers on LB page 229 alongside.
4. (a) to (b) See the answers on LB page 229 alongside.
   (c) I multiply the given input number by 2 and then subtract 4 from the answer.
5. See the completed table on LB page 229 alongside and LB page 230 on the next page.

(b) Describe by means of a formula how the input and output numbers are related. Use the letter y for the output numbers and x for the input numbers.
(c) Give a verbal description of how the input and output numbers are related.
(d) Themba wrote the formula $y = (x + 2)3$ to describe how the input and output numbers are related. Is Themba correct? Explain.

3. A certain function g is represented by means of the formula $y = 2(x - 4)$.
   (a) Copy and complete the table for g:

Input value	1	2	3	4	5	6	14	44	54
Output value	–6	–4	–2	0	2	4	20	80	100

   (b) Copy and complete the flow diagram for g (fill in the operators):

input value → [– 4] → [× 2] → output value

4. (a) Copy and complete the table for the relation given by the formula $y = 2x - 4$:

Input value	1	2	3	4	5	6	14	44	54
Output value	–2	0	2	4	6	8	24	84	104

   (b) Copy and complete the flow diagram (fill in the operators):

input value → [× 2] → [– 4] → output value

   (c) Give a verbal description of how to complete the table.

5. Copy and complete the following table:

Formula	Flow diagram	Table	Verbal description
$y = 4x$	→ [× 4] →	x: 0, 3,5, 7, 0,3 y: 0, 14, 28, 1,2	Multiply the input number by 4.
$y = x - 1$	→ [– 1] →	x: 2, 3, 4, 5 y: 1, 2, 3, 4	Subtract 1 from the input number.
$y = 4x - 1$	→ [× 4] → [– 1] →	x: 2, 3, 4, 5 y: 7, 11, 15, 19	Multiply the input number by 4 then subtract 1.

21.3 Completing more tables

LOOKING AT DIFFERENT FORMULAE AT THE SAME TIME

Teaching guidelines

Learners have mainly worked with linear formulae, for example $y = 2x + 1$. Now they will substitute values into a quadratic formula ($y = 10x^2$) and into an exponential formula ($y = 10^x$).

Learners may use a calculator, so if they use the toggle button to show the display output value for 10^{-7} they may see that it can be written as $\frac{1}{10\,000\,000}$. The decimal equivalent is 0,0000001.

Handling the powers with negative exponents in this way gives learners an intuitive understanding of the meaning of a negative exponent (a^{-n}).

Furthermore, learners can experience first-hand the effects of different formulae on the same input numbers.

Misconceptions

Learners incorrectly square both 10 and the value of x in an expression like $10x^2$.

Answers

1. See the answers on LB page 230 alongside and LB page 231 on the following page.

$y = 2(x + 1)$	+1 → ×2 →	x: −2, −1, 0, 1 y: −2, 0, 2, 4	Add 1 to the input number then multiply the answer by 2.
$y = 2x + 2$	×2 → +2 →	x: −2, −1, 0, 1 y: −2, 0, 2, 4	Multiply the input number by 2 and then add 2.
$y = 2x + 1$	×2 → +1 →	x: −2, −1, 0, 1 y: −3, −1, 1, 3	Multiply the input number by 2 and then add 1.

In Sections 21.1 and 21.2, you have used four different ways to represent functions, namely:

- a formula
- a table
- a flow diagram
- a verbal representation.

Later in this term you will represent functions by using **coordinate graphs**.

21.3 Completing more tables

LOOKING AT DIFFERENT FORMULAE AT THE SAME TIME

1. Copy the table. Use the given formulae in each column to complete the table. Some rows have been completed for you. You may use a calculator.

x	$y = 10x$	$y = 10x^2$	$y = 10^x$
−7	−70	490	0,0000001
−6	−60	360	0,000001
−5	−50	250	0,00001
−4	−40	160	0,0001
−3	−30	90	0,001
−2	−20	40	0,01
−1	−10	10	0,1

Teaching guidelines

You could use the answers to the table in question 3 to help learners understand that an expression like $-2x$ represents a number, therefore $-2x - 1$ is one less and $-2x + 1$ is one more than the number.

This means that the three numbers are consecutive numbers. If learners are asked to write three consecutive numbers, they should choose the one in the middle first and subtract one for the previous number and add one for the next number. For example, write consecutive numbers. Choose x for the one in the middle, then the numbers are:

$x - 2$ $x - 1$ x $x + 1$ $x + 2$

Answers

2. (a) Increases by equal amounts
   (b) Increases by greater amounts
   (c) Increases by greater and greater amounts
3. (a) See the answers on LB page 231 alongside.
   (b) The output numbers in the left column are always one less than those in the middle column. The output numbers in the right column are always one more than those in the middle column.

x	$y = 10x$	$y = 10x^2$	$y = 10^x$
0	0	0	1
1	10	10	10
2	20	40	100
3	30	90	1 000
4	40	160	10 000
5	50	250	100 000
6	60	360	1 000 000
7	70	490	10 000 000

2. In each case, choose the correct answer from those given in brackets.
   As the input value increases by equal amounts (say from 1 to 2, 2 to 3, 3 to 4, and so on), the output value for:
   (a) $y = 10x$ (increases by equal amounts/increases by greater and greater amounts)
   (b) $y = 10x^2$ (increases by equal amounts/increases by greater amounts)
   (c) $y = 10^x$ (increases by equal amounts/increases by greater and greater amounts)

3. (a) Copy and complete the table below. Some examples have been done for you.

x	$y = -2x - 1$	$y = -2x$	$y = -2x + 1$
−4	$-2 \times -4 - 1 = 7$	$-2 \times -4 = 8$	$-2 \times -4 + 1 = 9$
−3	$-2 \times -3 - 1 = 5$	$-2 \times -3 = 6$	$-2 \times -3 + 1 = 7$
−2	$-2 \times -2 - 1 = 3$	$-2 \times -2 = 4$	$-2 \times -2 + 1 = 5$
−1	$-2 \times -1 - 1 = 1$	$-2 \times -1 = 2$	$-2 \times -1 + 1 = 3$
0	$-2 \times 0 - 1 = -1$	$-2 \times 0 = 0$	$-2 \times 0 + 1 = 1$
1	$-2 \times 1 - 1 = -3$	$-2 \times 1 = -2$	$-2 \times 1 + 1 = -1$
2	$-2 \times 2 - 1 = -5$	$-2 \times 2 = -4$	$-2 \times 2 + 1 = -3$
3	$-2 \times 3 - 1 = -7$	$-2 \times 3 = -6$	$-2 \times 3 + 1 = -5$
4	$-2 \times 4 - 1 = -9$	$-2 \times 4 = -8$	$-2 \times 4 + 1 = -7$

   (b) Describe the relationships between the corresponding output numbers in the three columns.

Teaching guidelines

You could use the information in the table for question 4 to reinforce learners' knowledge about exponents:

- $2^{x-1} = 2^x . 2^{-1} = \frac{2^x}{2}$ or $\frac{1}{2} \times 2^x$ which is half of 2^x
- $2^{x+1} = 2^x . 2$ which is double 2^x.

Answers

4. (a) See the answers on LB page 232 alongside.
   (b) The output numbers in the left column are always half of those in the middle column. The output numbers in the right column are always double those in the middle column.

21.4 Solving some problems

LOOKING AT SOME SITUATIONS

Teaching guidelines

For question you could consider letting learners use squared paper to draw rectangles with area 24 squares on the paper. They may see that the possibilities they get with sides that are whole numbers will be determined by the factor pairs of 24. Learners could consider what the side length of a square would be if the area is 24 square units (answer: $\sqrt{24} = 2\sqrt{6}$).

Notes on the questions

The first question allows learners to use a formula to find input values from known output values and to find output values from known input values.

The second and third questions are important enough for you to spend some time helping learners to see the link between the functions and the geometrical situation the function describes.

Answers

1. (a) See the answers on LB page 232 alongside.
   (b) 22
   (c) For $x = 5$
   (d) $x = 1$
2. See the answers on LB page 232 alongside.

4. (a) Copy and complete the following table:

x	$y = 2^{x-1}$	$y = 2^x$	$y = 2^{x+1}$
–1	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$
0	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$
1	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$
2	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$
3	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$
4	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
5	$2^4 = 16$	$2^5 = 32$	$2^6 = 64$
6	$2^5 = 32$	$2^6 = 64$	$2^7 = 128$

   (b) Describe the relationships between the corresponding output numbers in the three columns.

21.4 Solving some problems

LOOKING AT SOME SITUATIONS

1. The formula $y = 38 - 2x$ describes the relationship between y and x in a certain situation.
   (a) Copy and complete the following table for this situation:

x	10	5	15	1	8	18
y	18	28	8	36	22	2

   (b) What is the output value if the input value is 8?
   (c) For what input value is the output value equal to 28?
   (d) Which input value makes the statement $36 = 38 - 2x$ true?

2. Consider rectangles which each have an area of 24 square units. The breadth of the rectangles (y) varies in relation to the length (x) according to the formula $xy = 24$.
   Copy and complete the following table to represent this situation:

Length (x)	1	2	3	4	6	8	12	24
Breadth (y)	24	12	8	6	4	3	2	1

3. Consider rectangles with a fixed perimeter of 24 units. The breadth of the rectangles (y) varies in relation to the length (x) according to the formula $2(x + y) = 24$.

Teaching guidelines

In question 2 the product $x \times y$ was constant (= 24) and in question 3 the sum $x + y$ is constant (= 12).

In order to answer question 5(c) learners should see from the last answer in question (b) that $y = 0$ when $x = 50°$, and realise that when $y = 0$, the gap is closed. Otherwise, if they do not notice that the answer has already been found, they can solve the equation $0 = 2{,}5 - 0{,}05x$.

Answers

3. See the answers on LB page 233 alongside.
4. (a) See the answers on LB page 233 alongside.
   (b) $180° - \frac{360°}{20} = 162°$ and $180° - \frac{360°}{120} = 177°$ respectively
   (c) 12 sides (from the table)
5. (a) See the answers on LB page 233 alongside.
   (b) $y = 2{,}5$ cm $y = 1{,}6$ cm $y = 2{,}6$ cm $y = 0$ cm
   (c) 50 °C

Copy and complete the following table to represent this situation:

x	1	2	3	4	6	7	8	9	10	11
y	11	10	9	8	6	5	4	3	2	1

4. The formula $b = 180° - \frac{360°}{n}$ gives the size b of each interior angle in degrees for a regular polygon with n sides (an n-gon).
   (a) Copy and complete the following table:

Number of sides (n)	3	4	5	6	10	12
Angle size (b)	60°	90°	108°	120°	144°	150°

   (b) What is the size of each interior angle of a regular polygon with 20 sides, and a regular polygon with 120 sides?
   (c) If each interior angle of a polygon is 150°, how many sides does it have?

5. As you may know, metals contract when temperatures are low and expand when temperatures are high. So, when engineers build bridges they always leave small gaps in the road between sections to allow for heat expansion.

   For a certain bridge, engineers use the formula $y = 2{,}5 - 0{,}05x$ to determine the size of the gap for each 1 °C rise in temperature, where x is the temperature in °C.

   (a) Copy and complete the following table to show the size of the gap at different temperatures:

Temperature (°C)	3	4	10	15	25	30	35
Gap size (cm)	2,35	2,30	2	1,75	1,25	1	0,75

   (b) What is the size of the gap at each of the temperatures shown below?
   0 °C 18 °C −2 °C 50 °C
   (c) At what temperature will the gap close completely?

6. The formula $y = 0{,}0075x^2$, where x is the speed in kilometres per hour and y is the distance in metres, is used to calculate the braking distance of a car travelling at a particular speed. Use a calculator for this question.

   The braking distance is the distance required for a vehicle travelling at a certain speed to come to a complete stop after the brakes have been applied.

Example

What is the braking distance if someone drives at 80 kilometres per hour?

On your scientific calculator you must punch in 0,0075 followed by × sign followed by (80) followed by x^2. The calculator will do the following:

$y = [0{,}0075 \times (80)^2] = (0{,}0075 \times 6\,400) = 48$

∴ The braking distance is 48 m.

Teaching guidelines

Learners work with multiples of 10, which they should know from previous work, therefore they should be able to do the calculations without a calculator.

Answers

6. (a) $y = 0{,}0075x^2 = [0{,}0075 \times (100)^2]$ m = $(0{,}0075 \times 10\,000)$ m = 75 m
   (b) $y = [0{,}0075 \times (60)^2]$ m = $(0{,}0075 \times 3\,600)$ m = 27 m
   (c) See the answers on LB page 234 alongside.
   (d) The car will hit the sheep because at a speed of 40 km/h it will travel for another 12 m before it comes to a stop.
   (e) $y = [0{,}0075 \times (90)^2]$ m = 60,75 m
   The driver needs 60,75 m to stop the car.
7. (a) See the answers on LB page 234 alongside.
   (b) $y = 1{,}06 \times 750$
   $y = 795$
   (c) $y = 1{,}06 \times 2\,500$
   $y = 2\,650$
   (d) See the answers on LB page 234 alongside.
   (e) $x = 583 \div 1{,}06 = 550$
   (f) $x = 954 \div 1{,}06 = 900$
   (g) $x = 1\,000$ (from the table)
   (h) $x = 500$ (from the table)
8. See the answers on LB page 234 alongside.

(a) What is the braking distance at a speed of 100 kilometres per hour?
(b) Calculate the braking distance at a speed of 60 kilometres per hour.
(c) Copy and complete the table below. Give answers to two decimal places where necessary.

Speed in km/h	10	20	30	40	50	60	100
Braking distance in m	0,75	3	6,75	12	18,75	27	75

Refer to the table in question (c) to answer question (d) below.
(d) A car travels at a speed of 40 kilometres per hour. A sheep, 7 m away on the side of the road, suddenly runs onto the road. Will the car hit the sheep or will the driver be able to stop the car before it hits the sheep? Explain.
(e) A car travels at a speed of 90 km/h in an area that has school children crossing the road. What distance does the driver need to stop the car so that it does not hit the children?

7. (a) Copy the table below. Use the formula $y = 1{,}06x$ to complete the table:

x	100	200	300	400	500	1 000	5 000	10 000
y	106	212	318	424	530	1 060	5 300	10 600

(b) What is the value of y if $x = 750$?
(c) What is the value of y if $x = 2\,500$?
(d) Represent the formula $y = 1{,}06x$ by means of a flow diagram.



(e) If $y = 583$, what is the value of x?
(f) If $y = 954$, what is the value of x?
(g) The statement $1\,060 = 1{,}06x$ is given. For what value of x is the statement true?
(h) The statement $530 = 1{,}06x$ is given. For what value of x is the statement true?

8. The formula $y = 0{,}1x + 5\,000$ is given. What is the value of y in each case below?
   (a) $x = 10$ $y = 0{,}1 \times 10 + 5\,000 = 5\,001$
   (b) $x = 100$ $y = 0{,}1 \times 100 + 5\,000 = 5\,010$
   (c) $x = 1\,000$ $y = 0{,}1 \times 1\,000 + 5\,000 = 5\,100$
   (d) $x = 10\,000$ $y = 0{,}1 \times 10\,000 + 5\,000 = 6\,000$

Teaching guidelines

Learners should apply proportional thinking. For example, from the table they know the VAT inclusive price (R4,56) for an item that costs R4, so for an item that costs R40 the VAT inclusive price will be 10 times more (R45,60) and for an item that costs R400 the VAT inclusive price will be 100 times more (R456,00), and so on.

Answers

9. (a) 1,14 × R38 = R43,32 and 1,14 × R50 = R57,00
   (b) See the completed table on LB page 235 alongside.
10. (a) 10 × R4,56 = R45,60
    (b) 100 × R4,56 = R456
    (c) 10 × R7,98 = R79,80
    (d) R456 + R79,80 = R535,80
11. (a) R10
    (b) R300
12. (a) $y = 75 - 0{,}1 \times 0$
    $y = 75 - 0$
    $y = 75$
    (b) $y = 75 - 0{,}1 \times 750$
    $y = 75 - 75$
    $y = 0$
    (c) See the completed table on LB page 235 alongside.
    (d) $x = 750$
    (e) $x = -250$

9. The formula $y = 1{,}14x$ is used to calculate the price y of goods including VAT in rands, where x is the price in rands before VAT.
   (a) How much will you pay at the counter for goods that cost R38 without VAT, and for goods that cost R50 without VAT? You may use a calculator.
   (b) Copy and complete the following table for the prices of goods with VAT:

x	1	2	3	4	5	6	7	8	9
y	1,14	2,28	3,42	4,56	5,70	6,84	7,98	9,12	10,26

10. Use your answers for question 9(b) to find the prices with VAT for goods with the prices before VAT was added (below). Do not use a calculator at all in this question.
    (a) R40 (b) R400
    (c) R70 (d) R470

11. (a) An article costs R11,40 with VAT included. What is the price before VAT was added?
    (b) An article costs R342 with VAT included. What is the price before VAT was added?

12. Consider the function represented by $y = 75 - 0{,}1x$. What is the value of y if:
    (a) $x = 0$? (b) $x = 750$?
    (c) Copy and complete the following table for the function represented by $y = 75 - 0{,}1x$:

x	0	10	20	50	100	200	500	700	800
y	75	74	73	70	65	55	25	5	–5

    (d) For what value of x is $75 - 0{,}1x = 0$?
    (e) For what value of x is $75 - 0{,}1x = 100$?

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
22.1 Revision	Setting up equations to describe situations; writing formulae using variables and constants; solving equations by inspection	Pages 236 to 238
22.2 Solving equations	Understanding the use of additive and multiplicative inverses in solving equations; solving exponential equations	Pages 238 to 239
22.3 Generating tables of ordered pairs	Using a formula to generate a table of values; testing formulae to match them with values in a table	Pages 240 to 241

CAPS time allocation	3 hours
CAPS content specification	Page 113

Mathematical background

We use mathematics to describe the world around us by setting up formulae and equations to describe situations. We then solve the equations and interpret the solutions to improve the descriptions if necessary. For example, a production process must be completed within ten days; let's say the planting of 1 500 apple trees. The number of apple trees planted in a day is given by x. So $10x = 1\,500$, to make a simple equation.

Another example could be: A farmer has 300 trees in an orchard and can plant 30 trees in a day. The number of trees in his orchard after x days is given by the formula $y = 30x + 300$. The formula can be translated into a table of values from which deductions can be made, for example, read off from the table how many trees there will be after 20 days, etc. This involves substituting input values to find output values.

Another question that can be asked is how many days it will take until he has 3 000 trees planted in the orchard. This involves solving the equation $3\,000 = 30x + 300$. There are various methods of doing this:

- using a table of values, but it would not be very efficient to count in units of thirty until you get to 3 000
- using inspection and trial and improvement
- using inverse operations to isolate the unknown.

This last method requires that one understands that the left-hand side of the equation has the same value as the right-hand side. When one changes a side (by adding a number or multiplying by a number) the same changes have to be made to the other side to keep the equality to be true.

The changes that are made will be to isolate the unknown value. We do this by reversing the flow diagram of the formula, for example, in $3x + 300$ the flow diagram was $x \longrightarrow \times 3 \longrightarrow + 300 \longrightarrow$. We reverse it by undoing each operation to get back to x. To undo the operations, we use the inverse operations: we added 300 so we subtract 300 (or add –300); we multiplied by 3 so we divide by 3 (or multiply by $\frac{1}{3}$).

The inverse of a number is that number that gives the identity element under the particular operation, for example, the additive inverse of a number x is $-x$ so that $x + (-x) = 0$ and the multiplicative inverse of x is $\frac{1}{x}$ so that $x \times \frac{1}{x} = 1$.

22.1 Revision

SETTING UP EQUATIONS TO DESCRIBE PROBLEM SITUATIONS

Teaching guidelines

The ability to set up equations from verbal descriptions of a situation is an important skill. Learners have to recognise the relationship between two quantities and decide which quantity is the dependent one.

Making a table can help them make sense of the information. From the table they can write a formula to describe the situation.

Ask questions that require solving of an equation, for example: After how many days will there be 1 000 apple trees in the orchard. Learners can get the answer by working with the table, which requires counting onwards in twenties. Explain that, although it is not wrong, this is not an efficient process and that mistakes could be made along the way.

The relationship between the quantities is known and is described by a formula, so if one of the quantities is known, we can use the relationship to work out the other, unknown one. Usually we find the dependant variable, in this case, the number of trees after x days. We substitute the value of x into the formula.

If we know the value of y and want to know what the corresponding value of x is, we have to solve an equation. This chapter is about learning that process.

At first learners can use the methods they learnt before, for example trial and improvement. Recording the results in a table is useful.

Answers

1. (a) $y = 100 + 20x$
   (b) $y = 250 + 10x$
   (c) $y = 250 + 10 \times 14 = 250 + 140 = 390$
   (d) 120; 140; 160; 180; 200; 220; 240; 260: after eight days

CHAPTER 22
Algebraic equations

22.1 Revision

SETTING UP EQUATIONS TO DESCRIBE PROBLEM SITUATIONS

1. Farmer Moola has already planted 100 apple trees and 250 orange trees on his fruit farm. He decides to plant 20 more apple trees every day, as can be seen in the following table:

Number of days (x)	0	1	2	3	4	5	6	7	8
Number of trees (y)	100	120	140	160					

He also decides to plant ten orange trees a day, as shown below:

Number of days (x)	0	1	2	3	4	5	6	7	8
Number of trees (y)	250	260	270	280					

(a) Write a rule for calculating the number of apple trees after x days. Write the rule in the form of a formula. Represent the number of trees with the letter symbol y.
(b) Write a formula for finding the number of orange trees after x days.
(c) How many orange trees are there on the fourteenth day?
(d) After how many days will Farmer Moola have 260 apple trees?

After how many days will farmer Moola have 1 000 apple trees in his orchard?

It will take some time to work this out by counting in twenties. You can easily make a mistake and not even be aware of it. Another way to find the information is to figure out for which value of x it will be true that $100 + 20x = 1\,000$. To do this you may try different values for x until you find the value that makes $100 + 20x$ equal to 1 000. It is convenient to enter the results in a table as shown below. Anna first tried $x = 10$ and saw that 10 is far too small. She next tried $x = 100$ and it was far too big. She then tried 50.

Number of days (x)	10	100	50		
Number of apple trees (y)	300	2 100	1 100		

Teaching guidelines

You could talk about the domain (without calling it that). There cannot be half of a fruit tree or half of a child in a crèche, or negative values, so the input values must be whole numbers in these situations.

You could also ask learners to calculate after how many years the number of children in the crèche, described in question 4, will have dropped to 0 if the trend continues. (The answer is eight years.)

Answers

2. Consider learners' answers, for example, a number smaller than 50, such as 40 or 45.
3. Learners may do this in a number of different ways:
   - by counting on in tens and keeping track
   - by trying different numbers of days with or without using the equation, $250 + 10x = 900$
   - by subtracting 250 from 900 and dividing by 10
   - by formally solving the equation.

   If such a variety of methods emerge in class, it may be good to expose the class to all the methods and to discuss them all. Note that subtracting 250 from 900 and dividing by 10 is the same process as to solve the equation $250 + 10x = 900$ by using inverse operations (section 22.3 of this chapter).
4. y is the number of children admitted to the crèche each year.
   5 is the decrease in the number of children each year and x is the number of years.
   40 is the initial number of children at the crèche.
5. y is the total number of children in a given year.
   20 is the number of children when the crèche started.
   3 is the increase in the number of children per year and x is the number of years.
6. (a) The total numbers of naartjie and lemon trees changes, and the number of each kind of tree planted on each day stays the same.
   (b) $67 + 10 \times 23 = 67 + 230 = 297$ naartjie trees and $128 + 10 \times 17 = 298$ lemon trees
   (c) $y = 67 + 23x$ and $z = 128 + 17x$ for naartjie and lemon trees respectively
   (d) You can calculate after how many days into the planting season there will be 500 naartjie trees.
   (e) $128 + 17x = 500$

2. What number do you think Anna should try next, in her attempt to solve the equation $100 + 20x = 1\,000$?
3. How many days after he had 250 orange trees, will farmer Moola have 900 orange trees on his farm?
4. In 2004, there were 40 children at Lekker Dag Crèche. From 2005 onwards, the number of children dropped by about five children per year. Explain what each number and letter symbol in the formula $y = 40 - 5x$ may stand for.
5. Cool Crèche started with 20 children when it was opened in 2008. The number of children in Cool Crèche increases by three children every year. Explain what each letter symbol and number in the formula $y = 20 + 3x$ stands for.
6. Farmer Thuni already has 67 naartjie trees and 128 lemon trees on his fruit farm. He decides to plant 23 new naartjie trees and 17 new lemon trees every day during the planting season.
   (a) Which quantities in this situation change as time goes by, and which quantities remain the same?
   (b) How many naartjie trees and how many lemon trees will he have in total, ten days after the planting season has started?
   (c) Write formulae that can be used to calculate the total numbers of naartjie and lemon trees after any number of days during the planting season. Use letter symbols of your own choice to represent the variables.
   (d) What information about the situation on farmer Thuni's farm can be obtained by solving the equation $67 + 23x = 500$?
   (e) Set up an equation that can be used to find out how many days into the planting season it will be when farmer Thuni has 500 lemon trees. Use a letter symbol of your own choice to represent the unknown number of days.

Quantities that change are called **variables**, and are represented with letter symbols in formulae and equations. Quantities that do not change are called **constants**, and are represented by numbers in formulae and equations.

SOLVING EQUATIONS BY INSPECTION

To **solve** an equation means to find the value(s) of the unknown for which an expression has a given value.

One method of solving an equation is to try different values of the variable until you find a value for which the expression is equal to the given value, or for which the two expressions have the same value. This is called **solving by inspection**.

SOLVING EQUATIONS BY INSPECTION

Teaching guidelines

Remind learners that a solution to an equation is a value of the unknown that makes the equation true, in other words, the two sides of the equation are equal when the solution is substituted in place of the unknown value.

Solving by inspection involves substituting a value and comparing the two sides of the equation. If they are equal, the substituted value is a solution, if not, try another value until the solution is found.

Answers

1. (a) $x = 5$ is the solution because $3 \times 5 + 1 = 16$
   (b) $x = 13$ is the solution because $7 \times 13 = 91$
   (c) No, because $10 \times 6 + 9 = 69$ and $7 \times 6 + 30 = 42 + 30 = 72$
   $x = 7$ is a solution
   (d) No, because $-10 \times 3 - 1 = -30 - 1 = -31$
   $x = -3$ is a solution
   (e) $x = 1$ is the solution because $7 + 2 \times 1 = 9$
2. See the answers on LB page 238 alongside.
3. See the answers on LB page 238 alongside.

22.2 Solving equations

ADDITIVE AND MULITPLICATIVE INVERSES

Teaching guidelines

The inverses are used when we use the reverse flow diagram method to solve equations.

If a number and its additive inverse are added, we get the identity of addition, namely 0, for example $7 + (-7) = 0$.

If a number and its multiplicative inverse are multiplied, we get the identity of multiplication, namely 1, for example $7 \times \frac{1}{7} = 1$.

Misconceptions

Learners confuse the operations and apply the additive inverse to a situation where the multiplicative inverse should be applied, for example, to isolate x in $5x$ a learner will write $5x - 5 = x$. This especially happens when they try to take shortcuts and not write down all the steps in the process of solving an equation.

The value of the variable for which an expression is equal to a given value, or for which two expressions have the same value, is called the **solution** or the **root** of the equation.

1. In each case, determine whether the value of x given in brackets is a root or solution of the equation or not. Justify your answer, in other words, say why you think the number is or is not a root of the equation. In cases where the given number is *not* a root or solution, find the solution by trying other values.
   (a) $3x + 1 = 16$ $(x = 5)$
   (b) $7x = 91$ $(x = 13)$
   (c) $10x + 9 = 7x + 30$ $(x = 6)$
   (d) $-10x - 1 = 29$ $(x = 3)$
   (e) $7 + 2x = 9$ $(x = 1)$
2. Find the solution of each equation by inspection:
   (a) $x - 1 = 0$ $x = 1$
   (b) $x + 1 = 0$ $x = -1$
   (c) $1 + x = 0$ $x = -1$
   (d) $1 - x = 0$ $x = 1$
3. In each case, check if the number in brackets makes the equation true. Explain your answer.
   (a) $8 + x = 3$ $(x = 5)$ $x = 5$ is not a solution because $8 + 5 \neq 3$
   (b) $8 + x = 3$ $(x = -5)$ $x = -5$ is a solution because $8 + (-5) = 3$
   (c) $8 - x = 3$ $(x = 5)$ $x = 5$ is a solution because $8 - 5 = 3$
   (d) $8 - x = 3$ $(x = -5)$ $x = -5$ is not a solution because $8 - (-5) = 13$
   (e) $8 - x = 13$ $(x = -5)$ $x = -5$ is a solution because $8 - (-5) = 13$
   (f) $8 - x = 13$ $(x = 5)$ $x = 5$ is not a solution because $8 - 5 = 3$

22.2 Solving equations

ADDITIVE AND MULTIPLICATIVE INVERSES

One way of thinking about the **additive inverse** of a number is to ask the question: *What do I need to add to the given number to get 0?*

1. What is the additive inverse of each of the following? Explain your answers.
   (a) 5 -5 because $5 + (-5) = 0$
   (b) -5 5 because $5 + (-5) = 0$
   (c) 17 -17 because $17 + (-17) = 0$
   (d) 0,1 $-0,1$ because $0,1 + (-0,1) = 0$
   (e) $\frac{5}{6}$ $-\frac{5}{6}$ because $\frac{5}{6} + \left(-\frac{5}{6}\right) = 0$
   (f) $-2\frac{1}{4}$ $2\frac{1}{4}$ because $2\frac{1}{4} + \left(-2\frac{1}{4}\right) = 0$

Teaching guidelines

We use the inverse operations so that we can isolate the unknown to find its value in the given situation.

The important principle when solving equations is first to decide what should be done to the unknown to get its coefficient 1 and then to do that on both sides of the equation. For example, to solve for x in $2x + 5 = 35$, we can think in terms of a reverse flow diagram, so the first operation to perform is to subtract 5 or add (–5), because $2x + 5 - 5 = 2x$. Next we multiply by $\frac{1}{2}$ (or divide by 2) to get x. But because the left-hand side equals the right-hand side of the equation $2x + 5 = 35$, we have to repeat every operation on the right-hand side as well. Refer to the explanation on LB page 239 alongside. Let learners use this method to work out the answers to question 3.

Answers

1. See LB page 238 on the previous page.
2. See the answers on LB page 239 alongside.
3. (a) $x = 6$ Check: $5 \times 6 + 2 = 30 + 2 = 32$
   (b) $x = -2$ Check: $3 \times (-2) - 5 = -6 - 5 = -11$
   (c) $x = 8$ Check: $5 \times 8 = 40$
   (d) $x = 8$ Check: $5 \times 8 - 12 = 40 - 12 = 28$
   (e) $x = 25$ Check: one fifth of 25 is 5; three fifths is 15

EXPONENTIAL EQUATIONS

Teaching guidelines

Show learners that in order to compare the left-hand side with the right-hand side of the equation, both sides have to be powers of the same base. Then we can reason: LHS equals RHS; the bases are the same, therefore the exponents have to be the same for the equality to hold.

