

Grade 6

**CONTENT BOOKLET:
TARGETED SUPPORT
MATHEMATICS**

Term 2

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Universalisation Programme and in its Provincialisation Programme.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

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INTRODUCTION:

THREE PRINCIPLES OF TEACHING MATHEMATICS

PRINCIPLE 1:

TEACHING MATHEMATICS DEVELOPMENTALLY

What is developmental teaching and what are the benefits of such an approach?

- The human mind develops through phases or stages which require learning in a certain way and which determine whether children are ready to learn something or not.
- If learners are not ready to learn something, it may be due to the fact that they have not reached that level of development yet or they have missed something previously.
- The idea that children's thinking develop from concrete to abstract, comes from Piaget (1969). We adopted a refined version of that idea though, which works very well for mathematics teaching, namely a "concrete-representational-abstract" classification (Miller & Mercer, 1993).
- It is not possible in all cases or for all topics to follow the "concrete-representational-abstract" sequence exactly, but at the primary level it is possible in many topics and is especially valuable in establishing new concepts.
- This classification gives a tool in the hands of the teacher to develop children's mathematical thinking but also to fall back to a previous phase if it is clear that the learner has missed something previously.
- The need for concrete experiences and the use of concrete objects in learning, may pass as learners develop past the Foundation Phase. However, the representational and abstract development phases are both very much present in learning mathematics at the Intermediate and Senior Phase.

How can this approach be implemented practically?

The table below illustrates how a developmental approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- Where applicable, the initial concrete way of teaching and learning the concept is suggested and educational resources provided at the end of the lesson plan or topic to assist teachers in introducing the idea concretely.
- In most cases pictures (semi-concrete) and/or diagrams (semi-abstract) are provided, either at the clarification of terminology section, within the topic or lesson plan itself or at the end of the lesson plan or topic as an educational resource.
- In all cases the symbolic (abstract) way of teaching and learning the concept, is provided, since this is, developmentally speaking, where we primarily aim to be when learners master mathematics.

PRINCIPLE 2:

TEACHING MATHEMATICS MULTI-MODALLY

What is multi-modal teaching and what are the benefits of such an approach?

- We suggest a rhythm of teaching any mathematical topic by way of “saying it, showing it and symbolising it”. This approach can be called multi-modal teaching and links in a significant way to the developmental phases above.
- Multi-modal teaching includes speaking about a matter verbally (auditory mode), showing it in a picture or a diagram (visual mode) and writing it in words or numbers (symbolic mode).
- For multi-modal teaching, the same learning material is presented in various modalities: by an explanation using spoken words (auditory), by showing pictures or diagrams (visual) and by writing words and numbers (symbolic).
- Modal preferences amongst learners vary significantly and learning takes place more successfully when they receive, study and present their learning in the mode of their preference, either auditory, visually or symbolically. Although individual learners prefer one mode above another, the exposure to all three of these modes enhance their learning.

How can this approach be implemented practically?

The table on p8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- The verbal explanations under each topic and within each lesson plan, provide the “say it” or auditory mode.
- The pictures and diagrams provide suggestions for the “show it” mode (visual mode).
- The calculations, exercises and assessments under each topic and within each lesson plan, provide the “symbol it” or symbolic mode of representation.

PRINCIPLE 3:

SEQUENTIAL TEACHING

What is sequential teaching and what are the benefits of such an approach?

- Learners with weak basic skills in mathematics will find future topics increasingly difficult. A solid foundation is required for a good fundamental understanding.
- In order to build a solid foundation in maths the approach to teaching needs to be systematic. Teaching concepts out of sequence can lead to difficulties in grasping concepts.
- Teaching in a systematic way (according to CAPS) allows learners to progressively build understandings, skills and confidence.
- A learner needs to be confident in the principles of a topic before he/she is expected to apply the knowledge and proceed to a higher level.
- Ongoing review and reinforcement of previously learned skills and concepts is of utmost importance.
- Giving learners good reasons for why we learn a topic and linking it to previous knowledge can help to remove barriers that stop a child from learning.
- Similarly, making an effort to explain where anything taught may be used in the future is also beneficial to the learning process.

How can this approach be implemented practically?

If there are a few learners in your class who are not grasping a concept, as a teacher, you need to find the time to take them aside and teach them the concept again (perhaps at a break or after school).

If the entire class are battling with a concept, it will need to be taught again. This could cause difficulties when trying to keep on track and complete the curriculum in the time stipulated.

Some topics have a more generous time allocation in order to incorporate investigative work by the learners themselves. Although this is an excellent way to assist learners to form a deeper understanding of a concept, it could also be an opportunity to catch up on any time missed due to remediating and re-teaching of a previous topic. With careful planning, it should be possible to finish the year's work as required.

Another way to try and save some time when preparing for a new topic, is to give out some revision work to learners prior to the start of the topic. They could be required to do this over the course of a week or two leading up to the start of the new topic. For example, in Grade 8, while you are teaching the Theorem of Pythagoras, the learners could be given a homework worksheet on Area and Perimeter at Grade 7 level. This will allow them to revise the skills that are required for the Grade 8 approach to the topic.

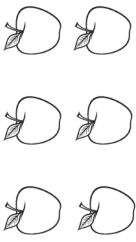
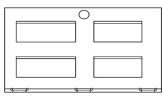
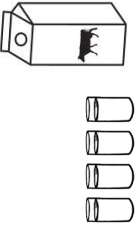
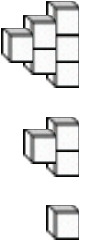
What does this look like in the booklet?

At the beginning of each topic, there will be a SEQUENTIAL TEACHING TABLE, that details:

- The knowledge and skills that will be covered in this grade
- The relevant knowledge and skills that were covered in the previous grade or phase (looking back)
- The future knowledge and skills that will be developed in the next grade or phase (looking forward)

Introduction: Three Principles of Teaching Mathematics

THREE-STEP APPROACH TO MATHEMATICS TEACHING: CONCRETE-REPRESENTATIONAL-ABSTRACT

CONCRETE: IT IS THE REAL THING		REPRESENTATIONAL: IT LOOKS LIKE THE REAL THING		ABSTRACT: IT IS A SYMBOL FOR THE REAL THING	
Mathematical topic	Real or physical For example:	Picture	Diagram	Number (with or without unit)	Calculation or operation, general form, rule, formulae
Counting	Physical objects like apples that can be held and moved			6 apples	$2 \times 3 = 6$ or $\frac{1}{2}$ of 6 = 3 or $\frac{2}{3}$ of 6 = 4
Length or distance	The door of the classroom that can be measured physically			80 cm wide 195 cm high	Perimeter: $2L + 2W = 390 + 160 = 550\text{cm}$ Area: $L \times W = 195 \times 80 = 15\,600\text{cm}^2 = 1.56\text{m}^2$
Capacity	A box with milk that can be poured into glasses			1 litre box 250 ml glass	$4 \times 250\text{ml} = 1\,000\text{ml} = 1\text{ litre}$ or $1\text{ litre} \div 4 = 0.25\text{ litre}$
Patterns	Building blocks			1: 3: 6...	$n \frac{(n+1)}{2}$
Fraction	Chocolate bar			$\frac{6}{12}$	$\frac{6}{12} = \frac{1}{2}$ or $\frac{1}{2}$ of 12 = 6
Ratio	Hens and chickens			4:12	$4:12 = 1:3$ Of 52 fowls $\frac{1}{4}$ are hens and $\frac{3}{4}$ are chickens. ie 13 hens, 39 chickens
Mass	A block of margarine			500g	$500\text{g} = 0.5\text{ kg}$ or calculations like $2\frac{1}{2}$ blocks = 1.25kg

Teaching progresses from concrete $\rightarrow$ to $\rightarrow$ abstract. In case of problems, we fall back $\leftarrow$ to diagrams, pictures, physically.

Inroduction: Three Principles of Teaching Mathematics

MODES OF PRESENTING MATHEMATICS WHEN WE TEACH AND BUILD UP NEW CONCEPTS

Examples	SPEAK IT: to explain the concept - Essential for introducing terminology in context - Supports learning through the auditory pathway - Important to link mathematics to everyday realities	SHOW IT: to embody the idea - Essential to assist <u>storing</u> and <u>retrieving</u> concepts - Supports understanding through the <u>visual</u> pathway - Important to <u>condense</u> a variety of information into a single image	SYMBOL IT: to enable <u>mathematising</u> - Essential to assist <u>mathematical thinking</u> about concepts - Supports the <u>transition</u> from situations to mathematics - Important to <u>expedite</u> calculation and problem solving
FP: Doubling and halving	“To double something, means that we make it twice as much or twice as many. If you got R50 for your birthday last year and this year you get double that amount, it means this year you got R100. If Mom is doubling the recipe for cupcakes and she used to use $2\frac{1}{2}$ cups of flour, it means she has to use twice as much this time.” “To halve something, means that we divide it into two equal parts or share it equally. If I have R16 and I use half of it, I use R8 and I am left with R8. If we share the 22 Astro’s in the box equally between the two of us, you get eleven, which is one half and I get eleven, which is the other half.”	1. Physical objects: Example: Double 5 beads Halve 12 beads 2. Pictures: Example: Double Halve 3. Diagrams:	$7 + 7 = \square$ $7 + \square = 14$ $\triangle + \bigcirc = 14$, but $\square + \square = 14$ 2 times $7 = 14$ double of 7 is 14 $14 - 7 =$ $14 - \square = 7$ 14 divided by 2 = 7 14 halved is 7

Introduction: Three Principles of Teaching Mathematics

IP: Geometric patterns	"If we see one shape or a group of shapes that is growing or shrinking a number of times, every time in the same way, we can say it is forming a geometric pattern. If we can find out how the pattern is changing every time, we can say we found the rule of the sequence of shapes. When we start working with geometric patterns, we can describe the change in normal language. Later we see that it becomes easier to find the rule if there is a property in the shapes that we can count, so that we can give a number value to each , or each term of the sequence" "You will be asked to draw the next term of the pattern, or to say how the eleventh term of the pattern would look, for example. You may also be given a number value and you may be asked, which term of the pattern has this value?"	Draw the next term in this pattern. T1 T2 T3 T4 Describe this pattern. What is the value of the 9th term of this pattern [T9]? To draw up to the ninth term of this pattern, is a safe but slow way. It is even slower to find out by drawing, which term has a value of 120 for example. One is now almost forced to deal with this problem in a symbolic way.	Note how important it is to support the symbolising by saying it out: T: 3: 6... T: 3: 6: 10... T: 3: 6:10:15 Inspecting the terms of the sequence in relation to their number values: T1: 1 = 1 The value of term 1 is 1 T2: 3 = 1+2 The value of term 2 is the sum of two consecutive numbers starting at 1 T3: 6 = 1+2+3 The value of term 3 is the sum of three consecutive numbers starting at 1 T4: 10 = 1+2+3+4 The value of term 4 is the sum of four consecutive numbers starting at 1 T5: 15 = 1+2+3+4+5 The value of term 5 is the sum of five consecutive numbers starting at 1 T9: 45 = 1+2+3+4+5+6+7+8+9 The value of term 9 is the sum of nine consecutive numbers starting at 1 We can see that the value of term n is the sum of n number of consecutive numbers, starting at 1.
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SP: Grouping the terms of an algebraic expression	“We can simplify an algebraic expression by grouping like terms together. We therefore have to know how to spot like terms. Let us say we have to sort fruit in a number of baskets and explain the variables or the unknowns in terms of fruits. Try to visualise the following pictures in your mind:”	Although not in a real picture, a mind picture is painted, or a mental image to clarify the principle of classification: • Basket with green apples (a) • Basket with green pears (b) • Basket with green apples and green pears (ab) • Basket with yellow apples (a^2) • Basket with yellow apples and green pears (a^2b) Or in diagrammatic form  a  b  ab  a^2  a^2b	$4b - a^2 + 3a^2b - 2ab - 3a + 4b + 5a - a - 2ab + 2a^2b + a^2b$ $= -3a + 5a - a + 4b + 4b - 2ab - 2ab - a^2 + 3a^2b + 2a^2b + a^2b$ $= a + 8b - 4ab - a^2 + 6a^2b$
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TOPIC 1: WHOLE NUMBERS - COUNTING, ORDERING, COMPARING, REPRESENTING AND PLACE VALUE