Answers

1. (a) $4^x = 4^3$, $x = 3$ (b) $3^x = 3^3$, $x = 3$ (c) $6^x = 6^3$, $x = 3$
   (d) $5^x = 5^3$, $x = 3$ (e) $2^x = 2^5$, $x = 5$ (f) $12^x = 12^2$, $x = 2$
2. (a) $4^{x+1} = 4^3$ $x + 1 = 3$ hence $x = 2$ Check: $4^2 + 1 = 4^3 = 64$
   (b) $3^{x-1} = 3^3$ $x - 1 = 3$ hence $x = 4$ Check: $3^{4-1} = 3^3 = 27$
   (c) $2^{x+5} = 2^5$ $x + 5 = 5$ hence $x = 0$ Check: $2^{0+5} = 2^5 = 32$

We can think of the **multiplicative inverse** of a number by asking the question: *What do I need to multiply the number by to get 1?*

2. What is the multiplicative inverse of each of the following? Explain.
   (a) 5 $\frac{1}{5}$ because $5 \times \frac{1}{5} = 1$ (b) -5 $-\frac{1}{5}$ because $-5 \times -\frac{1}{5} = 1$
   (c) $\frac{5}{6}$ $\frac{6}{5}$ because $\frac{5}{6} \times \frac{6}{5} = 1$ (d) $\frac{1}{8}$ 8 because $\frac{1}{8} \times 8 = 1$

You can solve the equation $2x + 5 = 45$ in the following way:

$2x + 5 = 45$

$2x + 5 - 5 = 45 - 5$ Subtract 5 from both sides to have only the term in x — This step can also be understood as $5 + (-5) = 0$

$2x + 0 = 40$

$\frac{2x}{2} = \frac{40}{2}$ Divide both sides by 2 to have x only — This step can also be understood as $2 \times \frac{1}{2} = 1$

$x = 20$

3. Solve the equations below. Check that the value of x that you get is the solution.
   (a) $5x + 2 = 32$ (b) $3x - 5 = -11$ (c) $5x = 40$
   (d) $5x - 12 = 28$ (e) $\frac{3}{5}x = 15$

EXPONENTIAL EQUATIONS

Example 1: Solve the equation $2^x = 8$.

Solution: $2^x = 2^3$ [Write 8 in terms of base 2, i.e. as a power of 2]

$x = 3$ [2 raised to the power of 3 is 8]

Solving exponential equations is the same as asking the question: *To which power must the base be raised for the equation to be true?*

1. Solve for x:
   (a) $4^x = 64$ (b) $3^x = 27$ (c) $6^x = 216$
   (d) $5^x = 125$ (e) $2^x = 32$ (f) $12^x = 144$

Example 2: Solve the equation $2^{x+1} = 8$.

Solution: $2^{x+1} = 2^3$

$x + 1 = 3$

$x = 2$

2. Solve for x:
   (a) $4^{x+1} = 64$ (b) $3^{x-1} = 27$ (c) $2^{x+5} = 32$

22.3 Generating tables of ordered pairs

PRELIMINARY ACTIVITY

Teaching guidelines

This activity should show learners that solving an equation means reversing what was done when substituting values into a formula.

The formula becomes an equation when an output value is known and the input value becomes the unknown. For example, if the formula is $y = 15x + 3$, then substituting values of the independent variable x gives values of the dependent variable y. As soon as a value of y is known, as in $15x + 3 = 48$, we have an equation.

A table of values may show the solution, but not necessarily.

Answers

1. See the answers on LB page 240 alongside.

2. (a) $15 \times 2 + 3 = 30 + 3 = 33$ (b) $15 \times \frac{1}{15} + 3 = 1 + 3 = 4$

3. (a) $15x + 3 - 3 = 33 - 3$
$15x = 30$
$x = 2$

(b) $15x + 3 - 3 = 4 - 3$
$15x = 1$
$x = \frac{1}{15}$

FROM A FORMULA TO A TABLE OF VALUES

Teaching guidelines

Applying the formula in each case, we generate pairs of values. The solutions to the equations can be found in the table at this stage.

Answers

1. to 3. See the answers on LB page 240 alongside.

22.3 Generating tables of ordered pairs

PRELIMINARY ACTIVITY

1. Copy and complete the following table for the given values of x:

x	–3	–2	–1	0	1	2	3	6	10
$-3x + 2$	11	8	5	2	–1	–4	–7	–16	–28

2. What is the value of $15x + 3$ for:
(a) $x = 2$? (b) x = multiplicative inverse of 15?

3. For what values of x are the following equations true?
(a) $15x + 3 = 33$ (b) $15x + 3 = 4$

In questions 1 and 2 you evaluated an expression for given values of x.

In question 3 you determined a value of x that makes the equation true. In other words, you solved for x.

In completing a table of values you may be confronted with questions similar to questions 1, 2 and 3.

FROM A FORMULA TO A TABLE OF VALUES

1. Copy and complete each of the tables below for the given formula.
(a) $y = x$

x	–9	–3	–2	–1	0	1	2	8	10	15
y	–9	–3	–2	–1	0	1	2	8	10	15

(b) $y = x + 2$

x	–7	–3	–2	–1	0	1	2	6	10	13
y	–5	–1	0	1	2	3	4	8	12	15

(c) $y = x^3$

x	–6	–3	–2	–1	0	1	2	5	6	10
y	–216	–27	–8	–1	0	1	8	125	216	1 000

2. Use the table in question 1(c) to solve for x in each case:
(a) $x^3 = -1$ $x = -1$ (b) $x^3 = 8$ $x = 2$ (c) $x^3 = 0$ $x = 0$

3. Copy and complete the table for $y = 2x$:

x	–7	–3	–2	–1	0	1	2	4	10	13
y	–14	–6	–4	–2	0	2	4	8	20	26

Teaching guidelines

Learners can use the tables to find solutions to the equations. For example, in question 6, they use the table in question 5 and in question 8, they use the table in question 7.

Show learners that in the table there are two solutions to $x^2 = 4$. Explain that this is because the x value is squared in the formula $y = x^2$, and both input values of, –2 and 2, will give 4 in this instance. Do not go into finding square roots to solve the equation at this stage.

In answering question 9 learners test the formulae by substituting x values from the table. Show learners how to eliminate the formulae that will not work. They substitute $x = 0$ in the formula. If it is a possible match, the calculated y value will be the same as that in the table, for example, the first formula gives $y = -2$ for $x = 0$, but if it were the formula that describes the table the y value would need to be 5. Therefore, we can eliminate it; as well as all the others except the third formula, as none of the others give 5 when 0 is substituted. If there is more than one possibility, as is for the table (b), then they have to test more values to find the correct formula.

Answers

4. (a) $x = 10$ (b) $x = 0$
5. See the answers on LB page 241 alongside.
6. (a) $x = -2$ (b) $x = -7$ (c) $x = 2$
7. See the answers on LB page 241 alongside.
8. (a) Two different numbers $x = -4$ and $x = 4$
   (b) $x = -3$ or $x = 3$
   (c) $x = -13$ or $x = 13$
   (d) $x = 2$ or $x = -2$ because $x^2 - 1 = 3$ is $x^2 = 4$
9. (a) $y = 2x + 5$
   (b) $y = 5x + 2$
   (c) $y = 3x + 2$
   (d) $y = -5x - 2$
   (e) $y = 2x - 5$

4. Use the table in question 3 to answer the following questions:
   (a) What value of x makes the equation $2x = 20$ true?
   (b) For what value of x is $2x = 0$?

5. Copy and complete the table for $y = -x - 2$:

x	–7	–3,5	–2	–1	0	1,2	2	6	6,9	13
y	5	1,5	0	–1	–2	–3,2	–4	–8	–8,9	–15

6. For what values of x are the following equations true?
   (a) $-x - 2 = 0$ (b) $-x - 2 = 5$ (c) $-x - 2 = -4$

7. Copy and complete the table for $y = x^2$:

x	–4	–3	–2	–1	0	1	2	3	4	13
y	16	9	4	1	0	1	4	9	16	169

8. Refer to the table in question 7 to answer the questions that follow:
   (a) Which different values of x make the equation $x^2 = 16$ true?
   (b) Solve for $x^2 = 9$.
   (c) Solve for $x^2 = 169$.
   (d) What is the solution of $x^2 - 1 = 3$?

9. Some tables of ordered pairs are given below. For each table, find out which of the following formulae was used to make the table. Write the number of the question and the correct formula.

   $y = -5x - 2$ $y = 5x + 2$ $y = 2x + 5$ $y = 2x - 5$
   $y = 2x - 5$ $y = -5x + 2$ $y = -3x + 2$ $y = 3x + 2$

(a)

x	–4	–3	–2	–1	0	1	2	3	4	5
y	–3	–1	1	3	5	7	9	11	13	15

(b)

x	–4	–3	–2	–1	0	1	2	3	4	5
y	–18	–13	–8	–3	2	7	12	17	22	27

(c)

x	–4	–3	–2	–1	0	1	2	3	4	5
y	–10	–7	–4	–1	2	5	8	11	14	17

(d)

x	–4	–3	–2	–1	0	1	2	3	4	5
y	18	13	8	3	–2	–7	–12	–17	–22	–27

(e)

x	–4	–3	–2	–1	0	1	2	3	4	5
y	–13	–11	–9	–7	–5	–3	–1	1	3	5

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
23.1 What we can tell with graphs	Interpreting graphs; comparing the information from two graphs of the same situation; choosing graphs to fit a situation; increases and decreases in graphs	Pages 242 to 246
23.2 More features of graphs	Maximum and minimum values on graphs; understanding what discrete and continuous values mean	Pages 247 to 248
23.3 Drawing graphs	Drawing global graphs incorporating knowledge about discrete and continuous values; graphing ordered pairs in linear and quadratic graphs using increasing and decreasing values; graphs that have maximum and minimum values	Pages 248 to 253

CAPS time allocation	9 hours
CAPS content specification	Page 114

Mathematical background

A graph is another way of representing the input and output values of a relationship, besides a table, a formula or a flow diagram.

Being able to interpret a graph is an important skill that can be used in all areas of life, not only mathematics.

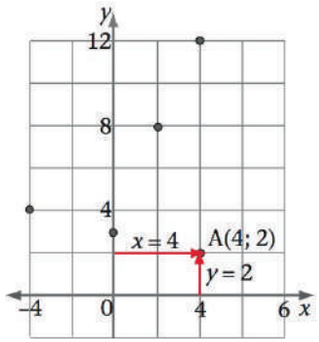
Some of the properties of graphs we need to interpret are:

- how graphs increase and decrease (at a constant rate – linear; not at a constant rate – curved; no increase or decrease – horizontal line)
- whether there are maximum or minimum values on the graph and what they imply
- whether the graph portrays discrete or continuous values and what it means. This point refers to the domain and range of a function.

Drawing graphs requires us to understand the points above and how to indicate them on a graph.

We can represent a relationship between two sets of numbers, x and y, on a graph using **ordered pairs**. The ordered pairs are generated by using the formula.

If we let a horizontal and a vertical number line cross each other at right angles at the point 0, we can describe the position of any point in terms of both number lines. We call these lines **axes** (singular: axis). The two axes form the **coordinate plane**. The point where the two axes cross is the **origin**, described by the ordered pair (0; 0).



The position of each point on the coordinate plane is described by an ordered pair. This means there is a first number and a second number and the order has meaning. The first number in an ordered pair is the x-coordinate and the second is the y-coordinate.

The x-coordinate describes the shortest distance between the point and the y-axis. We can also say that the point is x units from the origin in the horizontal direction.

The y-coordinate describes the shortest distance between the point and the x-axis. We can also say that the point is y units from the origin in the vertical direction.

23.1 What we can tell with graphs

INTERPRETING GRAPHS

Teaching guidelines

Interpreting graphs is a useful skill. The media often uses graphs to communicate information.

What is of interest is the general trend a graph shows, any high and low points, as well as any increases and/or decreases.

If learners can describe the general trend and answer the specific questions correctly, they are on their way to understanding how to interpret a graph.

Answers

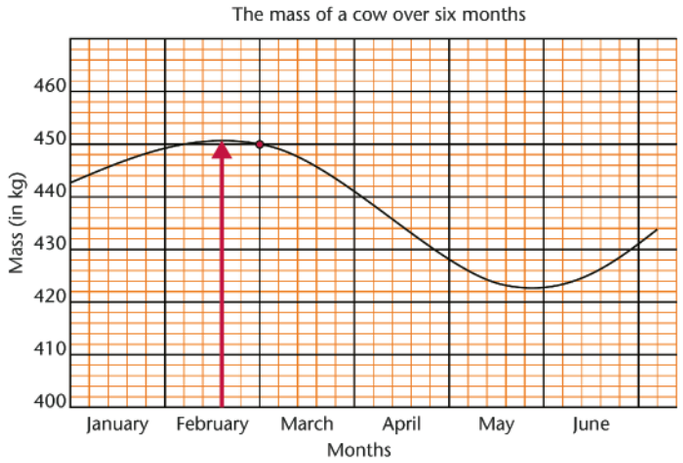
1. (a) Towards the end of May
   (b) May
   (c) January
   (d) March and April

CHAPTER 23
Graphs

23.1 What we can tell with graphs

INTERPRETING GRAPHS

1. Mrs Maleka is a dairy farmer. She cares for her cows and weighs all of them daily. Here is a graph of one cow's mass in kilograms over a period of six months. At the end of February, the mass of the cow was 450 kg, as shown by the red dot.



(a) The cow's mass reached a maximum a few days after the middle of February, as shown by the red arrow on the graph. When, in the period shown on the graph, did the cow's mass reach a minimum?
(b) During most of February the cow weighed slightly more than 450 kg. During which month did the cow weigh less than 430 kg, for the whole month?
(c) Throughout the month of June, the mass of the cow increased. During which other month did the mass of the cow also increase, right through the month?
(d) During which months did the mass of the cow decrease right through the month?

Teaching guidelines

Another skill is comparing two graphs that give information about similar situations.

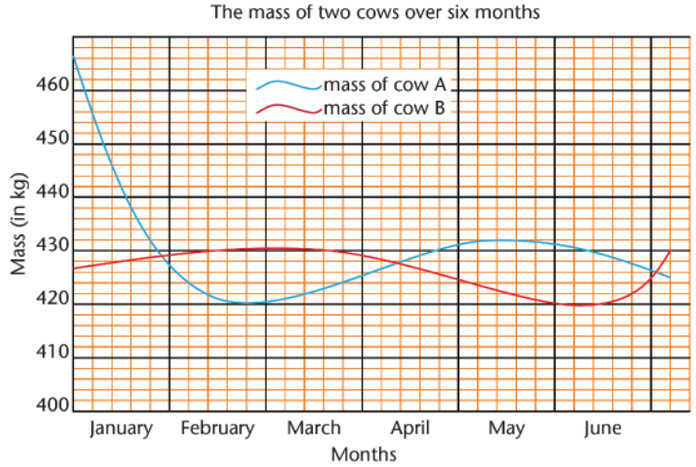
Learners should realise that if one curve or line is above another, the values shown by the first curve are more than those shown by the second curve for the same point on the horizontal axis. Where the curves intersect, the values are the same.

Let learners describe the general trend of the mass of each cow in their own words.

Answers

2. (a) Cow B
   (b) From the end of January until the second week in April, then again from early July.
   (c) January and June
   (d) Late in February
   (e) March
   (f) Just before the middle of April
   (g) In the middle of May
   (h) Early in July

2. The blue and red curves below are graphs that show how the mass of two cows varied over the same period of time.



   (a) Which cow was the heaviest at the end of February?
   (b) When was cow B heavier than cow A?
   (c) During which months did the mass of cow A decrease for the whole month?
   (d) When did cow A's mass start to increase again?
   (e) During which month did cow B's mass begin to decrease while cow A's mass increased for that whole month?
   (f) When did cow A's mass catch up with cow B's mass again?
   (g) When did cow A stop gaining weight and start losing weight again?
   (h) When did cow B's mass catch up with cow A's mass again?

3. A traffic department keeps track of the traffic density on different roads. Two traffic officers are posted somewhere along each main road and they count and record the number of cars that pass in each direction during each 15-minute interval. They use tally marks to do this, as you can see in the example below:

	//// ///| |||| ||||	/// ///| /// ///|	/// //// /// ///| /// ///|	/// /// /// ///| /// ///|	/// /// /// ///| /// ///|	/// /// /// ///| /// ///
Time	06:00 to 06:15	06:15 to 06:30	06:30 to 06:45	06:45 to 07:00	07:00 to 07:15	07:15 to 07:30
Cars	14	23	37	59	71	48

Teaching guidelines

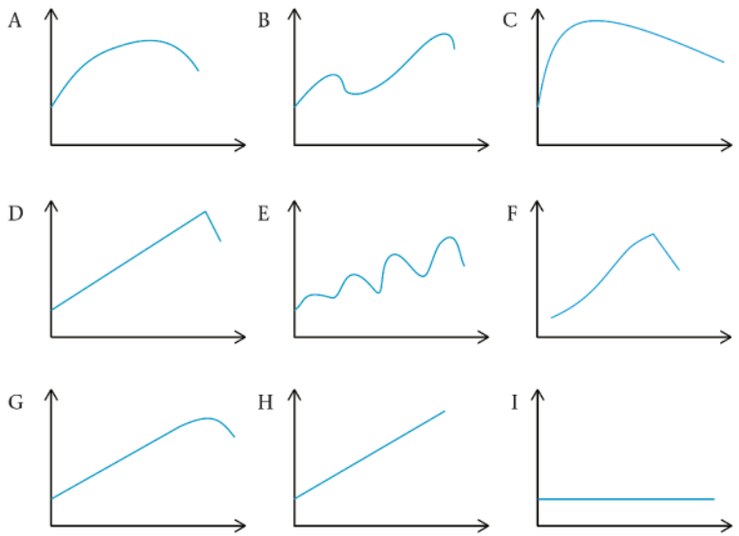
Let learners work in small groups or pairs and discuss the traffic flow representations. Let them discuss what is represented on each axis.

Give them pointers to look for, for example: maximums and minimums; slow or rapid increases; slow or rapid decreases; no change.

Answers

3. Graph F
4. Graph D is best for report (a) and graph C is best for report (b).
5. (a) January to mid-February; mid-March until early April; most of June
   (b) January to mid-February
   (c) June
   (d) In the second period (April to May) she gained weight much more slowly than in the first period (February to March).
   (e) Although her weight (mass) increases again after each period of weight loss, it does not reach the previous height before it starts to decrease again. The cow is possibly sick.

Which of the graphs below do you think is the best representation of the data at the bottom of page 243, on traffic flow?



4. Which of the graphs above is the best representation of each of these traffic flow reports?

(a)

Time	06:00 to 06:15	06:15 to 06:30	06:30 to 06:45	06:45 to 07:00	07:00 to 07:15	07:15 to 07:30
Cars	42	53	64	75	86	75

(b)

Time	06:00 to 06:15	06:15 to 06:30	06:30 to 06:45	06:45 to 07:00	07:00 to 07:15	07:15 to 07:30
Cars	42	123	158	147	136	124

5. Study the graph for another cow on the following page:
   (a) During which periods did the cow lose weight?
   (b) During which of these periods did the cow lose weight more slowly?
   (c) During which of the periods did the cow lose weight most rapidly?
   (d) Compare the two periods when the cow gained weight.
   (e) Is there anything else about the graph that may indicate that this cow has health problems?

HOW GRAPHS SHOW INCREASES AND DECREASES

Teaching guidelines

Explain that if there is a relationship between the variables, the independent variable is always shown on the horizontal axis and the dependent variable on the vertical axis. If time is part of the relationship, it is usually the independent variable and is shown on the horizontal axis.

Explain what it means if a graph increases (or decreases). Draw a system of axes on the board to help learners understand. We look at what happens to the values of the dependent variable as the independent variable increases. On the graph, the independent variable increases from left to right.

- If the dependent value becomes smaller as the independent value becomes bigger, the graph will be decreasing.
- If the dependent value becomes bigger as the independent value becomes bigger, the graph will be increasing.

Use rough graphs to illustrate and explain:

- a linear graph with a constant rate of change
- a curved graph with a rate of change that is not constant
- a constant graph that shows no change.

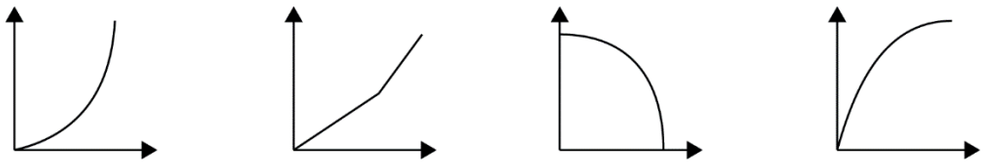
Misconceptions

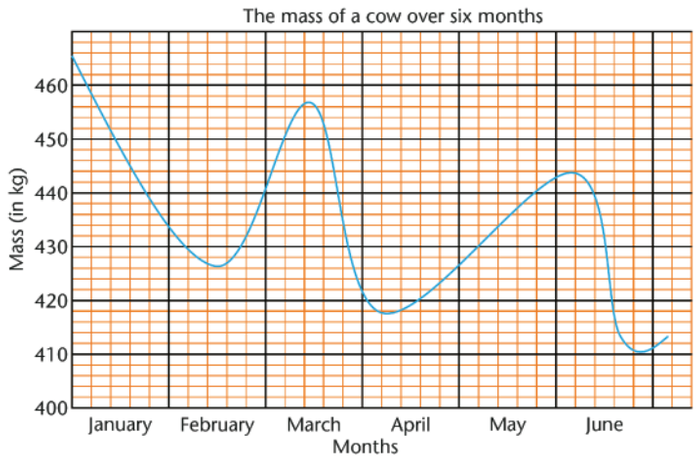
Often "increase" and "decrease" are just words that learners know the meaning of but cannot relate to graphs. Also, learners not having a clear understanding of the integers, can lead to misconceptions. Some learners think of the integers on a number line with 0 in the middle and the numbers growing bigger to either side of 0, instead of thinking of, for example, –1 000 as very small and the numbers growing larger as you move to the right on the number line. It is important that learners have the correct understanding in order to be clear about decreasing and increasing relationships.

Answers

1.

(a) (b) (c) (d)





HOW GRAPHS SHOW INCREASES AND DECREASES

A graph on a system of coordinates shows the way in which one quantity (called the **dependent variable**) changes when another quantity (called the **independent variable**) increases. A quantity can change in the following ways:

- It can increase or decrease.
- It can increase at a constant rate, for example the total amount saved if the same amount is saved every week or month.
- It can decrease at a constant rate, for example the length of a burning candle.
- It can increase (or decrease) at a varying rate, for example the increase in the area of a square as the side length increases.

When a quantity increases or decreases at a **constant rate**, it is called **linear** change or variation, and the graph is a **straight line**. When the rate of change is **not constant**, it is called **non-linear** change, and the graph is **curved**. If there is no change in the output variable, the graph is a horizontal straight line.

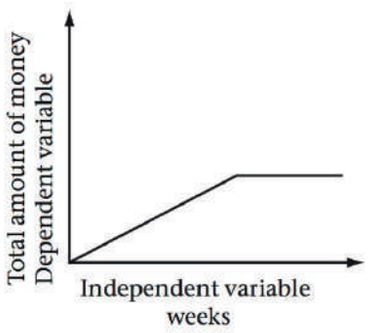
1. Draw a graph to match each of the following descriptions:
   (a) The quantity increases, and increases more rapidly as time progresses.
   (b) The quantity first increases slowly at a constant rate, and then increases at a faster, constant rate.
   (c) The quantity decreases faster and faster.
   (d) The quantity increases, and the rate of increase gradually diminishes.

CHAPTER 23: GRAPHS 245

Teaching guidelines

Talk about the meaning of a constant rate of change so that learners understand what it means. The example of saving the same amount every week and drawing a graph of the total amount saved is perhaps a good example to explain how the total amount increases by the same amount over the same time period.

You could change the situation so that the same amount is saved for a number of weeks after which no more is saved, but the money is left in the account. At that stage the graph will change to a horizontal line.

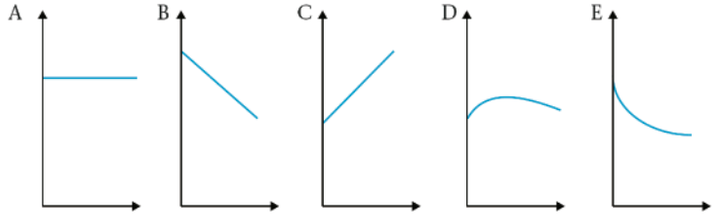


Answers

2. See the answers on LB page 246 alongside.
3. See the answers on LB page 246 alongside.
4. (a) B
   (b) C
   (c) A

2. Look at the statements and graphs about patterns of change in the petrol price per litre over a period. Match each statement with the appropriate graph given below. Time is represented on the horizontal axis in all these graphs, and petrol price on the vertical axis.

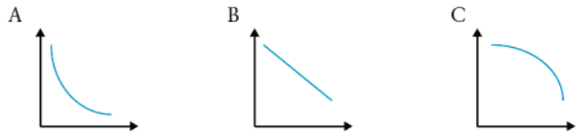
   (a) The price did not change. A
   (b) The price rose at a constant rate. C
   (c) The price decreased at a constant rate. B
   (d) The price dropped very fast at first and then at a slower rate. E
   (e) The price rose at a decreasing rate up to a point and then started to drop at an increasing rate. D



3. Copy and complete the following table in respect of the graphs in question 2:

Graph	Represents a linear or non-linear relation	Reason
A	linear	constant rate
B	linear	constant decrease
C	linear	constant increase
D	non-linear	increase and decrease
E	non-linear	varying decrease

4. (a) Which graph represents a quantity that decreases at a constant rate?
   (b) Which graph represents a quantity that decreases at an increasing rate?
   (c) Which graph represents a quantity that decreases at a decreasing rate?



23.2 More features of graphs

LOCAL MAXIMUM AND MINIMUM VALUES

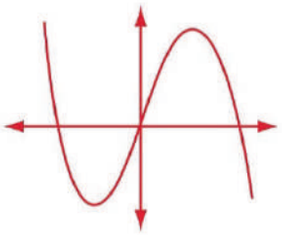
Teaching guidelines

Talk through the change from increasing to decreasing (or the other way around) with learners. Let them imagine walking along the horizontal axis in the direction of the increasing horizontal values, in other words from very small values (large negative values) through 0 to very big values. While walking, they imagine looking up or down, looking for the corresponding value of the relationship (the y value). If it becomes larger as they imagine walking and then changes and becomes smaller as they continue walking, the relationship has a maximum value. If it becomes smaller as they imagine walking and then changes and becomes bigger as they continue walking, the relationship has a minimum value. Refer to the drawings on LB page 247 alongside.

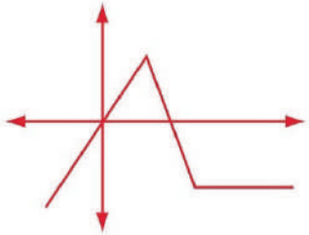
A relationship could have both a maximum and a minimum or even more.

Answers

1. (a) Graph 2
   (b) Graph 3
   (c) Graph 1
2. (a)



(b)

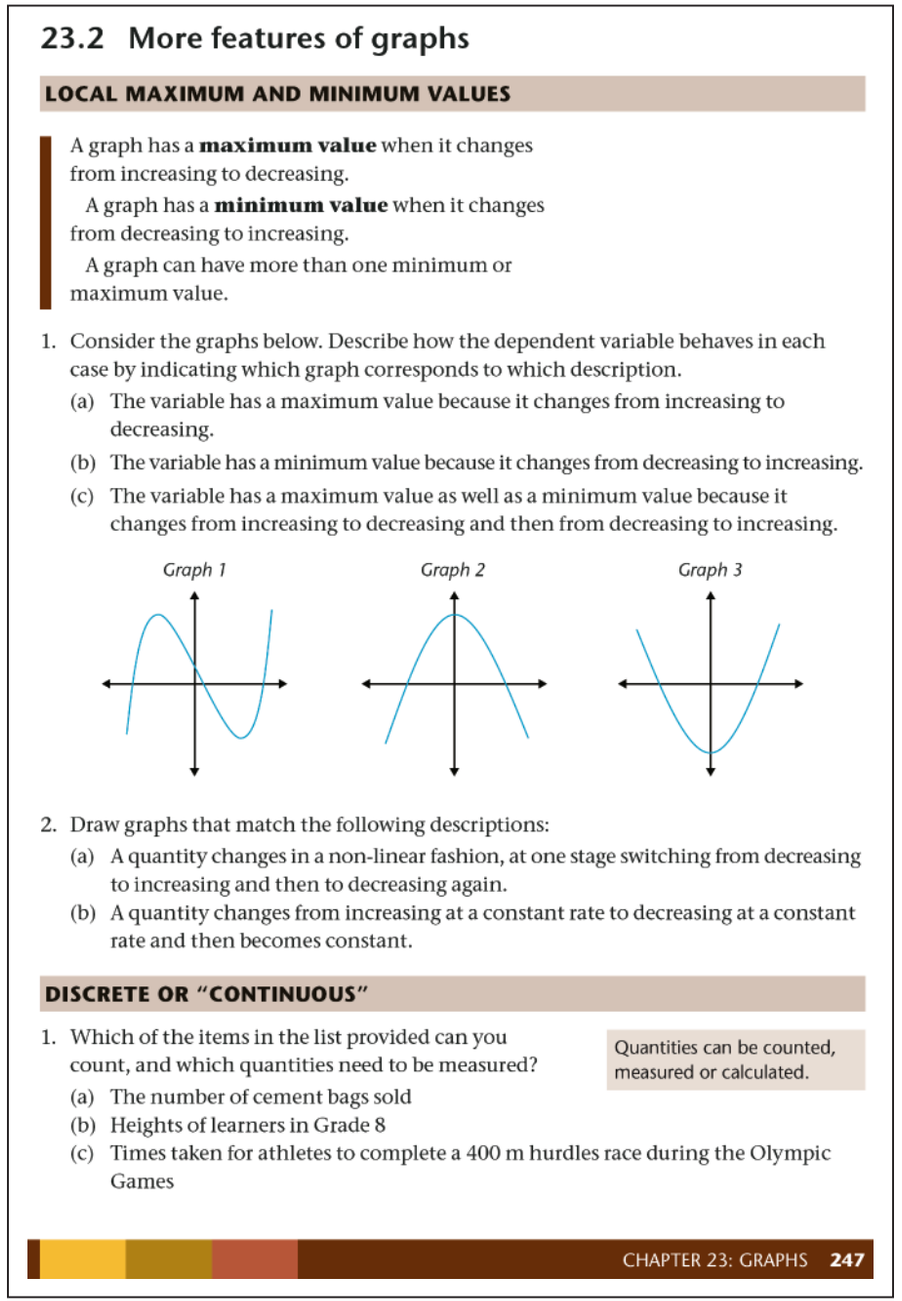


DISCRETE OR "CONTINUOUS"

Teaching guidelines

Discuss the difference between discrete (quantitative) data and continuous data.

Let learners name instances where they recognise discrete quantities and continuous quantities. For example, discrete quantities could be the number of learners in a class; the number of sandwiches or cool drinks sold in the tuck shop. Continuous data could be the height of a learner; the distance each learner travels to school, and so on.



23.2 More features of graphs

LOCAL MAXIMUM AND MINIMUM VALUES

A graph has a **maximum value** when it changes from increasing to decreasing.

A graph has a **minimum value** when it changes from decreasing to increasing.

A graph can have more than one minimum or maximum value.

1. Consider the graphs below. Describe how the dependent variable behaves in each case by indicating which graph corresponds to which description.
   (a) The variable has a maximum value because it changes from increasing to decreasing.
   (b) The variable has a minimum value because it changes from decreasing to increasing.
   (c) The variable has a maximum value as well as a minimum value because it changes from increasing to decreasing and then from decreasing to increasing.

Graph 1 Graph 2 Graph 3

2. Draw graphs that match the following descriptions:
   (a) A quantity changes in a non-linear fashion, at one stage switching from decreasing to increasing and then to decreasing again.
   (b) A quantity changes from increasing at a constant rate to decreasing at a constant rate and then becomes constant.

DISCRETE OR "CONTINUOUS"

1. Which of the items in the list provided can you count, and which quantities need to be measured?

   Quantities can be counted, measured or calculated.

   (a) The number of cement bags sold
   (b) Heights of learners in Grade 8
   (c) Times taken for athletes to complete a 400 m hurdles race during the Olympic Games

CHAPTER 23: GRAPHS 247

If we can count the quantity, we have discrete quantities and if we can measure, we have continuous quantities.

Answers

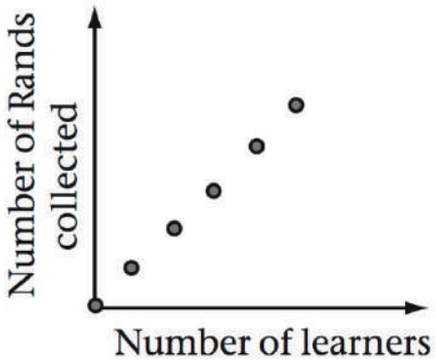
1. See the answers on LB page 248 alongside.
2. (a) This does not make sense. The number of people is always a whole number.
   (b) This does make sense. Distance can be expressed as a rational number.
   (c) This does not make sense. The number of cans is always a whole number.