INTRODUCTION

- This unit runs for 1 hour.
- This unit is part of the outcome 'Numbers, Operations and Relationships'. This counts for 50% of the final exam.
- This unit covers whole numbers up to 9 digits.
- The purpose of this section is to develop skills which will assist in preparing learners for algebra. New concepts and vocabulary are introduced and built on. Rounding is a vital skill that helps learners estimate answers with more accuracy and helps them to avoid unnecessary errors.
- Mental mathematics must be done daily as the basic skills that were taught in earlier grades and skills that were taught in the first term must be tested and built on in this new term. Most CAPS textbooks have examples of mental calculations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
- Order, compare and represent numbers to at least 6-digit numbers - Represent odd and even numbers to at least 1 000. - Recognize the place value of digits in whole numbers to at least 6 digit numbers. - Round off to the nearest 5, 10, 100 and 1 000	- Order, compare and represent numbers to at least 9-digit numbers - Represent prime numbers to at least 100 - Recognizing the place value of digits in whole numbers to at least 9-digit numbers - Round off to the nearest 5, 10, 100 and 1000. - Using a range of techniques to perform and check written and mental calculations of whole numbers including: - estimation - building up and breaking down numbers - rounding off and compensating - using addition and subtraction as inverse operations - adding, subtracting and multiplying in columns - long division - using multiplication and division as inverse operations	Revise the following done in Grade 6: - order, compare and represent numbers to at least 9-digit numbers - recognize and represent prime numbers to at least 100 - round off numbers to the nearest 5, 10, 100 or 1 000 - recognize and represent prime numbers to at least 100 - order, compare and represent numbers to at least 9-digit numbers - Revise prime numbers to at least 100 In the FET Phase: - Learners will apply their knowledge of numbers and operations across all contexts. The FET phase requires an increasing depth of the knowledge gained in all the previous phases so that application of knowledge can be used to solve problems in a large variety of contexts. - This is also extended into the algebraic concepts covered during the remainder of the senior phase and the FET phase. - Number sense is the key to Mathematical Literacy in learners.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Whole Numbers	The numbers in the set $\{0, 1, 2, 3, \dots\}$ are called whole numbers. Whole numbers are counting numbers including zero.
Place Value	The value of the digit depends on its order in the number. In 65 314 728, the 2 is in the 'tens' position, so it shows a value of 20.
Rounding Off	Numbers are either rounded up or down to the nearest 10, 100 or 1 000.
Digit	A single character used in a numbering system. In the decimal system, digits are 0 to 9.
Inverse Operations	Addition and subtraction are inverse operations. Multiplication and division are inverse operations.
Order Of Operation	BODMAS When there are multiple operations to be performed, there is a particular order that the operations should be performed in. 1. 'Brackets' [] 2. 'Of' means multiply or to apply an exponential increase 3. 'Division' and 'Multiplication': work left to right 4. 'Addition' and 'Subtraction': work left to right.
Estimation	The best guess arrived at after considering all the information given in a problem. To estimate is to find an answer that is as close as possible to the exact answer.
Million	A million is 1 000 000 or 1000 thousands.

SUMMARY OF KEY CONCEPTS

Counting in Millions, Ten Millions and Hundred Millions

There are many ways to practice counting, and it is very important to have learners count in the given interval such as millions, without always starting with the first multiple or with zero.

Counting can be done orally at the start of a lesson, or grids can be used. Learners can write number sets where they work with counting sets in various forms.



EXAMPLES:

Counting in millions:

1 000 000, 2 000 000, 3 000 000...

1 340 000, 2 340 000, 3 340 000...

Counting in ten millions:

10 000 000, 20 000 000, 30 000 000...

1 000 000, 11 000 000, 21 000 000 ...

12 300 000, 22 300 000, 32 300 000...

Counting in hundred millions:

100 000 000, 200 000 000, 300 000 000...

102 000 000, 202 000 000, 203 000 000...

Learners must be able to count forwards and backwards. If learners are finding this difficult, use place value tables and charts to help them see how to order the numbers correctly.

Topic 1 Whole Numbers - counting, ordering, comparing, representing and place value

Numbers up to One Hundred Million in Words and in Numeric Form

1. In Table Form:

	HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
Digits	5	6	5	3	1	4	7	2	8
What the digit means in terms of its position	This represents 5 hundred millions	This represents 6 ten millions	This represents 5 millions	This represents 3 hundred thousands	This represents 1 ten thousand	This represents 4 thousands	This represents 7 hundreds	This represents 2 tens	This represents 8 ones or units
Numeric	500 000 000	60 000 000	5 000 000	300 000	10 000	4000	700	20	8
How you would say it	Five hundred million	Sixty million	Five million	Three hundred thousand	Ten thousand	Four thousand	Seven hundred	Twenty	Eight
What the value of each digit is in the number	The digit 5 has a value of 500 000 000	The digit 6 has a value of 60 000 000	The digit 5 has a value of 5 000 000	The digit 3 has a value of 300 000	The digit 1 has a value of 10 000	The digit 4 has a value of 4000	The digit 7 has a value of 700	The digit 2 has a value of 20	The digit 8 has a value of 8

2. In Numerals: 565 314 728

3. In Words:

Five hundred and sixty five million, three hundred and fourteen thousand, seven hundred and twenty-eight.

4. Expanded Form:

$$500\,000\,000 + 60\,000\,000 + 5\,000\,000 + 300\,000 + 10\,000 + 4000 + 700 + 20 + 8$$

5. Identify the place and value of each digit in a number up to hundred millions (9 digits):

565 314 728

- The digit 5 is in the **hundred million** place.
- The digit 6 is in the **ten million** place.
- The digit 5 is in the **million** place.
- The digit 3 is in the **hundred thousand** place.
- The digit 1 is in the **ten thousand** place.
- The digit 4 is in the **thousand** place.
- The digit 7 is in the **hundred** place.
- The digit 2 is in the **tens** place.
- The digit 8 is in the **unit /ones** place.

Topic 1 Whole Numbers - counting, ordering, comparing, representing and place value



Teaching Tip: Learners often misread or write numbers incorrectly when they have a lot of zeros. This can be prevented by working with place value tables, and if educators emphasise the importance of zeros as place holders in the number. Learners must know that if hundred millions are mentioned, there will be at least 9 digits in the number. Learners can practice reading numbers in sets of three and making a point of stating that each space represents a word. Example: 234 567 123 is two hundred and thirty four million five hundred and sixty seven thousand one hundred and twenty three.

Compare Numbers within Hundred Millions (9 digits):

Learners must compare and order numbers according to their value. Working with place value tables and writing numbers directly below each other is a good strategy that will help learners make accurate comparisons.

1. Smaller than: <

Which number is smaller, 467 237 981 or 467 230 600?

When comparing numbers, look at the value of each digit starting from the left and gradually moving to the right of the number.

HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
4	6	7	2	3	7	9	8	1
4	6	7	2	3	0	6	0	0

0 thousands is **smaller** than **7** thousands. So, 467 230 600 is smaller than 467 237 981.

This must be written as $467\ 230\ 600 < 467\ 237\ 981$.



Teaching Tip: The best way to help learners remember the correct sign to use in these comparisons is to let them think of a crocodile. The open end of the sign can be an open crocodile mouth, and crocodiles are greedy and always want to eat the bigger share or number. This is a technique used in the foundation phase but it is very effective throughout the intermediate phase as well.

Topic 1 Whole Numbers - counting, ordering, comparing, representing and place value

2. Greater than: >

Which number is greater, 300 712 935 or 300 712 846?

HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
3	0	0	7	1	2	9	3	5
3	0	0	7	1	2	8	4	6

A table makes it easier to work out which number is bigger or smaller than.
Work from left to right.

If they are the same, continue to compare until the values of the digits are not the same.

The values of the digits in the hundreds place are not the same.

9 hundred is **greater** than 8 hundred. So 300 712 935 is **greater** than 300 712 846.
This must be written as $300\ 712\ 935 > 300\ 712\ 846$.

3. Arrange the numbers from smallest to biggest (ascending order).

324 688, 32 468, 3 246 880, 324 560 004

Look at which number has the **least** digits and this will become the first number. It will help if learners use a place value table and write numbers underneath each other.

Answer: 32 468, 324 688, 3 246 880, 324 560 004

4. Arrange the numbers from biggest to smallest (descending order)

324 688, 32 468, 3 246 880, 324 560 004

Look at which number has the **most** digits this becomes the first number in the sequence and is then followed by the next biggest number and so on.

Answer: 324 560 004, 3 246 880, 324 688, 32 468



Teaching Tip: If learners are still getting confused, then revise the rules for ordering numbers by always starting with smaller numbers, and working back to longer and bigger numbers, as the learners start to grasp the concepts of comparing and ordering

Rounding Off

This section is often difficult for learners to grasp, but remember the rules below and do a lot of oral work and mental maths with this concept.

1. Decide which digit is the last one you need to keep. You will know this because it is the digit in the place you are asked to round off to.
2. Leave it the same if the next digit is less than 5.
3. Round it up (increase by 1) if the next digit is greater than or equal to 5.



Example: Round 74 to the nearest 10

We focus on the 7 in the 10s place

The next digit is 4, which is less than 5, so no change is needed to 7

= 70

74 gets rounded down.



Example: Round 86 to the nearest 10

We focus on the 8 in the 10s place

The next digit is 6 which is 5 or more, so increase the 8 by 1 to 9

= 90

86 gets rounded up.

Learners must be able to round to the nearest 5, 10, 100 or 1000. These are skills that must be practised so that learners can perform rounding off without error.



Teaching Tip: Learners must be shown that some word problems will change the rounding off that is done. Sometimes the rule cannot be followed. Example: If 4, 5 people can fit in a car, this would be rounded down to 4 people, as you cannot squash an extra half a person into the car if there is no space.

Prime Numbers to at least 100

Learners must be able to identify all prime numbers between 0 and 100.

(RESOURCE 1 AT THE END OF THIS SECTION)

Learners must also know that numbers that are not prime are considered composite numbers, and this does not include the number 1, as it is the identity number.

(RESOURCE 2 AT THE END OF THIS SECTION)

Topic 1 Whole Numbers - counting, ordering, comparing, representing and place value

RESOURCES



RESOURCE 1

PRIME NUMBERS

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

RESOURCE 2

COMPOSITE NUMBERS

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

TOPIC 2: WHOLE NUMBERS – MULTIPLICATION

INTRODUCTION

- This unit runs for 5 hours.
- This unit is part of the outcome 'Numbers, Operations and Relationships'. This counts for 50% of the final exam.
- The unit covers multiplication of 4 digit and 3 digit numbers using a variety of techniques and skills. It is important that learners are extended to no longer use the technique of breaking up numbers when they multiply. Learners must be taught vertical multiplication as the methods used previously can become cumbersome and difficult to follow as the number sets become longer.
- This section is also vital in getting learners to evaluate the choice made when rounding in estimation as the rounding choice will have varying effects on the final answer.
- The purpose of this section is to prepare learners for the multiplication that will be used in the FET phase when products of algebraic expression must be found using methods such as FOIL and distribution.
- Mental mathematics must be practised daily as the basic skills that were taught in earlier grades and skills that were taught in the first term must be tested and built on in this new term. Most CAPS textbooks have examples of mental calculations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Multiplication of 3-digit whole numbers by 2-digit whole numbers	• Multiplication of 4-digit whole numbers by 3-digit whole numbers • Multiple operations on whole numbers with or without brackets	• Perform calculations using all four operations on whole numbers, estimating and using calculators where appropriate

GLOSSARY OF TERMS

Term	Explanation / Diagram
Factor	Factors are numbers we can multiply together to get another number: For example: 2 and 3 are factors of 6, because $2 \times 3 = 6$.
Multiple	The result of multiplying a number by an integer [not by a fraction]. For example: 12 is a multiple of 3, as $3 \times 4 = 12$
Multiplication	The basic idea of multiplication is repeated addition. For example: $5 \times 3 = 5 + 5 + 5 = 15$
Product	The result when two or more numbers are multiplied together.
Distributive Property	The Distributive property states that multiplying a number by a group of numbers added or subtracted is the same as doing each multiplication separately For example: $3 \times [2 + 4] = 3 \times 2 + 3 \times 4$

SUMMARY OF KEY CONCEPTS



Factors and Multiples:

Learners need to know the multiples and factors of 2 digit and 3 digit numbers. An understanding of prime factors is also required.



For example:

The factors of 24 are {1, 2, 3, 4, 6, 8, 12, 24}

The prime factors of 24 are 2 and 3.



For example:

The multiples of 24 are {24, 48, 72; 96...}. Multiples continue and are infinite -meaning they do not finish- with multiples being an open and continuing set.

Partitioning (breaking up) Numbers to Multiply Them

1. Using the distributive property to multiply.



For example:

How many bottles of water are there in 237 boxes, if each box contains 24 bottles?

The number 237 can be broken up into $200 + 30 + 7$; and now the distributive property can be used to complete the problem.

$$\begin{aligned}(200 \times 24) + (30 \times 24) + (7 \times 24) \\ = 4800 + 720 + 168 \\ = 5688\end{aligned}$$

The distributive property can be used further in the example, should learners need to work with smaller components to make multiplication easier.

Topic 2 Whole Numbers Multiplication



For example:

$$237 = 200 + 30 + 7 \text{ and } 24 = 20 + 4.$$

This would mean the calculation of the example would have far more parts.

$$\begin{aligned} & (200 \times 20) + (200 \times 4) + (30 \times 20) + (30 \times 4) + (7 \times 20) + (7 \times 4) \\ &= 4000 + 800 + 600 + 120 + 140 + 28 \\ &= 5688 \end{aligned}$$

Both methods result in the correct answer of 5688 bottles in total.

Long multiplication techniques also make use of the distributive property when calculating.

2. Using factors to multiply.

$$\begin{array}{r} 352 \\ \times 27 \\ \hline 2464 \\ 7040 \\ \hline 9504 \end{array}$$

This final step is the addition of two products using distribution.



For example:

There are 256 people at a soccer game. If each person spends R24, how much money was made?

The factors of 24 can be 6 and 4, and these can each be factored further where 4 is 2×2 and 6 is 2×3 .

This means multiplication can occur by doing the following:

$$\begin{aligned} & (256 \times 2) \times 2 \times 2 \times 3 \\ &= (512 \times 2) \times 2 \times 3 \\ &= (1024 \times 2) \times 3 \\ &= 2048 \times 3 \\ &= 6144 \end{aligned}$$

These are not new skills as learners have practiced them in previous grades. It is very important that learners practice decomposing numbers into factors, and if learners can decompose numbers into prime factors, then the final multiplication is much easier.

Topic 2 Whole Numbers Multiplication

Estimating an Answer to Help with Multiplication

The skill of rounding off must be revised before this method is attempted. Some learners may want to round everything off, however, it is important to stress that too much rounding has the potential to make the answer less accurate.



For example:

$$5325 \times 108 = 575\,100$$

$$5000 \times 108 = 540\,000$$

Not suitably rounded, as the answer is too far off.

$$5300 \times 108 = 572\,400$$

Rounding is more suitable, as it results in a more accurate answer.

$$5320 \times 108 = 574\,560$$

This is the best choice, but learners may find this calculation difficult.

$$5325 \times 100 = 532\,500$$

Not suitably rounded, as the answer is too far off.

$$5325 \times 110 = 585\,750$$

Not suitably rounded, as the answer is too far off.

Learners must understand the following: the smaller the rounding value is (rounding to the nearest 10, 100 or 1000), the closer the answer is going to be to the actual answer. Learners must understand that they choose the rounding value based on how accurate they wish to be, and on the numbers used in the calculation.

Topic 2 Whole Numbers Multiplication

The Column Method for Multiplication

This method of multiplication is suitable for the large numbers that learners should be able to multiply by the end of Grade 6. It also helps learners use acceptable methods that will empower them to perform horizontal multiplication calculations.(RESOURCE 1 AT THE END OF THIS SECTION)

M	Hth	Tth	Th	H	T	U	
			5	4	6	9	
				1	8	7	
		3	8	2	8	3	$\times 7$
	4	3	7	5	2	0	$\times 80$
	5	4	6	9	0	0	$\times 100$
1	0	2	2	7	0	3	Remember to add each of the products to give the final answer.

Learners must be comfortable with tables, factors and multiples. This method is best taught to learners using a board, as the numbers are far too large to teach this section practically. Learners can work on multiplying two digit by two digit numbers, and then gradually increase the size of each number, until learners are comfortable and can meet the requirement of multiplying four digit numbers by three digit numbers.



Teaching Tip: Learners can often forget to place zeros in the units columns when multiplying by the tens digit of a number, and two zeros when multiplying by the hundreds digit. Using the grid provided in the resource section on the next page will help learners remember the zeros.

Topic 2 Whole Numbers Multiplication

RESOURCES



RESOURCE 1

M	Hth	Tth	Th	H	T	U

TOPIC 3: PROPERTIES OF 3D SHAPES

INTRODUCTION

- This unit runs for 5 hours.
- This unit is part of the outcome 'Space and Shape'. This counts for 15% of the final exam.
- Learners must be able to identify and describe various 3D shapes. This becomes the foundation of geometric reasoning that is a requirement later in the senior and FET phases.
- The purpose of this unit is to develop an ability to work with 3D problems and understand these when working with real-life situations.
- Mental mathematics must be practised daily as the basic skills that were taught in earlier grades and skills that were taught in the first term must be tested and built on in this new term. Most CAPS textbooks have examples of mental calculations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
- Recognize, visualize and name 3-D objects in the environment and geometric settings, focusing on: - rectangular prisms and other prisms - cubes - cylinders - cones - pyramids - Similarities and differences between cubes and rectangular prisms - Describe, sort and compare 3-D objects in terms of - shape of faces - number of faces - flat and curved surfaces - Make 3-D models using cut out polygons - Cut open boxes to trace and describe their nets	- Recognize, visualize and name 3-D objects in the environment and geometric settings, focusing on - rectangular prisms - cubes - tetrahedrons - pyramids - Similarities and differences between tetrahedrons and other pyramids - Describe, sort and compare 3-D objects in terms of - number and shape of faces - number of vertices - number of edges - Make 3-D models using: drinking straws, toothpicks and other materials - Nets of shapes	- Describe, name and compare the 5 Platonic solids in terms of the shape and number of faces, the number of vertices and the number of edges - Recognize and describe the properties of: - spheres - cylinders - Use nets to create models of geometric solids, including: - cubes - prisms - pyramids - cylinders

GLOSSARY OF TERMS

Term	Explanation / Diagram
Three-dimensional (3D) Object	An object with three dimensions [such as height, width and depth] like any object in the real world.
Prism	A solid object with two identical ends and flat sides. The cross section is the same all along its length.
Polyhedron	A solid figure that has several polygon faces. A prism can also be known as a polyhedron.
Pyramid	A special type of polyhedron. It has a solid base that is a polygon, and the other faces are triangles that meet at an apex.*
Apex	The highest point of a pyramid or cone.
Tetrahedron	A solid made up of four triangular faces. It can also be considered a pyramid with a triangular base.
Platonic Solid	A 3D shape where each face is the same regular polygon and the same number of polygons meet at each vertex [corner]. There are 5 platonic solids.
Face	The flat surface or side of a solid shape.
Edge	The intersection of the faces of a 3D shape.
Vertex	The corner of a shape or solid. The place where the edges of a shape meet.
Net	A 2D pattern of a 3D shape that can be folded to result in the 3D object.
Sphere	A perfectly round ball. This shape is made only of curved edges. Every point on the surface of the sphere is the same distance away from the centre of the shape.
Cone	A solid [3-dimensional] object with a circular flat base joined to a curved side that ends in an apex point.
Cylinder	A closed solid that has two parallel circular bases connected by a curved surface.

SUMMARY OF KEY CONCEPTS

Identifying 3D Objects

Learners must be able to recognise and name 3D objects in their natural and cultural forms, and in geometric settings. Learners should be able to identify 3D shapes in everyday objects like food tins (cylinders), Smarties boxes (rectangular prisms) and a number of objects found around the home.

Ways in which to identify and group 3D objects:

1. Objects with a curved surface only – Spheres.

It is important to have both real-life and geometric examples for learners to use to familiarise themselves with the shape.



Sphere

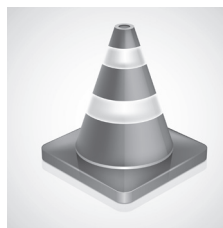
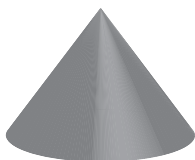


Remember that a every point on the surface of a sphere is an equal distance from the centre.

This means that a rugby ball is not an example of a sphere.

2. Objects with flat and curved surfaces – Cones and Cylinders.

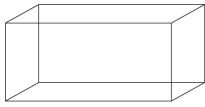
These shapes are important, and giving learners a variety of examples will help them identify the shape when necessary.



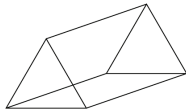
3. Objects with only flat surfaces – Prisms and Pyramids.

It is important for learners to discover and discuss the differences between various pyramids and prisms. Learners must also be confident in their own ability to find sensible reasons for the distinguishing between the shapes.

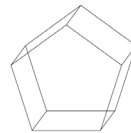
Prisms:



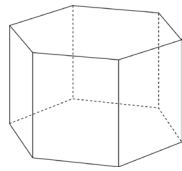
rectangular prism



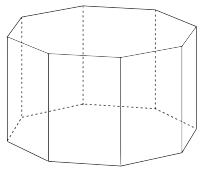
triangular prism



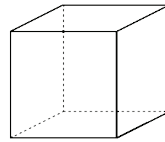
pentagonal prism



hexagonal prism

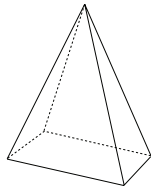


octagonal prism

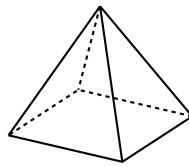


cube

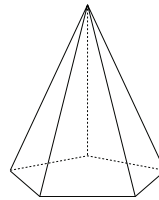
Pyramids:



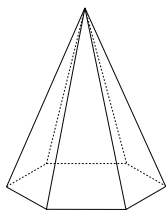
rectangular pyramid



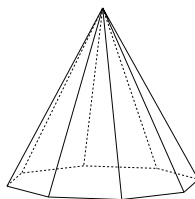
square pyramid



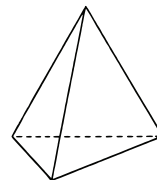
pentagonal pyramid



hexagonal pyramid



octagonal pyramid



Tetrahedron

Learners must be able to describe similarities and differences of prisms and pyramids. These shapes are unfamiliar in Grade 6, and it is important to help learners understand the fundamental differences and similarities between the shapes.



Teaching Tip: Learners may find it difficult to tell the difference between pyramids and prisms. It is vital to explain the definitions of these shapes, and to have models for the learners to see and handle, as it will assist them in getting a better conceptual understanding.

Topic 3 Properties of 3D Shapes

Constructing 3D Shapes

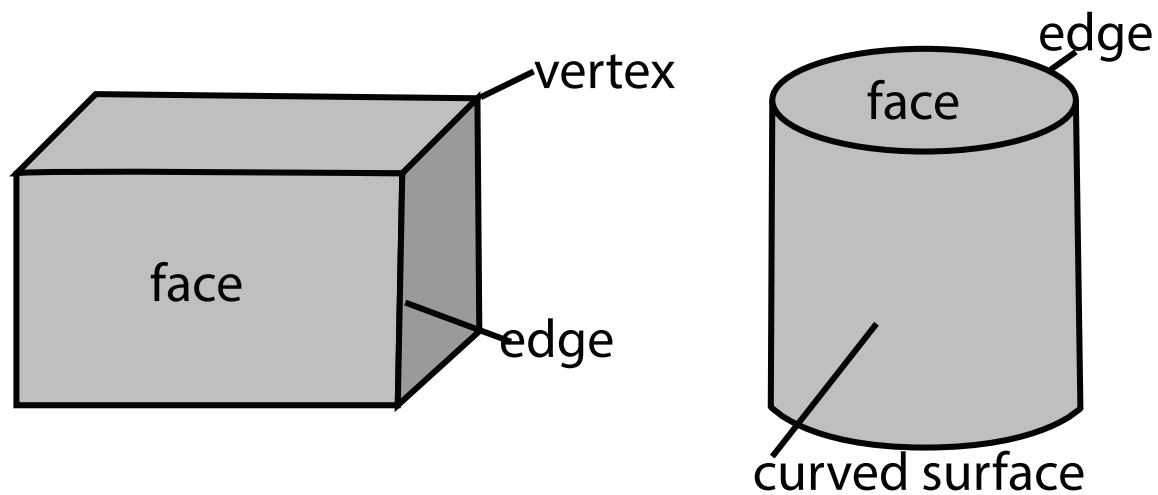
Learners can truly experience 3D shapes by constructing them out of paper. This will aid the learners in tasks that require classifying and sorting shapes. It gives the learners concrete ideas of the specific properties of the various shapes.

Included in the EDUATORS RESOURCES SECTION are nets that can be used to construct a variety of shapes.

Opening boxes that contain sweets will help learners investigate the nets of items that have a real-life connections, and will help to form a clearer contextual picture of the properties of the various shapes.

Describe, Sort and Compare 3D Shapes

In order to sort and compare 3D shapes accurately, learners should have a clear understanding of the parts labelled on the drawings below. These should be the criteria that is used by learners to sort and compare shapes.



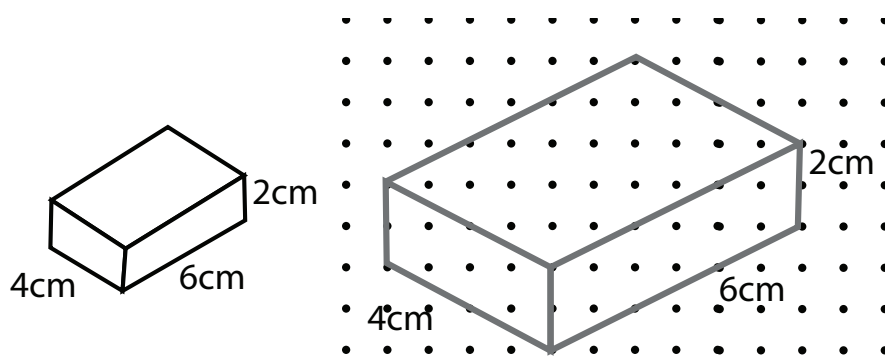
Learners can sort shapes in various ways:

1. Using the number of faces, edges and vertices (plural of vertex).
2. By sorting shapes through separating all shapes with flat edges together, those with flat and curved edges together; and then those with curved edges only together.
3. Learners could also group shapes according to the shape of the faces.

Draw 3D Shapes

Encouraging learners to draw a variety of 3D shapes is an important section of this unit. It helps learners to develop skills that can be used in the problem solving sections later in this phase, and the phases that follow. Learners should also be capable of determining if a drawn net will result in the desired shape (RESOURCE 1 PAGE 31).