23.3 Drawing graphs

DRAWING GLOBAL GRAPHS

Teaching guidelines

Learners should understand that discrete input values give discrete output values. For example, the number of R1 coins collected if each learner was asked to bring five R1 coins.



(d) The number of sweets in various 500 g bags
(e) The distance travelled by learners to school
(f) Cars passing at a scholar patrol crossing
(g) The cost of an exercise book in rands and cents
(h) Temperature

Copy the following table and write your answers in it:

Can only be counted	Can only be measured
(a) Number of bags of cement sold	(b) Heights of learners in Grade 8
(d) The number of sweets in various 500 g bags	(c) Times taken for athletes to complete a 400 m hurdles race during the Olympics
(f) Cars passing at a scholar patrol crossing	(e) The distance travelled by learners to school
(g) The cost of an exercise book in rands and cents	(h) Temperature

2. Say if the following make sense or not. Explain.
   (a) 501,3 learners attended a rugby match played by the senior team.
   (b) The distance from school to the nearest shopping mall is 10,75 km.
   (c) 2 004,75 cans of cola were sold during a fundraising event.

Quantitative data is numerical data such as a person's marks in a Mathematics test. Quantities that can be counted are sometimes said to be **discrete**: they do not allow values in between any two consecutive values. You cannot have 2,6 people for example. Quantities that allow many values between any two values are sometimes said to be **continuous**.

The terms "discrete" and "continuous" are used in different meanings than these in formal mathematics.

23.3 Drawing graphs

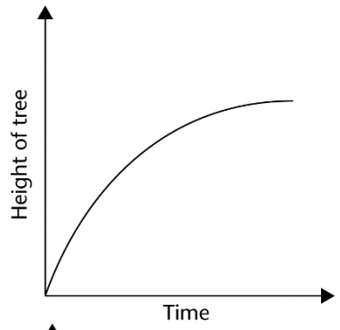
DRAWING GLOBAL GRAPHS

When we draw a graph of continuous data, it is a solid line or curve.

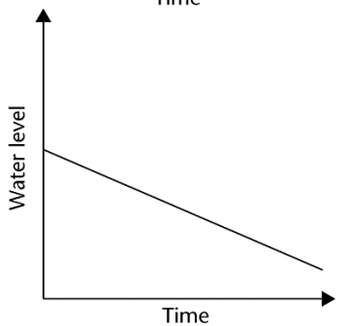
The graph of discrete data is a set of distinct points.

Answers

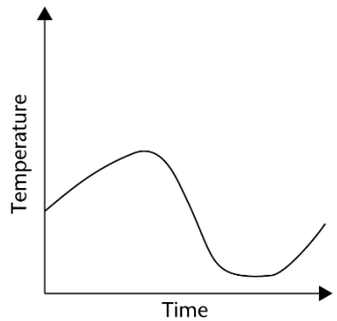
1. (a) Discrete data
   (b) Continuous data
   (c) Because it is a graph of continuous data.
   (d) Because it is a graph of discrete data.
2. Consider learners' graphs. The graphs below are examples of good answers.
   (a)



   (b)

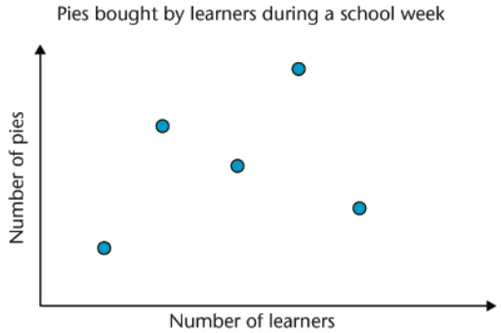


   (c)

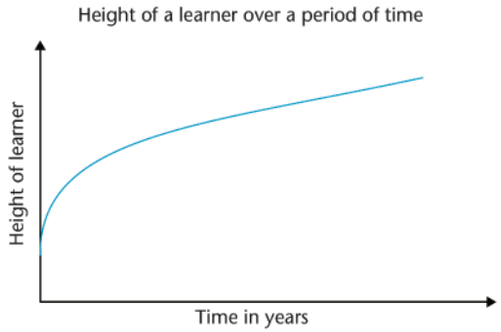


Consider the situations below.

Situation 1



Situation 2



1. (a) What type of data is graphed in situation 1?
   (b) What type of data is graphed in situation 2?
   (c) Why do you think the graph in situation 2 is a solid line?
   (d) Why are the points in situation 1 not joined?
2. Draw a rough graph for each of the following situations:
   (a) The height of a young tree and its age
   (b) The level of water in a dam, over a period without any rain
   (c) The temperature under a tree over a period of 24 hours

GRAPHS OR ORDERED PAIRS

Teaching guidelines

A set of ordered pairs defines a relation or a function. Explain to learners that the word "ordered" means there is a first and a second number.

- The first number is always the input number (or x, the independent variable).
- The second number in the ordered pair is always the output number (or y, the dependent variable).

A graph is another way of representing the relationship between the input and output values of a relationship.

If we let a horizontal and a vertical number line cross each other at right angles at the point 0, we can describe the position of any point in terms of both lines. We call these lines axes (singular: axis).

The position of each point on the coordinate plane created by the axes is described by an ordered pair. The first number is called the x-coordinate and the second is called the y-coordinate.

To describe the position of a point (3; 6), we find the numbers on the axes. On the horizontal axis the point lies in line with 3 and on the vertical axis it lies in line with 6. Hence, the position of the point being described by the ordered pair: (3; 6).

Misconceptions

Learners switch the order of the pair when plotting the points, reading the first coordinate on the vertical axis and the second on the horizontal instead of the other way around. Or they write down the coordinates in the wrong order and thus make a mistake.

Answers

1. See the answers on LB page 250 alongside.

GRAPHS OF ORDERED PAIRS

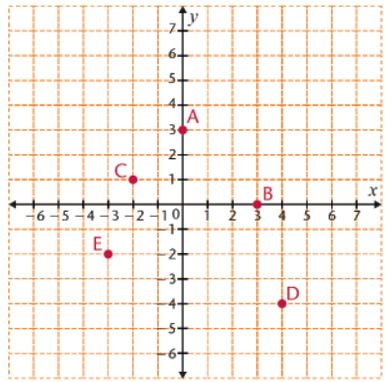
Input and output values can be written as a pair.
The first number in a pair represents the input number and the second number represents the output number.
We therefore say that the pair of numbers is ordered.

Making a graph of **ordered pairs** is another way to show how the input and output values are related.

When drawing a graph of ordered pairs, work as follows:

- First identify the input values (x) and output values (y). In most cases the input values will be given and the output values are calculated using the formula given.
- The output values are written on the y-axis (the vertical axis) and the input values are written on the x-axis (the horizontal axis).
- Plot the ordered pair. Suppose the ordered pair is (3; 6). To plot this pair, put your finger on the number 3 on the x-axis and another finger on the number 6 on the y-axis. Move your finger on the number 3 in a line straight up and move your finger on the number 6 straight across. Where your two fingers meet, make a point. You can describe this point with the ordered pair (3; 6).

1. On graph paper plot the ordered pairs given below:
   (a) A(0; 3)
   (b) B(3; 0)
   (c) C(–2; 1)
   (d) D(4; –4)
   (e) E(–3; –2)



2. (a) Copy and complete the table below for $y = x + 3$:

x	y	$(x; y)$
–4	–1	(–4; –1)
–3	0	(–3; 0)

Teaching guidelines

Discuss with learners that if these input values (for all the questions) were only integers, we would only get the points shown by red dots on the graph. This would be a discrete set of values.

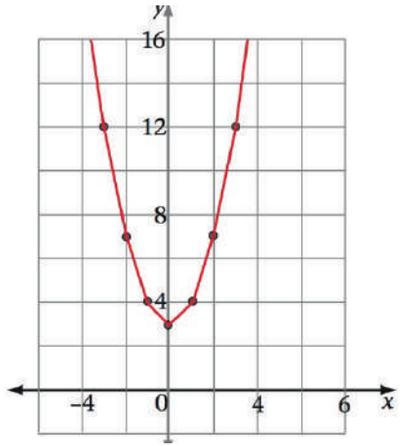
If the input values were also all the fractions between the integers, the set of values would be continuous and the graph would be a solid line.

Let learners decide if this is an increasing or decreasing graph and explain their answers.

Misconceptions

For the graph of question 3, learners do not have enough experience to realise that it is a curve and they will probably join the points with straight lines to get something like the graph in the drawing given here.

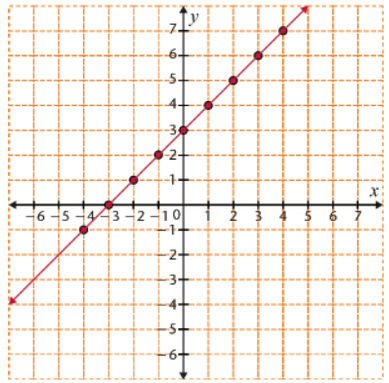
Explain that it should be a curve and show learners the example on the board.



Answers

2. (a) See the answers on LB page 250 on previous page and LB page 251 alongside.
   (b) See the answers on LB page 251 alongside.
   (c) See the answers on LB page 251 alongside.
   (d) Yes, because $y = 100 + 3 = 103$.
3. (a) See the answers on LB page 251 alongside.
   (b) See the answers on LB page 251 alongside.
   (c) Yes, because $y = (10)^2 + 3 = 100 + 3 = 103$.

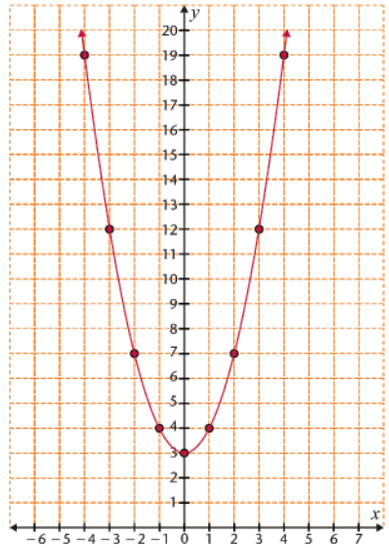
–2	1	(–2; 1)
–1	2	(–1; 2)
0	3	(0; 3)
1	4	(1; 4)
2	5	(2; 5)
3	6	(3; 6)
4	7	(4; 7)



(b) Copy the coordinates onto graph paper. Plot the ordered pairs on the given coordinate system.
(c) Join the points to form a graph.
(d) The ordered pair (1; 6) is not on the graph because when we substitute the value of x ($x = 1$) in the formula $y = x + 3$, we get 4 instead of 6 [$y = 1 + 3 = 4$]. Is the ordered pair (100; 103) on the graph? Explain.

3. (a) Copy and complete the table below for the formula $y = x^2 + 3$:

x	y	$(x; y)$
–4	19	(–4; 19)
–3	12	(–3; 12)
–2	7	(–2; 7)
–1	4	(–1; 4)
0	3	(0; 3)
1	4	(1; 4)
2	7	(2; 7)
3	12	(3; 12)
4	19	(4; 19)



(b) On graph paper, copy the axis system. Then plot the coordinates on the axis system. Join the points to form a graph.
(c) Is the point (10; 103) on the graph? Explain.

Teaching guidelines

Repeat the discussion about discrete and continuous values. By discussing this point learners are informally made aware of the domain and range. Discuss with learners that we do not know from which set of numbers the input values have been taken. If we look at the tables, it seems to be the set of integers. The output values will also be integers and the graph will then be the set of points indicated by the red dots. If we were to use any rational number, the graph could be the line that goes through all the points as we could then use any point between any two integers as an input value.

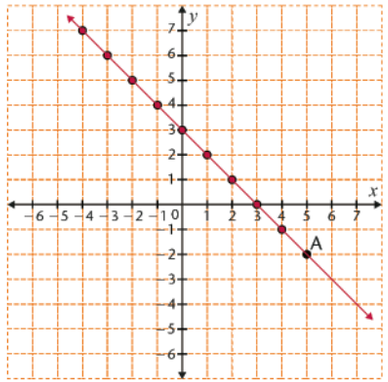
Let learners discuss whether these are increasing or decreasing graphs and explain their answers.

Answers

4. (a) to (c) See the answers on LB page 252 alongside.
   (d) A(5; –2)
5. (a) to (c) See the answers on LB page 252 alongside.

4. (a) Copy and complete the table below for the formula $y = -x + 3$:

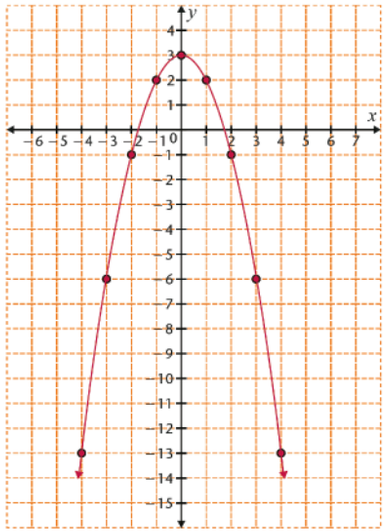
x	y	$(x; y)$
–4	7	(–4; 7)
–3	6	(–3; 6)
–2	5	(–2; 5)
–1	4	(–1; 4)
0	3	(0; 3)
1	2	(1; 2)
2	1	(2; 1)
3	0	(3; 0)
4	–1	(4; –1)



(b) On graph paper, copy the axis system. Then plot the ordered pairs on the axis system.
(c) Join the points to form a graph.
(d) What are the values of the ordered pair A on the graph?

5. (a) Copy and complete the table below for the formula $y = -x^2 + 3$:

x	y	$(x; y)$
–4	–13	(–4; –13)
–3	–6	(–3; –6)
–2	–1	(–2; –1)
–1	2	(–1; 2)
0	3	(0; 3)
1	2	(1; 2)
2	–1	(2; –1)
3	–6	(3; –6)
4	–13	(4; –13)



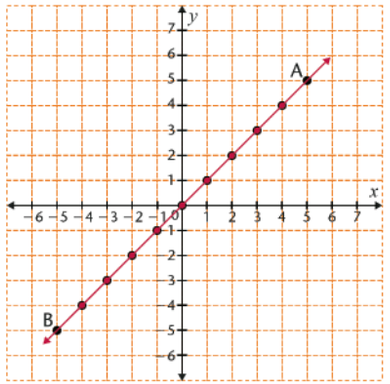
(b) On graph paper, copy the axis system and then plot the ordered pairs on the axis system.
(c) Join the points to form a graph.

Answers

6. (a) to (c) See the answers on LB page 253 alongside.
   (d) A(5; 5); B(–5; –5)

6. (a) Copy and complete the table below for the formula $y = x$:

x	y	$(x; y)$
–4	–4	(–4; –4)
–3	–3	(–3; –3)
–2	–2	(–2; –2)
–1	–1	(–1; –1)
0	0	(0; 0)
1	1	(1; 1)
2	2	(2; 2)
3	3	(3; 3)
4	4	(4; 4)



(b) On graph paper, copy the axis system. Then plot the ordered pairs on the axis system.
(c) Join the points to form a graph.
(d) Write down the values of the ordered pairs A and B on the graph.

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
24.1 Transformations and coordinate systems	Translation, reflection, rotation and centre of rotation; coordinate systems and quadrants	Pages 254 to 257
24.2 Translation on the coordinate system	Properties of translation; translation of points and triangles	Pages 257 to 258
24.3 Reflection on the coordinate system	Properties of reflection; reflection of points and triangles	Pages 259 to 261
24.4 Rotation on the coordinate system	Properties of rotation; rotation of points and triangles	Pages 261to 263
24.5 Enlargements and reductions	Scale factors; effect of enlargements and reductions on perimeter and area	Pages 264 to 268

CAPS time allocation	6 hours
CAPS content specification	Page 116

Mathematical background

- **Transformation geometry** deals with the operations that may be used on a figure to affect its position, size or shape, or any combination thereof.
- The **figure** (also referred to as the **object**) is the original shape before transformation is applied.
- The **image** is the shape which appears after transformation has been applied to the figure.
- Transformations that affect the **position** of a figure are translation, reflection and rotation:
  - During a **translation**, every point in the figure is shifted in the **same direction** over the **same distance**.
  - During a **reflection**, every point in the figure is flipped **perpendicularly** over a **line of reflection** (mirror line) so that the point and its image are the **same distance** from the line of reflection.
  - During a **rotation**, every point in the figure is turned **clockwise or anti-clockwise** through the same angle about a fixed point, the **centre of rotation**, so that the point and its image are the **same distance** from the centre of rotation.
- Transformations that affect the **size** of a figure are enlargements and reductions:
  - During an **enlargement**, every side of a figure is multiplied by a positive number bigger that 1 to produce an image larger than the figure.
  - During a **reduction**, every side of a figure is multiplied by a positive number smaller that 1 to produce an image smaller than the figure.
- Transformations that affect the **shape** of a figure are shears and stretches:
  - During a **shear**, all points along a fixed line remain fixed while all other points are shifted parallel to the fixed line (turning a square into a parallelogram).
  - During a **stretch**, all points along a fixed line remain fixed while all other points are stretched away from the fixed line (turning a square into a rectangle).

24.1 Transformations and coordinate systems

WHAT ARE TRANSFORMATIONS?

Background information

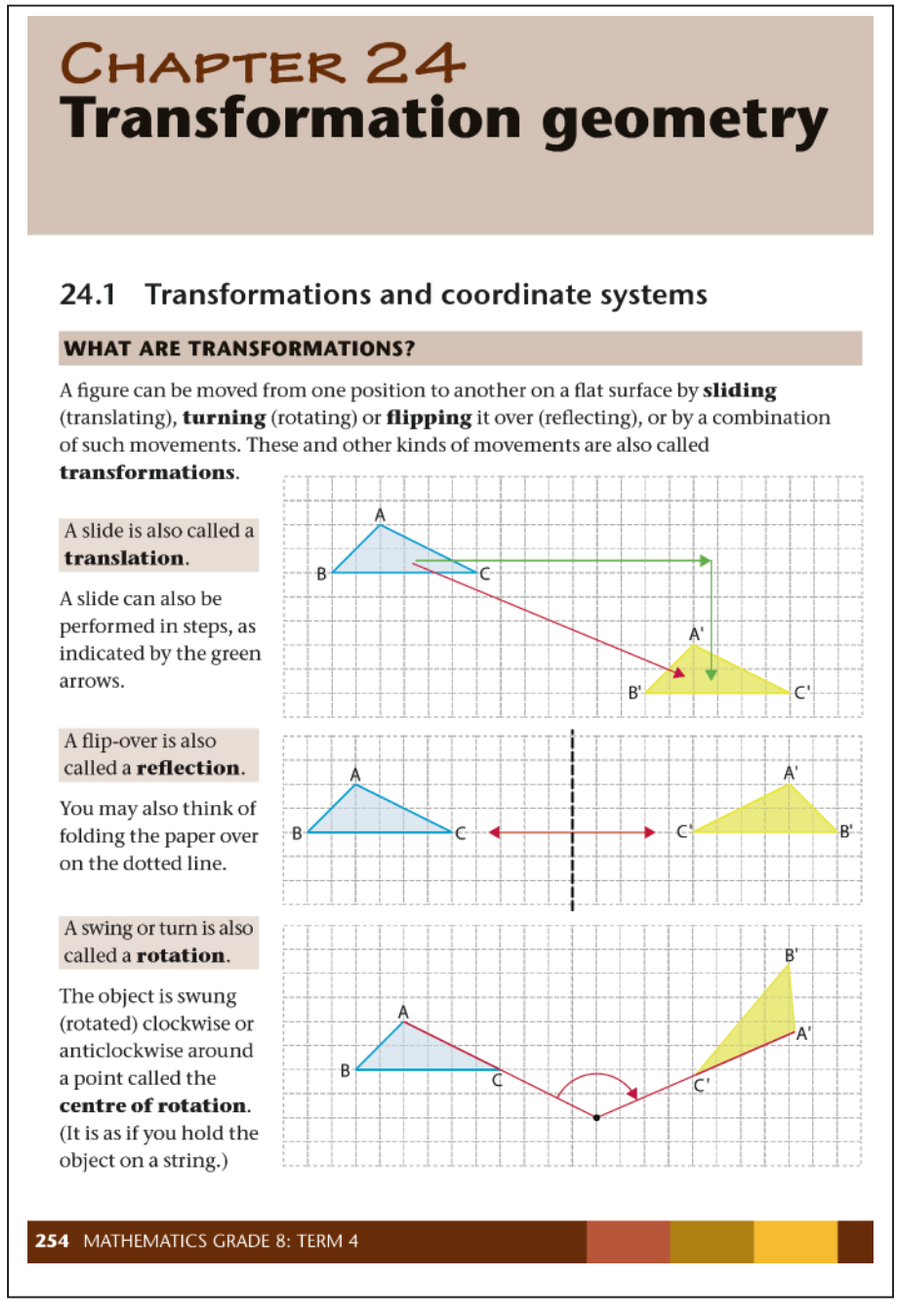
Transformations are operations that may be used on a figure to affect its position, size or shape, or any combination of those.

Figures can, **without changing their size or shape**, be moved around in three different ways:

- A **translation** is a "slide" in a certain direction over a certain distance.
- A **reflection** is a "flip-over" across a mirror line called the line of reflection.
- A **rotation** is a "swing" or "turn" about a fixed point called the centre of rotation.

Teaching guidelines

Learners recall the concepts listed above.



CHAPTER 24
Transformation geometry

24.1 Transformations and coordinate systems

WHAT ARE TRANSFORMATIONS?

A figure can be moved from one position to another on a flat surface by **sliding** (translating), **turning** (rotating) or **flipping** it over (reflecting), or by a combination of such movements. These and other kinds of movements are also called **transformations**.

A slide is also called a **translation**.

A slide can also be performed in steps, as indicated by the green arrows.

A flip-over is also called a **reflection**.

You may also think of folding the paper over on the dotted line.

A swing or turn is also called a **rotation**.

The object is swung (rotated) clockwise or anticlockwise around a point called the **centre of rotation**. (It is as if you hold the object on a string.)

254 MATHEMATICS GRADE 8: TERM 4

Background information (continued)

An **object** is the original figure before it is transformed.

The **image** is the figure which appears after transformation has been applied to the original figure.

When translation, reflection and rotation are performed, the original figure and its image are always **congruent**.

Images of a figure ABC are denoted by using the same letters but we add **prime symbols** after each letter, for example: A′B′C′; A″B″C″; A‴B‴C‴; and so on.

Teaching guidelines (continued)

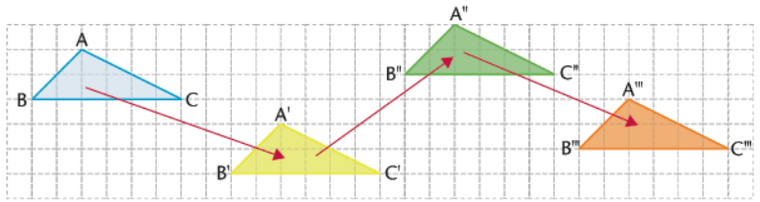
Revise the concepts listed above.

Answers

1. to 4. See the answers on LB page 255 alongside.

In its new position, the figure is called the **image** of the original figure. In the diagrams on page 254, the original figures are blue and the images are yellow. Slides, turns and flips do not change the shape or size of a figure. Hence, in these transformations, the original figure and its image are always congruent.

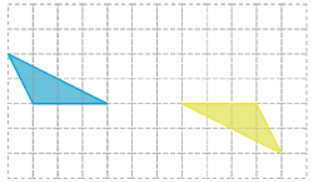
To name the image, we use the same letters as in the original figure, but we add the prime symbol (') after each letter. For example, the image of ΔABC is ΔA'B'C'. If there is a second image, we add two prime symbols, for example ΔA"B"C". If there is a third image, we use three prime symbols, for example ΔA'"B'"C'", and so on.



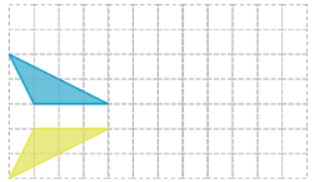
The grid in the background makes it possible to describe the different positions of the figure clearly. To do that, a **system of axes** can be drawn on the grid to form a **coordinate system**, as you will see on the next page. But first, answer the question below.

A coordinate system consists of numbered horizontal and vertical lines that are used to describe position.

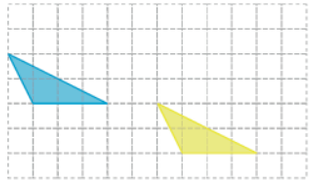
In each case, state whether the triangle was translated, reflected or rotated:

1. 

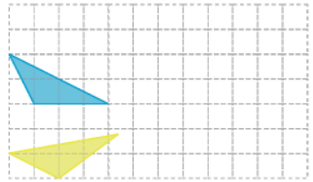
rotated

2. 

reflected

3. 

translated

4. 

rotated

Answers

5. See the answer on LB page 256 alongside.
6. See the answer on LB page 256 alongside.

COORDINATE SYSTEMS

Background information

A **coordinate system** consists of numbered horizontal and vertical lines that are used to describe position.

- The **point of intersection** of the numbered lines is called the **origin**.
- The **horizontal numbered line** is called the ***x*-axis**.
- The **vertical numbered line** is called the ***y*-axis**.
- The coordinate system is divided into **four quadrants** by the system of axes.

Teaching guidelines

The **position of a point** in a coordinate system is described by using an **ordered pair of coordinates (*x*; *y*)**. Refer to LB page 256 alongside.

The ordered pair of coordinates A(4; 3) indicates that:

- the value of the *x*-coordinate of point A is 4, which means that point A lies four units to the right of the *y*-axis, and
- the value of the *y*-coordinate of point A is 3, which means that point A lies three units above the *x*-axis.

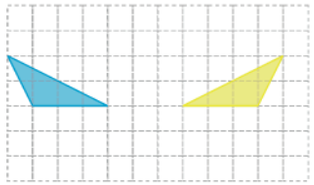
The ordered pair of coordinates B(4; –3) indicates that:

- the value of the *x*-coordinate of point B is 4, which means that point B lies four units to the right of the *y*-axis, and
- the value of the *y*-coordinate of point B is –3, which means that point B lies three units below the *x*-axis.

Answers

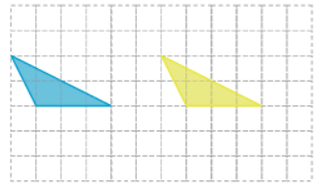
1. E (–6; –5) F (1; –5) G (0; 2)
   H (3; 0) I (–6; 4) J (1; –2)
2. See the points on the coloured coordinate system on LB page 256 alongside.

5.



reflected

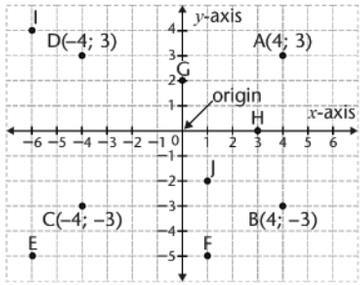
6.



translated

COORDINATE SYSTEMS

The position of any point on a system of coordinates can be described by two numbers, as demonstrated below for the points A, B, C and D.

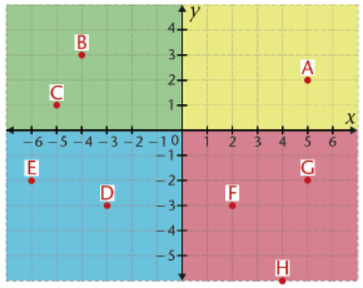


In honour of the French mathematician, Descartes, who invented it, a coordinate system is also called *a system of Cartesian coordinates*.

The horizontal axis on the coordinate system is called the *x*-axis and the vertical axis is called the *y*-axis. The ordered pair (4; 3) indicates that the value of the *x*-coordinate is 4 and the value of the *y*-coordinate is 3. A coordinate system is divided into four sections called **quadrants**.

1. For the coordinate system above, give the actual coordinates of points E, F, G, H, I and J.

The first quadrant is coloured yellow on the system on the right, the second quadrant green, the third quadrant blue and the fourth quadrant red.



2. Copy the coordinate system onto grid paper. Mark the following points on the coordinate system:
   A(5; 2) B(–4; 3)
   C(–5; 1) D(–3; –3)
   E(–6; –2) F(2; –3)
   G(5; –2) H(4; –6)

Teaching guidelines (continued)

- In the first quadrant both coordinates are positive: $x > 0$ and $y > 0$.
- In the second quadrant only the x-coordinate is negative: $x < 0$ and $y > 0$.
- In the third quadrant both coordinates are negative: $x < 0$ and $y < 0$.
- In the fourth quadrant only the y-coordinate is negative: $x > 0$ and $y < 0$.

Answers

3. (a) first quadrant (b) third quadrant
   (c) second quadrant (d) fourth quadrant

24.2 Translation on the coordinate system

TRANSLATING POINTS ON THE COORDINATE SYSTEM

Background information

These are the properties of translation:

- The line segments that connect any original point in the object to its image are all **equal in length**.
- The line segments that connect any original point in the object to its image are all **parallel**.
- When a figure is translated, the figure and its image are **congruent**.

Teaching guidelines

Remind learners of the properties of translation as learnt in grade 7.

Answers

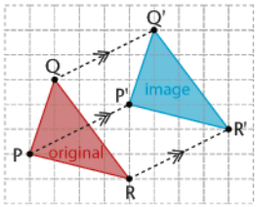
1. See the answers on the first coordinate system on LB page 257 alongside.
2. (a) A(–1; 2); B(1; –4); C(4; 1)
   (b) See the second coordinate system on LB page 257 alongside.
   (c) A'(–7; 6); B' (–5; 0); C' (–2; 5)
   (d) See the second coordinate system on LB page 257 alongside.
   (e) Yes

3. (a) In which quadrant are both coordinates positive?
   (b) In which quadrant are both coordinates negative?
   (c) In which quadrant is only the x-coordinate negative?
   (d) In which quadrant is only the y-coordinate negative?

24.2 Translation on the coordinate system

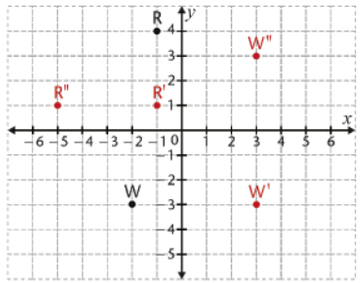
Revise the **properties of translation** from Grade 7:

- The line segments that connect any point in the original figure to its image are all equal in length. In the diagram: PP' = RR' = QQ'
- The line segments that connect any original point in the figure to its image are all parallel. In the diagram: PP'||RR'||QQ'
- When a figure is translated, its shape and size do not change. The original and its image are congruent.

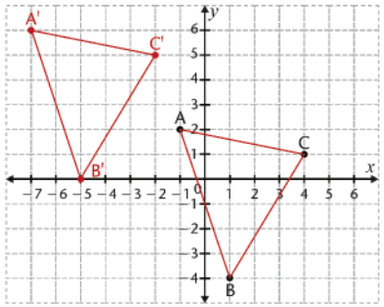


TRANSLATING POINTS ON THE COORDINATE SYSTEM

1. Copy the coordinate system onto grid paper. Plot the image of each of the following translations:
   (a) R is translated three units down to R'.
   (b) R' is translated four units to the left, to R".
   (c) W is translated five units to the right, to W'.
   (d) W' is translated six units up, to W".



2. (a) Copy the coordinate system onto grid paper. Write down the coordinates of points A, B and C.
   (b) Translate A, B and C six units to the left and four units up.
   (c) Write down the coordinates of points A', B' and C'.
   (d) Join points A, B and C to form a triangle. Do the same with points A', B' and C'.
   (e) Are ΔABC and ΔA'B'C' congruent?



Misconceptions

Learners often plot the coordinates of a point the wrong way around. The problem can be solved if the following strategy is followed:

- **ALWAYS start** at the **origin**.
- **From the origin**, perform the **horizontal translation**:
  If x is positive, move to the right. If x is negative, move to the left.
- **From that point on the x-axis**, perform the **vertical translation**:
  If y is positive, move up. If y is negative, move down.