Learners must be given the correct resources in order to be able to draw accurate 3D sketches of a variety of 3D shapes.



Topic 3 Properties of 3D Shapes

RESOURCES



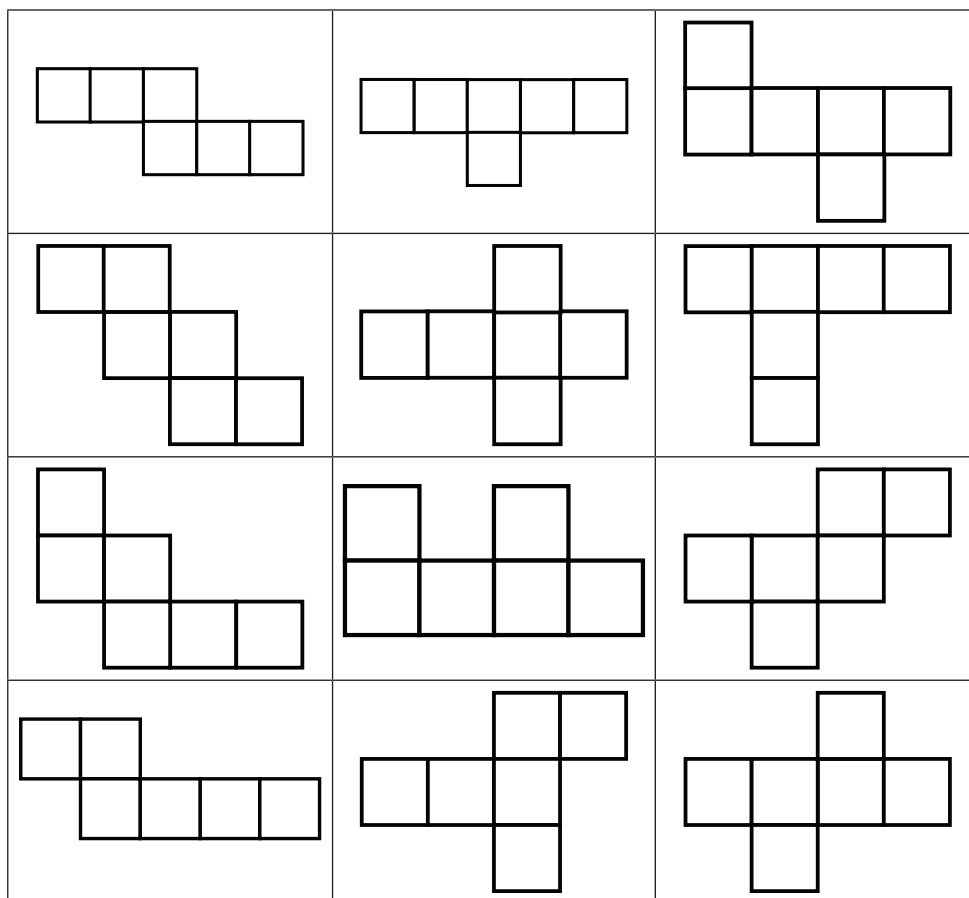
RESOURCE 1

Cube Nets

Which of these nets folds up to make a cube?

To find out, copy the nets onto squared paper. Cut them out and fold them along the lines.

When you have found a net that folds up into a cube, colour it in on the table.



Can you find any other nets that fold up into a cube shape?

Try out your ideas on squared paper, then draw your successful nets into your book.

TOPIC 4: GEOMETRIC PATTERNS

INTRODUCTION

- This unit runs for 6 hours.
- This unit is part of the 'Patterns, Functions and Algebra' section. This counts for 10% in the final exam.
- The unit covers the concept of Geometric Patterns, which is an extension of Number Patterns, and forms the foundation of the Algebra components in the senior and FET phases.
- The purpose of this section is to introduce learners to the concept of determining a general rule that can be applied in situations where a pattern develops. This is a vital building block in the development of algebraic skills. Mental mathematics must be practised daily, as the basic skills that were taught in earlier grades, and skills that were taught in the first term must be tested and built on. Most CAPS textbooks have examples of mental calculations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
- Investigate and extend geometric patterns looking for relationships or rules represented in diagram form and not limited to constant differences or ratios - Describe observed relationships or rules in own words - Determine input values, output values and rules for patterns and relations using flow diagrams - Determine equivalence of different descriptions of the same relationship or rule presented verbally, in a flow diagram or by a number sentence.	- Investigate and extend geometric patterns looking for relationships or rules represented in diagram form, in tables and not limited to constant differences or ratios - Describe observed relationships or rules in own words - Determine input values, output values and rules for patterns and relations using flow diagrams and tables - Determine equivalence of different descriptions of the same relationship or rule presented verbally, in a flow diagram, by a number sentence and in a table	- Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns: represented in physical or diagram form not limited to sequences involving a constant difference or ratio of learner's own creation represented in tables represented algebraically - Describe and justify the general rules for observed relationships between numbers in own words or in algebraic language - The FET phase expands this knowledge even further looking at series of sequences.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Sequence	A collection of terms following a rule or pattern.
Inspection	Using intuition and knowledge to recognise a familiar pattern leading to an immediate solution of a mathematical problem.
Flow Diagram	A drawing used to clarify the order in which operations must occur in order to achieve a desired outcome or result.
Geometric Pattern	Patterns made from 2D shapes and 3D objects that can be re-described using numeric patterns.
Rule	A given or determined method that results in the desired outputs when applied to the inputs of a pattern.
Equation	An equation is a statement that two expressions have the same value. It is a balanced calculation where the Left Hand Side [LHS] = the Right Hand Side [RHS].
Diagram	A picture showing the progression of a pattern.
Pattern	A special arrangement of numbers or shapes. Patterns repeat or change in a constant manner.
General Term	The rule that can be used to determine any term in a given pattern.

SUMMARY OF KEY CONCEPTS



Extend Patterns and Determine Rules

Learners should be capable of seeing and extending patterns, which takes a lot of practice.

There are several methods that can be used by learners to help them understand the concept of patterns.

1. Packing shapes:

Learners should investigate and expand patterns by using beads, matches, or any other small items. This should not be educator driven, but rather allow for learners to explore and make their own patterns of squares or triangles, and discover the “rule” of the pattern they have created.

2. Drawing a pattern:

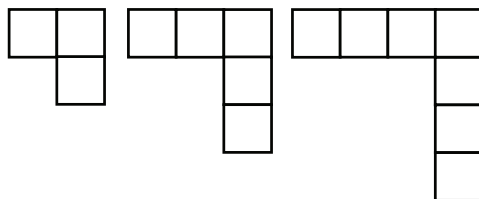
Learners must draw the patterns that they have created. Drawing the patterns allows for further investigation and comparison later.

3. Expanding and completing learner’s own patterns:

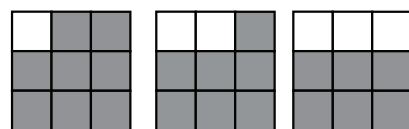
Activities that require completing a pattern or expanding up to 5 terms further.



EXAMPLE:



Draw the next three figures in the pattern above.



Draw the next three figures in the pattern above.

Learners can be asked to complete number patterns using beads as an aid to the problem.

Learners can be asked what would be required to produce the next figure in the geometric pattern



EXAMPLE:



How many matches are needed to produce the 4th figure in the pattern?

(13 matches)



Teaching Tip: Once learners have mastered square patterns, they can be given patterns that are similar except using hexagons, octagons etc. Learners can also be asked to investigate the relationships between the number of sides in the shape, and the number of matches required. This will become the first interaction with determining the rule of a pattern.

4. Describing patterns:

Describing a pattern in words is the fundamental base for finding the general term (formula). This is the foundation to writing rules that can be used to determine desired outputs in a pattern.



EXAMPLE:



Learners must be able to describe in their own words how a pattern is formed. This pattern is formed by adding three (3) more matches each time.

5. Designing own patterns

Learners can take physical objects such as squares or matches and physically lay out the patterns. It is vital to reinforce the names of the 2D shapes (done in Term 1) in this section.



Teaching Tip: A good task for learners in order to allow them to be creative would be to ask them to design their own wrapping paper or a pattern for a table cloth or even a design to be used for the school's soccer team shirt that uses repetitive patterns of squares, circles and other shapes

6. Developing tables from a rule:

Tables must be drawn up from a geometric pattern so that a rule can be determined.



EXAMPLE:



Topic 4 Geometric Patterns

No. of squares	1	2	3	4	5	10	20
No of matches needed.	4	7	10	?	?	?	?

The table can help find the rule that will be used to find terms in the pattern. It is easy to find consecutive terms, but learners should be able to determine the 100th term by using a simple rule. This will help learners understand that patterns are used to find general solutions, thus making it easier to find larger term numbers.

An easy way to help learners find a pattern is to use the constant difference, and the difference between the constant difference, and the first number in the pattern to make a rule.



EXAMPLE:

No. of squares	1	2	3	4	5	10	20
No of matches needed.	4	7	10	?	?	?	?
Constant difference is	3 [because 3 is added each time]						
Difference between the constant difference and the first number in the pattern.	3×1 [1st pattern which is ONE square] = 3. To get to 4 [number of matches] we need to add 1. 3×2 [2nd pattern/term] = 6. To get to 7 we need to add 1. Therefore the rule is: multiply by 3 and add 1.						

Rule = Constant difference (3) multiply by 'number of squares' and add 1.

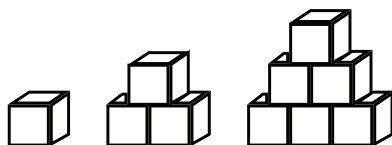
3D Geometric Patterns

At this stage, learners must start to develop a 3D pattern. Learners should know how to determine the actual pattern, and how to find the rule relating to the pattern. Learners should also be capable of completing a pattern comprised of 3D shapes.



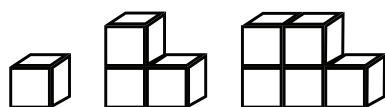
EXAMPLE:

Describe the next two figures in the given pattern:



The next pattern (4th) will have 4 cubes in the bottom row, 3 in the next row, 2 in the 3rd row then one at the top. The cubes are always centred on top of the previous row. In total there will be 10 cubes.

The 5th pattern will have 5 cubes on the bottom row with one less cube on each row so there will be 15 ($5 + 4 + 3 + 2 + 1$) cubes in total.



The 4th pattern will have 7 cubes. 4 on the bottom row and 3 on the next row. The three on the top row must be directly on top of the first 3 cubes in the bottom row.

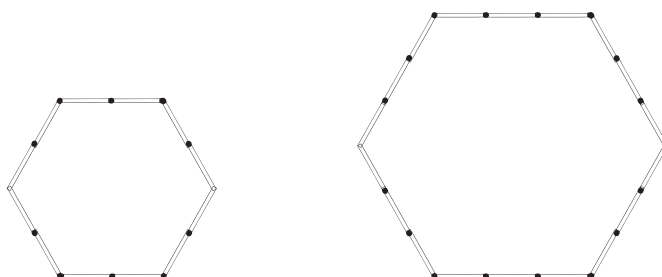
The 5th pattern will have 9 cubes. 5 on the bottom row and 4 on the top row.

Extending Patterns with a Constant Ratio

These patterns will usually represent shapes that are increasing or decreasing in size, and it is important for learners to develop skills relating to these kinds of patterns. Enlarged shapes will have a constant ratio that is greater than 1, which means that you will multiply by the same number to increase size. If the shape is getting smaller, the constant ratio is less than 1, meaning multiplication by a common fraction each time.



EXAMPLE:



This shape has increased to 1,5 times the original size. Use the length of the sides to help work this out. The larger hexagon has sides of 3 units long and the smaller hexagon has sides of 2 units long.

$$3 \text{ divided by } 2 = 1,5$$

Complete the table below for the number of matches used in this geometric pattern



No. of Triangles	1	2	3	4	5	6
No. of Matches	3	6	9	12	15	18

Learners need to be able to describe a pattern such as this one.



For example:

For every triangle needed there are 3 matches required. Therefore, if there are 10 triangles, 30 (10 x 3) matches will be required.

TOPIC 5: SYMMETRY

INTRODUCTION

- This unit runs for 2 hours.
- This unit is part of the outcome 'Space and shape'. This counts for 10% of the final exam.
- The unit covers the concept of symmetry in shapes. The focus is on line and rotational symmetry, and shapes should not only have vertical lines of symmetry.
- The purpose of this section is to develop skills where learners can see multiple lines of symmetry. This section broadens learners understanding of manipulation in mathematics, thus impacting on the ability to see patterns in algebra later in curriculum.
- Mental mathematics must be practised daily, as the basic skills that were taught in earlier grades, and skills that were taught in the first term must be tested and built on. Most CAPS textbooks have examples of mental calculations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Recognize, draw and describe line[s] of symmetry in 2-D shapes	• Recognize, draw and describe line[s] of symmetry in 2-D shapes	• Symmetry is not a stand alone topic in the Senior or FET phase. However, the understanding of symmetry will be used in topics such as Functions and Transformation geometry.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Line symmetry or Reflection symmetry	Shapes that have line symmetry can be divided into 2 perfectly identical parts, when cut or folded along the line of symmetry.*
Line Of Symmetry	A line that divides a figure into congruent [identical] parts, each of which is the mirror image of the other.
Infinite	This would mean that there is no end to the set, item or pattern.
Rotational Symmetry	A shape has rotational symmetry when it still looks the same after a rotation less than a full turn [complete rotation].
Order Of Rational Symmetry	This is the number of times a figure fits into itself in one complete rotation [full turn].

SUMMARY OF KEY CONCEPTS

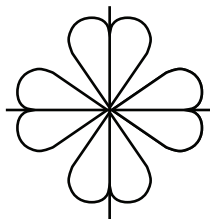


Line Symmetry

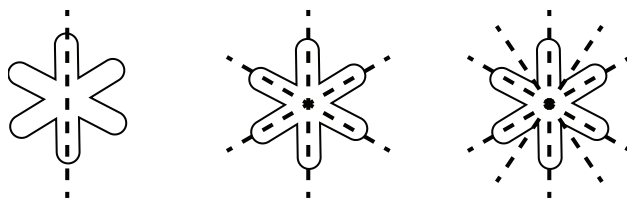
Learners must be able to indicate lines of symmetry in the presence of multiple lines of symmetry.



EXAMPLE:



This shape has both vertical and horizontal lines of symmetry

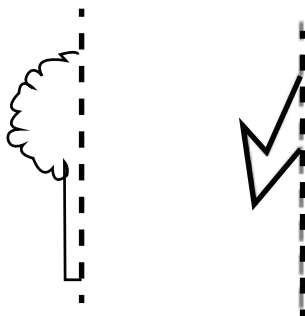


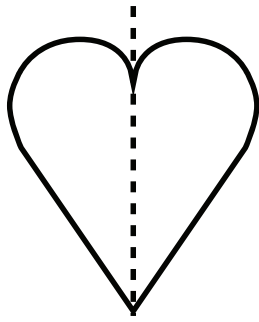
This shape has multiple lines of symmetry, and learners should be capable of determining all the lines of symmetry even along diagonal lines.

Learners should also be able to complete shapes when given a line of symmetry.

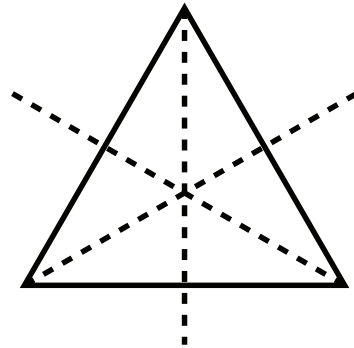


EXAMPLE:

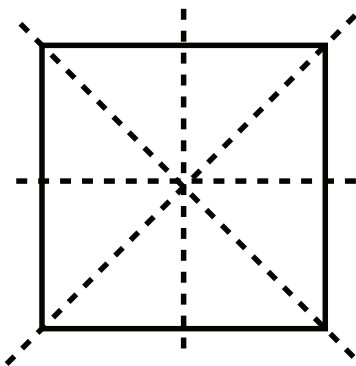




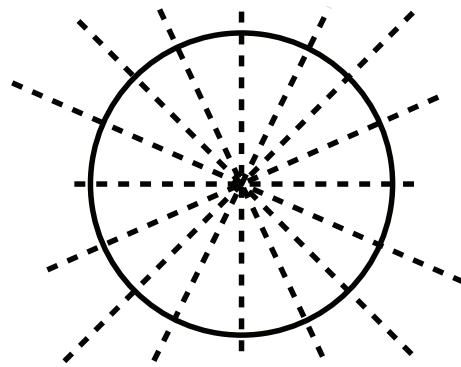
1 line of symmetry



3 lines of symmetry



4 lines of symmetry



infinite number of
lines of symmetry

Shapes like circles will have an infinite number of lines of symmetry.



Teaching Tip: Learners often struggle to complete shapes when given a line of symmetry. Using small mirrors could make it easier for learners to understand the reflective component of line symmetry.

Topic 5 Symmetry

Rotational Symmetry

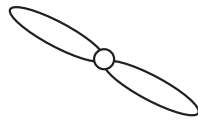
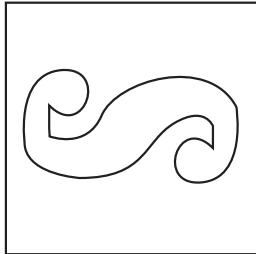
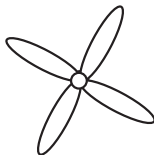
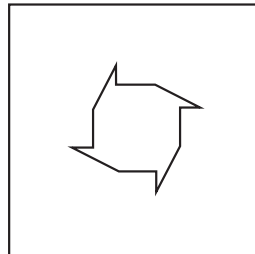
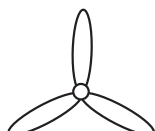
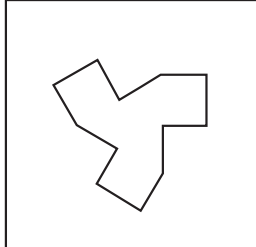
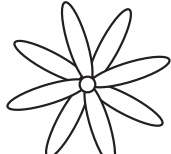
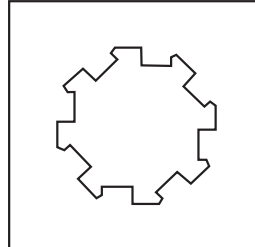
A shape has Rotational Symmetry when it still looks the same after a rotation.

How many times it matches back to its original shape when we go around is called order. Think of propeller blades as shown below to make it a little easier.

Learners may struggle with the concept of rotational symmetry as it is not always easy for them to visualise the shape being turned and also needing to keep track of how many times it has turned.

Cutting out shapes for the learners to hold and turn themselves would help them to grasp the idea better.

Rotation occurs through the centre of the shape.

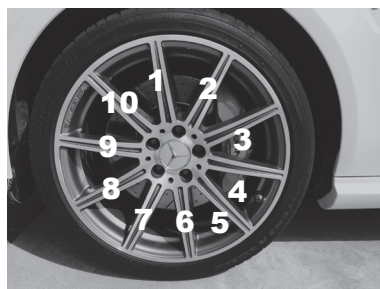
Order	Example shape				
Order 2			Order 4		
Order 3			Order 8		



The example below is a good real-life contextualization of rotational symmetry.



This wheel has rotational symmetry of order 5



This wheel has rotational symmetry of order 10

TOPIC 6: WHOLE NUMBERS – DIVISION

INTRODUCTION

- This unit runs for 8 hours.
- This unit is part of the outcome ‘Numbers, Operations and Relationships’. This counts for 50% of the final exam.
- The unit continues the concept of division. Learners have been taught methods suitable to division in previous grades, but must be able to apply these skills to the division of 4 digit numbers by 3 digit numbers. Learners are introduced to the concept of long division, prime factors and divisibility rules.
- This section develops skills needed for algebra.
- Mental mathematics must be practised daily, as the basic skills that were taught in earlier grades, and skills that were taught in the first term must be tested and built on. Most CAPS textbooks have examples of mental calculations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Division of at least 3-digit and 2-digit whole numbers • Using multiplication and division as inverse operations	• Division of at least whole 4-digit by 3-digit numbers • Using multiplication and division as inverse operations • Long division • Using a calculator	• Division of at least whole 4-digit by 2-digit numbers in shorter methods. • The FET phase is focused on problem solving in various contexts thus this section draws on the skills taught in all the previous phases and combines with new calculation skills and new sections of mathematics such as Trigonometry that require extensive problem solving within the related contexts. • Shorter division techniques are used later in the senior phase and even extend to algebraic contexts.

Topic 6 Whole Numbers - Division

GLOSSARY OF TERMS

Term	Explanation / Diagram
Share	To portion or divide something between varying numbers of groups.
Group	To place items together using criteria that relates the items to each other.
Prime Number	A positive integer that has only two factors, 1 and the number itself. This does not include 1, as it is an identity number.
Composite Number	Any number that has multiple factors.
Long Division	Long division is a process of division in arithmetic, usually used when the divisor is a large number, each step of the division is written out.
Ratio	The comparison of 2 or more values of the same kind. Usually written in the format 2:3
Remainder	What is left over when you try to share a whole out into a particular number of equal parts.
Divisor	The number we are dividing by in a division calculation.
Dividend	The number being divided.
Quotient	The answer to a division calculation.
Inverse	The opposite operation being performed. The inverse of division is multiplication.
Divisible	A number is divisible by another if there is no remainder after division.

SUMMARY OF KEY CONCEPTS



Working with Factors to Understand Division

Learners must recap the basics regarding factors that were taught earlier in Grade 6 and in previous grades.

Writing numbers in factored form is the basis of division.



EXAMPLE:

What are the factors of 125?

{1, 5, 25 and 125} are all the factors of 125.

In order to get these answers, division had to occur. Factors pair up to be multiplied and equal 125.

For example: $1 \times 125 = 125$ and $5 \times 25 = 125$

It is important in this section to teach learners the divisibility rules. These should have been covered in earlier grades, although it is always beneficial to remind learners of these rules.

Dividing by 2

Any number ending in 0, 2, 4, 6 or 8 (in other words an even number) is divisible by 2.

For example: 24 and 398 are both divisible by 2

Dividing by 3

If the sum of the digits of a number is divisible by 3 then that number is also divisible by 3.

For example:

123 is divisible by 3 because $1 + 2 + 3 = 6$ and 6 is divisible by 3.

42 105 is divisible by 3 because $4 + 2 + 1 + 0 = 7$ and 7 is not divisible by 3.

Dividing by 4

If the last 2 digits of a number are divisible by 4 then the entire number will also be divisible by 4.

For example:

724 is divisible by 4 because the last 2 digits (24) are divisible by 4

358 912 is divisible by 4 because the last 2 digits (12) are divisible by 4.

Topic 6 Whole Numbers - Division

Dividing by 5

Any number ending in 0 or 5 will be divisible by 5.

For example:

945 is divisible by 5

1020 is divisible by 5

Dividing by 6

If a number is divisible by 2 AND 3 then it will also be divisible by 6.

For example:

624 is divisible by 2 (it is even) and by 3 ($6 + 2 + 4 = 12$), therefore it will also be divisible by 6.

Dividing by 7 (2 Tests)

In order to test if a number is divisible by 7, perform the following 'test':

Take the last digit of the number you're testing and double it. Subtract this number from the rest of the digits in the original number. If this new number is either 0 or if it's a number that's divisible by 7, then you know that the original number is also divisible by 7.

For example:

357 is divisible by 7 because the final digit doubled is equal to 14 and $35 - 14 = 21$ which is divisible by 7.

2016 is divisible by 7 because the final digit doubled is 12 and $201 - 12 = 189$ which is divisible by 7. However, if that wasn't easy to see then the process must be repeated with 189. The final digit doubled is 18 and $18 - 18 = 0$ (the rest of the number) minus 18 is zero.

Dividing by 8

If the last 3 digits of a number is divisible by 8, then the entire number will be divisible by 8.

For example:

6008 is divisible by 8 because 008 (8) is divisible by 8.

1136 is divisible by 8 because 136 is divisible by 8. (This one is not so easy to 'see' - perhaps it would be quicker to just divide and check)

Dividing by 9

If the sum of the digits of a number is divisible by 9 then that number is also divisible by 9.

For example:

4374 is divisible by 9 because $4 + 3 + 7 + 4 = 18$ and 18 is divisible by 9.

Dividing by 10

If a number ends in zero it will be divisible by 10.

For example:

13 420 is divisible by 10 because it ends in zero.

Prime Numbers in Division

Learners are introduced to prime numbers as factors, and the benefit of breaking a large number into prime factors. Learners must be aware from the onset that 1 is neither a prime nor composite number.

Learners should recognise all prime numbers between 0 and 100. It is also helpful to use the Sieve of Eratosthenes:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

This grid is used to determine the prime numbers between 0 and 100 using the following steps:



Step 1:

Cross out all the multiples of 2, but do not cross out 2 itself.

Step 2:

Cross out all the multiples of 3, but do not cross out 3 itself.

Step 3:

Cross out all the multiples of 5, but do not cross out 5 itself.

Topic 6 Whole Numbers - Division

Step 4:

Cross out all the multiples of 7, but do not cross out 7 itself.

It may be an interesting discussion to have with the learners here as to why the multiples of 4, 6 and 8 were not part of the list of instructions. Remind them of the rules of divisibility - these numbers are all even and all of their multiples would have already been crossed out when the multiples of 2 were crossed out.

The numbers that are left are the prime numbers found between 1 and 100. The ladder method is a good tool to show learners how to write numbers as a product of their prime factors.



EXAMPLE:

Find the prime factors of 256 and 450

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
1	

2	450
3	225
3	75
5	25
5	5
	1

$$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

and

$$450 = 2 \times 3 \times 3 \times 5 \times 5$$

This skill will help learners practice basic division with smaller numbers, and therefore recap what they have learnt in previous grades on division.

Multiplication Used to Help with Division

For learners to use this method effectively, they should start by drawing up a clue board. This is a collection of multiplication facts related to the numbers they are working with. Learners then solve the division calculation by using approximation and subtraction, as well as through referring to the clue board for facts that are relevant.