TRANSLATING TRIANGLES ON THE COORDINATE SYSTEM

Teaching guidelines

Guide learners so that at the end of this activity they understand that:

- To translate ΔPQR six units to the right and two units down:
  - add six to the x-coordinates, and
  - subtract two from the y-coordinates of all its vertices.
- To translate ΔPQR four units to the left and three units up:
  - subtract four from the x-coordinates, and
  - add three to the y-coordinates of all its vertices.

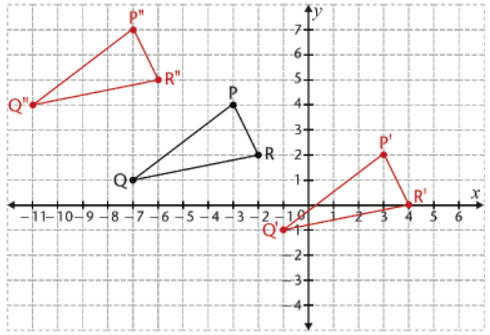
Answers

1. (a) P'(3; 2); Q'(−1; −1); R'(4; 0)
   (b) P''(−7; 7); Q''(−11; 4); R''(−6; 5)
2. (a) D'(−3; −5); E'(−1; −7); F'(1; −4)
   (b) D''(4; 1); E''(6; −1); F''(8; 2)
3. See the table on LB page 258 alongside.

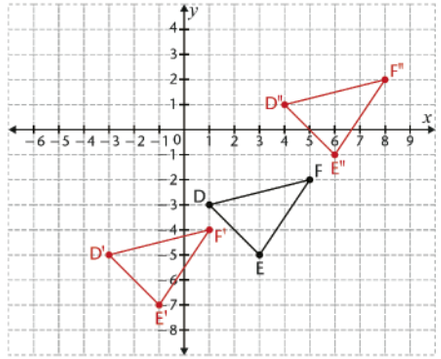
TRANSLATING TRIANGLES ON THE COORDINATE SYSTEM

When you plot the transformation of a shape, first plot the images of the vertices of the shape and then join the image points to create the shape.

1. (a) Copy the coordinate system onto grid paper. Translate ΔPQR six units to the right and two units down. What are the coordinates of the vertices of ΔP'Q'R'?
   (b) Translate ΔPQR four units to the left and three units up. What are the coordinates of the vertices of ΔP''Q''R''?



2. (a) Copy the coordinate system onto grid paper. Translate ΔDEF four units to the left and two units down. What are the coordinates of the vertices of ΔD'E'F'?
   (b) Translate ΔDEF three units to the right and four units up. What are the coordinates of the vertices of ΔD''E''F''?



3. Copy the table and write down the coordinates of the vertices of ΔKLM after each translation described in the following table:

Vertices of triangle	Translated five units to the right and two units down	Translated four units to the left and three units down	Translated two units to the right and three units up
K(−1; 3)	K'(4; 1)	K''(−5; 0)	K'''(1; 6)
L(−2; −3)	L'(3; −5)	L''(−6; −6)	L'''(0; 0)
M(4; 0)	M'(9; −2)	M''(0; −3)	M'''(6; 3)

24.3 Reflection on the coordinate system

REFLECTING POINTS IN THE *x*-AXIS OR IN THE *y*-AXIS

Background information

These are the properties of reflection:

- An object and its image lie on **opposite sides** of the line of reflection.
- The **distance** from the original point to the line of reflection is the same as the distance from the image point to the line of reflection.
- The line that connects the original point to its image point is always **perpendicular** to the line of reflection.
- When a figure is reflected, the figure and its image are **congruent**.

A line of reflection can run in **any direction**. However, in Grade 8, only reflections in the ***x*-axis** and ***y*-axis** are addressed.

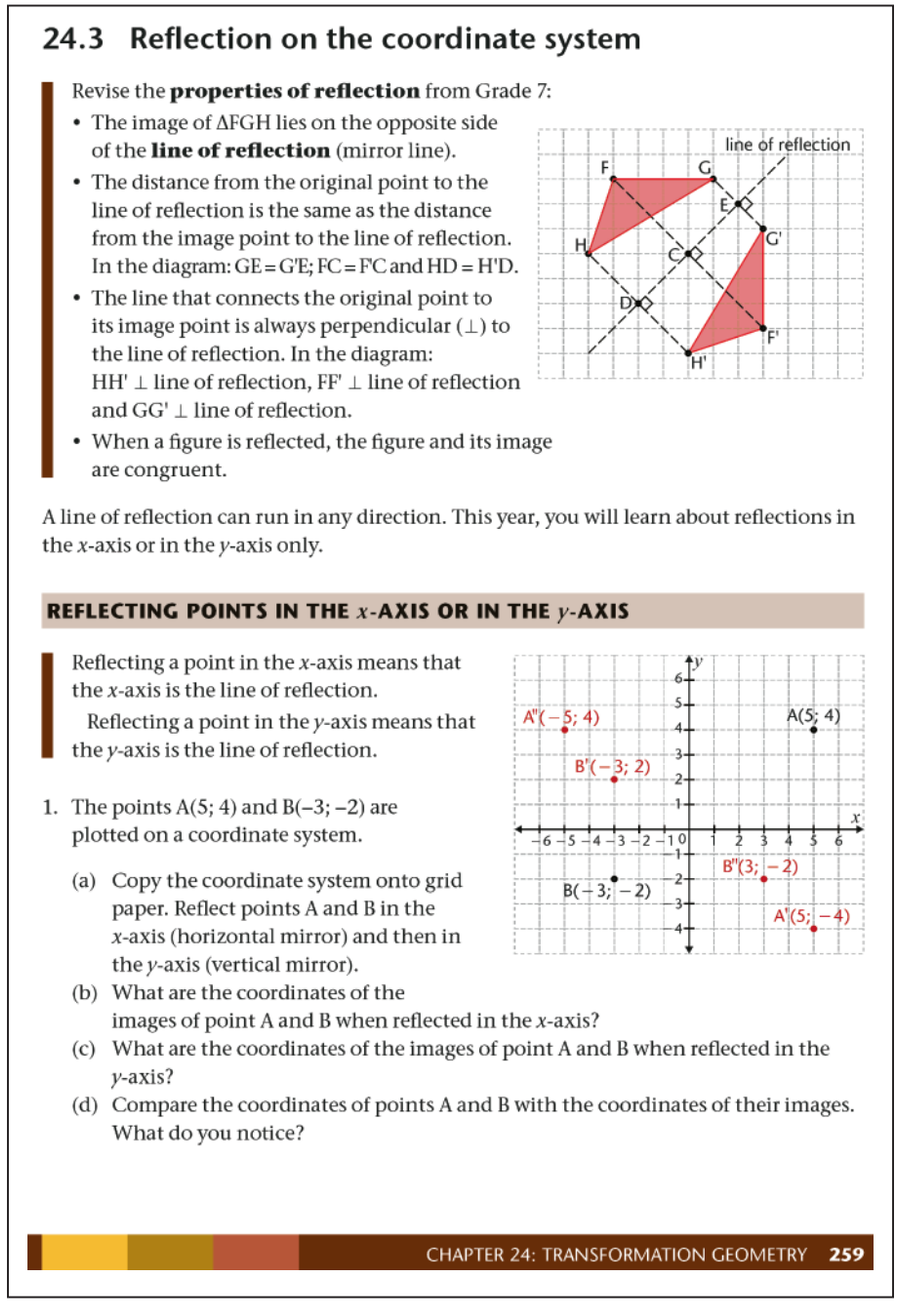
- Reflecting a point in the *x*-axis means that the ***x*-axis is the line of reflection**.
- Reflecting a point in the *y*-axis means that the ***y*-axis is the line of reflection**.

Teaching guidelines

- Remind learners of the properties of translations as learnt in grade 7.
- Recall the symbol for perpendicular (⊥).

Answers

1. (a) See the coordinate system on LB page 259 alongside.
   (b) A'(5; –4); B'(–3; 2)
   (c) A''(–5; 4); B''(3; –2)
   (d) If reflected in the *x*-axis, the *x*-coordinate stays the same and the *y*-coordinate has a different sign.
   If reflected in the *y*-axis, the *x*-coordinate has a different sign and the *y*-coordinate stays the same.



24.3 Reflection on the coordinate system

Revise the **properties of reflection** from Grade 7:

- The image of ΔFGH lies on the opposite side of the **line of reflection** (mirror line).
- The distance from the original point to the line of reflection is the same as the distance from the image point to the line of reflection. In the diagram: GE = G'E; FC = F'C and HD = H'D.
- The line that connects the original point to its image point is always perpendicular (⊥) to the line of reflection. In the diagram: HH' ⊥ line of reflection, FF' ⊥ line of reflection and GG' ⊥ line of reflection.
- When a figure is reflected, the figure and its image are congruent.

A line of reflection can run in any direction. This year, you will learn about reflections in the *x*-axis or in the *y*-axis only.

REFLECTING POINTS IN THE *x*-AXIS OR IN THE *y*-AXIS

Reflecting a point in the *x*-axis means that the *x*-axis is the line of reflection.

Reflecting a point in the *y*-axis means that the *y*-axis is the line of reflection.

1. The points A(5; 4) and B(–3; –2) are plotted on a coordinate system.
   (a) Copy the coordinate system onto grid paper. Reflect points A and B in the *x*-axis (horizontal mirror) and then in the *y*-axis (vertical mirror).
   (b) What are the coordinates of the images of point A and B when reflected in the *x*-axis?
   (c) What are the coordinates of the images of point A and B when reflected in the *y*-axis?
   (d) Compare the coordinates of points A and B with the coordinates of their images. What do you notice?

CHAPTER 24: TRANSFORMATION GEOMETRY 259

Answers

2. (a) K(–6; 3); M(–5; 0); T(2; –3)
   (b) K'(–6; –3); M'(–5; 0); T'(2; 3)
   (c) K''(6; 3); M''(5; 0); T''(–2; –3)
   (d) See the first coordinate system on LB page 260 alongside.
   (e) Yes

REFLECTING TRIANGLES IN THE *x*-AXIS OR IN THE *y*-AXIS

Teaching guidelines

Guide learners so that at the end of this activity they understand that:

To reflect ΔPQR in the *x*-axis:

- keep the *x*-coordinates, and
- change the signs of the *y*-coordinates of all its vertices.

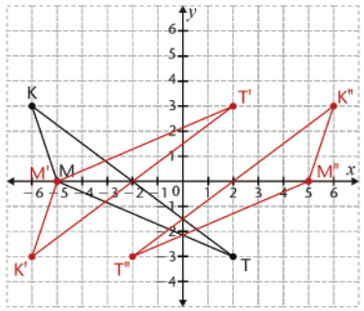
To reflect ΔPQR in the *y*-axis:

- change the signs of the *x*-coordinates, and
- keep the *y*-coordinates of all its vertices.

Answers

1. (a) See ΔP′Q′R′ on the second coordinate system on LB page 260 alongside.
   (b) See ΔP′′Q′′R′′ on the second coordinate system on LB page 260 alongside.
2. (a) See ΔD′E′F′ on the third coordinate system on LB page 260 alongside.
   (b) See ΔD′′E′′F′′ on the third coordinate system on LB page 260 alongside.

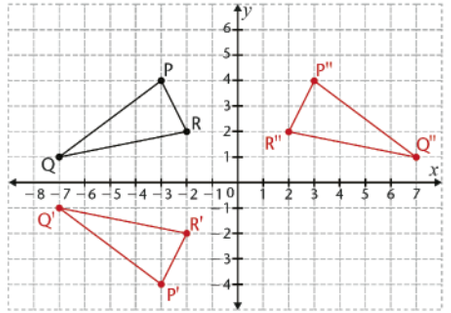
2. The points K, M and T are plotted on the coordinate system.
   (a) Write down the coordinates of points K, M and T.
   (b) Copy the coordinate system onto grid paper. Reflect each point in the *x*-axis and write down the coordinates of K', M' and T'.
   (c) Reflect points K, M and T in the *y*-axis and write down the coordinates of K'', M'' and T''.
   (d) Join points K, M and T to form a triangle. Do the same with points K', M' and T', and with points K'', M'' and T''.
   (e) Are all three triangles congruent?



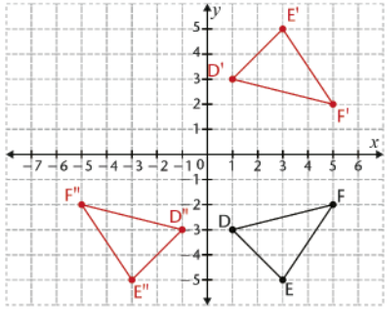
REFLECTING TRIANGLES IN THE *x*-AXIS OR IN THE *y*-AXIS

When you reflect a triangle, first reflect the vertices of the triangle and then join the reflected points.

1. (a) Copy the coordinate system onto grid paper. Reflect ΔPQR in the *x*-axis.
   (b) Reflect ΔPQR in the *y*-axis.



2. (a) Copy the coordinate system onto grid paper. Reflect ΔDEF in the *x*-axis.
   (b) Reflect ΔDEF in the *y*-axis.



Answers

3. (a) See the first table on LB page 261 alongside.
   (b) See the second table on LB page 261 alongside.
   (c) See the third table on LB page 261 alongside.

24.4 Rotation on the coordinate system

ROTATING POINTS AND FIGURES ABOUT THE ORIGIN

Background information

These are the properties of rotation:

- The **distance** from the centre of rotation to any point on the object is equal to the distance from the centre of rotation to the corresponding point on the image. This means that any point and its image are **equidistant** from the centre of rotation.
- The angle formed by the lines that connect a point and its image with the centre of rotation is the **angle of rotation**.
- When a figure is rotated, the figure and its image are **congruent**.

On the coordinate system, the centre of rotation can be any point. In Grade 8 only the **origin (0; 0)** is used as a centre of rotation.

A point, line segment or figure can be rotated **clockwise or anti-clockwise** through any number of degrees about the centre of rotation.

Teaching guidelines

- Assist learners to recall the properties of rotation as given in the introduction paragraph.
- Recall how to indicate an angle of 90° on a drawing.

3. The coordinates of the vertices of three triangles are given in the tables below. Copy the tables. For each vertex, write down the coordinates of its reflection in the x-axis or in the y-axis as required.

(a)

Vertices of triangle	Reflection in the x-axis
K(–4; 5)	K'(–4; –5)
L(2; –5)	L'(2; 5)
M(–5; –3)	M'(–5; 3)

(b)

Vertices of triangle	Reflection in the y-axis
X(–1; 3)	X'(1; 3)
Y(–2; –3)	Y'(2; –3)
Z(4; 1)	Z'(–4; 1)

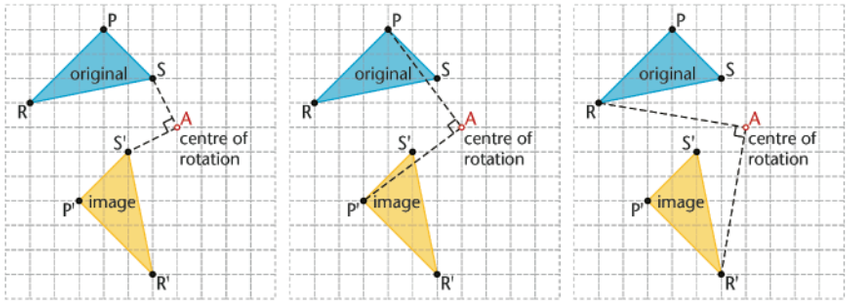
(c)

Vertices of triangle	Reflection in the y-axis	Reflection in the x-axis
D(–2; 5)	D'(2; 5)	D''(–2; –5)
E(0; –3)	E'(0; –3)	E''(0; 3)
G(2; 0)	G'(–2; 0)	G''(2; 0)

24.4 Rotation on the coordinate system

The distance from the centre of rotation to any point on the original image is equal to the distance from the centre of rotation to its corresponding point on the image. In the diagrams below: SA = S'A, PA = P'A and RA = R'A.

The angle that is formed between the line connecting an original point (S or P or R) to the centre of rotation A and the line connecting the image point (S', P', R') to the centre of rotation is equal to the angle of rotation. In the diagrams, the triangle was rotated through 90°, so SÂS' = 90°, PÂP' = 90° and RÂR' = 90°.



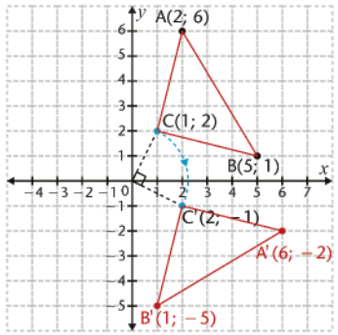
Answers

1. (a) See points A′ and B′ on the first coordinate system on LB page 262 alongside.
   (b) A'(6; –2); B'(1; –5)
   (c) See points ΔA′B′C′ on the first coordinate system on LB page 262 alongside.
   (d) Yes
   (e) The y-coordinate of the original point becomes the x-coordinate of its image, and the inverse of the x-coordinate of the original point becomes the y-coordinate of its image.
2. (a) K(–5; 7); L(–1; 3); M(3; 4)
   (b) See points K′, L′ and M′ on the second coordinate system on LB page 262 alongside.
   (c) K'(–7; –5); L'(–3; –1); M'(–4; 3)
   (d) See points K″, L″ and M″ on the second coordinate system on LB page 262 alongside.
   (e) K''(5; –7); L''(1; –3); M''(–3; –4)
   (f) A 180° angle is a straight line. Whether you rotate clockwise or anti-clockwise about 180°, you will be in the same spot, i.e. directly opposite in a straight line from where you were.

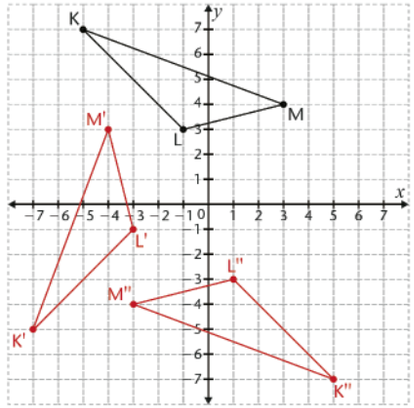
On the coordinate system, the centre of rotation can be any point. This year, you will focus on rotations about the point (0; 0), which is called the **origin**. A point, line segment or figure can be rotated clockwise or anticlockwise through any number of degrees about the centre of rotation.

ROTATING POINTS AND FIGURES ABOUT THE ORIGIN

1. In the diagram, point C has been rotated 90° clockwise about the origin. Copy the coordinate systems onto grid paper.
   (a) Rotate points A and B 90° clockwise about the origin.
   (b) Write down the coordinates of points A' and B'.
   (c) Join points A, B and C to form a triangle. Do the same with points A', B' and C'.
   (d) Are the triangle and its image congruent?
   (e) Compare the coordinates of points A, B and C with the coordinates of their images. What do you notice?



2. (a) Write down the coordinates of points K, L and M.
   (b) Rotate points K, L and M 90° anticlockwise about the origin.
   (c) Write down the coordinates of the image points.
   (d) Rotate points K, L and M 180° about the origin.
   (e) Write down the coordinates of K", L" and M".
   (f) Can you explain why there was no need to say "clockwise" or "anticlockwise" in question (d)?

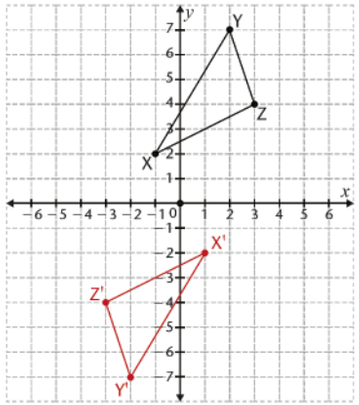


3. Copy the coordinate systems on the following page onto grid paper. Rotate the following triangles and write down the coordinates of the vertices of each triangle after the required rotation.

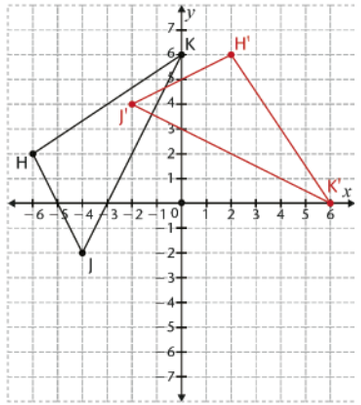
Answers

3. See the diagrams on LB page 263 alongside.
   (a) X' (1; –2); Y' (–2; –7); Z' (–3; –4)
   (b) H' (2; 6); J' (–2; 4); K' (6; 0)
   (c) D' (–1; –7); E' (4; –8); F' (3; –2)
   (d) R' (2; 3); S' (0; 5); T' (–2; –3)
4. See the diagrams on LB page 263 alongside.
   (a) K'(1; 0); C'(–1; –1); N'(–3; 2)
   (b) L'(3; –1); Z'(5; –5); F'(2; –4)
   (c) S'(4; 1); W'(0; 1); J'(4; 3)
   (d) V'(5; 3); A'(3; –1); G'(0; 3)

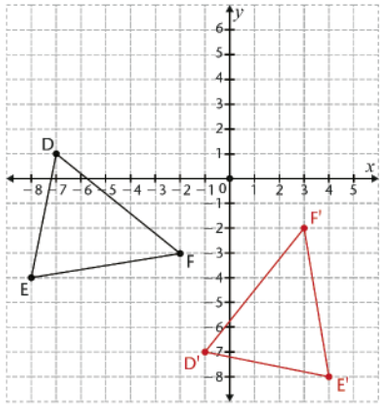
(a) 180° about the origin



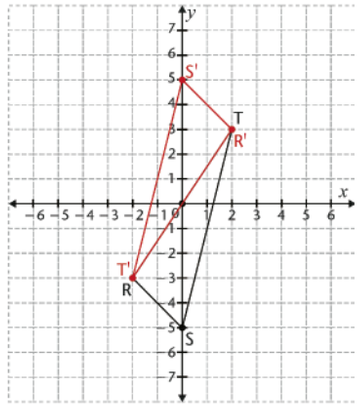
(b) 90° clockwise about the origin



(c) 90° anticlockwise about the origin



(d) 180° about the origin



4. Write down the coordinates of each image point after these transformations:
   (a) Rotation 180° about the origin: K(–1; 0); C(1; 1); N(3; –2)
   (b) Rotation 90° clockwise about the origin: L(1; 3); Z(5; 5); F(4; 2)
   (c) Rotation 90° anticlockwise about the origin: S(1; –4); W(1; 0); J(3; –4)
   (d) Rotation 180° about the origin: V(–5; –3); A(–3; 1); G(0; –3)

24.5 Enlargements and reductions

CALCULATE AND USE SCALE FACTORS

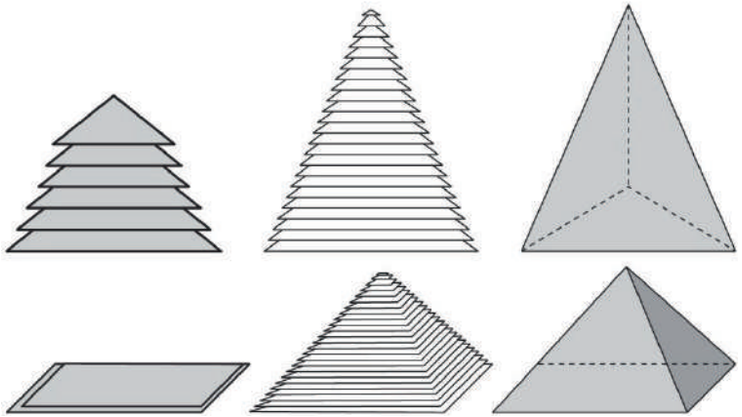
Background information

Some transformations **change the size but not the shape** of a figure.

- An **enlargement** is the image of a figure which has been made bigger by multiplying the lengths of all its sides by a number.
- A **reduction** is the image of a figure which has been made smaller by multiplying the lengths of all its sides by a number.

A figure is only an enlargement or reduction of another figure if the figures have the **same shape**. Such figures are called **similar figures**.

Similar figures which gradually decrease in size can be stacked to form a three-dimensional figure because they have the same shape.



When a figure with straight sides is enlarged or reduced, the **lengths of the sides** are increased or decreased.

To find the lengths of the sides of the new figure, the lengths of the sides of the original figure are all **multiplied** by the same number.

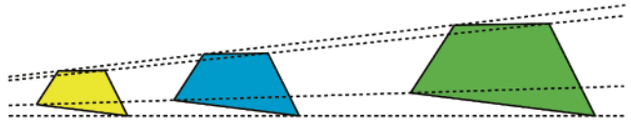
Teaching guidelines

Learners discuss the concepts of enlargement and reduction listed above.

24.5 Enlargements and reductions

CALCULATE AND USE SCALE FACTORS

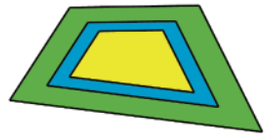
A figure may be made bigger or smaller without changing its shape.



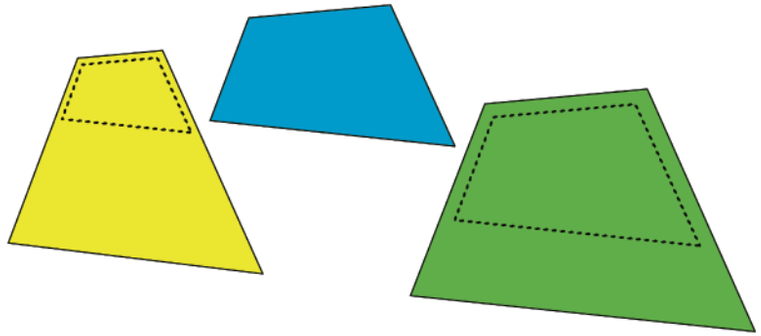
*The yellow figure is a **reduction** of the blue quadrilateral.*

*The green figure is an **enlargement** of the blue quadrilateral.*

A figure is only called an enlargement or reduction of another figure if the two figures have the **same shape**. The shapes can only be the same if all the corresponding angles are equal.



Even if the angles are equal, two figures may have different shapes. When the corresponding angles are equal, one figure is not necessarily an enlargement or reduction of the other.



Although the angles are equal, the yellow and green figures above are *not* enlargements of the blue figure.

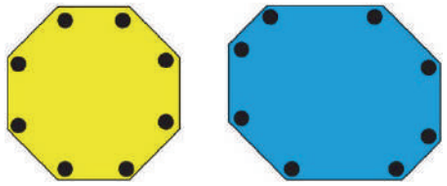
Background information (continued)

The number by which the sides of a figure are multiplied is called the **scale factor**.

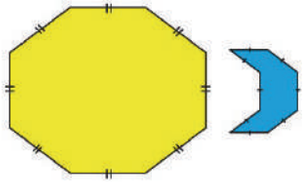
- If the scale factor is a **positive number bigger than 1**, the image is an **enlargement** of the original figure.
- If the scale factor is a **positive number smaller than 1**, the image is a **reduction** of the original figure.

A figure is an enlargement or reduction of another if **all pairs of corresponding angles are equal** AND the **ratios between the lengths of all pairs of corresponding sides are the same**. Both conditions must be satisfied because of the following:

- If the corresponding angles of two figures are equal, the figures do not necessarily have the same shape.



- If the ratios between pairs of corresponding sides of two figures are the same, the figures do not necessarily have the same shape.



When the corresponding points of a figure and its image are joined by straight lines, then all those lines will cross at a common point called the **centre of enlargement or reduction**.

Teaching guidelines (continued)

Learners continue to discuss the concepts of enlargement and reduction as listed above.

Answers

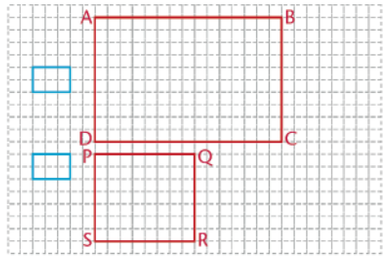
1. See rectangles ABCD and PQRS on the grid on LB page 265 alongside.

When a figure with straight sides is enlarged or reduced, the lengths of the sides are increased or decreased.

To find the lengths of the sides of the new figure, the lengths of the sides of the original figure are all multiplied by the same number. This number is called the **scale factor** of the enlargement or reduction.

The scale factor for an **enlargement** is bigger than 1. The scale factor for a **reduction** is smaller than 1.

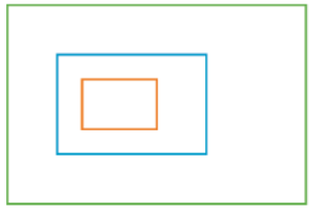
1. On grid paper, draw a bigger rectangle ABCD, with each side five times as long as the blue rectangle. Also draw another bigger rectangle PQRS, with each side five units longer than the blue rectangle.

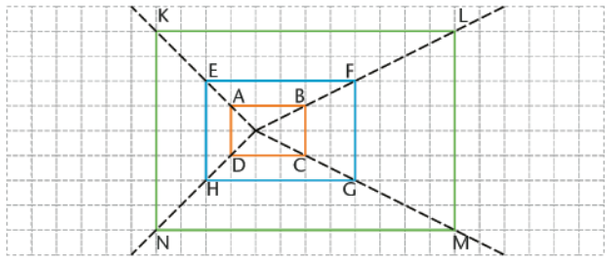


One figure is only called an enlargement or reduction of another figure if **the corresponding angles are equal** and **the ratio between the lengths of the corresponding sides is the same**, for all pairs of corresponding angles and sides in the two figures. This is demonstrated below.

The green rectangle on the right is an enlargement of the blue rectangle. The orange rectangle is a reduction of the blue rectangle.

In the following two diagrams, the same rectangles are shown on grids so that it is easy to compare the lengths of the corresponding sides and calculate the ratio between the lengths of the sides.





CHAPTER 24: TRANSFORMATION GEOMETRY 265

Background information

Ratios are used to:

- **compare the dimensions** of the original figure and its image, and
- **determine the scale factor** of an enlargement or reduction.

To determine the scale factor, the dimensions of the image should be stated before the dimensions of the original figure.

Teaching guidelines

Learners study the drawing and the accompanying information at the top of LB page 266 alongside.

Misconceptions

Learners might state the dimensions of the original figure before the dimensions of the image, which will result in the scale factor being calculated incorrectly.

Answers

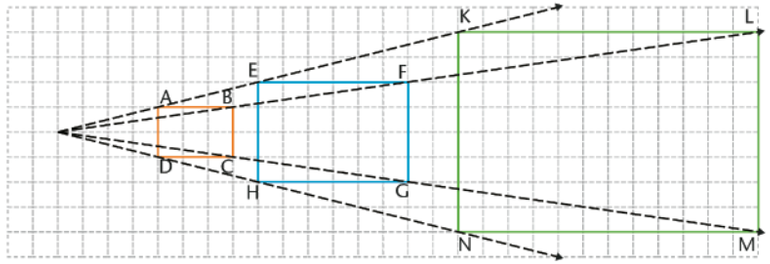
2. (a) 4
   (b) $\frac{1}{2}$
3. 15 mm wide and 20 mm long
4. (a) $\frac{HG}{DC} = \frac{6}{2} = 3$ and $\frac{EH}{AD} = \frac{18}{6} = 3$

   The ratio is constant and bigger than1.

   EFGH is an enlargement of ABCD.

   Perimeter of ABCD = 2(2 + 6) cm = 16 cm

   Perimeter of EFGH = 2(18 + 6) cm = 48 cm (which is three times the perimeter of ABCD)



KLMN is an enlargement of EFGH.

Note that $\frac{LM}{FG} = 8:4 = 2$, $\frac{MN}{GH} = 12:6 = 2$, $\frac{NK}{HE} = 8:4 = 2$ and $\frac{KL}{EF} = 12:6 = 2$.

The ratio between the lengths of corresponding sides is 2, for all four pairs of corresponding sides.

We say: The **scale factor** of the enlargement from EFGH to KLMN is 2.

To avoid confusion, mathematicians normally state the dimensions of the image first when forming ratios.

ABCD is a reduction of EFGH.