EXAMPLE:

$$3\ 450 \div 17$$

Clue Board

$$100 \times 17 = 1700$$

$$200 \times 17 = 3400$$

$$2 \times 17 = 34$$

$$3 \times 17 = 51$$

$$10 \times 17 = 170$$

Multiply	Subtract
$200 \times 17 = 3400$	$3450 - 3400 = 50$
$2 \times 17 = 34$	$50 - 34 = 16$

Therefore $3\ 450 \div 17 = 202$ remainder 16



Teaching Tip: This method is a good introduction to the benefits of long division. It also works well if learners can work on rough paper, or on white boards, because it is a trial and error method that does not take long. Therefore learners should not do too many of these. This strategy assists learners should they continue to struggle with long division. Always let learners check their answers by using multiplication and reversing the process. The size of certain numbers can make methods previously taught very difficult to apply.

Long Division

This traditional method of division remains the best method used to determine the quotient when dividing. It also works well on very large numbers.

$$\begin{array}{r}
 24\text{remainder } 59 \\
 146 \overline{) 3563} \\
 \underline{292\downarrow} 2 \times 146 \\
 643 \\
 \underline{584} 4 \times 146 \\
 59
 \end{array}$$

Topic 6 Whole Numbers - Division

Learners can be taught a simple rhyme that reminds them of the steps they need to follow to complete long division.

Daddy – Divide the dividend by the divisor.

Mommy – Multiply by the divisor the factor.

Sister – Subtract the numbers.

Brother – Bring down the next number in the dividend.

Encourage learners to check answers using a calculator, until they are confident and feel that they have grasped long division completely.

Ratio – Comparing Quantities of the Same Type

The concept of ratio links well to division.

Use the phrase 'for every' when introducing ratios to learners.



For example:

In a bunch of flowers there are 10 red flowers and 2 yellow flowers.

This can be written as 10 : 2 (read as 10 to 2)

10 : 2 can be simplified to 5 : 1

This can be described as: For every 5 red flowers there is one yellow flower in the bunch.

It is important to point out to learners that in a ratio of 5 : 1 there are 6 items in total. Therefore, the ratio actually means that 5 out of every 6 flowers are red and 1 out of every 6 flowers are yellow.



TEACHING TIP:

It would be a good idea to bring in a bottle of dilute cool drink (it doesn't need to be full) to show learners that what is written on the label is a ratio. This tells us how many part cool drink to have with how many parts water. Discuss then how many parts there will be in total in your glass.





EXAMPLE:

Susan buys 100 small apples to feed the 2 donkeys on her plot. One donkey is much bigger than the other and she decides to share the apples in a ratio of 7: 3. How many apples does each donkey get?

Total parts is $7 + 3 = 10$

$100 \div 10 \text{ parts} = 10$

Big donkey gets $7 \times 10 = 70$

Small donkey gets $3 \times 10 = 30$

Therefore, if Susan shares the apples in the ratio of 7 : 3, one donkey will get 70 apples and the other donkey will get 30 apples.

Learners must check that the numbers they get still add up to the total amount that the person is initially sharing.



Teaching Tip: Learners often write ratios the wrong way round, therefore it often helps to have them write the words out, and then underneath start solving the problem. Encourage learners to show all their steps and this will help them to not make unnecessary errors.



EXAMPLE:

Big Donkey's apples: Small Donkey's apples will help them make sure that their answer is the correct way round.

TOPIC 7: DECIMAL FRACTIONS

INTRODUCTION

- This unit runs for 9 hours.
- This unit is part of the outcome 'Numbers, Operations and Relationships'. This counts for 50% of the final exam.
- The unit covers the concept of decimal fractions. Learners are being introduced to decimals for the first time, so this is an important section as foundations are being laid.
- The purpose of this section is to develop skills needed for algebra. New concepts and vocabulary are introduced. Learners will also need these skills in problem solving that relate to money and other financial calculations, which are covered later in the senior and in the FET phase.
- Mental mathematics must be practised daily, as the basic skills that were taught in earlier grades, and skills that were taught in the first term must be tested and built on. Most CAPS textbooks have examples of mental calculations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
Basic comprehension and understanding of division and multiplication by 10, 100 or 1000.	- Count forwards and backwards in decimal fractions to at least two decimal places - Compare and order decimal fractions to at least two decimal places - Place value of digits to at least two decimal places - Addition and subtraction of decimal fractions with at least two decimal places - Multiply decimal fractions by 10 and 100 - Solve problems in context involving decimal fractions - Recognize equivalence between common fraction and decimal fraction forms of the same number - Recognize equivalence between common fraction, decimal fraction and percentage forms of the same number	All the skills are revised in the senior phase and applied to algebraic contexts and extended throughout the FET phase.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Decimal, Decimal Fraction and Decimal Number	These are all words that mean the same thing. They refer to fractions that have a power of 10 in the denominator. This means that the denominator is either 10, 100 or 1000. This explanation is at Grade 6 level, but will be made clearer from Grade 7 onwards.
Decimal Places	The position of a digit to the right of a decimal point. Each successive position to the right has a denominator of an increased power of 10.
Notation	A writing system used for recording concepts in mathematics. Symbols are regularly used.
Value	What something is worth.
Terminating Decimals	This is a decimal that ends. It has a denominator that is a power of 10.

SUMMARY OF KEY CONCEPTS



Introducing Decimal Fractions

As far as possible, work with money when introducing decimal fractions. Emphasize that the decimal refers to a whole rand as well as to a fraction of a rand (but a whole number of cents)



For example:

R1,25 is one whole rand and 25 hundredths of a rand which is 25 cents.

R2,06 is two rands and 6 hundredths of a rand which is 6 cents.

Show learners how placing zeros at the end of a decimal fraction does not change the value:



For example: $0,3 = 0,30 = 0,300$ (because $\frac{3}{10} = \frac{30}{100} = \frac{300}{1000}$)

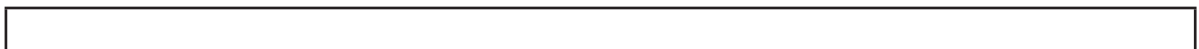
This is useful when needing to compare decimals as well as when adding or subtracting decimals and is dealt with under those sub-topics.

The following method of introducing decimal fractions requires a good understanding of common fractions (in this case particularly those with a denominator of 10).

If you think the learners need to review these skills then that would be the best way to start this section.

Once you feel learners are ready to tackle this new topic, follow these steps:

1. Draw a long rectangle on the board.



2. Split it into ten sections.

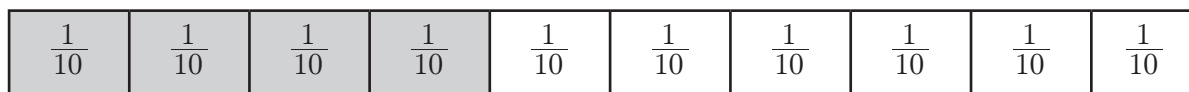


3. Ask the learners how we can label each section of the rectangle ($\frac{1}{10}$) then write this into each section

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
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4. Remind learners how each of these makes a whole ($\frac{10}{10} = 1$)

5. Colour or shade some of the sections in. For the sake of this example, we will shade 4 sections.



6. Ask the learners what fraction of the rectangle has been coloured. In this case $\left(\frac{4}{10}\right)$.
7. Explain that can be written in another way - 0,4 (no units and four tenths). Show on the rectangle why there are clearly no units (the learners need to remember that we started with one large rectangle that was split up into 10 equal parts).
8. Explain the features of the notation:
- The comma in between the 0 and the 4 is called the DECIMAL COMMA and we use it to separate the units from the tenths.
 - We always write in the 0 before the decimal point because it reminds us that the whole number is less than one.
 - We say this number as “zero comma four” or “zero point 4”.

Using the resource at the end of the section you can give each learner a few of these strips and call out how many blocks to shade in then ask questions similar to above. Learners could also work in pairs and ‘test’ each other. The strips each learner uses could be stuck into their books for them to look back on before an assessment.

If photocopying is not possible, learners can draw two of their own (in pen), then shade in pencil so it can be erased to try a new example.

Counting in Decimals

This is an excellent mental maths activity for this particular unit. Learners should be able to count forwards and backwards in various decimal forms.

Start off with easy amounts to count up or down in. 0,1 will be the best place to start.

Ask learners to count from 0,1 and go up in tenths (in other words add on 0,1 each time)



For example:

0 ; 0,1 ; 0,2 ; 0,3 ; 0,4 ; 0,5....

Next build up to counting in decimals of 0,2 and 0,3 etc.

Once learners have mastered this, use the same decimals and ask learners to count backwards from a given number.



For example:

Count down in decimals of 0,2 from 4,6 :
4,6 ; 4,4 ; 4,2 ; 4,0 ; 3,8 ; 3,6...

Now learners may be ready to count in hundredths (in other words decimal fractions with 2 decimal places).



For example:

Counting in decimals of 0,05.



Learners are tempted to write this as 0,010.

Important teaching note: Remind learners that 0,10 can be written as 0,1

because $\frac{10}{100} = \frac{1}{10}$



Mental maths can be assessed using number chains, such as the one used in the example above. Learners should not be spending too much time counting verbally.

Teaching Tip: Learners often have trouble when counting across a number bridge if the decimal they are counting in does not fall on a whole number. This can be overcome if learners work in a table and understand that they should always proceed to the place value left of the digit once they have reached the end of that value. This is the same strategy that learners would have used for whole number counting techniques.

Topic 7 Decimal Fractions

Reading and Writing Decimal Fractions

Learners must be able to correctly read and write decimals. That is, they should know the correct name and placement according to the place value of the digit they are given.



EXAMPLE:

Numbers such as 345,23 and 34,06.

Thousands	Hundreds	Tens	Units	Decimal comma	tenths	hundredths
	3	4	5	.	2	3
		3	4	.	0	6



Teaching Tip: Learners may think that a decimal with only one decimal place is smaller than one with two decimal places. It is a good idea to let learners write decimals with an equivalent number of decimal places using zero to fill missing decimals. The learners must understand the significance of the digits after the comma and the portion they represent. Using value tables, such as the one above will be helpful.

Learners must know how to read decimal numbers correctly.



EXAMPLES:



Numbers such as 324,67 (READ AS: Three hundred and twenty-four comma six seven) and 320,07 (READ AS: Three hundred and twenty comma zero seven).



Teaching Tip: Learners often try to read the numbers after the comma as normal two digit numbers, and this must be corrected. Make it a point that learners ought to remember to read numbers after the comma as separate digits.

Rounding off of Decimal Fractions

Rounding is an important concept that learners need to master.

When rounding a decimal, the interest is in the digit to the right of where the rounding requirement is- so if required to round to 2 decimal places, the digit in the 3rd place is of importance.

If that digit is 5 or higher, the digit in the correct position is rounded UP. If that digit is 4 or smaller, the digit remains as it is.



EXAMPLE:

0,547

Rounded to 2 decimal places: 0,55

(7 is in the 3rd (thousandth) position and as it is larger than 5, the 4 in the 2nd (hundredth) position will be rounded up to 5)

Rounded to 1 decimal place: 0,5

(4 is in the 2nd (hundredth) position and as it is less than 5, the 5 in the position of interest will remain the same)

Rounded to a whole number: 1

(5 is in the 1st position so round the digit in the whole number's position '0' up to 1).

Equivalent Forms

Learners must be able to change common fractions into their decimal form and vice versa. It is vital that learners understand the importance of the denominator being 10 or 100 at this stage. Remember that this is a new section, so it is a good idea to relate the section back to the division topic covered previously.



EXAMPLE:

To change $\frac{6}{25}$ into a decimal fraction:

1. Since the fraction being dealt with has a denominator that is a factor of 10, 100 or 1000, equivalent fractions can be used to change it into a fraction with a power of 10 as a denominator which can then be changed into a decimal fraction. $\frac{6}{25} = \frac{24}{100}$



2. Once the denominator is a power of 10, change into a decimal fraction ensuring that the number of decimal places matches the denominator. In other words, if the denominator is 10, there should be ONE decimal place and if the denominator is 100 there should be TWO decimal places etc.

3. In this case $\frac{24}{100} = 0,24$

Topic 7 Decimal Fractions

Converting Decimals into Fractions

1. Read the decimal aloud. In this case, zero comma two eight.
2. Note the number of decimal places. In this case, two.
3. Use the rule equating number of decimal places to a denominator of 10, 100, 1000 etc. In this case, two decimal places means a denominator of 100.
4. Write the decimal as a fraction with the decimal places (28) as the numerator and the power of 10 (100) as the denominator

Solution:

$$0,28 = \frac{28}{100}.$$

5. Simplify the fraction if possible by finding common factors to divide into both the numerator and denominator.

Final solution:

$$0,28 = \frac{7}{25}$$



EXAMPLE

0,35

Reads as zero comma three, five or “thirty- five hundredths”.

$$0,35 = \frac{35}{100}$$

Simplified this would be: $\frac{(35 \div 5)}{(100 \div 5)} = \frac{7}{20}$

Addition and Subtraction of Decimal Fractions

The simplest way to add or subtract decimals is in columns (vertically). The decimal signs must be exactly underneath each other so the place values match. (Tenths underneath tenths, hundredths underneath hundredths, and so on). If need be, zeroes can be placed at the end to make the decimal fractions the same ‘length’.



EXAMPLES:

i. $5,1 + 1,35$

$$\begin{array}{r} 5,10 \\ + 1,35 \\ \hline 6,45 \end{array}$$

ii. $4,29 + 5,641$ (carrying over required)

$$\begin{array}{r} 4,290 \\ + 5,641 \\ \hline 9,931 \end{array}$$

iii. $5,12 - 2,452$ (borrowing is required)

$$\begin{array}{r} 5,120 \\ -2,452 \\ \hline 2,668 \end{array}$$



Teaching Tip: Learners often do not place the decimals correctly underneath each other when they need to perform various operations. It helps to use a place value table, or to get learners to draw columns until they have mastered this technique. Learners must ensure that decimals are the same length when adding them or subtracting them as this will eliminate unnecessary errors.

Multiplication of Decimal Fractions

Multiplying by powers of 10

This is very easy to do - for every 10 being multiplied by, the digits move one place to the left.

Remember to check that the number has become larger, as it should when multiplying.



EXAMPLES:

$$54,21 \times 10 = 542,1$$

$$406,156 \times 100 = 40615,6$$



Teaching Tip: Learners must realise that the comma is a FIXED point, and that only the digits move, and not the comma, when being multiplied by multiples of 10. The digits move to the left of the comma as the number gets bigger. It is also helpful to remind learners that divide and multiply are inverse operations, and that they can check their answers by using the opposite operation to revert back to the question.

Topic 7 Decimal Fractions

Compare and Order Decimal Fractions

Use decimal fraction place value columns to compare decimal fractions.

EXAMPLE:

Compare 0, 45 and 0,045.

Units	Decimal comma	tenths	hundredths	Thousandths
0	.	4	5	0
0	.	0	4	5

Looking at the Units the numbers seem similar, but looking at the tenths, hundredths and thousandths reveals that 0,450 is the bigger decimal number.

The numbers must be compared from the left to the right.

Teaching Tip: Decimals can only be compared if they contain the same number of decimal places.

Remind learners that by putting another zero at the end of a decimal fraction does not change its value ($0,4 = 0,40$ because $\frac{4}{10}$ is the same as $\frac{40}{100}$).

'Topping up' a decimal fraction by adding in zeroes so that the decimals match each other in length will help in all calculations without changing the value of the decimal fraction. Learners often forget to put a zero if it is required at the end of a decimal, such as in calculations involving money. This must be reinforced as that zero is important. This section can easily be covered by referring learners to simple mathematical calculations like comparing the cost of items in a shop

RESOURCES



Introduction to Decimal Fractions

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

TOPIC 8: CAPACITY AND VOLUME

INTRODUCTION

- This unit runs for 5 hours.
- This unit is part of the outcome 'Measurement'. This counts for 15% of the final exam.
- The unit covers the concept of volume and capacity. Learners are taught that although the two concepts are related, they are not the same concept.
- The purpose of this section is to develop skills in measurement that will be used later in the senior and FET phase, where the algebraic concept of scale factor is introduced.
- Learners doing both Mathematics and Mathematical Literacy will need these measurement skills.
- Mental mathematics must be practised daily, as the basic skills that were taught in earlier grades, and skills that were taught in the first term must be tested and built on. Most CAPS textbooks have examples of mental calculations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
- Practical measuring of 3D objects by estimating, measuring, recording, comparing and ordering - Calculations and problem solving involving capacity and volume which can include conversions	- Practical measuring of 3D objects by estimating, measuring, recording, comparing and ordering - Calculations and problem solving involving capacity and volume which can include conversions - Conversions should include fraction and decimal forms	- Use appropriate formulae and conversions between SI units to solve problems and calculate the surface area, volume and capacity of: - cubes - rectangular prisms - triangular prisms - cylinders - Investigate how doubling any or all the dimensions of right prisms and cylinders affects their volume - Use and convert between appropriate SI units, including: $\text{mm}^3 \leftrightarrow \text{cm}^3$ $\text{cm}^3 \leftrightarrow \text{m}^3$ - Use equivalence between units when solving problems: $1 \text{ cm}^3 \leftrightarrow 1 \text{ ml}$ $1 \text{ m}^3 \leftrightarrow 1 \text{ kl}$

GLOSSARY OF TERMS

Term	Explanation / Diagram
Capacity	This is the amount a container can hold. It is related to volume, but remains a separate concept. The units for capacity include ml, <i>l</i> and <i>kl</i> .
Volume	The amount of space taken up by a 3D object. It is related to capacity, but uses a different set of SI units. 3D objects have 3 dimensions and therefore volume is recorded as cm cubed or m cubed etc [Note: the cubed needs to be the exponent '3'] .
Convert	This is an important concept referring to the changes from one unit of measurement to another.
Kilolitre	This is one of the SI units for capacity. $1000\text{l} = 1\text{kl}$
Litre	This is another SI unit for capacity. $1000\text{ml} = 1\text{l}$.
Error of Parallax	This is a scientific concept that integrates well into this section as learners must be able to read measurements from calibrated measuring instruments correctly. This error occurs when learners do not look at liquid inside a jug correctly and thus do not read measurements accurately.
Measuring Instrument	These instruments include measuring cylinders, jugs and any other calibrated container used to determine the capacity of a liquid.
Calibration	This is a system to be exact.
Graduation lines	Numbered intervals on a measuring instrument.

SUMMARY OF KEY CONCEPTS



Comparing Volume and Capacity

Learners must have a clear understanding of the difference between volume and capacity.

1. Volume is the amount of space taken up by a 3D object, while capacity is the measure of an object's ability to hold a substance (the focus here will be on liquid)
2. Volume is measured in cubic units, while capacity (of liquid) is measured in millilitres, litres etc
3. Volume is calculated by multiplying the length, breadth and height of an object.



Teaching Tip: Give learners a cup filled with water. Tell them the capacity of the cup is 250 ml, but have them observe that the cup cannot be filled to the brim because it will mess. This means the volume of the water in the cup will not be 250 ml, and as they drink the water, the volume will decrease. They must notice that although the volume of the water in the cup is changing, the capacity of the cup stayed the same.

Kilolitres, Litres and Millilitres

1. Basic conversions

$$1 \text{ Kl} = 1\,000 \text{ l}$$

$$1 \text{ l} = 1\,000 \text{ ml}$$

$$\frac{1}{2} \text{ l} = 500 \text{ ml}$$

$$\frac{1}{4} \text{ l} = 250 \text{ ml}$$

$$\frac{1}{5} \text{ l} = 200 \text{ ml}$$

$$\frac{1}{10} \text{ l} = 100 \text{ ml}$$

$$1000\text{ml} \div 2 = 500\text{ml}$$

$$1000\text{ml} \div 4 = 250 \text{ ml}$$

$$1000\text{ml} \div 5 = 200 \text{ ml}$$

$$1000\text{ml} \div 10 = 100 \text{ ml}$$

Topic 8 Capacity and Volume

It is important for learners to be familiar with everyday items such as teaspoons, spoons, cups, jugs; and bottles like milk cartons, drinks containers and any other item that has a capacity occupied by a volume of liquid or solid. (RESOURCE 1 AT THE END OF THIS SECTION)



If possible, gather as many different kinds of bottles and containers as you can and allow the learners to work with these.

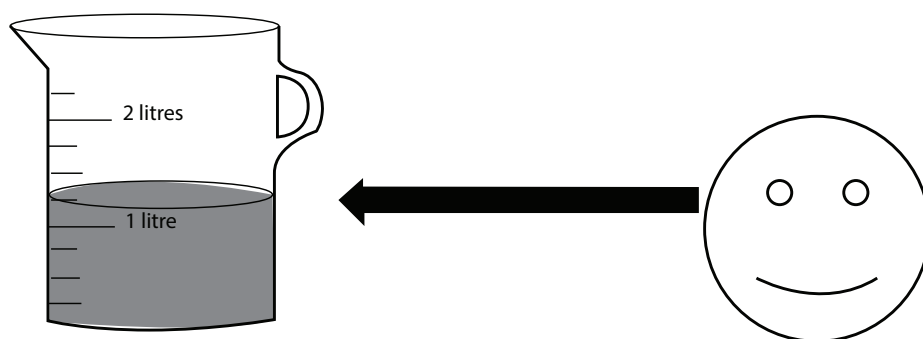
Example:

See how many medicine teaspoons you need to fill a 500ml bottle of cool drink, or how many cups of water to fill a 2 litre drinks container, or even how many tablespoons it would take to put water in a cup.

Learners must use estimation skills and actual measurement skills to determine the relationship between various containers.

Reading Scale Accurately- The Error of Parallax

1. When measuring an amount of liquid in a measuring jug or beaker, take care to read the amount correctly.
2. To do this, the eye must be at the same level as the top of the liquid.
3. Take care to understand the scale on the measuring cylinder. Sometimes not all spaces are marked. For instance, the litre point may be marked and then spaces below this may be marked too.
4. Look at how many marks there are and then divide that number by 1000ml.
5. This will give you the amount measured at each mark.



In the example above there are 4 divisions so: $1000ml \div 4 = 250ml$.

This means that each graduation line represents 250 ml and that the jug contains 1250 ml or 1 litre 250 ml or 1,250 l.



Teaching Tip: Learners often struggle to find the scale of graduation lines. It is good practice to give learners many examples of graduation lines with one or two intervals not filled in. This is an excellent mental maths activity for this section. Encourage learners to use a ruler to help them read the correct graduation interval when they are reading measurements from a picture. This is especially helpful if the graduations are quite close together, or if the value lies between two graduation lines on a beaker or jug.

Rounding Off

Learners must be able to extend their knowledge of rounding throughout different topics, as the rules never change, with learners instead starting to round numbers that are larger. Learners must be able to round to the nearest 5, 10 or 100

Conversions: *ml* and *l* and *Kl*

Use the following to convert:

- $1\text{ l} = 1\,000\text{ ml}$
- $1\,000\text{ ml} = 1\text{ l}$
- $1\,000\text{ l} = 1\text{ Kl}$
- $1\,000\,000\text{ ml} = 1\text{ Kl}$

1. Converting litres to millilitres (x 1000):

9 l to ml

$$9\text{ l} = 9\,000\text{ ml} \quad (9 \times 1000\text{ml})$$

2. Converting litres and millilitres to millilitres:

2l 453ml

$$= 2\,453\text{ ml}$$

3. Converting fractions of a litre to millilitres:

$$\frac{1}{2}\text{ l} = 500\text{ ml} \quad (1\,000 \div 2 = 500)$$

$$\frac{1}{4}\text{ l} = 250\text{ ml} \quad (1\,000 \div 4 = 250)$$

$$\frac{1}{5}\text{ l} = 200\text{ ml}$$

$$\frac{1}{10}\text{ l} = 100\text{ ml}$$

4. Converting millilitres to litres ($\div 1000$):

$$1\,250\text{ml} = 1\text{ l } 250\text{ml}$$

5. Convert to a fraction of a litre:

$$6500\text{ml} = 6\frac{1}{2}\text{ l}$$

6. Convert a fraction of a litre to millilitres:

$$3\frac{1}{2}\text{ l} = 3500\text{ml}$$

7. Using conversions to compare capacities:

Capacities can be given in different forms and learners will need to convert in order to compare and insert an equal sign, less than sign or greater than sign.

Topic 8 Capacity and Volume



For example:

$$6\frac{1}{2}l * 6500ml$$

$$6\frac{1}{2}l = 6500ml$$

$$3\frac{1}{4}l * 3300ml$$

$$3\frac{1}{4}l < 3300ml$$

8. Conversions involving *kilolitres*:

$$5000l = 5KL$$

$$7KL = 7000l$$

When converting between measurements that are two units apart, it is easier to first convert to the next measurement before finally converting to the one required.

In other words, when converting from *millilitres* to *kilolitres* rather first change the *millilitres* to *litres* first and then change into *kilolitres*.

For example:

Change 4KL into *millilitres*.

$$4KL = 4000l = 4000000ml$$

Change 6 500 000ml into KL

$$6500000ml = 6500l = 6,5KL$$

9. Using conversions to arrange measurements in ascending or descending order:

For example:

Arrange the following in ascending order:



$$1,35l; 1475ml; 1\frac{1}{2}l; 1,450l$$



First write all the measurements in the same unit of measurement. In this case, *millilitres* may be the easiest.

$$1,35l = 1,350l = 1350ml$$

$$1475ml$$

$$1,5l = 1,500l = 1500ml$$

$$1,450l = 1450ml$$

Now rearrange but remember to use the original units given when writing them in either ascending or descending order.

$$\text{Ascending: } 1,35l; 1,450l; 1475ml; 1\frac{1}{2}l$$



Teaching Tip: Learners must always convert units to the same unit before they can compare or order the measurement. It is useful to show them using real-life examples, like comparing who gets more if one is given 2 *litres* of water versus 10 cups of water, if each cup holds 250 *ml* of water. Learners also get confused as to when they should multiply and when to divide. A good way to overcome this is to remind them that if they are converting from a larger unit to a smaller one, they are making many smaller pieces and should therefore multiply. When learners are converting from a smaller unit to a larger unit, they are combining small units into a larger unit and should therefore divide.