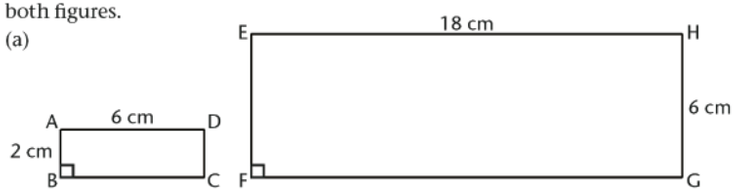
Note that $\frac{BC}{FG} = 2:4 = \frac{1}{2}$, $\frac{CD}{GH} = 3:6 = \frac{1}{2}$, $\frac{DA}{HE} = 2:4 = \frac{1}{2}$ and $\frac{AB}{EF} = 3:6 = \frac{1}{2}$.

The ratio between the lengths of corresponding sides is $\frac{1}{2}$, for all four pairs of corresponding sides. The scale factor of the reduction from EFGH to ABCD is $\frac{1}{2}$.

2. (a) What is the scale factor of the enlargement from ABCD to KLMN?
   (b) What is the scale factor of the reduction from KLMN to EFGH?
3. A rectangular shape on a photograph is 3 mm wide and 4 mm long. The photograph is enlarged with a scale factor of 5. What is the width and length of the rectangular shape on the enlarged photograph?

We work out the scale factor by calculating the ratios of the lengths of corresponding sides of the two figures. If the ratios are equal, we say that the corresponding sides are **in proportion**. This means that the second figure (the image) is a reduction or an enlargement of the first figure (the original).

4. Determine whether the second figure in each of the following pairs is an enlargement, a reduction, or neither of the two. Also work out the perimeters of both figures.
   (a)



Answers

4. (b) $\frac{XY}{TU} = \frac{5}{10} = \frac{1}{2}$; $\frac{XZ}{TV} = \frac{7}{12}$ and $\frac{YZ}{UV} = \frac{9}{15} = \frac{3}{5}$

The ratios are all different.

It is neither a reduction nor an enlargement.

Perimeter of triangle TUV = 10 cm + 12 cm + 15 cm = 37 cm

Perimeter of triangle XYZ = 5 cm + 7 cm + 9 cm = 21 cm

(c) $\frac{SR}{ML} = \frac{5}{20} = \frac{1}{4}$ and $\frac{QR}{KL} = \frac{7}{28} = \frac{1}{4}$

The ratio is constant and smaller than 1.

PQRS is a reduction of JKLM.

The perimeter JKLM = 2(20 + 28) cm = 96 cm

The perimeter of PQRS = 2(7 + 5) cm = 24 cm

5. It is not an enlargement.

EFFECT OF ENLARGEMENTS OR REDUCTIONS ON PERIMETER AND AREA

Background information

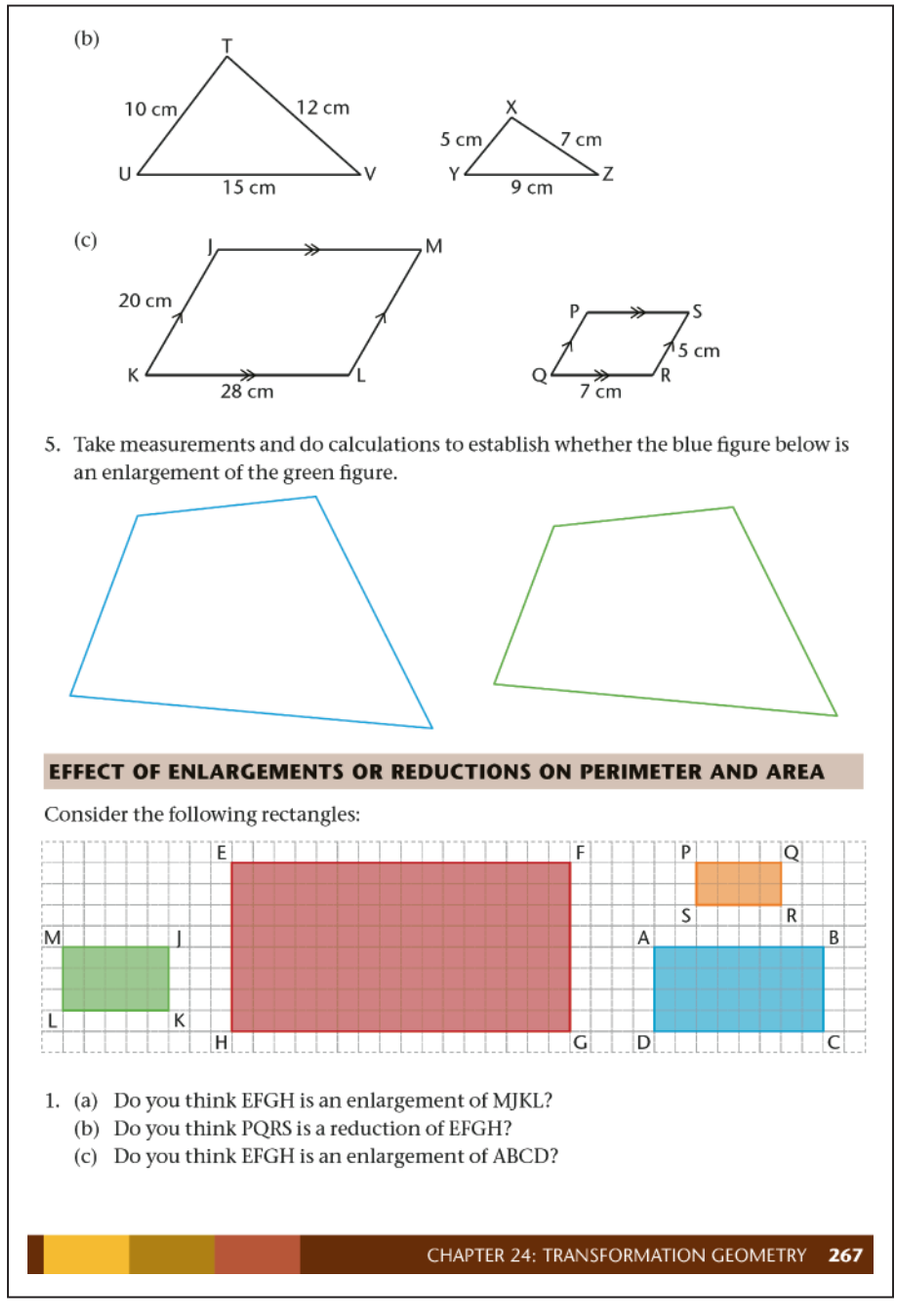
- The **perimeter** of an enlargement or reduction is equal to the **product of the perimeter of the original figure and the scale factor**.
- The **area** of an enlargement or reduction is equal to the **product of the area of the original figure and the square of the scale factor**.

Teaching guidelines

At the end of question 5 learners should be able to understand and conclude what is listed in the background information points above.

Answers

1. Learners should be allowed to explore the hypotheses they form here.


(b)

(c)

5. Take measurements and do calculations to establish whether the blue figure below is an enlargement of the green figure.

1. (a) Do you think EFGH is an enlargement of MJKL?
   (b) Do you think PQRS is a reduction of EFGH?
   (c) Do you think EFGH is an enlargement of ABCD?

Answers

2. (a) $\frac{EF}{ML} = \frac{16}{5} = 3\frac{1}{5}$; $\frac{FG}{JK} = \frac{8}{3} = 2\frac{2}{3}$; $\frac{GH}{KL} = 3\frac{1}{5}$; $\frac{HE}{LM} = 2\frac{2}{3}$
   (b) No
   (c) Not an enlargement
3. (a) $\frac{PQ}{EF} = \frac{4}{16} = \frac{1}{4}$; $\frac{QR}{FG} = \frac{2}{8} = \frac{1}{4}$; $\frac{RS}{GH} = \frac{1}{4}$; $\frac{SP}{HE} = \frac{1}{4}$ (b) Yes (c) $\frac{1}{4}$
4. (a) $\frac{EF}{AB} = \frac{16}{8} = 2$; $\frac{FG}{BC} = \frac{8}{4} = 2$; $\frac{GH}{CD} = 2$; $\frac{HE}{DA} = 2$ (b) Yes (c) 2
5. (a) Agree (b) Agree

CALCULATING PERIMETERS AND AREAS OF ENLARGED OR REDUCED FIGURES

Teaching guidelines

Learners apply their knowledge of enlargements and reductions to solve some problems.

Answers

1. Perimeter D'E'F'G' = 20 cm × 3 = 60 cm
   Area D'E'F'G' = 16 cm^2 × $(3)^2$ = 16 × 9 = 144 cm^2
2. Perimeter △J'K'L' = 120 cm × 0,5 = 60 cm
   Area △J'K'L' = 600 cm^2 × $(0,5)^2$ = 600 × 0,25 = 150 cm^2
3. Perimeter P'Q'R'S' = 30 mm × 0,2 = 6 mm
   Area P'Q'R'S' = 50 mm^2 × $(0,2)^2$ = 50 × 0,04 = 2 mm^2
4. Perimeter △S'T'U' = 51 cm ÷ 3 = 17 cm
   Area △S'T'U' = 12 cm^2 ÷ $(3)^2$ = 12 ÷ 9 = $\frac{12}{9} = \frac{4}{3}$ cm^2
5. (a) That means the scale factor = 2. Therefore, perimeter = 48 m × 2 = 96 m
   (b) Four times
6. The ratio is 21 ÷ 7 = 3. The scale factor is 3 and the area of the enlargement △D'E'F' is nine times the area of △DEF.
7. The ratio is 13 ÷ 26 = 0,5. The scale factor is 0,5 and the area of the reduction A'D'F'S' is 0,25 ($\frac{1}{4}$) times the area of ADFS.

2. (a) Calculate $\frac{EF}{MJ}$, $\frac{FG}{JK}$, $\frac{GH}{KL}$ and $\frac{HE}{LM}$.
   (b) Is rectangle EFGH an enlargement of rectangle MJKL?
   (c) If EFGH is an enlargement of MJKL, what is the scale factor?
3. (a) Calculate $\frac{PQ}{EF}$, $\frac{QR}{FG}$, $\frac{RS}{GH}$ and $\frac{SP}{HE}$.
   (b) Is rectangle PQRS a reduction of rectangle EFGH?
   (c) If PQRS is a reduction of EFGH, what is the scale factor?
4. (a) Calculate $\frac{EF}{AB}$, $\frac{FG}{BC}$, $\frac{GH}{CD}$ and $\frac{HE}{DA}$.
   (b) Is rectangle EFGH an enlargement of rectangle ABCD?
   (c) If EFGH is an enlargement of ABCD, what is the scale factor?
5. Do you agree or disagree with the following statements?
   (a) Perimeter of enlargement/reduction = perimeter of original × scale factor
   (b) Area of enlargement/reduction = area of original × (scale factor)^2

CALCULATING PERIMETERS AND AREAS OF ENLARGED OR REDUCED FIGURES

1. The perimeter of rectangle DEFG = 20 cm and its area = 16 cm^2. Find the perimeter and area of the enlarged rectangle D'E'F'G' if the scale factor is 3.
2. The perimeter of ΔJKL = 120 cm and its area = 600 cm^2. Determine the perimeter and area of the reduced ΔJ'K'L' if the scale factor is 0,5.
3. The perimeter of quadrilateral PQRS = 30 mm and its area is 50 mm^2. Find the perimeter and area of quadrilateral P'Q'R'S' if the scale factor is $\frac{1}{5}$.
4. The perimeter of ΔSTU = 51 cm and its area is 12 cm^2. Calculate the perimeter and area of ΔS'T'U' if the scale factor is $\frac{1}{3}$.
5. The perimeter of a square = 48 m.
   (a) Write down the perimeter of the square if the length of each side is doubled.
   (b) Will the area of the enlarged square be twice or four times that of the original square?
6. The perimeter of ΔDEF = 7 cm and ΔD'E'F' = 21 cm. What is the scale factor of enlargement? How many times larger is the area of ΔD'E'F' than the area of ΔDEF?
7. The perimeter of quadrilateral ADFS = 26 cm and the perimeter of quadrilateral A'D'F'S' = 13 cm. How many times larger is the area of quadrilateral A'D'F'S' than the area of quadrilateral ADFS?

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
25.1 Revision: 3D objects	General properties of 3D objects: surfaces (flat or curved), edges, vertices; polyhedra; prisms and pyramids; Euler's formula for polyhedra	Pages 269 to 276
25.2 Nets and models of prisms and pyramids	Quick, informal ways to make prisms and pyramids; nets for polyhedra, recognising nets, drawing nets and constructing 3D models of pyramids; understanding how nets work; using circles to construct nets for pyramids	Pages 277 to 293
25.3 Platonic solids	Investigating polyhedra with identical faces and equal edges; defining regular polygons and recognising solids with all faces regular polygons as Platonic solids; naming the solids; using given nets to build the Platonic solids	Pages 293 to 299

CAPS time allocation	7 hours
CAPS content specification	Page 116

Mathematical background

A **3D object** takes up space. There are different types of 3D objects. You can identify the object by its features. Some objects have:

- only curved surfaces, for example, a ball
- only flat surfaces, for example, a box (rectangular prism) – flat surfaces are called faces (2D shapes with straight sides, for example, a triangle or a pentagon)
- curved and flat surfaces, for example, a cylinder or a cone.

A 3D object with flat surfaces only is called a **polyhedron** and has 2D shapes (polygons) as faces, an edge where two faces meet and a vertex where three or more edges meet. Two particular polyhedra are prisms and pyramids.

- **Prisms** have two identical parallel faces, called the bases, and between them lateral faces that join the two bases. The bases can be any polygon. The lateral faces are parallelograms. A right prism is one where the lateral faces are rectangles that are perpendicular to the base.
- **Pyramids** have only one base that can be any polygon. The lateral faces are triangles that meet in a point called the apex. A right pyramid is one where all the faces are isosceles triangles.

Euler's formula for polyhedra gives the relationship between the number of faces (F), vertices (V) and edges (E): $F + V - E = 2$ or $F + V = 2 + E$.

A **net** of an object is the 2D pattern obtained when an object is cut open and flattened out. There may be more than one net for a given polyhedron.

A net can be designed for a polyhedron, for example a pyramid, using circles to construct the triangles on the base. In a net for a pyramid there is a triangle on each side of the polygon that forms the base. The side of each triangular face must be the same length as the closest side of the triangle next to it. Circles can be used to construct these sides so that they are equal. **Platonic solids** are polyhedra that have all their faces as identical regular polygons. There are only five Platonic solids.

25.1 Revision: 3D objects

THINK OF SPACE WHILE YOU LOOK AT PICTURES AND DRAWINGS

Teaching guidelines

Remind learners of the following points:

- 3D objects can have only curved surfaces
- 3D objects can have curved and flat surfaces
- 3D objects can have flat surfaces only.

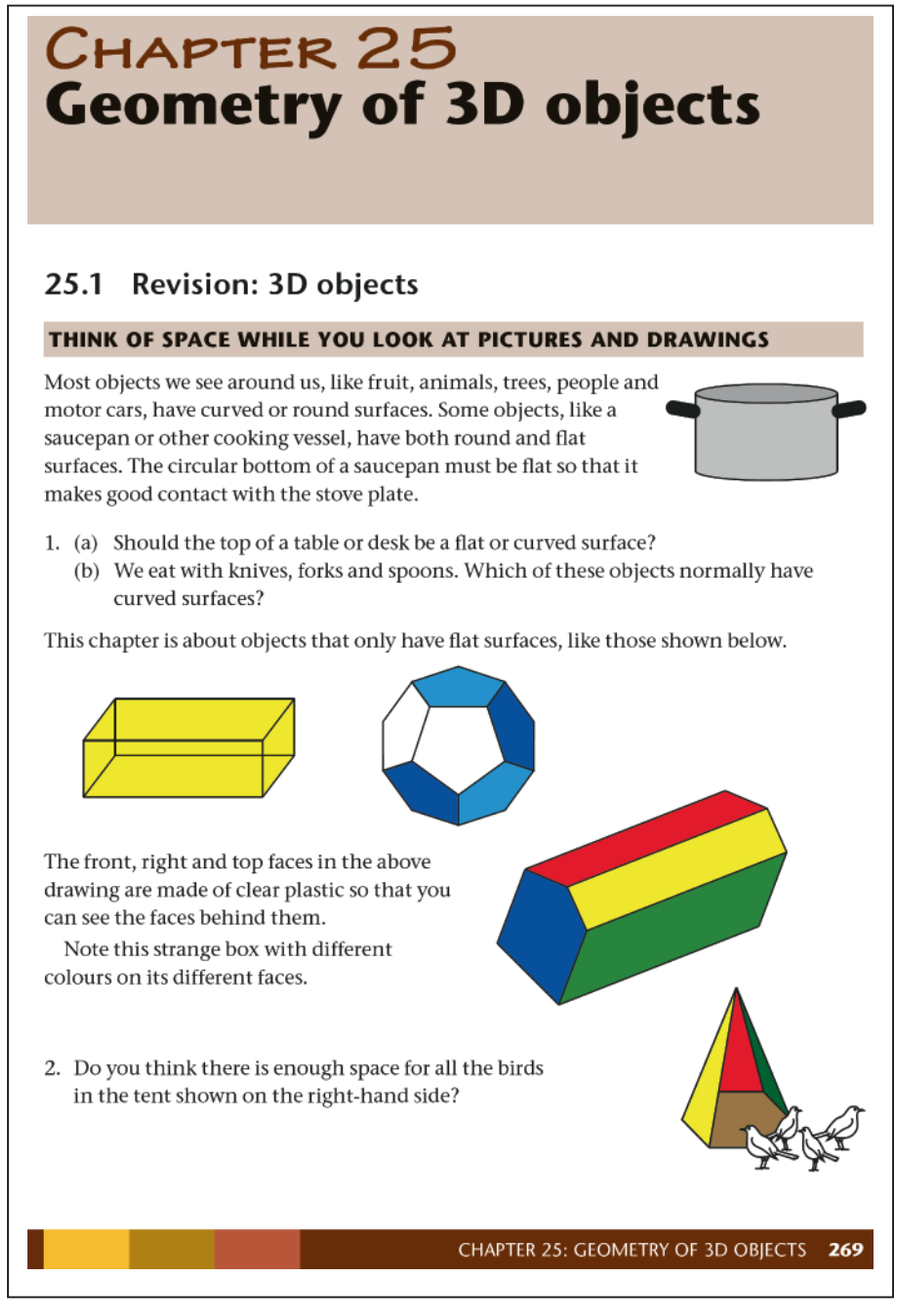
If a 3D object has flat surfaces only it is called a polyhedron.

A polyhedron has:

- **edges** where two flat surfaces meet
- a **vertex** where three or more edges meet.

Answers

1. (a) Flat
   (b) Forks and spoons
2. The answer is not relevant. The purpose of the question is to get learners to think three dimensionally when looking at drawings like these.



CHAPTER 25
Geometry of 3D objects

25.1 Revision: 3D objects

THINK OF SPACE WHILE YOU LOOK AT PICTURES AND DRAWINGS

Most objects we see around us, like fruit, animals, trees, people and motor cars, have curved or round surfaces. Some objects, like a saucepan or other cooking vessel, have both round and flat surfaces. The circular bottom of a saucepan must be flat so that it makes good contact with the stove plate.

1. (a) Should the top of a table or desk be a flat or curved surface?
   (b) We eat with knives, forks and spoons. Which of these objects normally have curved surfaces?

This chapter is about objects that only have flat surfaces, like those shown below.

The front, right and top faces in the above drawing are made of clear plastic so that you can see the faces behind them.

Note this strange box with different colours on its different faces.

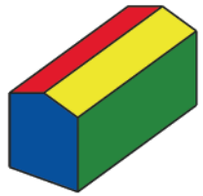
2. Do you think there is enough space for all the birds in the tent shown on the right-hand side?

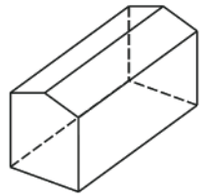
CHAPTER 25: GEOMETRY OF 3D OBJECTS 269

Answers

3. (a) 7
   (b) 3
   (c) 5
   (d) 2
4. (a) 15
   (b) 10
5. B C E F G

3. The unusual box, shown on the previous page, with flat faces (surfaces) only is shown again below. In the drawing of the same box on the right, dotted lines are used to indicate edges and surfaces that are hidden in the coloured drawing.





(a) How many faces (different flat surfaces) does this object have altogether?
(b) How many faces cannot be seen in the coloured drawing on the left?
(c) How many of the faces are rectangles?
(d) How many of the faces are pentagons?

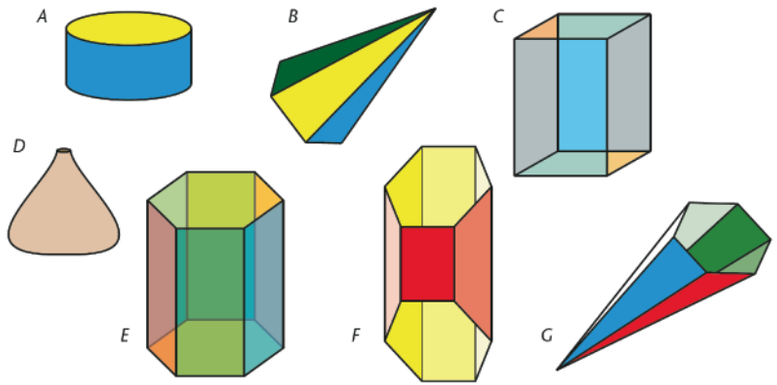
A 3D object with **flat faces** (**surfaces**) **only** is called a **polyhedron** (plural: **polyhedra**).

A straight **edge** is formed where two flat surfaces meet. The point where two or more edges meet is called a **vertex** (plural: **vertices**).

The word **polyhedron** means "many-seated" and describes the shape of such an object with many flat faces.

4. (a) How many edges does the coloured polyhedron in question 3 have?
   (b) How many vertices does it have?

5. Which of the objects below are polyhedra?



TWO SPECIAL TYPES OF POLYHEDRA

Teaching guidelines

Two special types of polyhedra to be discussed are prisms and pyramids.

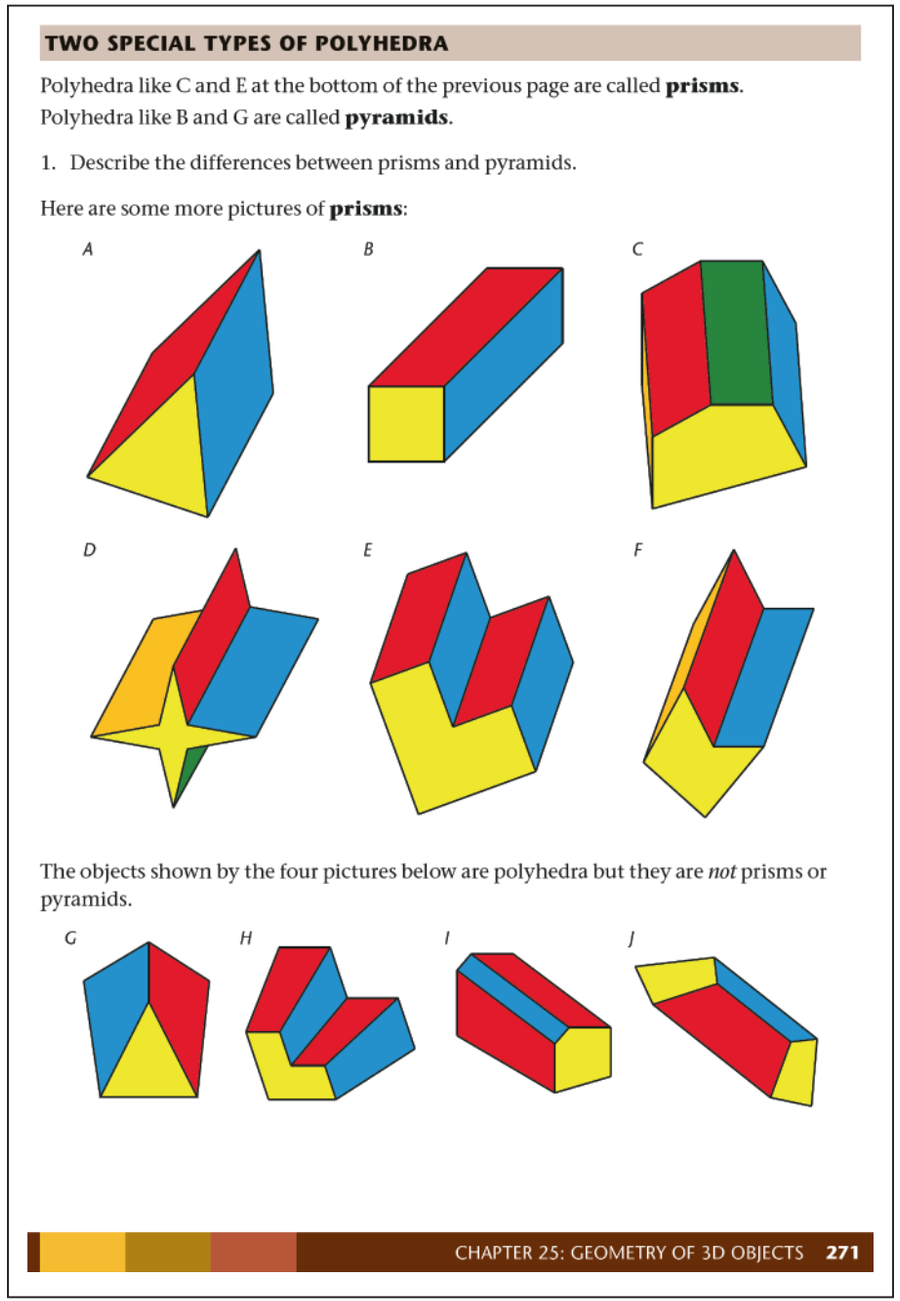
- **Prisms** are 3D objects with two flat surfaces that are parallel to each other and joined by parallelograms as lateral faces.

  A special class of prisms is the right prisms. These right prisms have the property that the lateral faces are perpendicular to the bases. The shape of the base can be any polygon.
- **Pyramids** are 3D objects on any polygonal base with lateral faces that are all triangles and that meet in a point, called the apex.

  If a polyhedron is cut open and its faces laid down flat, the 2D shape formed is called a net. A net can be folded up to give the polyhedron.
- A **cylinder** has two identical, flat, circular bases and a cone has a flat circular base.

Answers

1. Learners' answers should not be rigorously assessed. The purpose of the question is simply to get them to look at the pictures carefully. They may only mention that pyramids have triangles and prisms have rectangles, and this would be adequate for now.



TWO SPECIAL TYPES OF POLYHEDRA

Polyhedra like C and E at the bottom of the previous page are called **prisms**.
Polyhedra like B and G are called **pyramids**.

1. Describe the differences between prisms and pyramids.

Here are some more pictures of **prisms**:

A B C

D E F

The objects shown by the four pictures below are polyhedra but they are *not* prisms or pyramids.

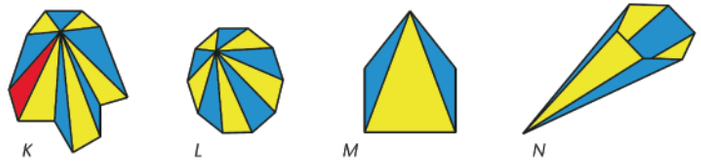
G H I J

CHAPTER 25: GEOMETRY OF 3D OBJECTS 271

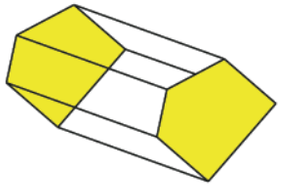
Answers

2. (a) C and F
   (b) E
   (c) D
3. N

Here are some pictures of **pyramids**. More pictures of pyramids are shown at the bottom of this page and also on the next page.



A **prism** has two identical, parallel faces (called **bases**) that are connected by parallelograms (called **lateral faces**). In the case of right prisms, the lateral faces are perpendicular to the bases and the lateral faces are rectangles.



A prism with pentagonal bases like this one is called a **pentagonal prism** because the base is a pentagon.

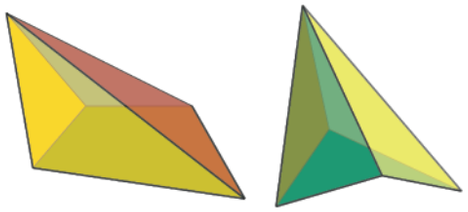
2. (a) Which pictures on the previous page also show pentagonal prisms?
   (b) Which picture on the previous page shows a hexagonal prism?
   (c) Which picture on the previous page shows an octagonal prism?

A **pyramid** has only one base. The lateral faces of a pyramid are triangles that meet at the **apex**.



The pyramid on the right is called a **hexagonal-based pyramid**.

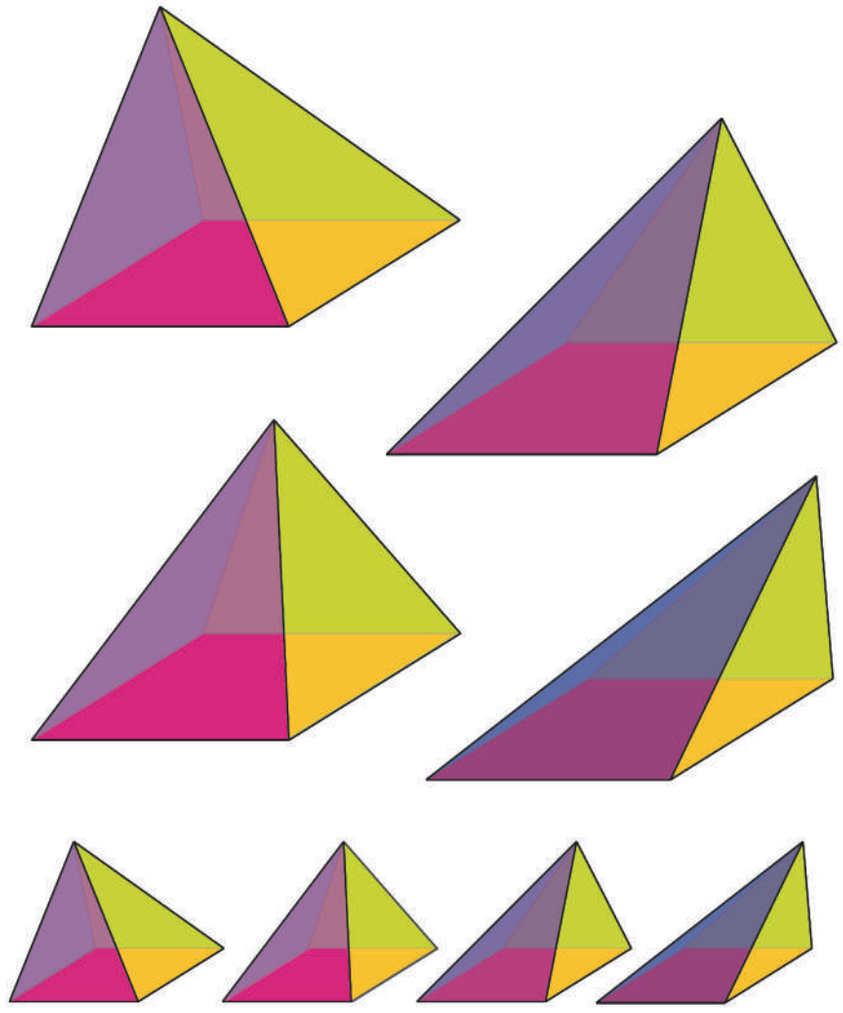
The two pyramids below have quadrilaterals as bases and are called **quadrilateral-based pyramids**.



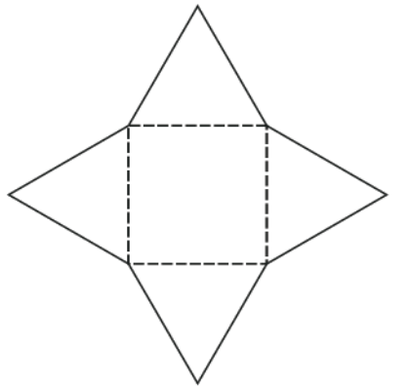
A triangular-based pyramid is also called a **triangular pyramid**; a square-based pyramid is also called a **square pyramid**; a hexagonal-based pyramid is also called a **hexagonal pyramid**, etc.

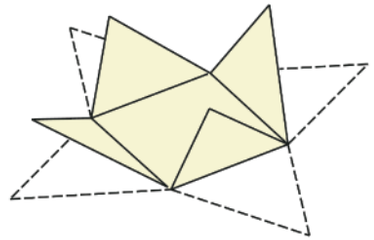
3. Which picture at the top of this page shows a hexagonal pyramid?

Pictures of different **square-based pyramids** are shown below.

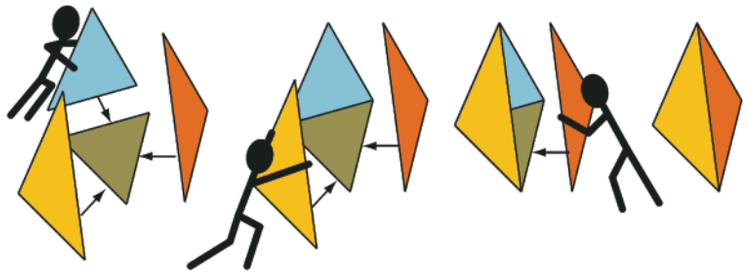


You can make a square-based pyramid by drawing and cutting out a diagram like the one on the left below, and folding the triangles up on the dotted lines, as shown on the right.



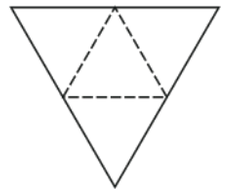


These men are building a **triangular-based** pyramid:



A triangular-based pyramid is also called a **tetrahedron**, which literally means "four-face".

A tetrahedron with four identical faces that are equilateral triangles is called a **regular tetrahedron**.



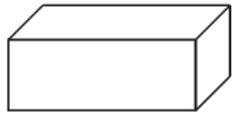
If you copy and cut out a figure like the one on the right, and fold the triangles up on the dotted lines, you can make a regular tetrahedron. A diagram like this, that can be cut out and folded to make a model of a polyhedron, is called a **net**.

A **regular polyhedron** has identical faces that are regular polygons; that is, with all sides and angles equal.

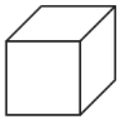
Answers

4. See the answers on LB page 275 alongside.
5. See the answers on LB page 275 alongside.

A rectangular prism is also called a **cuboid**.



A cuboid with square faces is also called a **cube**.



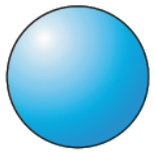
An object with two identical circular bases and one curved surface is called a **cylinder**.



A "pyramid" with a round base is called a **cone**.

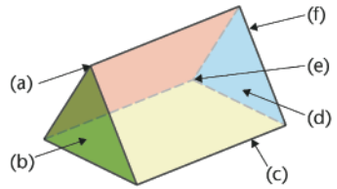


An object with the shape of a ball, in other words one curved surface with every point on its surface the same distance from its centre, is called a **sphere**.



Cylinders, cones and spheres are *not* polyhedra since they have curved surfaces. Remember, a polyhedron has faces, edges and vertices. The faces are the flat surfaces. An edge is a line along which two faces of a 3D object meet, and connects two vertices. A vertex is the point where the edges meet.

4. Write down and name the labels for parts (a) to (f) on the following figure:
   (a) vertex
   (b) face
   (c) edge
   (d) face
   (e) vertex
   (f) edge



Triangular prism

5. Grade 8 learners made 3D objects from cardboard. Can you say which kind of figure the following three learners made?
   (a) Adam's object had eight vertices and 12 edges. cube or rectangular prism
   (b) Lea's object had four vertices and four faces. triangular pyramid/tetrahedron
   (c) Mary's object had 12 edges and six congruent faces. cube

Teaching guidelines

Page 267 in the LB develops the concept of Euler's formula which applies to polyhedra: $F + V - E = 2$

> "The number of faces plus the number of vertices
> minus the number of edges equals two."

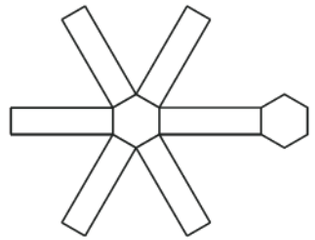
Learners can test the polyhedra they work with to make sure they can use the formula and understand it.

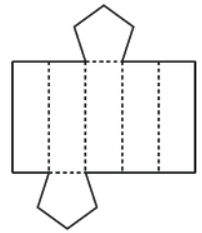
Answers

6. See the answers on LB page 276 alongside.
7. See the answers on LB page 276 alongside.
8. Yes, it is.
9. Yes, it is.

6. Copy and complete the table for prisms. Count the bases as faces too. If you find this difficult, it may help you to make quick rough sketches of nets for some prisms, like the sketches given below the table.

Number of sides in each base	Number of faces	Number of vertices	Number of edges	Faces + vertices	Edges + 2
3	5	6	9	11	11
4	6	8	12	14	14
5	7	10	15	17	17
6	8	12	18	20	20
8	10	16	24	26	26
10	12	20	30	32	32





7. Copy and complete the table for pyramids. Count the bases as faces too.

Number of sides in each base	Number of faces	Number of vertices	Number of edges	Faces + vertices	Edges + 2
3	4	4	6	8	8
4	5	5	8	10	10
5	6	6	10	12	12
6	7	7	12	14	14
7	8	8	14	16	16
9	10	10	18	20	20

8. Consider your answers for questions 6 and 7.
Is the following statement true for both prisms and pyramids?

This statement is called **Euler's formula** for polyhedra.

Statement: *The number of faces + the number of vertices = 2 + the number of edges*

9. Is Euler's formula true for the polyhedra G, H, I and J on page 271?

25.2 Nets and models of prisms and pyramids

A QUICK WAY TO MAKE PRISMS AND PYRAMIDS

Teaching guidelines

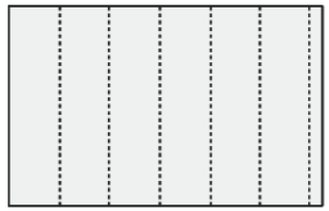
Let learners try to fold the prisms on this page. Work through the instructions with the learners.

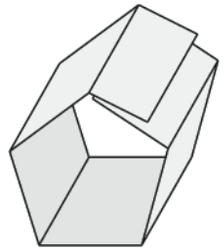
25.2 Nets and models of prisms and pyramids

A QUICK WAY TO MAKE PRISMS AND PYRAMIDS

Fold sections about two fingers wide on a sheet of A4 paper, more or less as shown by the dotted lines in the sketch on the right.

Fold the sheet into a "tube" with five or six faces along its length, as shown below.

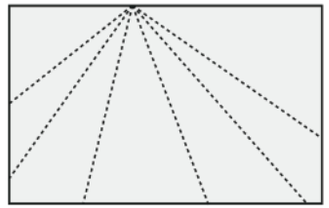


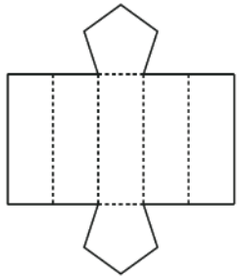


With a little extra work, you can now make a paper prism. You need to cut out two bases so that they fit well.

You can make prisms with triangular, square, rectangular, hexagonal and other shaped bases in this way.

You can make a pyramid in the same way, but it is more difficult. Draw dotted lines on a sheet of A4 paper, as shown in the sketch on the right. Fold the paper along the dotted lines. It is quite difficult to know where and how to cut so that the base is a flat surface.





Apart from the difficulty of getting the base of the pyramid flat, the above method has the disadvantage that you have to use separate pieces of paper or other material to make one object. It would be better to make the whole object by folding one piece of paper. For example, a prism with a pentagonal base can be made by drawing, cutting out and folding a sheet of paper, as shown on the left. This diagram is called a **net** of a prism with a regular pentagonal base.

NETS FOR DIFFERENT POLYHEDRA

Teaching guidelines

Remind learners that the net will be folded along the dotted lines and the flat surfaces fold up to form a 3D object. It is recommended that learners be allowed to build these objects from copied templates.

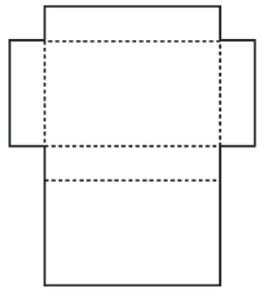
Answers

1. See answers on LB page 278 alongside.

NETS FOR DIFFERENT POLYHEDRA

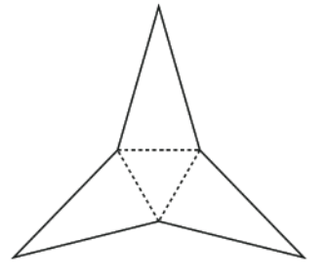
1. Name the polyhedron that can be made from each of the following nets:

(a)



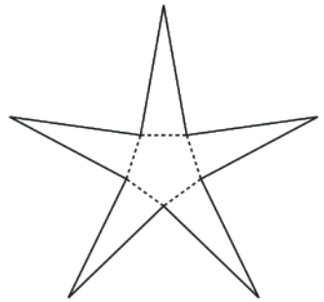
rectangular prism

(b)



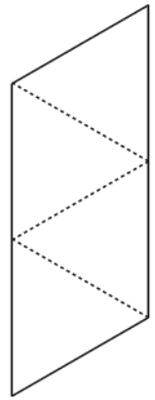
triangular pyramid

(c)



pentagonal pyramid

(d)



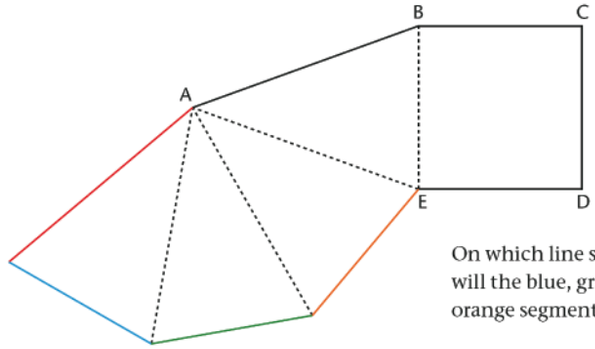
triangular pyramid/tetrahedron

Answers

2. (a) Square-based pyramid
   (b) orange on DE
   green on DC
   blue on BC
3. (a) Triangular prism
   (b) red on DE
   blue on HD
   green on AB

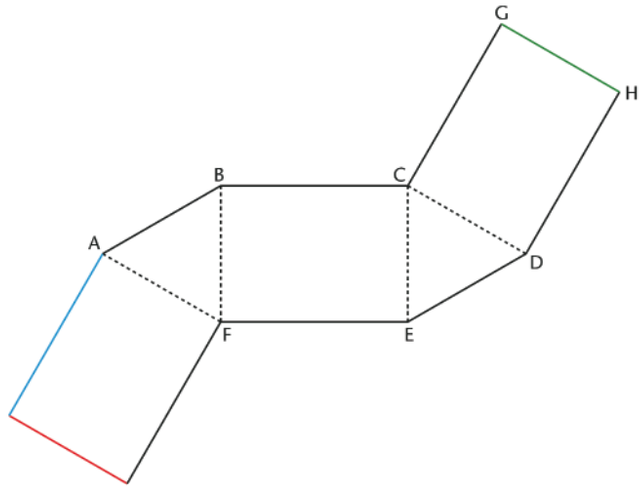
Some learners may find this very challenging. You could suggest that they copy the diagram, cut it out and fold it to make the prism.

2. (a) Name the polyhedron that can be formed by cutting out the diagram below on the solid lines and folding it on the dotted lines.
   (b) When this net is folded to make a polyhedron, the red line segment will fall on AB to form an edge.



On which line segments will the blue, green and orange segments fall?

3. (a) Name the polyhedron that can be formed by cutting out the diagram below on the solid lines and folding it on the dotted lines.
   (b) On which segments will the red, blue and green segments fall to form edges of this polyhedron?



Answers

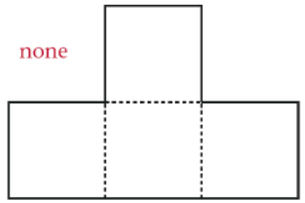
4. See the answers on LB page 280 alongside.

4. Some of the diagrams below and on the next page are the nets for the following objects:
   - a square-based pyramid
   - a hexagonal pyramid
   - a cuboid
   - a triangular prism
   - a hexagonal prism
   - a cube

Under each diagram, write the name of the object for which the diagram is a net. There may be more than one net for some of the objects. Write "none" if the diagram is not a net for any prism or pyramid.

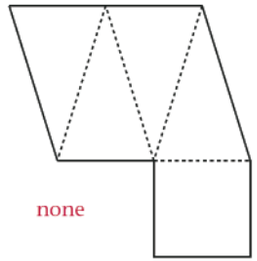
A diagram is only called a **net** of an object if the cut-out diagram can be folded to form **all** the faces of the object.

(a)



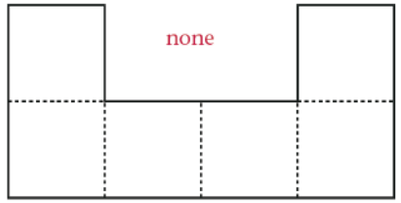
none

(b)



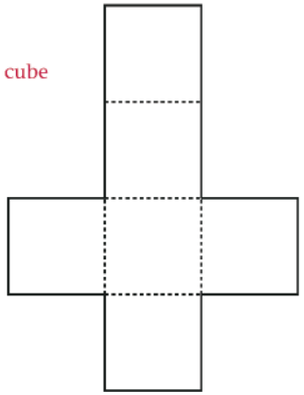
none

(c)



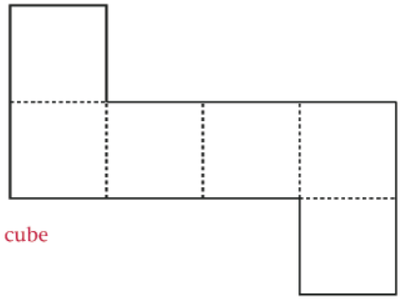
none

(d)



cube

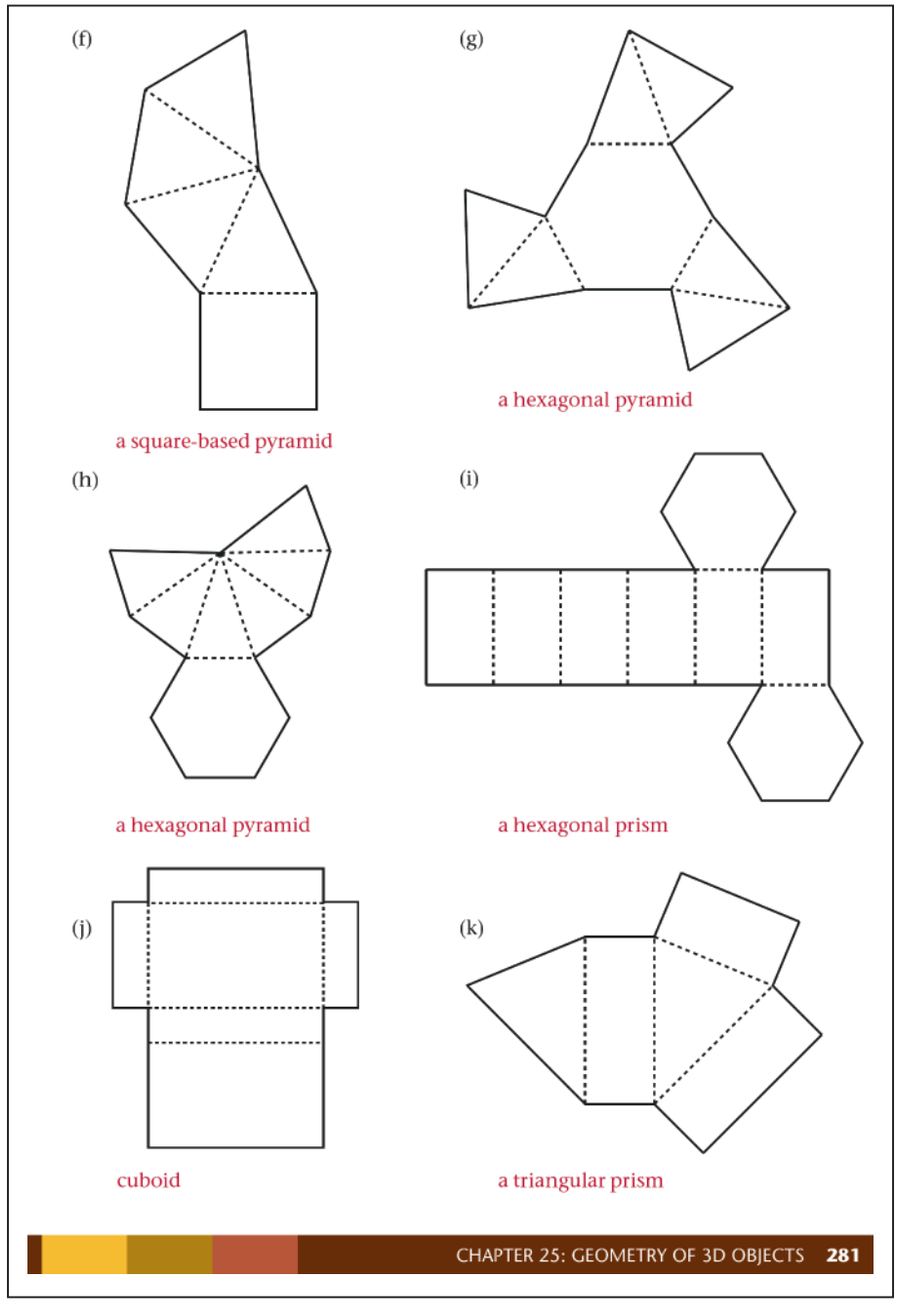
(e)



cube

Answers

4. See the answers on LB page 281 alongside.

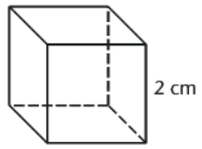


Answers

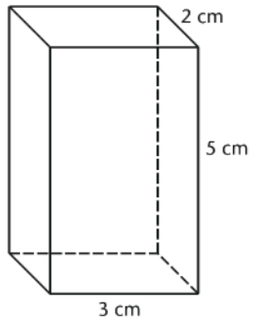
5. Different nets are possible. Consider learners' drawings.
6. Allow learners to build the polyhedra.

5. Draw a net for each of the following objects. Be accurate in your measurements.

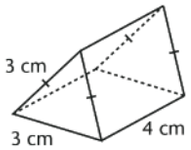
(a) Cube



(b) Rectangular prism



(c) Triangular prism



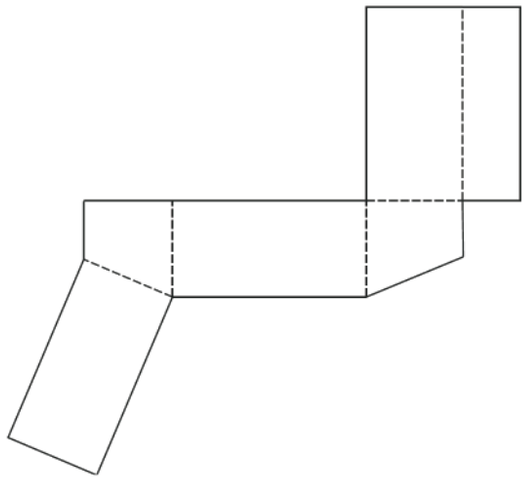
6. (a) Copy the nets in question 5 onto cardboard, but multiply the length of each side by 2. Be accurate in your constructions.
   (b) Cut out, fold and use sticky tape to paste the nets to build the 3D models.

Answers

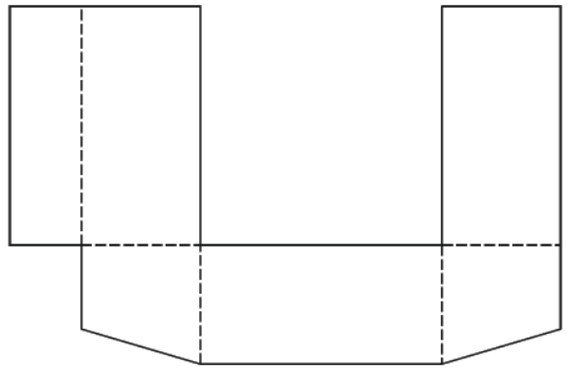
7. Only diagram (d) on LB page 284 will work as a net for the prism.

   Allow those learners who think some of the other diagrams will work to cut out the nets and try to fold them into objects. They will benefit from this.

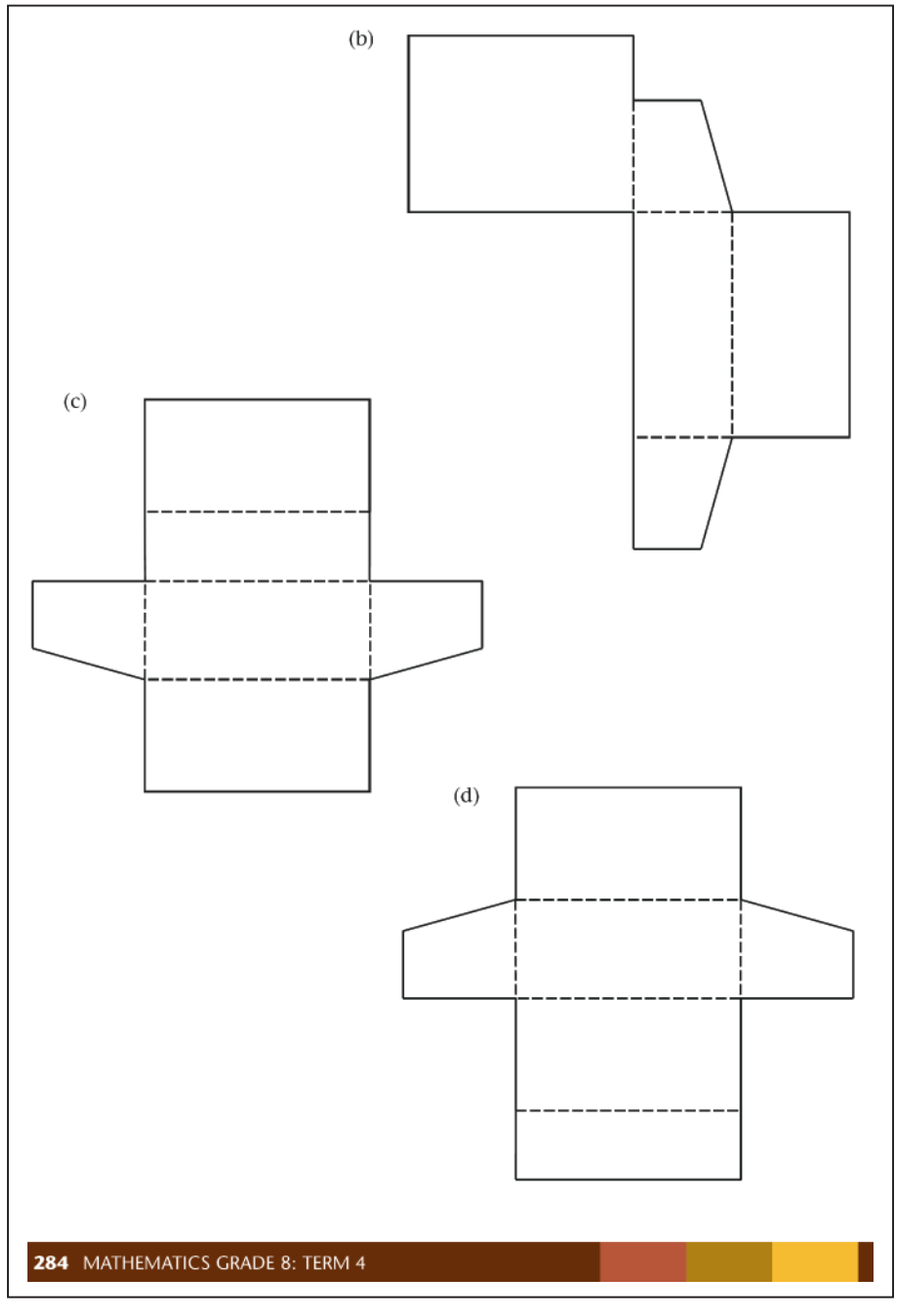
7. The first diagram below is a net for a prism with quadrilateral bases. Which of the diagrams (a), (b), (c) and (d) are nets for the same prism, and which diagrams are not nets for the prism?



(a)



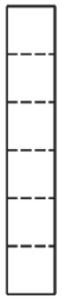
Answers (continued)



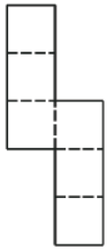
Answers

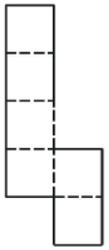
8. See the answers on LB page 285 alongside.

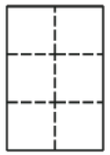
8. Which of these diagrams will work as nets for a cube?

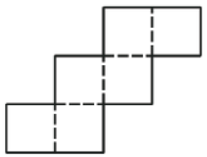
(a) 
will not work

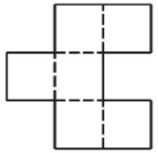
(b) 
will not work

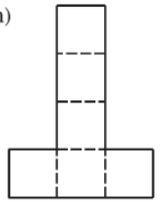
(c) 
will work

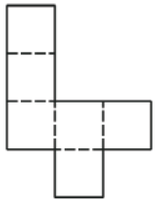
(d) 
will not work

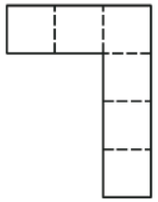
(e) 
will not work

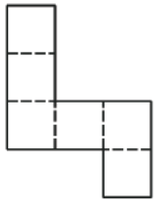
(f) 
will work

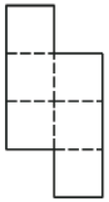
(g) 
will not work

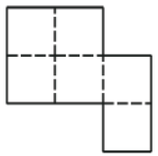
(h) 
will work

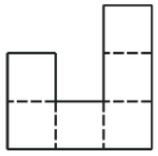
(i) 
will not work

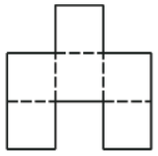
(j) 
will not work

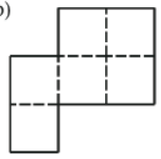
(k) 
will not work

(l) 
will not work

(m) 
will not work

(n) 
will not work

(o) 
will not work

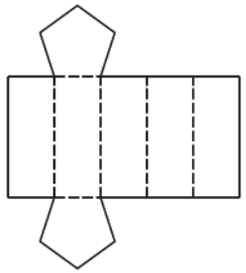
(p) 
will not work

Answers

9. See the answers on LB page 286 alongside.

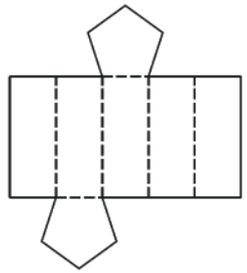
9. In each case below, state whether the diagram will work or not work as a net for making a pentagonal prism. The base need not be a regular pentagon. In the cases where the diagram will not work, explain why.

(a)



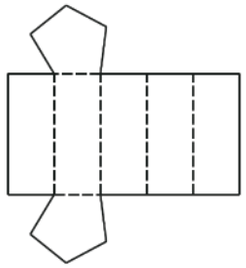
will work

(b)



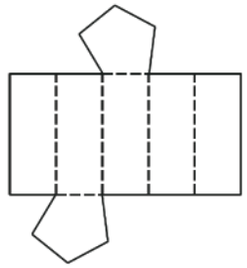
will work

(c)



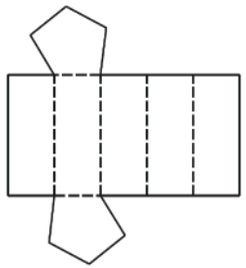
will work

(d)



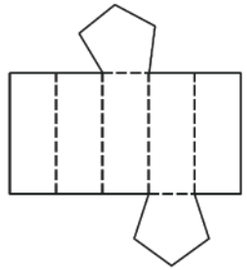
will not work (bases will not match)

(e)



will not work (bases will not match)

(f)



will not work (bases will not match)

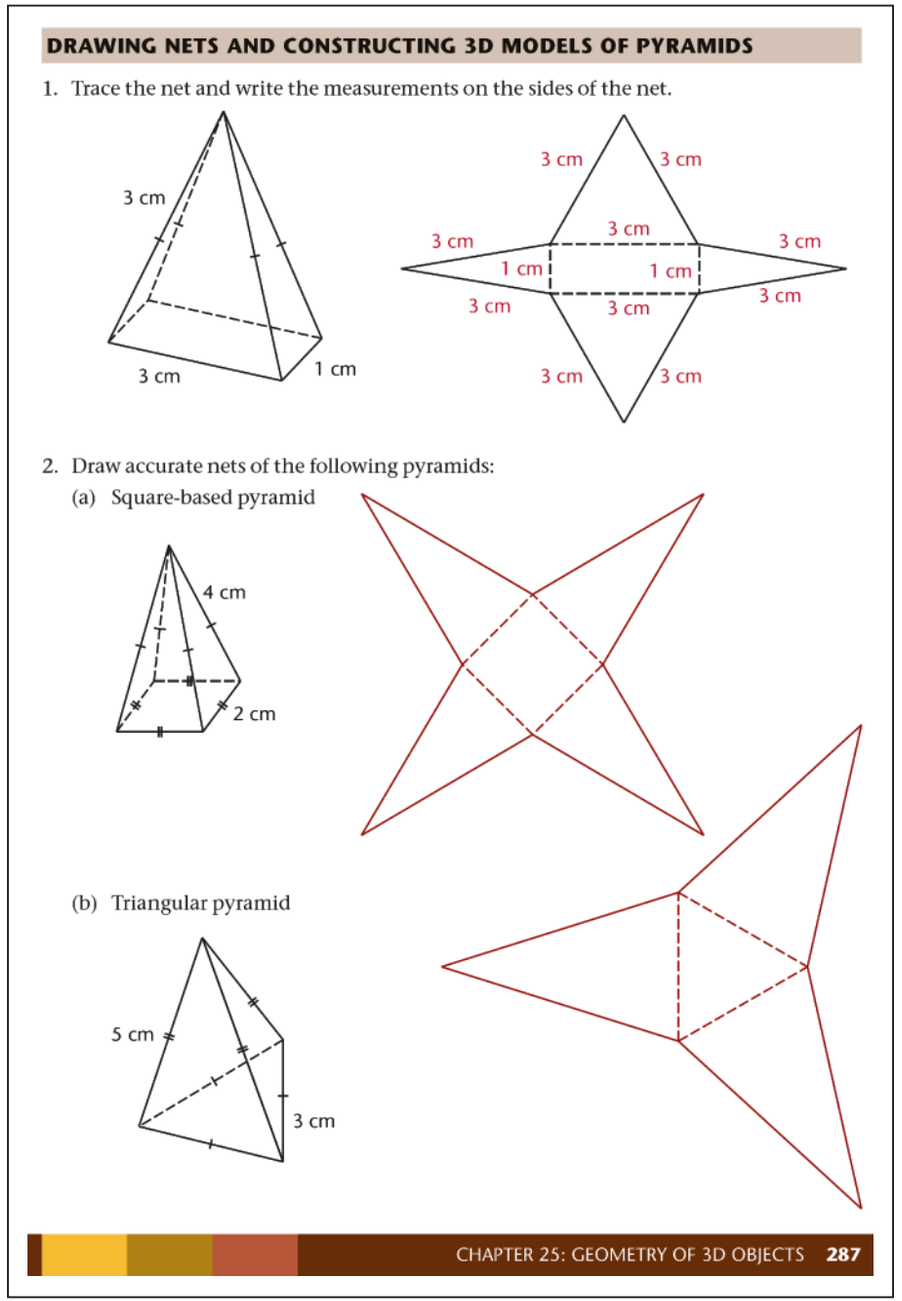
DRAWING NETS AND CONSTRUCTING 3D MODELS OF PYRAMIDS

Teaching guidelines

Let learners work in pairs to construct the nets.

Answers

1. See the answers on LB page 287 alongside.
2. (a) See the answers on LB page 287 alongside.
   (b) See the answers on LB page 287 alongside.
3. Learners build the polyhedra.