Calculations with Capacity

Convert before calculating, if necessary:

1. $250\text{ ml} + 1\,457\text{ ml} = \underline{\hspace{2cm}}$

$$\begin{array}{r} 250 \\ + 1\,457 \\ \hline 1\,707 \end{array}$$

2. $4\text{ l} + 25\text{ l} = \underline{\hspace{2cm}}$
 $\hspace{1.5cm} = 29\text{ l}$

3. $346\text{ ml} + 1,647\text{ l}$

Convert to ml first

$$\begin{array}{r} 346 \\ + 1\,647 \\ \hline 1\,993\text{ ml} \end{array}$$

Note: There is no need to change this back into litres unless specified in the question

4. $\frac{5}{8}\text{ l} + \frac{2}{8}\text{ l}$
 $\hspace{1.5cm} = \frac{7}{8}\text{ l}$

5. $48\text{ ml} \times 7 = \underline{\hspace{2cm}}\text{ ml}$

$$\begin{array}{r} 48 \\ \times 7 \\ \hline = 336\text{ ml} \end{array}$$

6. $925\text{ ml} \div 5 = \underline{\hspace{2cm}}\text{ ml}$

$$\begin{array}{r} 185\text{ ml} \\ 5 \overline{) 925} \end{array}$$

Topic 8 Capacity and Volume

RESOURCES



RESOURCE 1

Useful capacities used every day

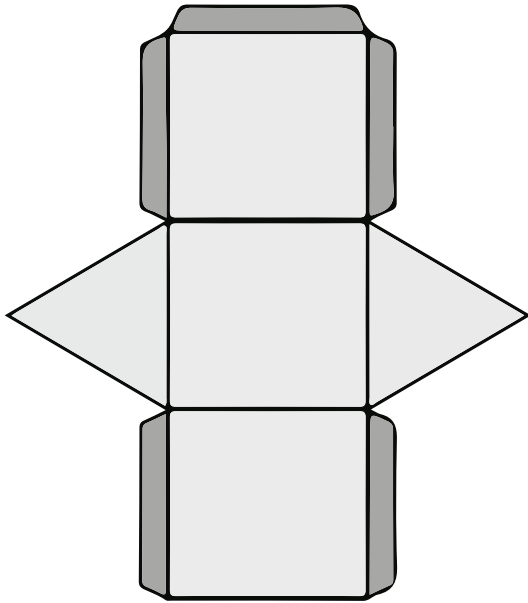
<i>Teaspoon (Tsp.)</i>	<i>5ml</i>
<i>Tablespoon (Tbsp.)</i>	<i>15ml</i>
<i>1 Cup</i>	<i>250ml</i>
$\frac{1}{4}l$	<i>1 Cup</i>
$\frac{1}{2}l$	<i>500ml</i>
$\frac{3}{4}l$	<i>750ml</i>
<i>1l</i>	<i>1000ml</i>

EDUCATOR RESOURCES

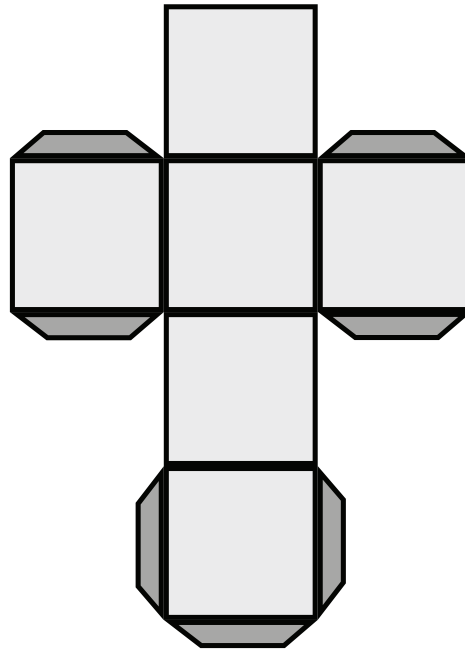


Properties of 3D Objects - nets to assist in constructions

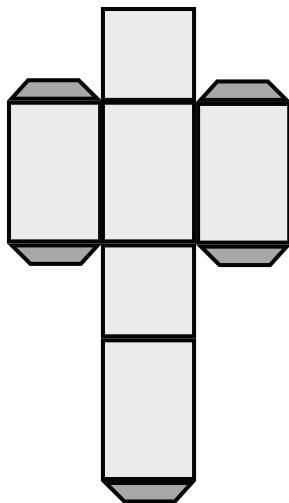
Triangular Prism



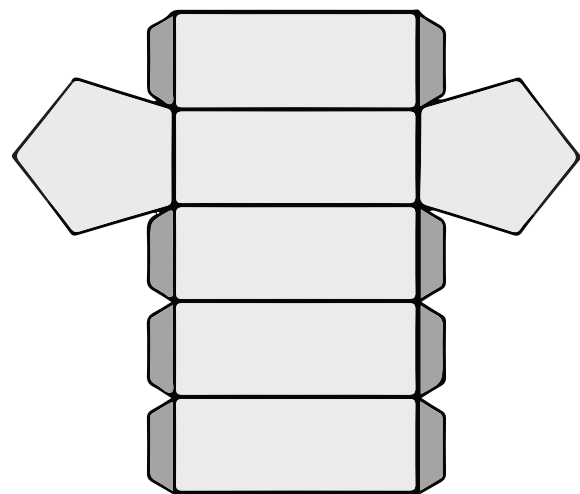
Cube



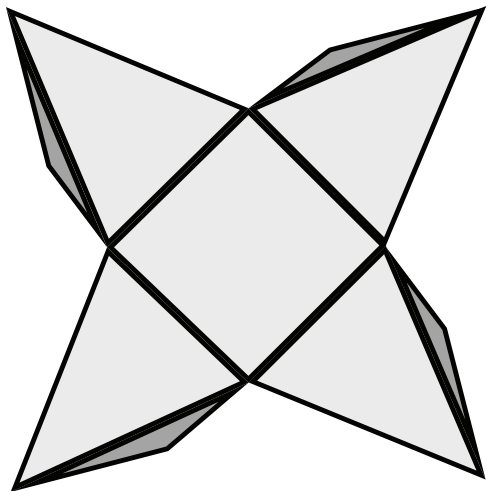
Rectangular Prism



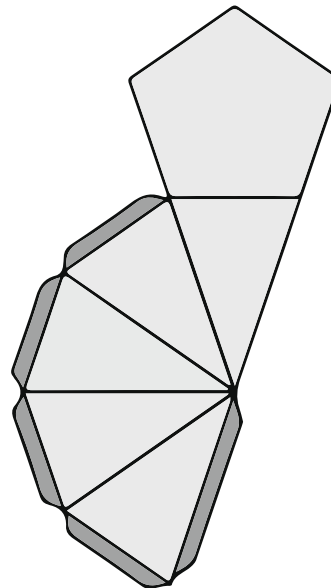
Pentagonal Prism



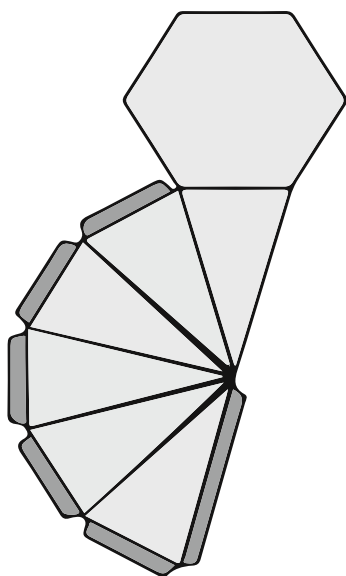
Square Base
Pyramid



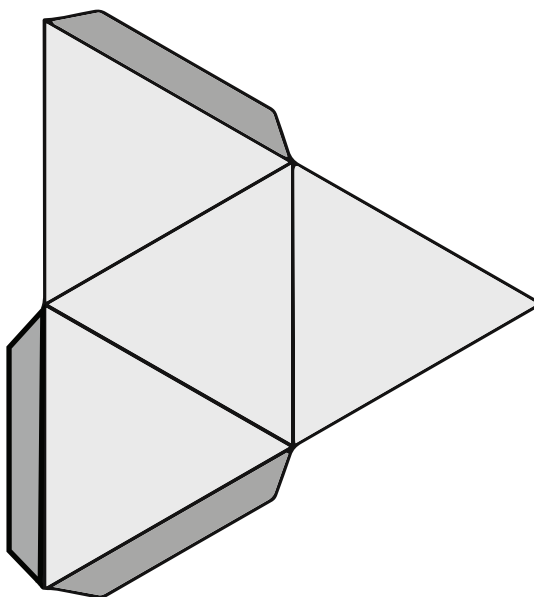
Pentagonal Base
Pyramid



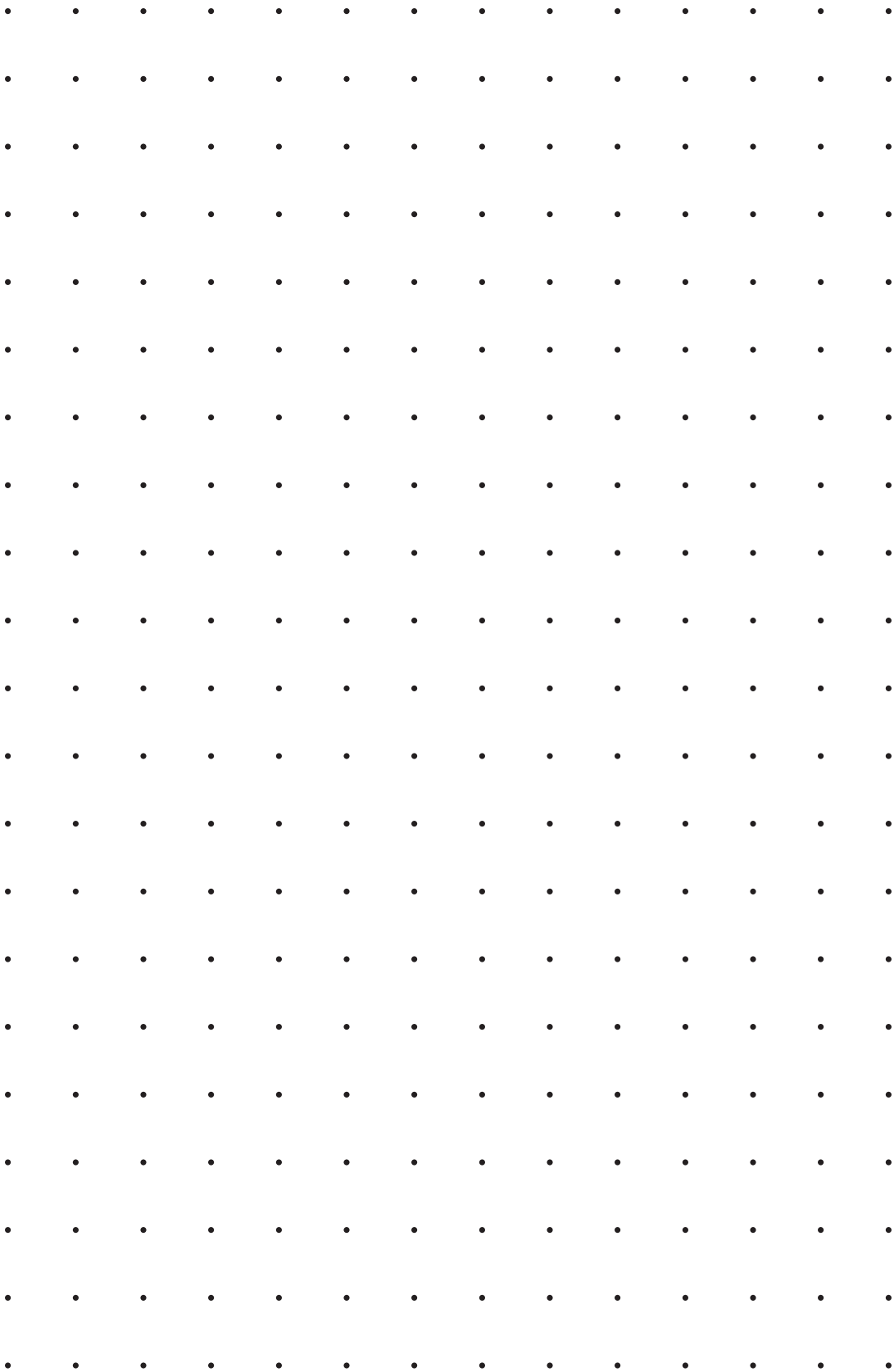
Hexagonal Base
Pyramid



Tetrahedron



Dot paper to assist learners in drawing 3D Objects



Resource for Decimal Fractions

Converting decimals to fractions and fractions to decimals.

Show how you converted the following fractions and decimals.

Decimal	Showing your conversion	Fraction
		$\frac{8}{10}$
		$\frac{2}{10}$
0.25		
0.065		
		$\frac{15}{100}$
0.8		