DRAWING NETS AND CONSTRUCTING 3D MODELS OF PYRAMIDS

1. Trace the net and write the measurements on the sides of the net.

3 cm
3 cm
1 cm

3 cm 3 cm
3 cm 3 cm 3 cm
1 cm 1 cm
3 cm 3 cm 3 cm
3 cm 3 cm

2. Draw accurate nets of the following pyramids:
   (a) Square-based pyramid

4 cm
2 cm

(b) Triangular pyramid

5 cm
3 cm

CHAPTER 25: GEOMETRY OF 3D OBJECTS 287

WHAT MAKES A NET WORK?

Teaching guidelines

Work through the next two pages with the learners. Make sure that learners understand that the sides of triangles on adjacent sides of the base have to have the same length for the net to work.

Answers

1. See the answer on LB page 288 alongside.
2. See the answer on LB page 288 alongside.

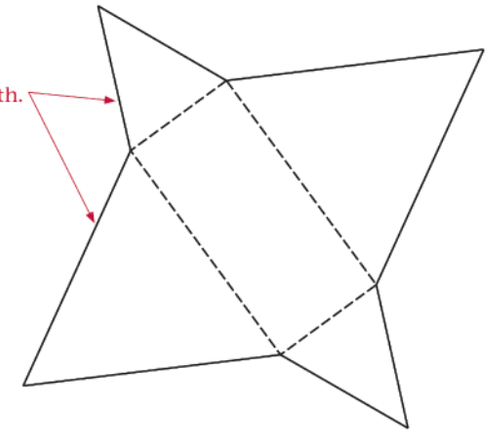
3. (a) Copy the nets you have drawn in question 2 onto cardboard or paper, but multiply the measurements by 2.
   (b) Then cut out, fold and paste the net to make a model of each 3D object.

WHAT MAKES A NET WORK?

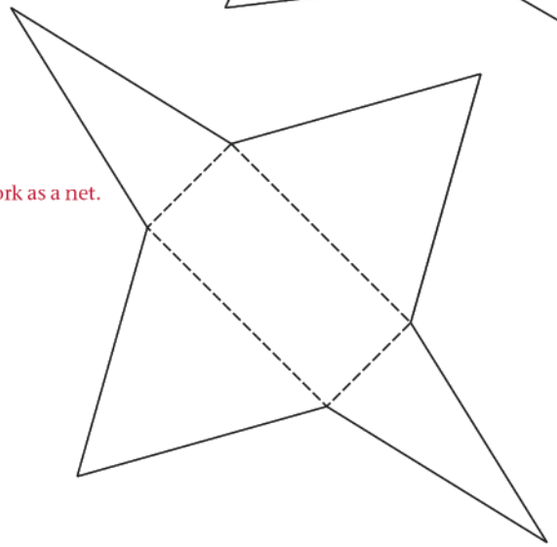
Which of the diagrams below will not work as nets to make a rectangular pyramid? You may have to take measurements to be sure in some cases, or make a copy and cut the diagram out and fold it. In each case where you say no, explain why you think it will not work.

1.

This will not work.
These sides must be equal in length.



2.



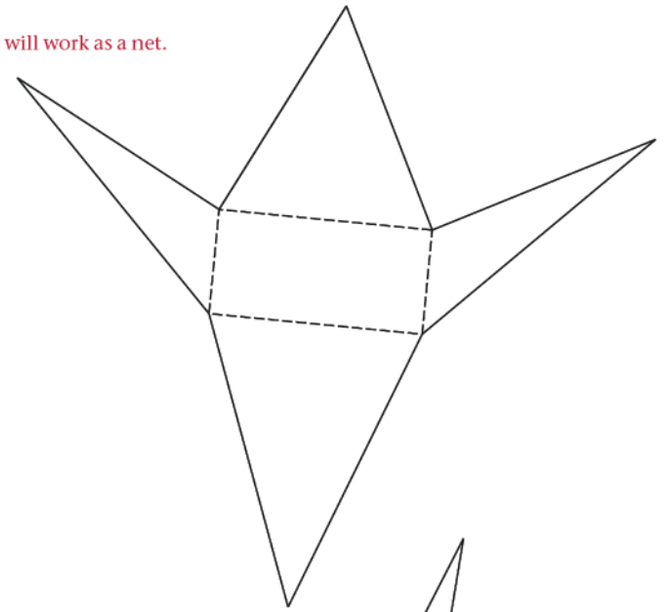
This will work as a net.

Answers

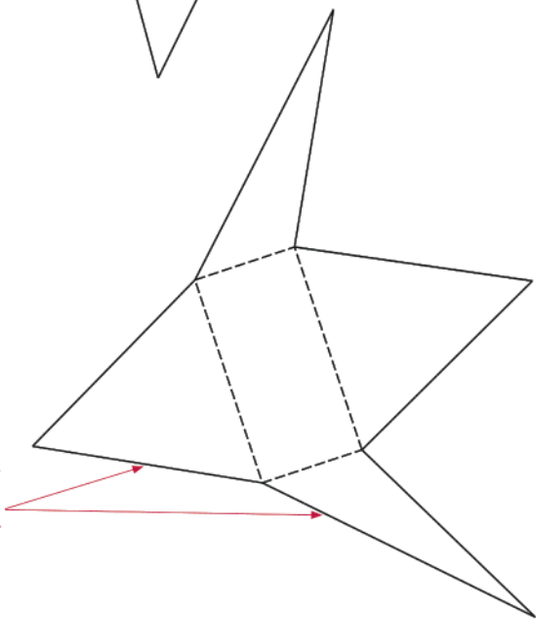
3. See the answer on LB page 289 alongside.
4. See the answer on LB page 289 alongside.

3.

This will work as a net.



4.



This will not work.
These sides must
be equal in length.

CIRCLES AND PYRAMIDS

Teaching guidelines

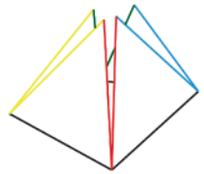
Work through the next three pages with the learners. They may find the required constructions challenging.

Answers

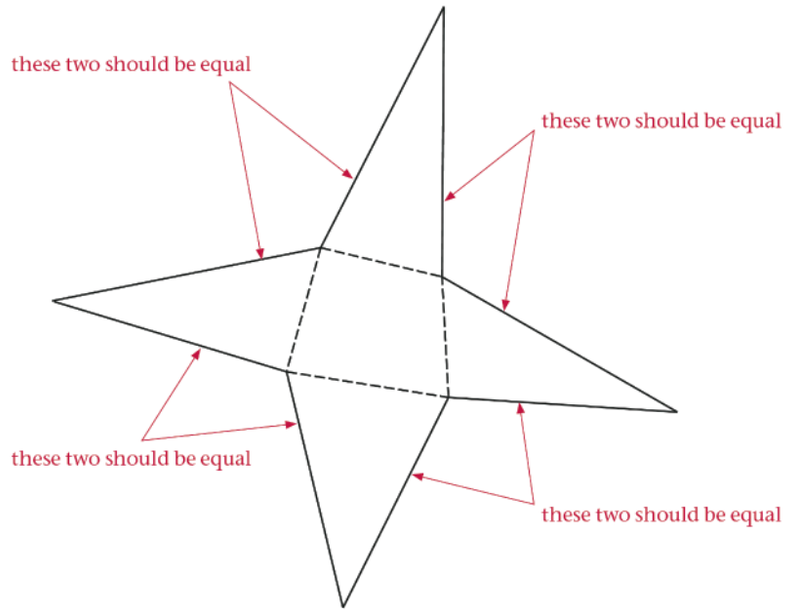
1. See the answers on LB page 290 alongside.
2. Learners can copy and build the pyramid.

In order to meet at the apex, one side of each triangular face of a pyramid must be the same length as the closest side of the triangle next to it.

This means that certain line segments in the net of a pyramid must be equal.



1. Trace the diagram below. Mark the line segments that should be equal in the diagram, so that a pyramid can be made by folding a cut-out of the diagram on the dotted lines.

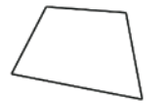


2. Make an accurate copy of the above diagram on stiff paper or cardboard. Cut it out and fold along the dotted lines. See if you can make a pyramid in this way.

Teaching guidelines

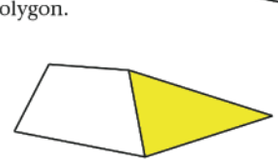
Work through the steps on this page to show learners how they can construct a net for a pyramid.

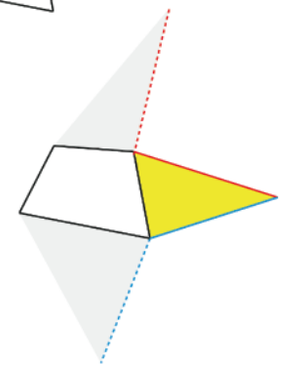
Any polygon can form the base of a pyramid.



If you want to draw a net for a pyramid, you can start by drawing a polygon.

Then draw any triangle on one side of the polygon.



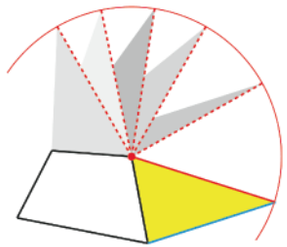


The triangles that will be adjacent to the first one must each have one side equal to the matching side of the first triangle, as indicated with the solid and dotted red and blue line segments in the sketch on the right.

The dotted line segments can be in other positions too, as long as they have the same lengths as the coloured sides of the first triangle.

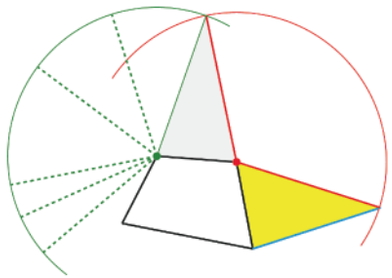
This means that once the first triangular face is drawn, there are many different possibilities for each of the two triangles that will be adjacent to it on the pyramid.

The circle with the red dot as midpoint, on the sketch on the right, shows the possibilities for one triangular face.



Once a triangle on the upper edge of the base is chosen, a circle can be drawn around the green vertex to indicate the possibilities for the third triangular face. The radius of this circle is the length of the second leg of the second triangle, shown in solid green on the sketch.

Any line segment drawn from the green circle to the green vertex can be a side of the third triangular face.



Answers

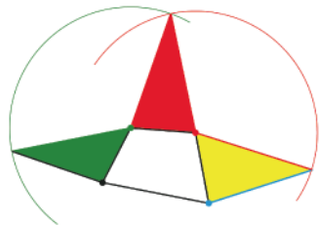
3. Consider learners' rough drawings.

4. (a) The third vertex of the blue triangle must lie on the circle with the black dot as midpoint and radius equal to the length of the black side of the green triangle.

   (b) The third vertex of the blue triangle must lie on the circle with the blue dot as midpoint and radius equal to the length of the blue side of the yellow triangle.

Only one triangular face remains to be drawn now. We will refer to it as the blue triangle.

The black and blue dots on the sketch show where two vertices of the blue triangle should be.

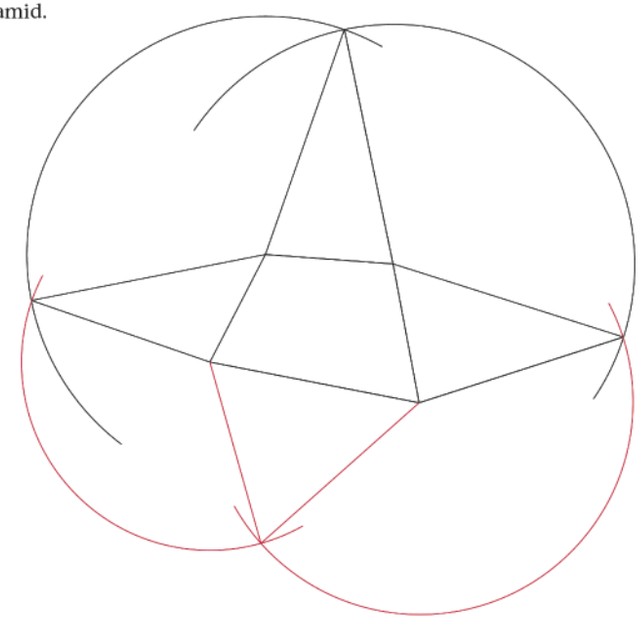


3. Copy the above sketch into your book and roughly draw the fourth triangular face for a pyramid. Think how long the sides should be so that the diagram will work precisely as a net to make a pyramid.

4. (a) How can the black dot and the green triangle help you to get some idea as to where the third vertex of the blue triangle should be?

   (b) How can the blue dot and the yellow triangle help you to get some idea as to where the third vertex of the blue triangle should be?

5. An enlargement of the sketch given at the top of this page is given below. Copy the enlargement into your book and use your pair of compasses to find the third vertex of the face that is not yet drawn, and complete the net for the pyramid. Then make a copy of the diagram on stiff paper or cardboard, cut it out and fold it to see if it forms a pyramid.



25.3 Platonic solids

MAKING POLYHEDRA WITH IDENTICAL FACES AND EQUAL EDGES

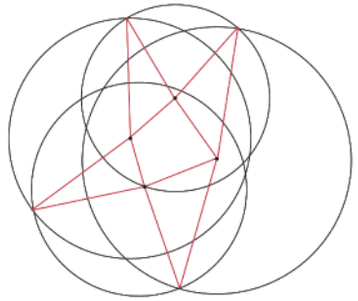
Teaching guidelines

Learners get to know polyhedra with identical faces and equal edges. This knowledge leads to an investigation into the Platonic solids.

Answers

1. Eight
2. A regular tetrahedron. Learners may or may not succeed. It is fine if they do not. Do not allow more than ten minutes for this question.
3. (a) to (c) Learners can cut out the shape and make the "container".

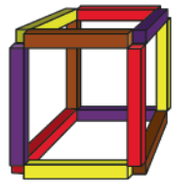
6. Join points on this sketch to draw a net for a pyramid.



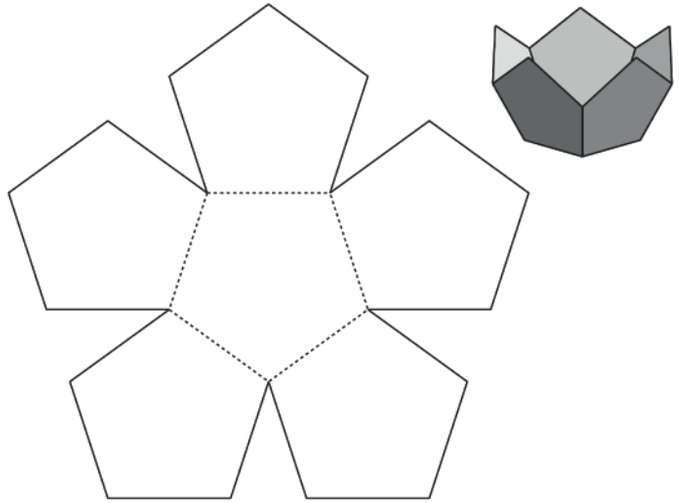
25.3 Platonic solids

MAKING POLYHEDRA WITH IDENTICAL FACES AND EQUAL EDGES

A cube is a special type of polyhedron. It has six identical faces, and its 12 edges are all equal in length.



1. How many vertices does a cube have?
2. Can you think of an object which has faces that are identical triangles, and all its edges are equal? Try draw a rough sketch of the net for such an object.
3. (a) Make a copy of the diagram below. Cut it out and fold it along the dotted lines. Attach the faces with sticky tape to make an open container with pentagonal faces.

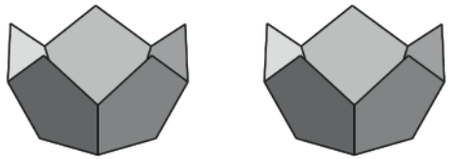


CHAPTER 25: GEOMETRY OF 3D OBJECTS 293

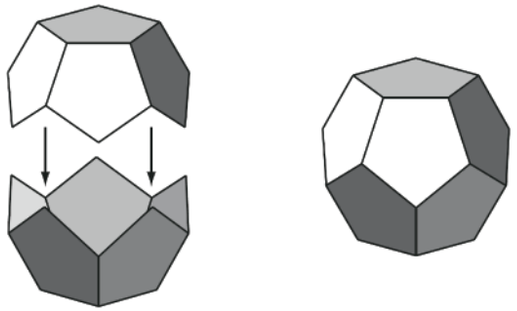
Answers

3. (d) 12
   (e) Identical regular pentagons
   (f) Thirty equally long edges

(b) Make another copy of the pentagon diagram, and make a second container.



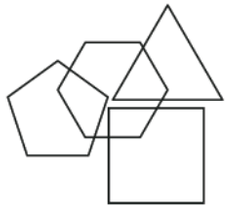
(c) Turn one of your containers upside down and put the two together to form a polyhedron.



(d) How many faces does your polyhedron have?
(e) Are the faces identical, and what shape are they?
(f) Are the edges equally long, and how many edges are there?

A polyhedron of which all the faces are identical regular polygons is called a **Platonic solid**, because the Greek philosopher Plato was fascinated by such objects.

A **regular polygon** is a polygon with equal sides and equal angles.

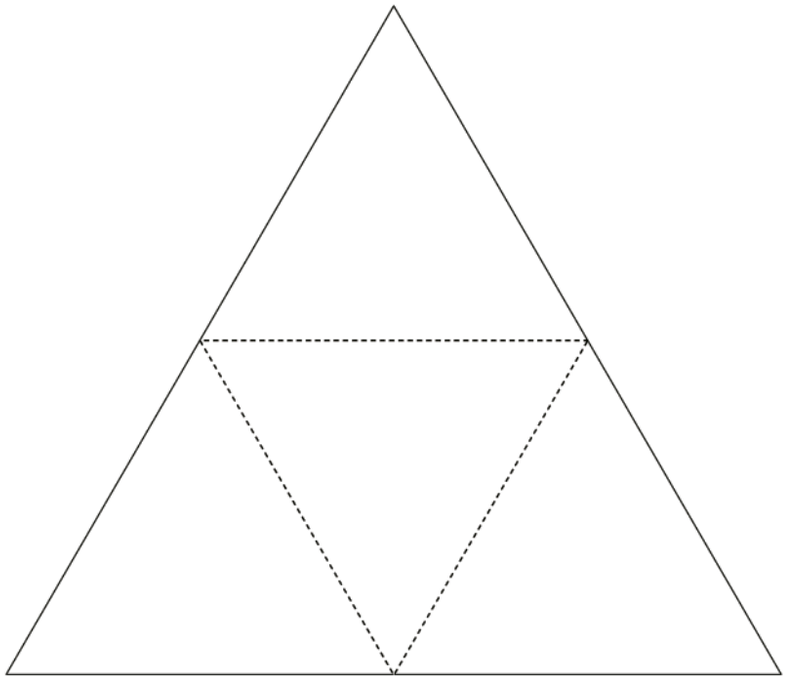


Answers

4. Given their recent experiences, learners should easily observe that a triangular pyramid can be folded from the given net. The real challenge in the question is to relate to the definition of Platonic solids given on the previous page.

   A regular tetrahedron can be made, a pyramid with an equilateral triangle as the base and three equilateral triangles as the other faces. The four faces are identical, equilateral triangles, so the object is a Platonic solid: it has four regular faces, four vertices and six edges.

4. Do you think the diagram below can be used as a net to make a Platonic solid? If it is possible, how many faces, how many edges and how many vertices will the polyhedron have?



5. (a) Make two accurate copies of the above diagram. Cut out on the solid lines and fold on the dotted lines to form two identical polyhedra. Polyhedra like this are called **regular tetrahedra**.
   (b) Try to combine your two regular tetrahedra to make another solid, with six faces, nine edges and five vertices.

THE PLATONIC SOLIDS

Teaching guidelines

There are only five platonic solids. Learners may wonder why only five. There are different ways of explaining this. One way has to do with the size of the angles in a regular polygon and the fact that at a vertex at least three, and sometimes more, faces meet.

The sum of the angles at a vertex must be less than 360° for the net to be folded up, in other words, if the polygons are laid flat with their edges touching, there must be a gap, otherwise it cannot be folded. The only possible polygons that can be used are a triangle (60°), a square (90°) and a pentagon (108°), keeping in mind that we need at least three at a vertex.

The next polygon is a hexagon, but three next to each other allows for no gap as the sum of the angles is 360°, so it will not fold up to make a solid with all its faces as regular hexagons.

Explain the different solids and their names to learners.

Misconceptions

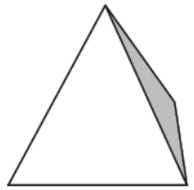
Learners don't realise or tend to forget that the faces of each solid are congruent and are all regular polygons.

Answers

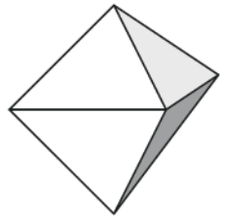
1. See the answers next to each net on LB pages 297–299.
2. It is true.

THE PLATONIC SOLIDS

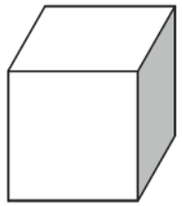
The Platonic solids have special names, and these are given below.
There are only five Platonic solids.



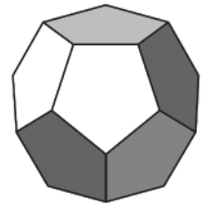
A **tetrahedron** consists of four equilateral triangles. It has six edges and four vertices.



An **octahedron** consists of eight equilateral triangles. It has 12 edges and six vertices.

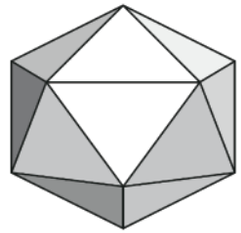


A **hexahedron** (also known as a **cube**) consists of six squares. It has 12 edges and eight vertices.



A **dodecahedron** consists of 12 regular pentagons. It has 30 edges and 20 vertices.

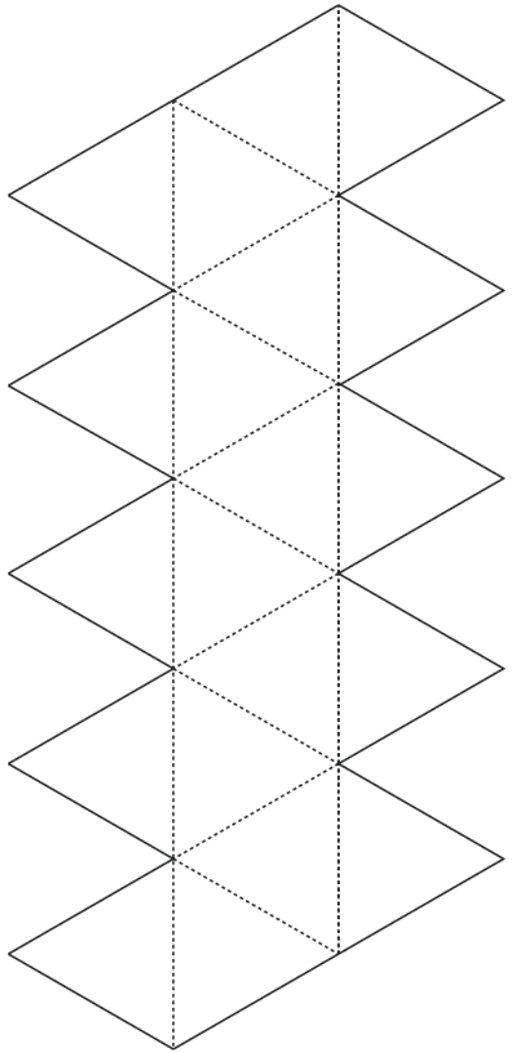
An **icosahedron** consists of 20 equilateral triangles. It has 30 edges and 12 vertices.



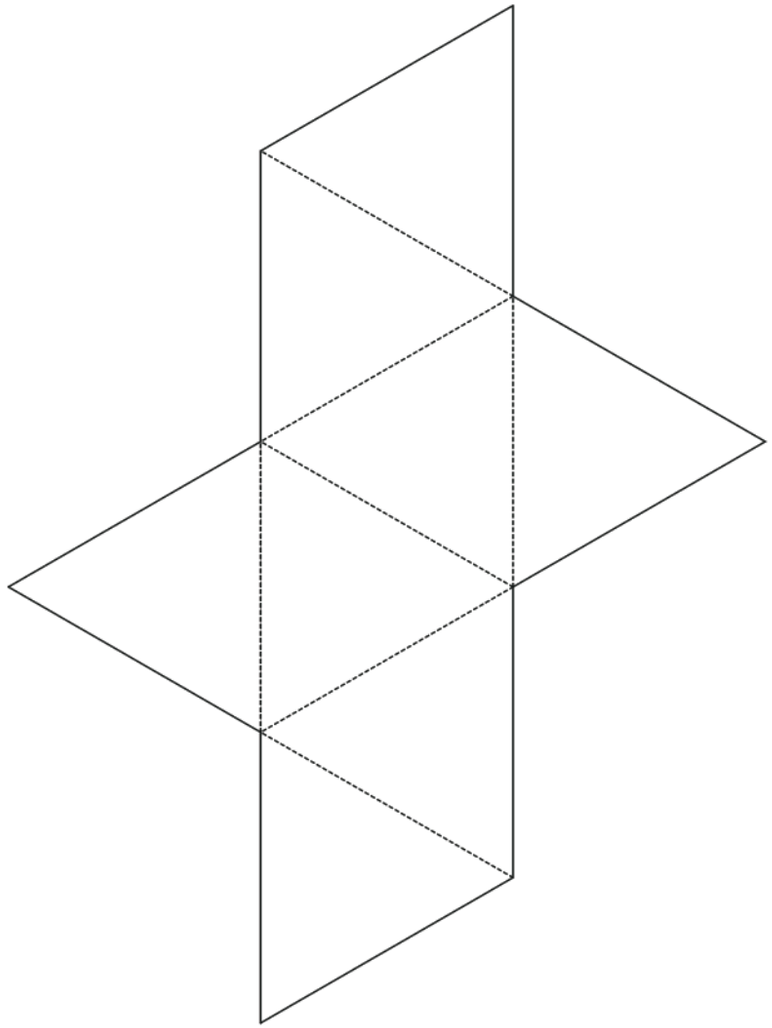
1. Nets for some of the Platonic solids are given on the following pages. Write the names of the objects next to the nets that can be used to make them.
2. Investigate whether Euler's formula is true for the Platonic solids.
   (Euler's formula: the number of faces + the number of vertices = 2 + the number of edges)

Answers (continued)

(a) icosahedron

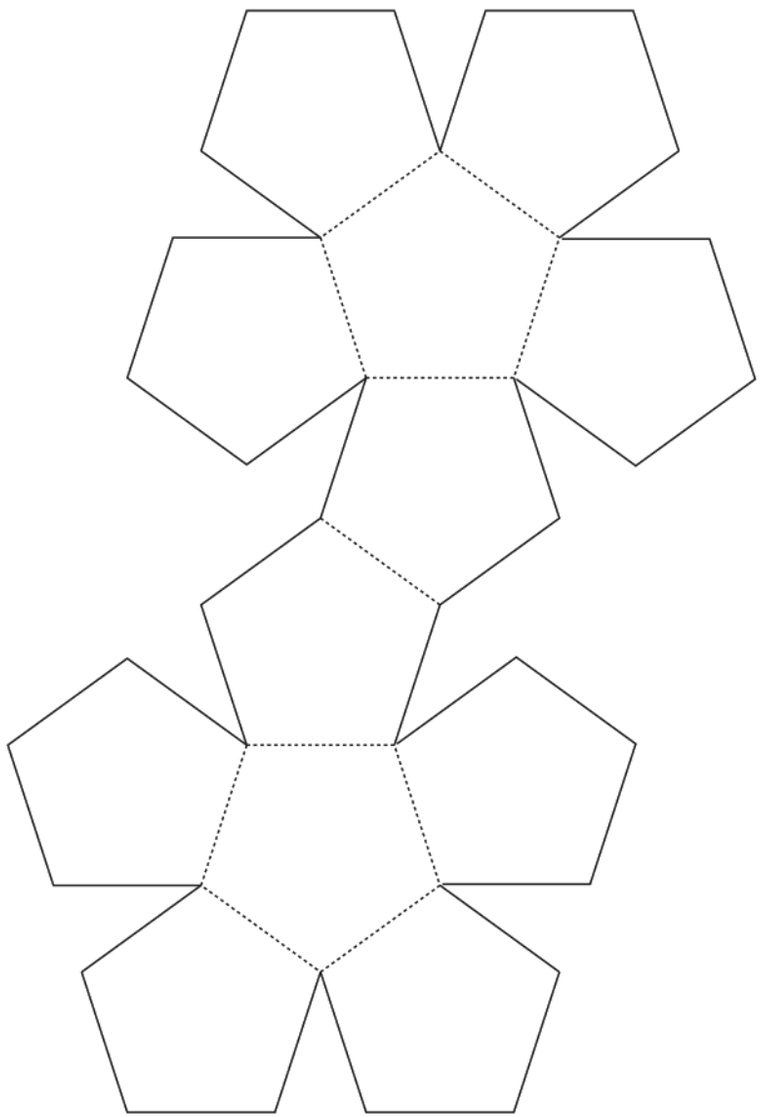


(b) octahedron



Answers (continued)

(c) dodecahedron



Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
26.1 How often different things can happen	Frequency, relative frequency and range of relative frequencies; trials and experiments; events and outcomes; equally likely outcomes; expected frequency and actual frequency	Pages 300 to 306
26.2 Probability	Possible outcomes; favourable outcomes	Pages 306 to 308

CAPS time allocation	4,5 hours
CAPS content specification	Page 117

Mathematical background

- An **event** is something that may or may not happen when some action is performed.
- A **trial** is an action which may lead to some result, for example, rolling a die.
  - A **possible outcome** is any of the possible results of a trial, for example, scoring a 1, 2, 3, 4, 5 or 6 when a die is rolled.
  - A **favourable outcome** is any of the possible outcomes which favour a specific event, for example, scoring a factor of 6 when a die is rolled has four favourable outcomes: 1, 2, 3 or 6.
  - An **actual outcome** is the actual result of a single trial, for example, rolling a die and scoring 3.
  - **Equally likely outcomes** of an event are outcomes which are as likely to occur as any other, for example, scoring less than 7 when rolling a die.
- An **experiment** is a series of trials performed one after the other, for example, rolling a die ten times in a row.
- The **frequency of an outcome** is the number of times it happens, for example, the number of times 3 was scored when a die was rolled ten times.
  - The **expected frequency of an outcome** is the number of times one expects the outcome to occur during an experiment.
  - The **actual frequency of an outcome** is the number of times the outcome actually occurs during an experiment.
- The **probability of an outcome** is a measure of how likely that outcome is.
  - The **(theoretical) probability** of an outcome is given by the ratio $\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$, for example, the theoretical probability of scoring a prime number when rolling a die is $\frac{3}{6} = \frac{1}{2}$, or 0,5 or 50% because there are three favourable outcomes (2, 3, 5) and six possible outcomes (1, 2, 3, 4, 5, 6).
  - The **experimental probability (relative frequency) of an outcome** is given by the ratio $\frac{\text{number of times the outcome happens}}{\text{total number of trials}}$ and can be expressed as a fraction, a decimal number or a percentage.
  - It takes many trials before the relative frequency of an outcome approaches the probability of the outcome.

26.1 How often different things can happen

DIFFERENT FRACTIONS OF A WHOLE NUMBER

Background information

- A **possible outcome** is any of the possible results of an event or trial.
- **Equally likely outcomes** have an equal chance of occurring.
- The **relative frequency of an outcome** is a fraction of the number of trials which have a specific outcome.
- The **range of the relative frequencies** of all the different outcomes of an experiment is the difference between the largest and smallest relative frequencies.

Teaching guidelines

- Ensure that learners understand the concepts listed above.
- At the end of question 5 learners should understand what is meant by the range of the relative frequencies of the different outcomes of an experiment.

Answers

1. 0 fish, 1 fish, 2 fish, 3 fish, 4 fish and 5 fish
2. (a) 1; 2; 3; 4; 5; 6
   (b) Yes
   (c) No
3. (a) 9
   (b) $\frac{9}{60} = \frac{3}{20}$
   (c) $\frac{12}{60} = \frac{1}{5}$
4. (a) $\frac{10}{60} = \frac{1}{6}$
   (b) $\frac{11}{60}$

CHAPTER 26
Probability

26.1 How often different things can happen

DIFFERENT FRACTIONS OF A WHOLE NUMBER

Jayden lives close to the sea. He goes fishing every day. Some days he catches no fish, but on some days he catches several fish. He never catches more than five fish in a day.

He has decided that he will always stop fishing when he has caught five fish in one day.

1. What are the different possible outcomes of each of Jayden's daily fishing trips?
2. Jayden rolls a dice just once each day before he goes fishing.
   (a) What are the possible outcomes of rolling a dice once?
   (b) Are the six outcomes of rolling a dice equally likely?
   (c) Is there any reason for Jayden to believe that the outcome of his fishing trip on a day will be one less than the number that came up when he rolled the dice on that day?



3. Jayden keeps a record of the outcomes of his daily dice rolls. Here is a summary of his record for 60 consecutive days:

Outcome	1	2	3	4	5	6
Frequency	9	9	12	11	10	9

   (a) How many times was the outcome a 6?
   (b) What fraction is this of the total of 60 events?
   (c) On what fraction of the days was the outcome a 3?

The fraction of a number of events which have a specific outcome is called the **relative frequency** of that outcome.

4. What is the relative frequency of a:
   (a) 5 in Jayden's series of 60 dice rolls?
   (b) 4 in Jayden's series of 60 dice rolls?

The **range** of a set of numbers is the difference between the smallest and largest numbers in the set.

Answers

5. The range is $\frac{3}{60}$, which is 5%.
6. The purpose of the question is simply to put the idea in learners' minds that different outcomes may not be equally likely in all situations, and that it may in fact be difficult or even impossible to know whether outcomes are equally likely in a given situation.
7. (a) See the table on LB page 301 alongside.
   (b) $\frac{68}{200} - \frac{4}{200} = \frac{64}{200} = 32\%$

HOW OFTEN CAN WE EXPECT SOMETHING TO HAPPEN?

Background information

- A **trial** takes place when an action is performed, for example, selecting a button from a bag.
- An **outcome** is any result of a trial, for example, selecting a button from a bag and noting that the button is red.

Teaching guidelines

At the end of question 3 learners should be able to provide more examples of a trial and the outcomes of that trial.

Answers

1. (a) No
   (b) Learners' discussion
2. (a) yellow, green, pink, blue, red
   (b) five
3. (a) yellow, green, pink, blue or red
   (b) yellow, green, pink, blue or red
   (c) yellow, green, pink, blue or red
   (d) yellow, green, pink, blue or red

5. What is the **range** of the relative frequencies of the different outcomes in Jayden's series of dice rolls? Express the range as a fraction in sixtieths, and as a percentage.
6. Do you think the six possible outcomes of Jayden's daily fishing trips are equally likely? Give reasons for your answer.
7. (a) Jayden has a book in which he keeps a record of the outcomes of his daily fishing trips. A summary of his record for a period of 200 consecutive days is given in the table below. Copy the table and write the relative frequencies of the different outcomes in the table, with each expressed as two hundredths.

Outcome	0	1	2	3	4	5
Frequency	30	32	68	54	12	4
Relative frequency	$\frac{30}{200}$	$\frac{32}{200}$	$\frac{68}{200}$	$\frac{54}{200}$	$\frac{12}{200}$	$\frac{4}{200}$

   (b) What is the range of the relative frequencies in this case? Express the range as a common fraction and as a percentage.

HOW OFTEN CAN WE EXPECT SOMETHING TO HAPPEN?



Imagine that you have five coloured buttons as shown above in a paper bag.

1. Imagine that you put your hand into the bag without looking inside, and grab one of the buttons.
   (a) Can you say which colour that button will be?
   (b) Discuss this with some classmates.
2. (a) What are the different possible colours of buttons that you could draw from the bag?
   (b) How many different possibilities are there?
3. Read the passage below, then answer the questions that follow.
   When you draw a button from the bag, we say you perform a ***trial****. The colour you draw is called the* ***outcome*** *of the trial.*
   (a) What are the different possible outcomes of the trial if you draw one button out of the bag?
   (b) Imagine that you put the first button back into the bag. If you now draw one button from the bag again, what are the possible outcomes of this new trial?
   (c) Imagine that you repeat the event a third time. What are the possible outcomes of this new trial?
   (d) Imagine that you perform many trials. What are the possible outcomes of each repetition?

Background information

Outcomes are said to be **equally likely** when there is no reason to believe that any outcome will occur more often than any other outcome.

Teaching guidelines

At the end of question 5 learners should be able to interpret the meaning of "equally likely outcomes".

Answers

4. (a) There is no reason to believe that one colour will be drawn more often.
   (b) Learners' discussion
5. (a) Approximately 12 times would be a good answer.
   (b) Approximately 12 times would be a good answer.
   (c) Learners' discussion
6. (a) Learners use their own interpretation of equally likely outcomes to complete the first table on LB page 302 alongside.
7. Answers A, D and F are poor, because the different colours are equally likely outcomes.

   Answer G makes some sense, but it is quite unlikely that things will work out so precisely.

   Answers C and E are poor, because they do not correspond to the total number of trials (40).

   Answers B and H are good, because they indicate that the outcomes are equally likely, and that the total number of trials is 40.

   The intention with this and the ensuing questions, is to provide learners with opportunities to develop intuitions and thoughts that will enable them to make sense of a formal definition of probability that is given in section 26.2. A sound understanding of the formal concept of probability involves the notion that answer G above makes sense although it will hardly ever happen in practice, but will be approached closer and closer as more trials are performed.

4. (a) When you draw one of the five buttons many times and put it back each time, do you think you will draw one colour more often than the others?
   (b) Discuss this with some classmates.
5. (a) Imagine that you draw a button out of the bag with five buttons and put it back, and repeat this 60 times. Approximately how many times do you think you will draw the red button?
   (b) Approximately how many times do you think you will draw the pink button?
   (c) Discuss this with some classmates.

When there is no reason to believe that any outcome will occur more often than any other outcome, the outcomes are said to be **equally likely**.

6. Susan decides to perform 160 trials on the bag with five buttons. In each trial she will draw one button from the bag, note its colour, and put it back. Lebogang decides she will perform 60 trials and Archie decides to perform 40 trials.

   Approximately how many times do you think each of them will draw each of the buttons? Copy the table below and enter your expectations in it:

					
Susan					
Lebogang					
Archie					

7. Here are the answers that eight different people gave for Archie, with his 40 trials.

"Close to 6" means it can be 6 or another number close to 6, for example 5 or 7 or 4 or 8.

					
Answer A	close to 5	close to 5	close to 5	close to 5	close to 20
Answer B	7, 8 or 9	7, 8 or 9	7, 8 or 9	7, 8 or 9	7, 8 or 9
Answer C	6, 7 or 8	6, 7 or 8	6, 7 or 8	6, 7 or 8	6, 7 or 8
Answer D	close to 7	close to 9	close to 8	close to 10	close to 6
Answer E	close to 6	close to 6	close to 6	close to 6	close to 6
Answer F	6	9	7	8	10
Answer G	8	8	8	8	8
Answer H	close to 8	close to 8	close to 8	close to 8	close to 8

Background information

- The **frequency of an outcome** is the number of times that specific outcome is obtained during a series of trials.
- When the different possible outcomes of an event are **equally likely**, it is reasonable to expect that when the event is repeated many times, the frequencies for the different outcomes will be almost equal.

Teaching guidelines

At the end of question 11 learners should understand the concepts listed above.

Answers

8. (a) 32, 12 and 8
   (b) Consider the learners' answers.
9. (a) one fifth
   (b) one fifth
10. (a) one seventh
    (b) one seventh
11. one twelfth
12. See the answers on LB page 303 alongside.

Which answers do you think are good answers, and which do you think are poor answers? For each answer, explain why you think it is good or poor.

8. (a) How much is one fifth of 160, one fifth of 60 and one fifth of 40?
   (b) Look at your own answers for question 6 again. Do you still agree with your answers? If you want to give different answers now, do so and explain what made you change your position.
9. Willem has decided to perform as many trials as he can in an afternoon, drawing one button each time out of a bag containing five coloured buttons.
   (a) In close to what fraction of the trials can he expect to get yellow as the outcome?
   (b) In close to what fraction of the trials can he expect to get red as the outcome?
10. Manare has decided to perform as many trials as he can in an afternoon, drawing one button each time out of a bag containing seven different coloured buttons.



   (a) In close to what fraction of the trials can he expect to get blue as the outcome?
   (b) In close to what fraction of the trials can he expect to get grey as the outcome?
11. Miriam has decided to perform as many trials as she can in an afternoon, drawing one button each time out of a bag containing 12 different coloured buttons.
    In close to what fraction of the trials can Miriam expect to get each specific colour as the outcome?

The number of times that a specific outcome is obtained during a series of trials is called the **frequency** of the outcome.

12. What is the frequency for each of the following colours in answer F, in question 7 on the previous page?
    (a) red 10 (b) pink 7
    (c) yellow 6 (d) blue 8

When the different possible outcomes of an event are equally likely, it is reasonable to expect that when the event is repeated many times, the frequencies for the different outcomes will be almost equal.

AN INVESTIGATION

Background information

- The **expected frequency of an outcome** is the number of times one expects the outcome to occur during an experiment.
- The **actual frequency of an outcome** is the number of times the outcome actually occurs during an experiment.

Teaching guidelines

Learners should have a clear understanding of the difference between the expected frequency (probability) and the actual frequency (relative frequency) of an outcome.

Misconceptions

Learners often confuse the concepts of "expected frequency" and "actual frequency" of an outcome.

Answers

1. 5
2. (a) 25
   (b) 125
3. Learners find the actual frequencies during an experiment of 40 trials and use the results to complete the first table on LB page 304 alongside.
4. Learners copy their actual frequencies on a slip of paper, like the second table on LB page 304 alongside.
5. Learners form a group with four other classmates.
6. Learners use the frequencies on the five slips of paper in the group to complete rows 1, 4, 7, 10 and 13 of the table on LB page 305 on the following page.
7. Learners follow the instruction on LB page 304 alongside.
8. Learners follow the instruction on LB page 304 alongside.
9. Learners follow the instruction on LB page 304 alongside.

AN INVESTIGATION

1. Make eight small cards or pieces of paper. On each card write a different letter. Use the letters A, B, C, D, E, F, G and H. Put the cards in a paper bag. Imagine that you draw a card out of the bag, note the letter and put it back. Imagine that you perform 40 such trials, noting the outcome each time. Then you find the frequency for each letter. To what number do you think each of the frequencies will be close? The number you think of may be called the **expected frequency**.
2. What will be the expected frequencies for each letter if:
   (a) 200 trials are performed?
   (b) 1 000 trials are performed?
3. Now actually do the experiment described in question 1. Record your results with tally marks in a table like the one below. When you have finished, count the tally marks to find the **actual frequencies**.

	A	B	C	D	E	F	G	H
Tally marks								
Actual frequency								
Expected frequency	5	5	5	5	5	5	5	5

4. Write your actual frequencies on a slip of paper, in a table like as follows:

	A	B	C	D	E	F	G	H
Actual frequency								

5. The next step is to collect the slips of four different classmates. Copy the table on page 305, and write their frequencies in rows 1, 4, 7, 10 and 13 of the table, together with your own frequencies. **Do not do it yet**. When you put the five sets of results together, and add them up, you will have the actual frequencies out of 200 trials. You will write these in row 16. In the row for expected frequencies, write the numbers to which you think the frequencies will be close.
6. Now work with your four classmates, and complete rows 1, 4, 7, 10 and 13.
7. In the first empty row after each actual frequency row, express the frequency as a fraction of the total number of outcomes in the experiment, which was 40 in each case. You need not simplify the fractions in rows 2, 5, 8, 11 and 14 of the table.
8. In rows 17 and 20, express the frequencies of rows 16 and 19 as fractions of 200.
9. In the remaining empty rows, express the fractions as percentages.

Background information

- The more trials you perform during an experiment, the smaller the **difference** between the actual frequency and the expected frequency of an outcome.
- The **probability of an outcome** is the expected relative frequency of that outcome.

Teaching guidelines

- Revise what is meant by the range of the relative frequencies of the different outcomes of an experiment.
- Point out that the expected relative frequency of an outcome can be referred to as the probability of that outcome.

Answers

10. Learners calculate the ranges of the numbers in rows 3, 6, 9, 12, 15 and 18 of the table on LB page 305 alongside.
11. It will be row 18. Learners may not be able to give a good explanation now. An important purpose of this investigation is to provide learners with an experience of the phenomenon that the actual relative frequencies get closer to the probabilities as the number of trials increase.
12. It will be row 18.

10. Calculate the ranges of the numbers in rows 3, 6, 9, 12, 15 and 18.

		A	B	C	D	E	F	G	H
1	Actual frequencies								
2									
3									
4	Actual frequencies								
5									
6									
7	Actual frequencies								
8									
9									
10	Actual frequencies								
11									
12									
13	Actual frequencies								
14									
15									
16	Total actual frequencies								
17									
18									
19	Expected frequencies	25	25	25	25	25	25	25	25
20		$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$	$\frac{25}{200}$
21		12,5%	12,5%	12,5%	12,5%	12,5%	12,5%	12,5%	12,5%

11. In which row is the range the smallest? Try to explain why this is the case.
12. In which of the rows in question 10 are the numbers closest to the expected percentages in row 21?

The expected relative frequency of an outcome is called the **probability** of the outcome.

Answers

13. (a) 10 (b) $\frac{1}{10}$ (c) No

(d) The purpose of the question is not to elicit a "right" answer from the learners, but to make them think about the situation to help them to make sense of the definition of probability given further on.

26.2 Probability

Background information

- The **probability of an outcome** is the expected relative frequency of an outcome.
- The **probability** of an outcome is given by the following ratio:

  $$\frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

  Example: The probability of scoring a prime number when rolling a die is $\frac{3}{6}$ or $\frac{1}{2}$, or 0,5 or 50% because there are three favourable outcomes (2, 3, 5) and six possible outcomes (1, 2, 3, 4, 5, 6).
- The **relative frequency** of an outcome is given by the following ratio:

  $$\frac{\text{number of times the outcome happens}}{\text{total number of trials}}$$
- The probability and relative frequency of an outcome can be expressed as a **fraction**, a **decimal number** or a **percentage**.

Teaching guidelines

Revise the difference between the following concepts:

- Actual relative frequency (**experimental probability**): the number of times an outcome happens divided by the number of trials performed.
- Expected relative frequency (**theoretical probability**): the number of favourable outcomes divided by the number of possible outcomes.

Answers

1. (a) $\frac{6}{10}$ or $\frac{3}{5}$ (b) $\frac{3}{10}$ (c) $\frac{1}{10}$

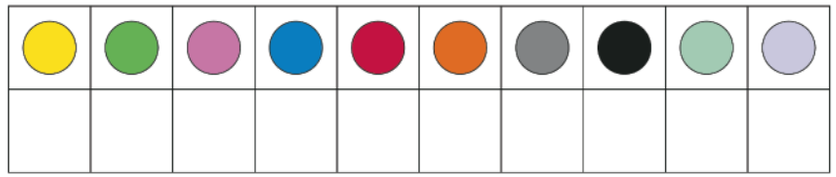
13. Imagine that you have ten different coloured buttons in a bag, as shown below.



(a) Imagine that you draw one button out of the bag.
How many different equally likely outcomes are there for this trial?

(b) Imagine you draw one button out of the bag, look at the colour and make a tally mark in the column for that colour on a table like the one below. Imagine you do it many times. Approximately what fraction of the total number of tally marks do you expect to be in each column?

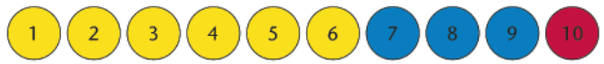
(c) The fraction you have specified in (b) is the probability of the outcome for each of the columns. Would you expect to get precisely that fraction in each column?



(d) Hashim says he expects to have **approximately** ten tally marks in each column, because the outcomes are equally likely. Do you agree with Hashim? Give reasons for your answer.

26.2 Probability

In this activity you have to think about the following situation:
There are ten coloured, numbered buttons in a bag: six yellow buttons, three blue buttons and one red button.



1. (a) What fraction of the total number of buttons is yellow?
   (b) What fraction of the total number of buttons is blue?
   (c) What fraction of the total number of buttons is red?

Answers

2. (a) It is unlikely that blue will be drawn more often than yellow, because there are more yellow buttons than blue buttons.
   (b) It is probable, but not necessarily.
   (c) One cannot say what colour the button will be. The purpose of the question is not to assess the learners, but to encourage or enable them to think about situations with different possible outcomes.
   (d) Learners' discussion
3. (a) 6
   (b) 3
   (c) 1
4. (a) The purpose of the question is merely to get learners to think about random events again.
   (b) Learners' discussion
   (c) The purpose of the question is to get learners to think about random events. Learners who think that what was described in (c) will happen rather than what was described in (a) demonstrate a sound intuition about probability.
   (d) Learners' discussion
5. (a) Learners follow the instructions on LB page 307 alongside.
   (b) Learners follow the instructions on LB page 307 alongside .
   (c) Learners follow the instructions on LB page 307 alongside .
   (d) Learners follow the instructions on LB page 307 alongside .

2. Suppose you put your hand into the bag without looking inside, take one button out and note its colour, and then put it back into the bag.

   If you repeat this trial many times, you will sometimes get a yellow, sometimes a blue button and sometimes a red button.

   (a) Do you think you will get blue more often than yellow? Explain your answer.
   (b) Do you think you will draw yellow about twice as often as blue?
   (c) Can you be certain which colour will be drawn? Explain your answer.
   (d) Share your ideas with two classmates.

Here is an experiment that you will do later. **Do not do it now**.

Put ten buttons like those on page 308, or pieces of paper or cardboard with the names of the colours written on them, in a bag. Put your hand into the bag without looking inside, and take one button out. Check what colour it is, copy the table below and make a tally mark in the column for that colour, and put the button back into the bag. Do this ten times.

Yellow	Blue	Red

Each time you perform a trial, a certain **event** takes place, and there are three possible events:

A. The event of the colour being yellow
B. The event of the colour being blue
C. The event of the colour being red

3. (a) In how many different ways can event A be achieved in one trial?
   (b) In how many different ways can event B be achieved in one trial?
   (c) In how many different ways can event C be achieved in one trial?

4. (a) Suppose you do the experiment and make ten trials. Do you think event A will happen three times or maybe four times, event B will happen three times or maybe four times and event C will happen three times or maybe four times?
   (b) Share your ideas with two classmates.
   (c) Do you rather think event A will happen six times (or maybe five or seven times), event B will happen three times (or maybe two or four times), and event C will happen once (or maybe twice or not at all)?
   (d) Share your ideas with two classmates.

5. (a) Do the experiment that is described before question 3. Copy the table on the next page, and write the results in the second row of the table.
   (b) Repeat the experiment and write the results in the third row of the table.
   (c) Repeat the experiment three more times and enter the results in the table.
   (d) Complete the last two rows of the table.

Teaching guidelines

Revise the concepts of trial and possible outcomes of a trial.

Answers

6. (a) 6
   (b) 3
   (c) 1
7. (a) $\frac{6}{10}$
   (b) $\frac{3}{10}$
   (c) $\frac{1}{10}$
8. (a) $\frac{3}{10}$
   (b) Consider learners' answers.

Outcome	Yellow	Blue	Red
Frequency of each colour during the first ten trials			
Frequency of each colour during the second ten trials			
Frequency of each colour during the third ten trials			
Frequency of each colour during the fourth ten trials			
Frequency of each colour during the fifth ten trials			
Total frequencies out of 50 trials			
Total frequencies divided by 5			

When you did the experiment for the first time in question 5(a), you performed ten **trials**: you took a button out of the bag and inspected the colour.

Each time, there were three **possible outcomes** for the trial: the button could be **yellow**, it could be **blue** or it could be **red**.

We can also say that three different events were possible: yellow, blue and red. But if we consider the numbers on the buttons, ten different outcomes are possible.

6. (a) How many different outcomes (numbered buttons) will produce the event yellow?
   (b) How many different outcomes will produce the event blue?
   (c) How many different outcomes will produce the event red?
7. (a) What fraction of the ten possible outcomes will produce the event yellow?
   (b) What fraction of the ten possible outcomes will produce the event blue?
   (c) What fraction of the ten possible outcomes will produce the event red?

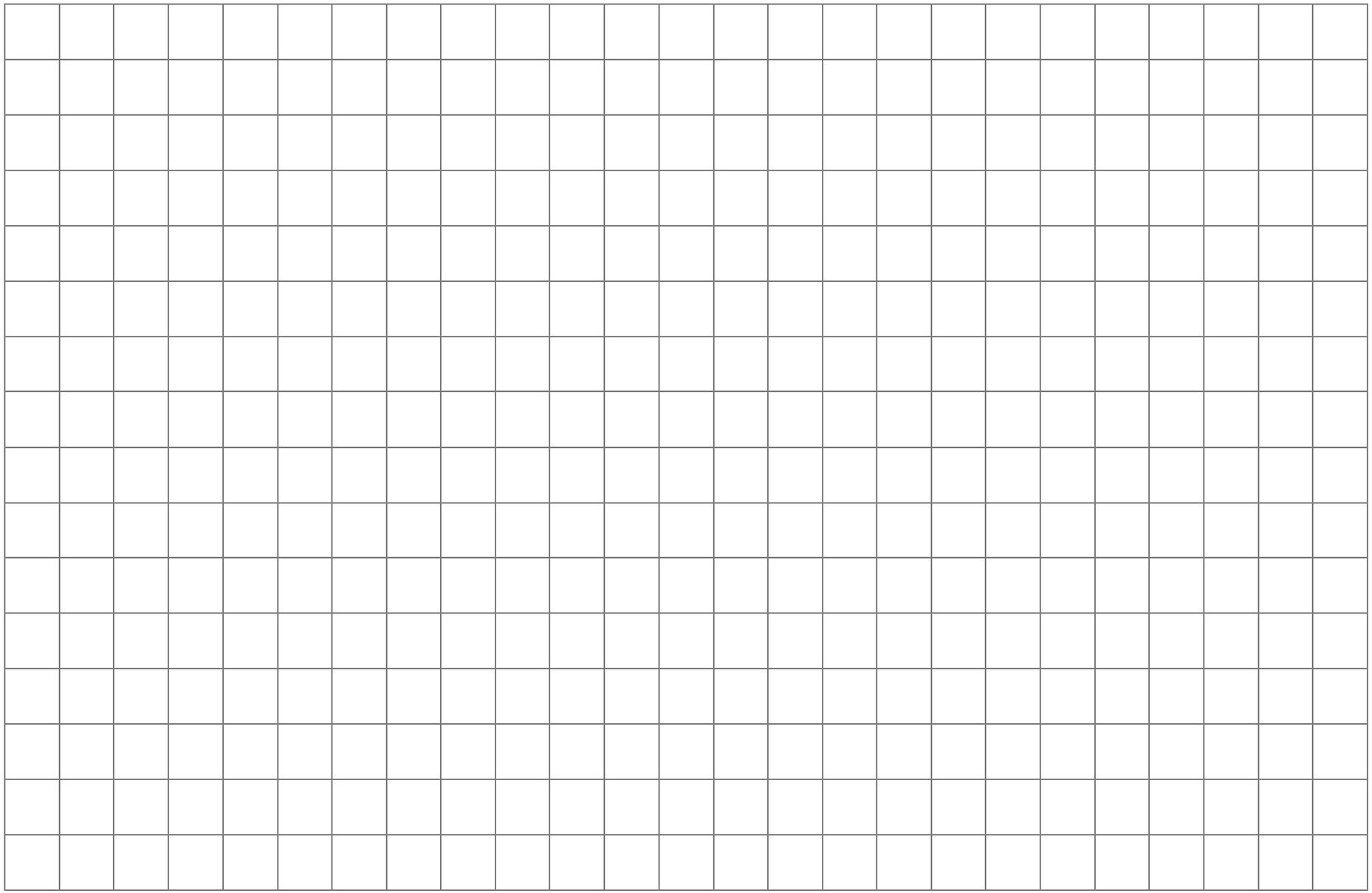
The fractions you have given as answers for question 7 are the **probabilities** of the three different events.

8. (a) What is the probability of getting blue when one of the buttons below is drawn from a bag?

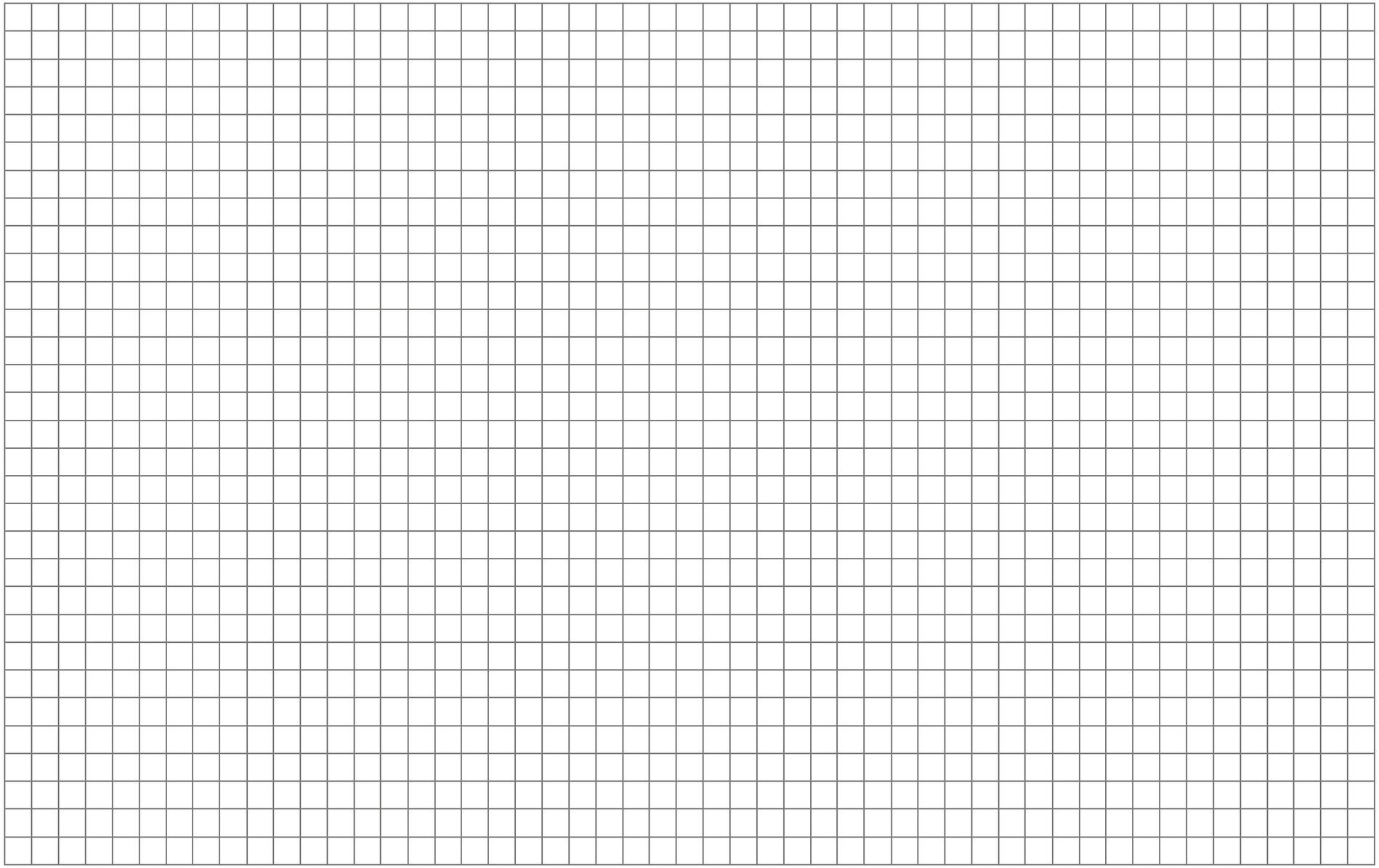


   (b) Describe in your own words what is meant by saying the probability of an event is $\frac{3}{20}$.

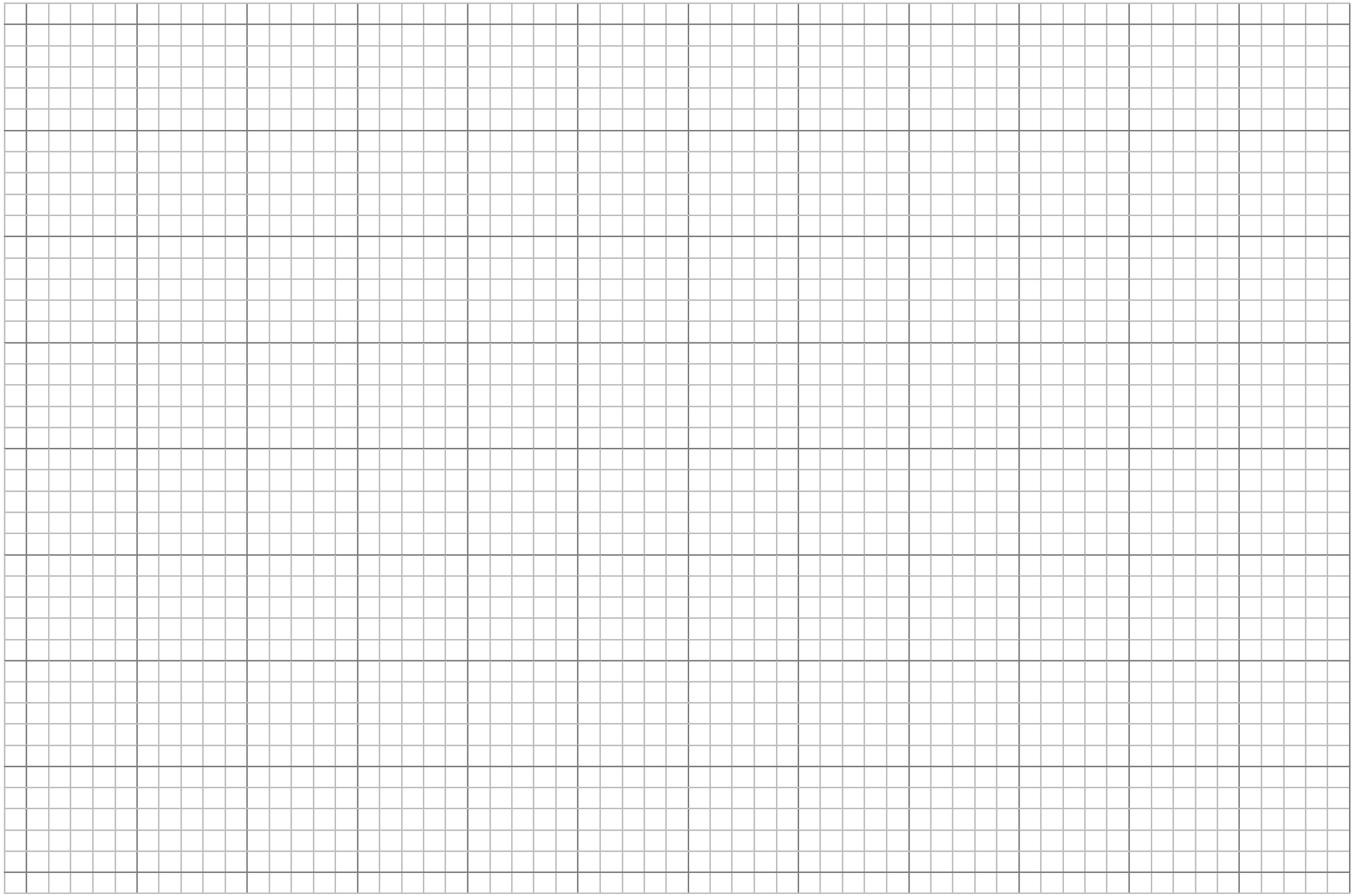
Square grid paper (1 cm × 1 cm)



Graph paper / Square grid paper (0,5 cm × 0,5 cm)



Graph paper



Dotted paper

